Committing to being Irresponsible: Deficit Spending to Escape a Liquidity Trap

Gauti B. Eggertsson*

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Abstract

This paper explores the peculiar credibility problem that a zero bound on the short-term nominal interest rate, the liquidity trap, poses to monetary and fiscal policy. We present a rational expectations model in which the zero bound on short-term nominal interest rates is binding due to deflationary shocks. When the zero bound is binding the Central Bank best achieves its objectives by generating inflation expectations to lower the real rate of interest and stimulate aggregate demand. A discretionary Central Bank that is independent from fiscal policy, however, cannot credibly commit to inflation. The result is a liquidity trap that is characterized by excessive deflation and a negative output gap. This "deflation bias" is the opposite of the "inflation bias" analyzed by Barro/Gordon (1983) and Kydland/Prescott (1977). Turning to fiscal policy, our model implies that if the Central Bank is independent then Ricardian equivalence holds and deficit spending, i.e. tax cuts and debt accumulation, has no effect. Our proposed solution involves abolishing the independence of the Central Bank. If fiscal and monetary policies are coordinated, Ricardian Equivalence fails, and the government can credibly commit to future inflation by deficit spending. As a result it lowers the real rate of return, curbs deflation and increases output. Finally we address what coordination of fiscal and monetary policy might entail in practice. We review the applicability of our model to the current situation in Japan. We then discuss the extent to which the successful policies pursued in Japan during the Great Depression can be rationalized by our model.

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1 Introduction

"To paraphrase Clemenceu, monetary policy is much too serious a matter to be left to the central bankers."
Milton Friedman

For several decades inflation has been considered the main threat to monetary stability. In the aftermath of the double digit inflation of the 70's there was a movement to separate monetary policy from fiscal policy and vest it in the hands of "independent" central bankers whose primary responsibility was to prevent inflation. This development was reinforced by important contributions on the theoretical level, most notably by Kydland/Precott (1977) and Barro/Gordon's (1983) illustration of the "inflation bias" of a discretionary government. It is easy to forget that in the aftermath of the Great Depression, when deflation was the norm, the discussion at the political and theoretical level was quite the opposite. Paul Samuelson claimed that the Federal Reserve was "the prisoner of its own independence" during the Great Depression, exaggerating the slump by its inability to fight deflation.[1] Even better known is Milton Friedman's argument against independent central bankers, part of which is quoted above.[2]

The low inflation rates in several countries in recent years, together with the current problems in Japan, have once again made the threat of deflation a topic of current concern. Deflationary pressures pose hard questions to macroeconomics. Battling deflation can be even more problematic for central banks than bringing down inflation. When large deflationary shocks hit the economy the zero bound on short term nominal interest rates can be binding, paralyzing the Central Bank's principal policy instrument. The challenge is to illustrate how to use non-standard policy instruments to curb deflation when the zero bound is binding. While Central Bank independence may be an effective way of achieving price stability under normal circumstances, we argue that during a liquidity trap it can thwart the government's ability to curb deflation. Our central conclusion is that the government can control the price level, even if the zero bound is binding, by coordinating monetary and fiscal policy. The principal policy tool we discuss is deficit spending. We also discuss how our analysis can be extended to address policy options such as foreign exchange interventions or open market operations in long-term bonds. We argue that to understand the effectiveness of these policies it is essential to analyze the nature of the cooperation between fiscal and monetary authorities.

We propose a simple rational expectation model to analyze the problem the zero bound on the nominal interest rates, i.e. the liquidity trap, poses to fiscal and monetary policy. What we mean by a liquidity trap in this paper is a situation in which the short-term nominal interest rate is zero. We build from micro foundations an extended version of the Kydland/Prescott and Barro/Gordon model (KP/BG). There is a Treasury that selects real taxes to minimize tax distortions and a Central Bank that selects the nominal interest rates to minimize the output gap and inflation.

[1] See Mayer, Thomas (1990) p. 6.

[2] from Free to Choose (1980). Friedman argues against independent central bankers on several different occations see e.g. Capitalism and Freedom (1962).

Social welfare is determined by a loss function that includes tax distortions, inflation and the output gap. The Central Bank is independent if it minimizes its objectives regardless of social welfare. In this paper the zero bound is binding because of large shocks that make the Central bank unable to lower the nominal interest rate enough to prevent deflation and a deleterious decline in output. We show that in the presence of these shocks there is instead a "deflation bias" of a discretionary independent Central Bank. This deflation bias can be viewed as the inverse of the inflation bias analyzed by KP/BG. In a liquidity trap the Central Bank would best achieve its goals if it could commit to moderate future inflation in order to maintain price stability and keep employment close to potential. If it is a discretionary maximizer it cannot, however, do this because its announcements are not credible. The result is a liquidity trap characterized by excessive deflation and undesirably low output.

The source of this dynamic inconsistency problem is not unique to the model we use. The key idea is that aggregate demand depends on the level of current and future real interest rates. Even if the zero bound is binding, monetary policy can still lower the real rate of interest, and thus aggregate demand, by influencing inflation expectations. This is why many believe that announcing a positive inflation target is an attractive policy option in a liquidity trap.[3] To increase inflation expectations, however, is problematic in our model. Although it is in the interest of the Central Bank to promise future inflation in a liquidity trap in order to increase aggregate demand, it has an incentive to renege on its promise once time comes to match deeds to words.

The Central Bank "cooperates" with the Treasury if monetary and fiscal policies are jointly determined to maximize social welfare. In this paper we consider fiscal policy of the simplest nature. We assume that real government spending is exogenously given. Thus fiscal policy determines the evolution of taxes and debt. We make this assumption to focus the analysis on *deficit spending* as opposed to *real spending,* the latter is discussed in Eggertsson (2000). We introduce a budget constraint for a Treasury that can issue one period nominal debt and levy distorting taxes. In choosing between debt and taxes, the Treasury seeks to minimize collection costs of taxes as in Barro's (1979) classic tax smoothing analysis. The introduction of taxes and nominal debt in this model gives the government an additional instrument that can be essential in a liquidity trap. If the Central Bank is independent, however, fiscal policy is completely ineffective due to Ricardian equivalence and thus the Central Banks inability to commit itself is particularly problematic.[4]

Cooperation between the Central Bank and the Treasury can be valuable because it enables the government to credibly commit to future inflation by cutting taxes and issuing debt. This in

[3] There is an extensive literature that assumes that central banks are able commit to monetary policy rules and illustrates how the choice of the optimal monetary policy rule is affected by the zero bound. Contributions include Summers (1991), Fuhrer and Madigan (1997), Woodford and Rotemberg (1997), Wolman (1998), Reifschneider and Williams (1999) and references there in. Since monetary policy rules arguably become credible over time these contributions can be viewed as illustration of how to *avoid* a liquidity trap rather than a prescription of how to escape them once trapped as stressed by Svensson (2001).

[4] By Ricardian equivalence in this paper we mean that the Treasury's choice of taxes versus debt has no effect on aggregate demand.

turn reduces the real rate of interest, increases output and curbs deflation in a liquidity trap. In the presence of tax distortions, the inflation target is credible because the real value of outstanding debt and the real rate of return paid on this debt would increase if the government deviates from the target. Since fiscal policy is impotent in the absence of cooperation between the Treasury and the Central Bank, our result is not merely a roundabout way of reaching Keynes' famous conclusion that the government should use deficit spending to get out of a liquidity trap. In our setting, deficit spending will *only* increase output and prices if the Central Bank and the Treasury cooperate to maximize social welfare. In particular, the Central Bank must take into account the fiscal consequences of its actions. The way out of a liquidity trap proposed in this paper, therefore, involves deficit spending *and* abolishing the independence of the Central Bank.

There are considerable payoffs from modeling the effects of taxes and debt on the equilibrium outcome beyond considering the effects of deficit spending. In particular, policy options such as foreign exchange rate interventions, open market operations in long term bonds or more exotically, dropping money from helicopters, can be addressed as natural extensions of our model.[5] We argue that the effects of all of these policies can be understood, in one way or another, through their influence on government debt, thus changing the inflation incentives of the government.[6]

After largely vanishing from the economic research agenda, liquidity traps have once again become a topic of current concern. Two pictures and one table from Japan can explain this. Figure (1) shows that the short-term nominal interest rate went to zero in 1999 and has stayed close to that level to present day. Figure (2) shows that during the same time real money balances have increased substantially. Although the Bank of Japan (BOJ) has aggressively increased the money supply by buying short-term nominal bonds and supplying liquidity to the banking sector, deflation and slump still persist as is illustrated by several macroeconomic indicators in Table (1). This has led Japanese Central Bankers to conclude that a further monetary easing is not useful. Kazuo Ueda, a member of the BOJ policy board stated in the Wall Street Journal (09/21/1999) that banks were already holding large excess reserves with a substantial proportion of them being held as idle cash balances at the BOJ. According to Ueda this was "sufficient evidence that banks do not need any more liquidity." This is hardly surprising since money and bonds are perfect substitutes at zero interest rates. Standard open market operations in short term bonds are not, therefore, going to have much effect apart from increasing cash reserves of banks.

Krugman (1998) suggests that even though increasing the *current* money supply is ineffective in Japan today, committing to increasing the *future* money supply raises output and the price level. In order to accomplish this, Krugman proposes that the Central Bank should announce an inflation target. The inflation target policy proposal was greeted skeptically by the BOJ. Kunio Okina, director of the Institute for Monetary and Economic Studies at BOJ, responded in Dow Jones

[5]Several authors have argued for currency depriciation as a way out of a liquidity trap including McCallum (1999), Meltzer (1999), McKinnon (1999), Svensson (1999,2001), and Bernanke (2000).

[6]We will not address here other non-traditional ways of escaping a liquidity trap such as tax on currency discusses by Buiter and Panigirtzoglou (1999) and Goodfriend (1999) or the purcases of options discussed by Clouse, Henderson, Orphanides, Small and Tinsley (2000).

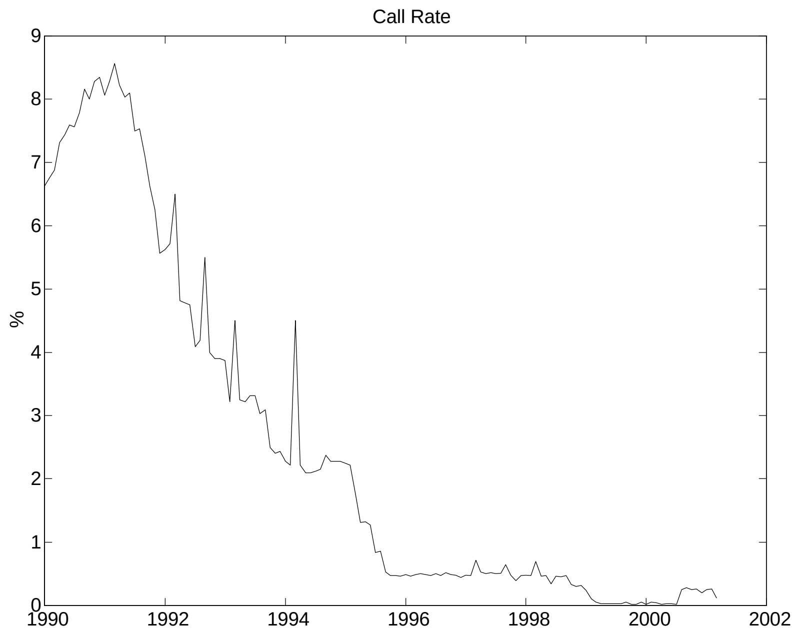

Figure 1: Uncollateralized overnight interest rates in Japan.

News (08/11/1999): "Because short-term interest rates are already at zero setting an inflation target of, say, 2 percent, wouldn't carry much credibility."[7] Our analysis of the deflation bias of an independent Central Bank indicates Okina's objections can be rationalized. Our solution, however, specifies direct actions that the government can take to implement Krugman's proposal and make it credible.

The main problem faced by an independent Central Bank seeking to announce a credible inflation target is that it is only that: an announcement. Announcing an inflation target in a liquidity trap requires no action! Since nominal interest rates are already at zero the Central Bank has no traditional tools at its disposal to manifest its appetite for inflation. When monetary and fiscal policies are coordinated this problem is eradicated. If the Central Bank and Treasury cooperate and announce an inflation target, the government can take direct *actions* to make this target credible: cutting taxes and accumulating debt until inflation expectations rise. We discuss what lessons can be drawn from our model for policy makers in Japan today. Finally we explore how cooperation has worked in practice by briefly discussing monetary and fiscal policy in Japan during the Great Depression and possible interpretations of this historical episode in the context of our model.

[7] Dominguez (1998), Svensson (1999,2001) and Woodford (1999) also criticize an inflation target in a liquidity trap on similar grounds.

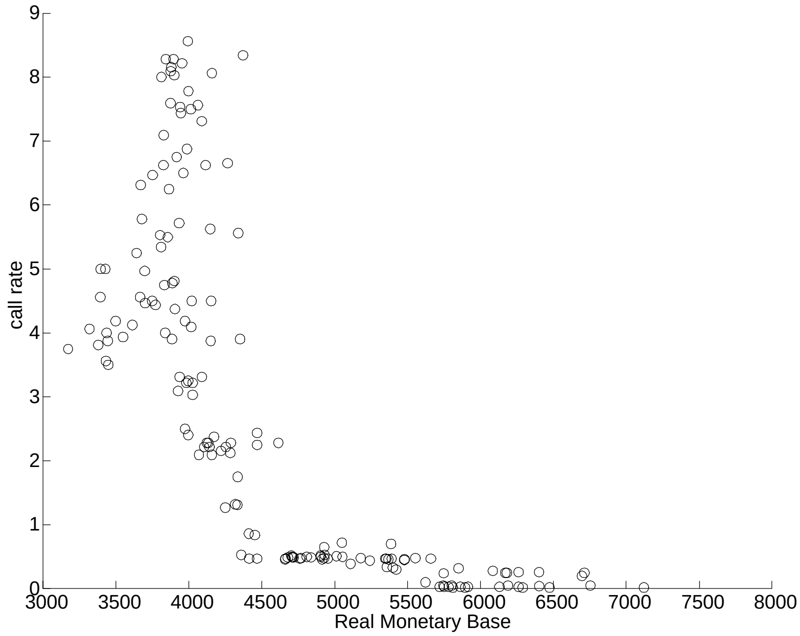

Figure 2: Real Money Balances in Japan 1990-2001.

	GDP deflator	Nikkei Comm.Pr deflator	CPI-National deflator	WPI deflator	Nom. wage index	Real GDP
90	2.59%	1.45%	3.09%	2.07%	NaN	5.26%
91	2.94%	-5.01%	3.21%	-0.65%	4.38%	3.09%
92	1.84%	-7.07%	1.76%	-1.58%	1.99%	0.86%
93	0.50%	-9.41%	1.22%	-2.92%	0.21%	0.55%
94	0.10%	-5.66%	0.70%	-2.04%	1.54%	0.98%
95	-0.40%	-0.71%	-0.10%	-0.89%	1.11%	1.56%
96	-0.70%	4.02%	0.10%	0.10%	1.10%	3.41%
97	0.30%	1.34%	1.70%	1.50%	1.58%	1.90%
98	-0.10%	-10.23%	0.69%	-1.57%	-1.27%	-1.04%
99	-1.41%	-7.48%	-0.29%	-3.40%	-1.28%	0.81%
0	-1.63%	3.77%	-0.68%	0.00%	0.40%	1.61%

Table 1: Percentage change in a few macoreconomic indicators in Japan.

2 A Simple Model

In this section we derive a simple rational expectations model from micro foundations. The Aggregate Supply (AS) equation derived, to a linear approximation, is equivalent to the AS equation assumed by Kydland/Prescott and Barro/Gordon. We also derive an "IS equation" or Euler equation that introduces the short-term nominal interest rate into the model, allowing us to address the problem of the zero bound.

2.1 The Private sector

2.1.1 Households

We assume there is a representative household that maximizes expected utility over the infinite horizon:

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t[u(C_t,\xi_t)-v(h_t,\xi_t)]\right\} \tag{1}$$

where C_t is the consumption, ξ_t is a vector of exogenous shocks and h_t are hours worked. $u(.)$ is assumed to be concave and strictly increasing in C_t for any possible value of ξ and $v(.)$ is assumed to be an increasing and convex for any possible value of ξ.

The budget constraint of the representative household is:

$$\frac{B_t}{P_t}=\frac{A_t}{P_t}+h_t\frac{W_t}{P_t}-\tau_t-C_t+\int_0^{N_t}Z_t(j)dj \tag{2}$$

where B_t is the nominal value of the end of period bond portfolio, W_t is the nominal wage rate, A_t is the beginning of period nominal wealth, τ_t is net real tax collections by the government, $Z_t(j)$ is the real profit from firm j and N_t is the number of firms.[8] The consumption plan of the representative household must satisfy a borrowing limit that rules out Ponzi schemes:[9]

$$A_{t+1}\geq-\sum_{T=t+1}^{\infty}\left[E_{t+1}R_{t,T}(h_t\frac{W_t}{P_t}+\int_0^{N_t}Z_t(j)dj-\tau_t)\right] \tag{3}$$

where $R_{t,t+1}$ is a nominal stochastic discount factor[10] and $R_{t,T}\equiv\prod_{s=t+1}^{T}R_{s-1,s}$. Condition (6) says that the household can never borrow more than the net present value of expected income.

At time t there is a fixed number N_{t+1} of labor contracts offered by firms for the next period. The household chooses how many contracts, n_{t+1}, to accept facing the market wage W_{t+1} that it takes as exogenous. We assume that W_{t+1} is determined at time t so as to clear the labor market. In period $t+1$ the firms are free to choose the hours worked at the given wage rate. Thus at time

[8] We assume no monetary frictions. Therefore, money does not enter the utility function nor is there a cash-in-advance constraint. Thus at any positive interest rate the household holds no money.

[9] For a detailed discussion of this borrowing limit and its interpretation see Woodford (2001a).

[10] $R_{t,t+1}$ has the property that the price of a bond portfolio with a random value A_{t+1} in the following period is given by $E_t[R_{t,t+1}A_t]$

$t+1$ the representative household supplies the labor $h_{t+1} = \frac{n_{t+1}}{N_{t+1}} L_{t+1}$ where L_{t+1} is aggregate labor demand of firms. The problem of the household is as follows: at every time t the household takes A_t, h_t and $\{W_t, R_{t,T}, P_t, \tau_t, Z_t(j), N_t, L_t, \xi_t; j \geq t\}$ as exogenously given and maximizes (1) subject to (6) and (2) by choice of $\{B_j, n_{j+1}, C_j; j \geq t\}$.

2.1.2 Firms

The representative firm has the production function:

$$y_t = F(l_t) \tag{4}$$

where F is a concave function and l_t is labor used by the firm in terms of hours worked. We abstract from capital dynamics. In every period t each firm offers one labor contract for the next period. The problem of the firm is as follows: at every time t the firm takes P_t and W_t as given and maximizes profits subject to (4).[11]

2.1.3 Private Sector Equilibrium Conditions: AS and IS Equations

In this subsection we describe necessary conditions for equilibrium that stem from the maximization problems of the private sector. These conditions must hold for *any* government policy. In the next subsection we describe government instruments and policy preferences. We assume that there is a continuum of households and firms of measure 1 in the economy. The first order conditions of the household maximization imply an Euler equation of the form:

$$\frac{1}{1+i_t} = E_t\{\frac{\beta u_c(C_{t+1}, \xi_{t+1})}{u_c(C_t, \xi_t)} \frac{1}{1+\pi_{t+1}}\} \tag{5}$$

where π_t is inflation. Assuming market clearing we can replace consumption in the expression above with $Y_t - I_t$ where I_t is exogenous aggregate spending. The resulting equation is often referred to as the IS equation in the literature. It is useful to define the expectation variable $f_t^e \equiv E_t \frac{u_c(C_{t+1}, \xi_{t+1})}{1+\pi_{t+1}}$ as the part of the nominal interest rates that is determined by the expectations of the private sector formed at time t. The IS equation can be written as:[12]

$$1 + i_t = \frac{u_c(Y_t, \xi_t)}{\beta f_t^e}$$

The optimal consumption plan of the representative household must also satisfy a transversality condition:[13]

$$\lim_{T \to \infty} \beta^T E_t(u_c(C_T, \xi_T) \frac{B_T}{P_T}) = 0 \tag{6}$$

[11] Several model with similar stuctural characteristics for the labor market have been expored in the literature before see e.g. Taylor (1980) and Levin (1990).

[12] To simplify notation we have supressed I_t from $u_c(Y_t, \xi_t)$ as the stochastic component of I_t is already assumed to be contained in the vector of shocks ξ_t.

[13] For a detailed discussion of how this transversality condition is derived see Woodford (2001a).

Aggregate demand and supply of labor are obtained from the first order conditions of the households and firms. An equilibrium relationship between inflation and output is found by equating the wages implied by these two equations:[14]

$$F'(F^{-1}(Y_t)) = \frac{1}{1+\pi_t}\frac{E_{t-1}(\tilde{v}_y(Y_t,\xi_t)F^{-1}(Y_t))}{E_{t-1}(u_c(Y_t,\xi_t)\frac{F^{-1}(Y_t)}{1+\pi_t})} \tag{7}$$

Equation (7) implicitly defines equilibrium output as a nonlinear function of three variables:

$$Y_t = S(\pi_t, u^e_{t-1}, v^e_{t-1}) \tag{8}$$

where it is convenient to define the expectation variables $u^e_{t-1} \equiv E_{t-1}u_c(Y_t,\xi_t)\frac{F^{-1}(Y_t)}{1+\pi_t}$ and $v^e_{t-1} = E_{t-1}\tilde{v}_y(Y_t,\xi_t)F^{-1}(Y_t)$. Equation (8) is what we refer to as the AS equation.

It is useful to define output that would be produced in absence of any contracting frictions. Following Friedman (1968) we call this the *natural rate of output.*

Definition 1 *The natural rate of output, Y^n_t, at time t is the output produced if wage setting for that period could occur at time t.*

Then Y^n_t solves the equation:

$$F'(F^{-1}(Y^n_t)) = \frac{\tilde{v}_y(F^{-1}(Y^n_t),\xi_t)}{u_c(Y^n_t,\xi_t)} \tag{9}$$

We define the output gap, x_t, as percentage deviation of output from the natural rate i.e. $x_t \equiv \frac{Y_t - Y^n_t}{Y^n_t}$. It is useful to define the real rate of interest that is necessary for an equilibrium in which output is equal to the natural rate. Following Woodford (2001a) we call this the *natural rate of interest.*

Definition 2 *The natural rate of interest, R^n_t , is the real interest rate that is necessary for output to be equal to the natural rate of output at all times*

The natural rate of interest satisfies:

$$\frac{1}{1+R^n_t} = E_t\frac{\beta u_c(Y^n_{t+1},\xi_{t+1})}{u_c(Y^n_t,\xi_t)} \tag{10}$$

2.1.4 Interpretation of the IS and the AS Equation

When characterizing the government's optimization problem in coming sections we use the nonlinear equations derived in the last subsection. For interpretation, however, it is useful to linearize the IS and the AS equation around a zero inflation rate and zero output gap. The linearized AS equation is:

$$\pi_t = \kappa\hat{x}_t + E_{t-1}\pi_t + \epsilon_t \tag{11}$$

[14] To simplify notation we have replaced the function $v(L_t,\xi_t)$ with a simple transformation $\tilde{v}(Y_t,\xi_t) \equiv v(F^{-1}(Y_t),\xi_t)$.

where $\kappa \equiv -\frac{F''\bar{Y}}{F'^2} > 0$ and ϵ_t is an exogenous shock given by $\epsilon_t = \kappa(\hat{Y}_t^n - E_{t-1}\hat{Y}_t^n)$.[15] There are three reasons why it is interesting to illustrate our result with AS equation of this nature. First, our AS equation is, to a linear approximation, equivalent to the Phillips curve used by KP/BG, allowing us to compare the deflation bias to the inflation bias derived by these authors. Second, Krugman's (1998) assumption about price setting can be seen as a special case of our pricing assumption.[16] Our conclusions therefore apply directly to his model. Third, we obtain simple closed form solutions that illustrate the basic ideas in a transparent way.

The linearized IS equation is:

$$\hat{x}_t = E_t\hat{x}_{t+1} - \sigma(\hat{\imath}_t - E_t\pi_{t+1} - \hat{r}_t^n) \tag{12}$$

where $\sigma \equiv -\frac{u_c}{u_{cc}\bar{Y}}$. This equation can solved recursively forwards to yield:

$$\hat{x}_t = -\sigma E_t \sum_{j=0}^{\infty}(\hat{r}_{t+j} - \hat{r}_{t+j}^n) \tag{13}$$

where $\hat{r}_t = \hat{\imath}_t - E_t\pi_{t+1}$ is the real rate of interest. This IS equation has a familiar interpretation: the output gap today depends on expected long-term real rates, which in turn depend upon expected future short rates. The output gap does not only depend on the level of the current and expected real rate of interest, it depends on the difference of the real rate and the natural rate of interest.

The natural rate of output and interest are both a functions of exogenous shocks in our model.[17] A linear approximation of the natural rate of output, the natural rate of interest and the implied value of ϵ_t are given by:

$$\hat{Y}_t^n = \theta g_t \tag{14}$$

$$\begin{aligned} \hat{r}_t^n &= \sigma^{-1}E_t[(g_t - \hat{Y}_t^n) - E_t(g_{t+1} - \hat{Y}_{t+1}^n)] \\ &= \sigma^{-1}(1-\theta)E_t(g_t - g_{t+1}) \end{aligned} \tag{15}$$

$$\epsilon_t = \kappa\theta(g_t - E_{t-1}g_t) \tag{16}$$

where $\omega \equiv \frac{v_{yy}\bar{Y}}{v_y}$ and $0 < \theta \equiv \frac{\sigma^{-1}}{\kappa+\sigma^{-1}+\omega} < 1$. The variable $g_t \equiv -\frac{u_{c\xi}}{u_{cc}\bar{Y}}\xi_t$ is a linear combination of the shocks in the vector ξ_t and summarizes all the disturbances in our model economy.[18] It has

[15] The hats on the varibles indicates that they refer to percentage deviation of each of the variable from the constant solution we linearize around.

[16] Krugman assumes that prices are set one period ahead. If the production is linear in labor, prices are set one period ahead in our model and $\kappa = 0$ in the AS equation above.

[17] There is work in progress by the author that makes the natural rate of interest endogenous.

[18] Note that the $u_{c\xi}^T$ is a vector of the same dimension as ξ_t. We have assumed that the shocks in ξ_t have no effect on the disutility of working. It is straight forward to extend the model to consider shocks that affect the disutility of working but not central to our analysis.

been suggested by several authors that a necessary condition for the zero bound to be binding is a negative natural rate of interest. Shocks that lower the natural rate of interest, i.e. negative values for g_t, are the source of the liquidity trap in our model. How large these shocks need to be for the zero bound to be binding, however, depends on what assumptions we make about government behavior.[19]

2.2 The Government

2.2.1 Central Bank Preferences and Instruments

The Central Bank's loss function is:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda_x x_t^2] \right\} \tag{17}$$

Minimizing (17) subject to the linearized Phillips curve (11) is a classic problem in macroeconomics and what Sargent et al (2001) call the *Phelps problem* (due to Phelps (1967)).[20] In this paper, however, we will use the nonlinear version of all the other economic constraints (including the AS equation) when solving the minimization problems of the government. This is important because there are critical non-linearities in the government budget constraint (which we introduce in next subsection). The quadratic deviation of output from its efficient level in the loss function can be derived by taking a second order approximation of the representative household's utility. The term involving inflation in the classic Phelps problem is *ad hoc* but could represent disutility the representative household suffers from price movements (in so far as they do *not* affect labor or consumption choices). If a New Keynesian model for price setting is assumed, Woodford (2001a) demonstrates that both terms in the loss function above can be justified by a second

[19] A simple example of a shock that lowers g_t is a shock to exogenous aggregate spending I_t. If this is the only shock in the economy then $g_t = \hat{I}_t$. Here $\hat{I}_t$ denotes the absalute deviation of exogenous spending from the constant solution we linearize around over aggregate output. Thus all we need for a liquidity trap in our model is a sufficiently large collapse in exogenous spending. We can think of variety of reason why spending might temporarily collapse. An obvious candidate for Japan is that due to the asset market price collapse in the 90's and the following balance sheet problems by firm's investment temporarily declined. Another explanation would be that firms "over invested" during the late 80's because they expected a higher growth path for the economy than was realized in the 90's leading to a temporarily decline in investment. We will not try to model such stories explicitly but assume that this collapse in spending is purely exogenous in our model. This is of course exactly the same type of thought experiment that has long been popular in elementary macroeconomic textbooks where there is some "exogenous" shift in spending that causes the IS curve to shift. Other examples of shocks that lowers g_t are exogenous shifts in preferences such as an increase in the propensity to save. The interpretation given to movements in g_t in our exercises is not of principal importance in this paper. All that matters is that these shocks are treated as exogenous. That allows us to model the peculiar credibility problem of the government in a liquidity trap in a straightforward fashion.

[20] Other contributions include Kydland and Prescott (1977), Barro and Gordon (1983) and an extensive literature that followed. Note that these authors assumed that the Central Bank would desire output to be above its efficient level due to some distortions. In our model there are no distortions in the economy that justify this assumption since the wage level that would result in the absence of frictions is socially optimal. Thus we assume that the Central Bank wants output to be equal to the natural rate of output at all times.

order expansion of the utility of the representative household. Many of our results are unchanged in that framework and are discussed elsewhere.[21] Apart from being a classic starting point in addressing dynamic inconsistency problems, our framework has the advantage over the New Keynesian model that it delivers simple closed form solutions.

The Central Bank determines the nominal interest rate in every period. The nominal interest rate cannot, however, be less than zero.

$$i_t \geq 0 \tag{18}$$

Even in the absence of monetary frictions, the Central Bank can still control the nominal interest rate. For example, this could be done by varying the interest rate paid on balances held at the Central Bank. These would still be held even in the absence of frictions — there would simply no longer be a spread between money-market interest rates and the interest rate paid by the Central Bank (Hall (1999), Woodford (1999a, 2001a)).[22] If holders of central-bank balances have the right to convert them into non-interest-earning currency, then the Central Bank's ability to set the interest rate on overnight balances is limited by the fact that it cannot force the interest rate to be negative. In an equilibrium without monetary frictions, currency is never held in periods when $i_t > 0$, and it does not matter whether anyone chooses to hold it when $i_t = 0$. (Currency and central-bank balances are in that case equivalent assets.) Note that central-bank balances are equivalent to other (short-term, nominally denominated) government debt, as far as the budget constraints of both households and the government are concerned. Thus we do not need to introduce any notation below for the supply of central-bank balances; we only need to track the evolution of total short-term government debt. If the Central bank is independent it uses its instrument to minimize (17) regardless of the evolution of fiscal variables and social welfare. We define the problem of the Central Bank in the next two sections under several assumptions about government behavior.

2.2.2 Treasury Preferences and Instruments

The Treasury's loss function is:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \tau_t^2 \right\} \tag{19}$$

[21] The result regarding the deflation bias in particular still holds, see e.g. Eggertsson (2000).

[22] This is not the system of monetary control actually used in Japan, but the difference is of little consequence in the case of equilibria where nominal interest rates are at or near zero, as in this paper. Introducing monetary frictions, so that the Central Bank's operating target for overnight interest rates is not necessarily identical to the interest rate on overnight balances at the Central Bank, would not change the nature of the zero bound. Nor would it affect the rest of our analysis, except in relatively minor ways; for example, there would be a small contribution of seignorage revenues to the government budget constraint, abstracted from in this paper. These additional factors can in principle be arbitrarily small regardless of the size of the equilibrium spread between the interest paid on central-bank balances and money-market interest rates, as shown in Woodford (1998).

The Treasury determines real taxes in every period. We suppose that the Treasury can only issue one period nominal bonds. We discuss an extension for bonds with longer maturity in Section 8 and in Eggertsson (2001). The budget constraint can be written as:

$$B_t = (1 + i_{t-1})B_{t-1} + G_t P_t - \tau_t P_t \tag{20}$$

where B_t refers to one period nominal government debt. We impose a borrowing limit on the government that rules out Ponzi schemes. It is convenient for our coming discussion to define the borrowing limit in terms of a variable $\tilde{b}_t \equiv \frac{B_t}{P_t} u_c(Y_t, \xi_t)$:

$$\tilde{b}_t \leq \bar{b} < \infty \tag{21}$$

where $\bar{b}$ can be an arbitrarily high finite number.[23] It is easy to show that (21) is a sufficient condition for (6) to hold at all times. Minimizing (19) subject to (20) and (21) is another classic problem in macroeconomics and was first analyzed by Barro (1979).[24] As in Barro (1979), we interpret this loss function as representing tax collection costs and assume that the level of real government spending is an exogenous process $\{G_t\}_{t=0}^{\infty}$. For simplicity we assume real government spending is constant at all times so that $G_t = G$. We make this assumption in order to focus the analysis on the effect of fiscal policy through taxes and nominal debt, i.e. *deficit spending*, as opposed to the effect of fiscal policy through *real spending*. The effect of real spending by the government, however, is important and is studied in Eggertsson (2000). We define the problem of the Treasury in the next two sections under several assumptions about government behavior.

2.2.3 Social Objectives and Cooperation

We assume that the social objective is a weighted average of the preferences of the Treasury and the Central Bank, i.e.

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda_x x_t^2 + \lambda_\tau \tau_t^2] \right\} \tag{22}$$

We define cooperative solutions as equilibria that result when the Treasury and the Central Bank coordinate their instruments to minimize social losses given by (22). In contrast we define non-cooperative solutions as equilibria that result when the Treasury and the Central Bank use their instruments to minimize their own loss functions.

By an "independent Central Bank" we mean a Central Bank that does not coordinate its instruments with the Treasury to maximize overall social welfare. It only cares about minimizing its own objectives. Since we assume that taxes are lump sum, they can only affect the equilibrium

[23] A more sophisticated borrowing limit would constrain the ability of the government to borrow to the expected net present value of the real tax base. This specification would not change our results but add some technical details that are not central to our analysis.

[24] Barro's original formulation was $\frac{\tau_t^2}{Y_t}$ but this specification would not affect our qualitative conclusion but complicate the algebra slightly.

through tax collection costs, i.e. through their presence in (19). An appealing feature of this assumption is that if the Central Bank is independent, it will determine the path of output and inflation independently of fiscal variables. In this case fiscal policy has no effect on output and prices due to Ricardian equivalence. When the fiscal policy and monetary policy are coordinated, on the other hand, fiscal policy becomes non-Ricardian and has powerful effects. Due to Ricardian equivalence, we do not have to consider strategic interactions between the Central Bank and the Treasury when the Central Bank is independent.[25] We can first analyze the Central Bank's problem to give us the evolution of all of the endogenous variables apart from debt and taxes. We can then do a separate analysis of the behavior of the Treasury that treats the nominal interest rate and inflation as exogenously given.

By assuming that the government cannot increase real spending or vary taxes that influence the natural rate of output (such as labor taxes or any other taxes that distort prices), the scope for effective fiscal policy is severely limited. Thus our *assumptions* are effectively stacking the cards *against* fiscal policy. Yet, despite these strong assumptions, we show that in the presence of quite moderate tax distortions (captured by collection costs) fiscal policy can have powerful effects in a liquidity trap.

2.3 Equilibrium for Arbitrary Public Policy

We now define an equilibrium given arbitrary monetary and fiscal policy.

Definition 3 *A Private Sector Equilibrium at date* $t \geq 0$ *is a set of sequences* $\{Y_t, Y_t^n, C_t, A_t, B_t, R_t, R_t^n, Z_t, W_t, h_t, i_t, \pi_t, \tau_t, \xi_t\}$ *that satisfy the firm and the household maximization problem, market clearing and initial conditions and (18),(20) and (21).*

We can eliminate the nominal interest rate from our constraints by combining the zero bound and the IS equation to yield:

$$u_c(Y_t, \xi_t) \geq \beta f_t^e \tag{23}$$

The budget constraint and the IS equation are similarly combined to yield:

$$b_t = \frac{u_c(Y_{t-1}, \xi_{t-1})}{\beta f_{t-1}^e} \frac{b_{t-1}}{1+\pi_t} + G - \tau_t \tag{24}$$

where $b_t \equiv \frac{B_t}{P_t}$. It simplifies our discussion to assume that the Central Bank sets inflation directly in every period subject to the economic constraints rather than selecting the short term nominal interest rate. The zero bound compels the Central bank to select inflation rates that are consistent with positive nominal interest rates.

With the model and possible equilibria in hand we now specify behavioral assumptions for the government and the resulting equilibria.

[25] Which are in any event not related to the main point we want to make.

3 The Problem of an Independent Central Bank: The Deflation Bias

We first consider the equilibrium outcome when the Central Bank is independent. This is what we call the non-cooperative solution. Then each government agency minimizes its own loss function without consideration of overall social objectives. The result of KP/BG was that a discretionary Central Bank creates excessive inflation without any gains in employment, whereas the commitment solution involves no inflation. We show that in the presence of certain shocks that create a liquidity trap this result is reversed. The discretion solution for an independent Central Bank involves excessive deflation compared to the commitment solution. In addition, the discretionary solution involves excessive output losses. When the Central Bank is independent, the Treasury treats inflation, output and the interest rate as exogenously given. As it seeks to minimize tax collections over time it increases debt when the real rate is low and decrease it when it is high.

3.1 Equilibria under Commitment when the Central Bank is Independent

We first consider the optimal policy when the Treasury and the Central Bank are able to commit to any paths of taxes and inflation.[26]

Definition 4 *The optimal commitment solution under non-cooperation is a Private Sector Equilibrium that satisfies: (i) The Central Bank Problem. The sequence $\{\pi_t\}$ minimizes (17) subject to (8), (23) and initial conditions given an exogenous process $\{\xi_t\}$. (ii) The Treasury Problem. The sequence $\{\tau_t\}$ minimizes (19) subject to (21), (24) and initial conditions where the set of sequences $\{\pi_t, Y_t, f_t^e, \xi_t\}$ is exogenously given.*

We can characterize the Central Bank's optimal commitment solution by a Lagrangian minimization problem. It is useful to think of the Central Bank as choosing the expectation variables f_t^e, v_t^e and u_t^e in addition to the endogenous variables π_t, Y_t and τ_t at each time t. The choice of the expectation variables is subject to the condition that expectations must be rational in equilibrium. Thus when writing the Lagrangian for the optimal commitment solution, in addition to the constraints listed above, we have three rational expectation constraints (where the three

[26] Note that the Treasury only chooses between taxes and debt. Due to Ricardian equivalence there are therefore no strategic interaction between the Treasury and the Central Bank. Note that the borrowing constraint is a *constraint on fiscal policy* when the Central Bank is independent. The Central Bank can thus choose any paths for π_t and Y_t that satisfy the IS and AS equations and the zero bound. Fiscal policy will guarantee that the borrowing constraint is satisfied at all times as well and thus the transversality condition of the maximizing household will be satisfied as well (as pointed out in Section 2.2.2 the borrowing constraint of the government implies that the transversality condition must hold).

rational expectation constraints are written last):

$$
\begin{aligned}
L_s &= E_s \sum_{t=s}^{\infty} \beta^t [\frac{1}{2}\pi_t^2 + \frac{1}{2}\lambda_x (\frac{Y_t - Y_t^n}{Y_t^n})^2 \\
&\quad + \eta_t (Y_t - S(\pi_t, u_{t-1}^e, v_{t-1}^e)) + \psi_t (1 - \frac{u_c(Y_t, \xi_t)}{\beta f_t^e}) + \phi_t^1 (u_t^e - u_c(Y_{t+1}, \xi_{t+1}) \frac{F^{-1}(Y_{t+1})}{1 + \pi_{t+1}}) \\
&\quad + \phi_t^2 (v_t^e - \tilde{v}_y(Y_{t+1}, \xi_{t+1}) F^{-1}(Y_{t+1})) + \phi_t^3 (f_t^e - \frac{u_c(Y_{t+1}, \xi_{t+1})}{1 + \pi_{t+1}})] \qquad (25)
\end{aligned}
$$

There are 6 nonlinear first-order conditions that result from this minimization problem, including two complementary slackness conditions. They are relegated to Appendix A. The Treasury problem can similarly be written as a Lagrangian:

$$
L_s = E_s \sum_{t=s}^{\infty} \beta^t \left[\frac{1}{2}\lambda_\tau \tau_t^2 + \mu_t (b_t - \frac{u_c(Y_{t-1}, \xi_{t-1})}{\beta f_{t-1}^e} \frac{b_{t-1}}{1 + \pi_t} - G + \tau_t) + \gamma_t (u_c(Y_t, \xi_t) b_t - \bar{b}) \right]
$$

The first-order condition for this problem are shown in Appendix A.

3.2 Equilibria under Discretion when the Central Bank is Independent

3.2.1 Structure of the Game

What we mean by a discretion equilibrium in this paper is a Markov-perfect solution. Thus we only consider equilibria in which the strategies of the players depend on a well-defined "minimum" set of variables that are directly relevant to current market conditions. We do not consider any equilibria built on reputation. Consider a repeated game between the Treasury, the Central Bank and the private sector. Each round of the game begins at the end of time $t-1$ when the private sector forms expectations. A major convenience of defining the game in this fashion is that it only involves two state variables. They are $\tilde{b}_{t-1} \equiv u_c(Y_{t-1}, \xi_{t-1}) b_{t-1}$ and the vector of exogenous shocks ξ_{t-1} that is assumed to follow a Markov process. The Treasury's actions must satisfy the borrowing limit (21) and the government budget constraint that can be rewritten in terms of $\tilde{b}_t$:

$$
\frac{\tilde{b}_t}{u_c(Y_t - G, \xi_t)} = \frac{1}{\beta f_{t-1}^e} \frac{\tilde{b}_{t-1}}{1 + \pi_t} + G - \tau_t \qquad (26)
$$

The Central Bank's actions are constrained by the AS equation (8) and the IS/ZB inequality (23). The sequence of actions is as follows:

1) Each round starts with initial values for the state variables $\tilde{b}_{t-1}$ and ξ_{t-1}. $\tilde{b}_{t-1}$ influences the equilibrium outcome by entering the budget constraint and thereby effecting the Treasury's choices at time t. The private sector uses ξ_{t-1} to form expectations about $\xi_{t.}$

2) The private sector forms expectations f_{t-1}^e, u_{t-1}^e and v_{t-1}^e. The variables u_{t-1}^e and v_{t-1}^e influence the equilibrium through the AS equation since it is the ratio $\frac{v_{t-1}^e}{u_{t-1}^e}$ that determines the nominal wage rate on the labor market. f_{t-1}^e effects the equilibrium outcome through the budget constraint. It is the part of nominal interest rate at time $t-1$ that is determined by expectations about future marginal utility and inflation.

3) The vector of shocks ξ_t is realized.

4) The Central Bank chooses π_t to minimize (17) and the Treasury τ_t to minimize (19) which in turn determine $\tilde{b}_t$ and Y_t. $\tilde{b}_t$ and ξ_t give the initial values for the next round of the game.

3.2.2 Strategy Functions of the Players

When the Central Bank is independent it determines inflation and output independently of the level of debt. In this case the strategy functions of the private sector are only a function of ξ_{t-1} and independent of the value of the other state variable $\tilde{b}_{t-1}$ (when the Central Bank and the Treasury cooperate this is not, however, true):

$$u_{t-1}^e = u^e(\xi_{t-1}) \tag{27}$$

$$v_{t-1}^e = v^e(\xi_{t-1}) \tag{28}$$

$$f_{t-1}^e = f^e(\xi_{t-1}) \tag{29}$$

The IS/ZB inequality and the AS equation indicate that Central Banks policy depends on $u_{t-1}^e, f_{t-1}^e, v_{t-1}^e$ and ξ_t. Given the strategy functions of the private sector we can define the strategy functions of the Central Bank as:

$$\pi_t = \pi(\xi_t, \xi_{t-1}) \tag{30}$$

The government budget constraint indicates that the Treasury's policy depends on $Y_t, \xi_t, \tilde{b}_{t-1}$ and f_{t-1}^e. Given the strategy functions of the private sector and the Central Bank and the AS equation we can write the strategy function of the Treasury as:

$$\tau_t = \tau(\tilde{b}_{t-1}, \xi_t, \xi_{t-1}) \tag{31}$$

3.2.3 Characterizing Government Policy

The Central Bank's problem is particularly simple under discretion when it is independent. At any time t we can characterize the problem of the Central Bank as a one period minimization problem:

$$\min_{\pi_t} [\pi_t^2 + \lambda_x x_t^2] \tag{32}$$

s.t. (21),(8),(23) and (27)-(29).

The Treasury is unable to commit to any future policies apart from honoring the nominal value of its debt.[27] The problem of the Treasury is to choose taxes in every period t to minimize

[27] This type of "partial" commitment has a long tradition in macroeconomics, see e.g. Lucas and Stockey (1983).

the expected value of its losses given by (19). We can characterize the minimization problem of the Treasury in period t as:

$$\min_{\tau_t}[\tau_t^2 + \beta V(\tilde{b}_t, \xi_t)] \tag{33}$$

s.t. (21),(26),(27)-(29) and (30).

Here $V(\tilde{b}_t, \xi_t)$ is the value function of the Treasury, i.e. the expected value at time t of its losses in period $t+1$ onwards. This value is calculated under the expectations that the Treasury will minimize under discretion from period $t+1$ onwards. The value function satisfies the Bellman equation:

$$V(\tilde{b}_{t-1}, \xi_{t-1}) = E_{t-1}\min_{\tau_t}[\lambda_\tau \tau_t^2 + \beta V(\tilde{b}_t, \xi_t)] \tag{34}$$

s.t. (21),(26),(27)-(29) and (30).

Definition 5 *An optimal discretion solution under non-cooperation is a Private Sector Equilibrium, private sector strategy functions (27)-(29), government strategy functions (30)-(31) and a value function $V(\tilde{b}_t, \xi_t)$ that satisfy: (i) Central Bank minimization. The strategy function of the Central Bank solves the minimization problem (32) given an exogenous process $\{\xi_t\}$. (ii) Treasury minimization. The strategy function of the Treasury solves the minimization problem (33) and the value function $V(\tilde{b}_t, \xi_t)$ satisfies the Bellman equation (34) given an exogenous process $\{\xi_t\}$. (iii) Private sector maximization. The strategy functions of the private sector satisfy the household and firm maximization problem taking the government strategy functions and the exogenous process $\{\xi_t\}$ as given. (iv) Rational expectations and initial conditions.*

Appendix A illustrates the first order conditions for the minimization problems defined in Definition 5.

3.3 Solutions Paths for an Approximate Solution

In this subsection we illustrate an approximate solution to the equilibria defined in Definitions 4 and 5. We approximate the constraints and the first-order conditions by a first-order Taylor expansion. The point we expand around is a constant solution defined in Appendix A. The resulting system of equations, which we also illustrate in Appendix A, cannot be solved with standard methods for linear rational expectation models. This is due to the inequality constraint stemming from the zero bound on the nominal interest rate. To illustrate solution paths we make assumptions about the stochastic process for the vector ξ_t that contains all the shocks in our model. In the approximate solution these shocks can be summarized by a single disturbance g_t defined in Section 2.1.4. We first consider the most simple process for g_t to obtain closed form solutions. We then consider a simple stochastic process and show some numerical results.

Case 1 (C1) *In period zero there is an unexpected shock to g_0. In period $t > 0$ $g_t = 0$. There is perfect foresight from period 0 onwards.*

As discussed in footnote 19 in Section 2.1.4 movements in g_0 can be interpreted as exogenous shifts in spending (e.g. an exogenous collapse in spending for a negative value of g_0) or exogenous shifts in preferences (e.g. a temporary increase in the propensity to save for a negative value of g_0).

3.3.1 The Central Bank

When the Central Bank is independent it determines the set of variables $\{\pi_t, x_t, i_t\}$ independently of fiscal policy. As a starting point we solve the commitment and discretion problem when the zero bound is not binding.

Case 1A (C1A) $g_0 > g_L = -\sigma\frac{\kappa^2+\lambda_x}{\kappa^2+(1-\theta)\lambda_x}(1-\beta)$

Proposition 1 *Suppose C1A and that the Central Bank is independent. Then the solution for optimal monetary policy under discretion and commitment is identical and the zero bound is not binding.*

Proof: The first-order conditions (58)-(68) and (78)-(87) under non-cooperation are identical when $\psi_t = 0$. The linearized condition (69)-(77) in Appendix A can be solved for each of the endogenous variables when $\psi_t = 0$ under C1A. The implied value for the nominal interest rate is:

$$i_0 = \frac{1-\beta}{\beta} + \frac{\kappa^2 + (1-\theta)\lambda_x}{\lambda_x + \kappa^2}\sigma^{-1}\beta^{-1}g_0 \tag{35}$$

Then the zero bound is binding when:

$$g_0 \leq g_L = -\sigma\frac{\kappa^2 + \lambda_x}{\kappa^2 + (1-\theta)\lambda_x}(1-\beta) \tag{36}$$

Case 1B (C1B) as Krugman (1998) $g_0 < g_L = -\sigma\frac{\kappa^2+\lambda_x}{\kappa^2+(1-\theta)\lambda_x}(1-\beta)$

We now illustrate the optimal solution under commitment when the zero bound is binding so that $g_0 < g_L$. By (15) $\hat{r}_t^n = \sigma^{-1}(1-\theta)g_0$. Thus our assumption is equivalent to Krugman's (1998) assumption.[28] He supposes that the natural rate of interest is negative for one period and then positive from that point onwards. If we assume C1B, the discretion solution is different from the commitment solution. This stems from the fact that the latter involves a commitment to future policy that is dynamically inconsistent. It is only a matter of algebra to show that equation (69)-(77) given in Appendix A yield the commitment solution for inflation in period 1 onwards:[29]

$$\pi_1^C = -\frac{(1-\theta)\lambda_x + \kappa^2}{\lambda_x + \kappa^2 + \beta\sigma^{-2}}\sigma^{-1}(g_0 - g_L) > 0,\ \pi_t^C = 0 \ \ \forall \ \ t > 1 \tag{37}$$

where the superscript C refers to the commitment equilibrium when the Central Bank is independent.

[28] If we make the assumption $\kappa = 0$ which would reduce our model to Krugman's specification.

[29] This can be seen as a special case of the solution method presented in Appendix B for a more general stochastic process.

Proposition 2 ***Optimal Inflation Target in a Liquidity Trap.*** *Suppose C1B. Optimal monetary policy of an independent Central Bank under commitment results in expected inflation.*

Proof: See equation (37)

This result is exactly what is suggested by Krugman (1998). Optimal monetary policy in a liquidity trap under commitment involves *expected* inflation. The logic behind this result is simple. When the natural rate of interest is negative a negative real rate of return, $\hat{r}_t = \hat{\imath}_t - E_t\pi_{t+1}$, is required to prevent an excessive negative output gap and deflation. This cannot be achieved through the nominal interest rates due to the zero bound, but the real rate can still be lowered by expected inflation.

The solution for discretion is found by solving (88)-(92) in Appendix A. The solution for inflation from period 1 onwards is:[30]

$$\pi_t^D = 0 \;\forall\; t > 0 \tag{38}$$

where the superscript D refers to the discretionary equilibrium when the Central Bank is independent. Although optimal policy under commitment mandates inflation in period 1 a discretionary Central Bank cannot commit to positive inflation. The result is excessive deflation and output gap in period zero. That can be shown by solving (69)-(77) and (88)-(92) in Appendix A:[31]

$$\pi_0^D - \pi_0^C = \kappa\frac{(1-\theta)\lambda_x + k^2}{\lambda_x + \kappa^2 + \beta\sigma^{-2}}(g_0 - g_L) < 0 \tag{39}$$

$$x_0^D - x_0^C = \frac{(1-\theta)\lambda_x + k^2}{\lambda_x + \kappa^2 + \beta\sigma^{-2}}(g_0 - g_L) < 0 \tag{40}$$

Proposition 3 ***The Deflation Bias.*** *Suppose C1B. An independent Central Bank that is discretionary will provide less inflation than a Central Bank that can commit in a liquidity trap. A discretionary Central Bank will experience more negative output gap and deflation than if it could commit to optimal policy.*

Proof: See equations (39) and (40).

The Deflation bias proved in the proposition above is illustrated in figure (3) for illustrative coefficients of the model.[32] Optimal policy under commitment involves *promising future inflation*

[30] This can be seen as a special case of the solution method presented in Appendix B for a more general stochastic process.

[31] This can be seen as a special case of the solution method presented in Appendix B for a more general stochastic process.

[32] As should be clear from the discussion above, the result do not in any way depend on these particular values of the parameters or the special functional form chosen. To draw the pictures we assume that the length of labor contracts is 3 years so that a period in the model corresponds to 3 years. This is done to get larger real effects from the shock. When we discuss the stochastic version of the model that allows multiple period traps we halve the duration of the labor contracts. The value of inflation in the figure above refer to annualized value of inflation.

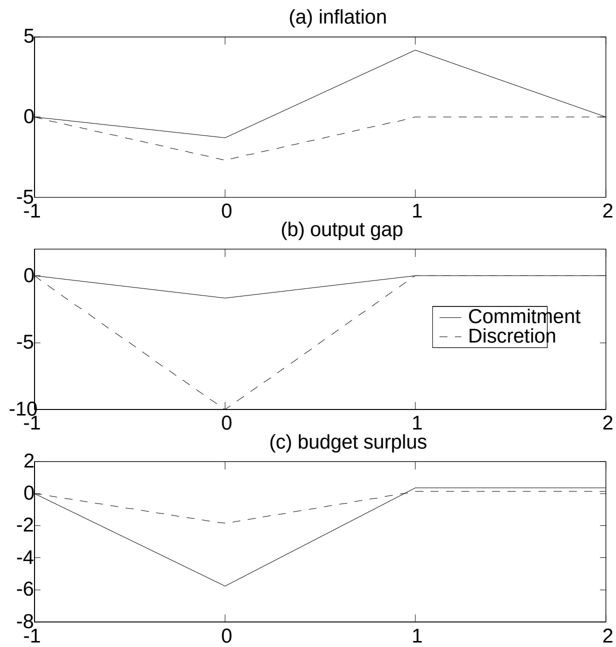

Figure 3: The Deflation Bias of an independent Central Bank

once in the trap. This is not credible for the Central Bank if it optimizes under discretion. The Central Bank has incentives to promise inflation once in the trap. If the private sector believes this promise the equilibrium in period zero will be given by the optimal commitment solution in period 0. However once out of the trap (the natural rate of interest is positive again) in period 1, the Central Bank has incentives to renege on its promise in order to achieve lower inflation. This, however, cannot be a rational expectations equilibrium. Under discretion the only rational expectations solution is given by the discretion path in figure (3) and is characterized by excessive deflation and loss in output. This is the deflation bias of an independent discretionary Central Bank. This dilemma is as old as the mountains. In essence it is the same "dynamic inconsistency" problem as described in several fables by Aesop thousands of years ago. The moral of the story can be summarized as follows: A lion is trapped in a deep hole. A fox passes by and the lion asks it to pass down a tree branch. The lion makes many promises about the reward it will give the fox if it escapes from the trap. The fox understands that the lion is hungry and that once escapes

---

We use simple functional forms to motivate the values we choose. We assume the production function takes the form $y_t = h_t^{\alpha}$. We assume that labor share is 2/3. This implies that $\kappa = \frac{1-\alpha}{\alpha} = 0.5$. We assume that government spending is $G = 1/3$ so that the share of consumption in output is 2/3. The consumer is assumed to have a log-utility function and the dis-utility of working is quadratic i.e. of the power $s = 2$. This implies $\sigma = -\frac{u_c}{u_{cc}C}\frac{C}{Y} = 2/3$ and $\omega = \frac{s}{\alpha} - 1 = 2$. We assume that the rate of time preferences is $\beta = 0.98\hat{}3$ implying an equilibrium real rate of return in steady state of 2% per year. Note that these calibration values imply that $\theta = 2/7$. For a given year we assume that the output gap and (annualized) inflation have the same weight in the loss function. The shock corresponds to a natural rate of interest of $-5\%/\beta$ (thus a 5% inflation would be needed to close the output gap). This implies a value of $g_0 = -0.22$.

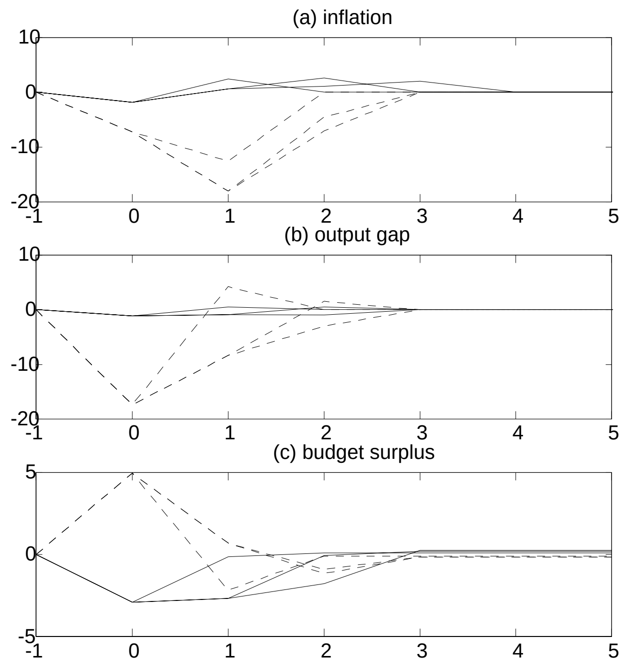

Figure 4: The Deflation Bias in a stochastic setting. The figure shows optimal monetary policy under discretion (dashed lines) and commitment (solid lines).

the trap it will simply eat it. Once the lion is free from the trap it has no incentive to fulfill its promise but has every reason to make the fox its meal.

These results are not unique to the simple process we assumed for g_t. The result can be generalized by considering a stochastic process for g_t.

Case 2 (C2) *In period zero there is an unexpected shock to* g_0. *Conditional on that* $g_{t-1} \neq 0$ *in every period* $t > 0$ *there is a probability* $\alpha_t > 0$ *that the vector of shocks* ξ_t *return back to zero so that* $g_t = 0$. *Let us call the stochastic date* g_t *returns to zero* T. *In periods* $t > T$ *there are no further shocks to the economy.*

Appendix B illustrates a general solution method for the stochastic process in C2. Figure (4) illustrates a solution for a simple stochastic process that is a special case of C2. Now we allow for the possibility that g_t does not return to zero with certainty in period 1. We suppose that in period 0 there is a 2/3 probability that the shocks return back to their zero in period 1. Similarly, conditional on being in the trap in period 1, there is a 2/3 probability that the shocks return back to their zero in period 2. Finally we suppose that the shocks are back to zero in period 3 with probability 1. Once the shocks return back to zero there are no further shocks to the economy.[33] In the figure we suppose that g_t follows a path so that in each of the periods the economy is trapped, the natural rate of interest is $-3/\beta\%$ (so that 3% expected inflation would

[33] We assume the same parameter value in this figure as in the one period trap apart from that now we assume that the labor contract is one and a half year. We assume a path for g_t that would imply a negative natural rate

be required to close the output gap). The figure shows all the different contingencies given this simple stochastic process. If the liquidity trap lasts over several periods, the optimal commitment policy does not only involve expected inflation, there will also be inflation during the trap. This can for example be seen by the line that illustrates the contingency that $r_t^n = -3/\beta\%$ in period 0,1 and 2. Again, although the optimal commitment solution involves expected inflation in the trap, a discretionary policy maker cannot commit to positive inflation.

Two aspects of a liquidity trap render the deflation bias a particularly acute problem and possibly a more serious one for policy makers than the inflation bias analyzed by KP/BG. First, if the Central Bank announces a higher inflation target in a liquidity trap it involves no direct policy action - since the short term nominal interest rate is at zero it cannot lower them any further. The Central Bank has therefore no means to manifest its appetite for inflation. Thus announcing an inflation target in a liquidity trap may be less credible than under normal circumstances when the Central Bank can take direct actions to show its commitment. Second, unfavorable shocks create the deflation bias. If these shocks are infrequent (which is presumably the case given the few examples of a binding zero bound in economic history) it is hard for the Central Bank to acquire any reputation for dealing with them. To make matters worse, optimal policy in a liquidity trap involves committing to inflation. In an era of price stability the optimal policy under commitment is fundamentally different from what has been observed in the past.

3.3.2 The Treasury

We will now consider the optimal policy of the Treasury when the Central Bank is independent. In this case, the Treasury's optimization problem reduces to the same optimization problem as was analyzed by Barro (1979). It is easy to verify that the debt dynamics satisfy the same equations for commitment and discretion in Appendix A if the Central Bank is independent.

Proposition 4 *If the Central Bank is independent the discretion solution for the Treasury is identical to the commitment solution.*

The logic behind this proposition is simple. If the Central Bank is independent the process for output, prices and the nominal interest rate are determined independently from the evolution of fiscal variables. In this case the expectations of the private sector about future fiscal policy have no effect on the equilibrium outcome at any given time. Since there is no feedback between expectations about future fiscal policy and current outcomes there is no advantage to commitment over discretion for the Treasury.

By equation (71) and (72) in Appendix A the approximate solution for the Treasury satisfies:

$$\hat{\tau}_t = E_t\hat{\tau}_{t+1} + \hat{\imath}_t - E_t\pi_{t+1}$$

of interest of $-3/\beta\%$ when in the trap (thus a 3% inflation target would be needed to close the output gap). The paths for the shocks g_t that would imply this evolution for the natural rate of interest for all of the 3 periods is [-0.11,-0.10,-0.07].

If we assume as Barro (1979) that the price level and the real rate of return is constant we get his well known tax-smoothing result that $\hat{\tau}_t = E_t\hat{\tau}_{t+1}$. As we assume that the real rate of return is not constant we obtain the result that a low real rate of return induces the government to reduce taxes at time t relative to taxes at time $t+1$.

Solving this for C1 results in:

$$\hat{\tau}_0 = -G^{-1}b_0 = \beta^2\hat{r}_0$$

The amount of taxes collected and debt issued depends on future real rates of interest, which in turn depends on whether the Central Bank minimizes under discretion or commitment. Figure (3) illustrates the case of a one period trap. If the Central Bank is able to commit, the Treasury will lower taxes further than it would if the Central bank is discretionary. This stems from the fact that the real rate of return will be lower under commitment than under discretion, since the Central Bank can commit to future inflation. Thus the benefits of cutting taxes and accumulating debt are higher when the Central Bank commits. Figure (4) demonstrates that the difference between commitment and discretion is more dramatic when we consider C2. This stems from the fact that in the stochastic case the real rate of return becomes positive under discretion. This gives the Treasury incentives to accumulate assets as opposed to debt giving rise to *tax increases* and budget surpluses during the first periods of the trap.

4 Gains of Cooperation: Committing to being Irresponsible

We have demonstrated that a discretionary Central Bank that is independent cannot commit to the inflation target that is mandated by optimal policy. Since the short-term nominal interest rate is bounded by zero it cannot take any action to credibly commit. The inability of the Central Bank to commit to future inflation resulted in excessive deflation and a negative output gap. The challenge for policy makers is thus to select a course of action that will render a higher inflation target credible. The Central Bank must "commit to being irresponsible" in the word of Krugman (1998). Here we illustrate a simple solution to this credibility problem. Suppose the independence of the Central Bank is abolished and the government coordinates fiscal and monetary policies. Then the government maximizes the social objectives illustrated in (22) taking into account the losses due to tax distortions, inflation and the output gap. In this case, the government can use fiscal policy to effectively commit the Central Bank to future inflation by cutting taxes and issuing nominal debt.

4.1 Equilibria under Discretion when there is Cooperation

4.1.1 Structure of the Game

The structure of the game is exactly the same as described when the Central Bank is independent except for that in this case the Treasury and the Central Bank coordinate their policy instruments to minimize social losses given by (22).

4.1.2 Strategy Functions of the Players

The strategies of the private sector are a function of both $\tilde{b}_{t-1}$ and ξ_{t-1} when the Treasury and the Central Bank cooperate since now the Central Bank takes into account the fiscal consequences of its actions. Thus we define strategy functions for the private sector as:

$$u^e_{t-1} = u^e(\tilde{b}_{t-1}, \xi_{t-1}) \tag{41}$$

$$v^e_{t-1} = v^e(\tilde{b}_{t-1}, \xi_{t-1}) \tag{42}$$

$$f^e_{t-1} = f^e(\tilde{b}_{t-1}, \xi_{t-1}) \tag{43}$$

Equation (26) and the IS and the AS equation indicate that government policy depends on $u^e_{t-1}, f^e_{t-1}, v^e_{t-1}$ and ξ_t. Given the strategy functions of the private sector we can define the strategy functions of the government as:

$$\pi_t = \pi(\tilde{b}_{t-1}, \xi_t, \xi_{t-1}) \tag{44}$$

$$\tau_t = \tau(\tilde{b}_{t-1}, \xi_t, \xi_{t-1}) \tag{45}$$

4.1.3 Characterizing Government Policy

The problem of the government is to select taxes and inflation subject to the constraints in every period t to minimize the expected value of the social loss function (22). It minimizes:

$$\min_{\pi_t, \tau_t} [\pi_t^2 + \lambda_x (\frac{Y_t - Y_t^n}{Y_t^n})^2 + \lambda_\tau \tau_t^2 + \beta V(\tilde{b}_t, \xi_t)] \tag{46}$$

s.t. (8),(21),(23),(26),(41)-(43).

Here $V(\tilde{b}_t, \xi_t)$ is the value function of the government, i.e. the expected value at time t of its losses in period $t+1$ onwards. This value is calculated under the expectations that the government will minimize under discretion from period $t+1$ onwards. The value function satisfies the Bellman equation:

$$V(\tilde{b}_{t-1}, \xi_{t-1}) = E_{t-1} \min_{\pi_t, \tau_t} \{\pi_t^2 + \lambda_x (\frac{Y_t - Y_t^n}{Y_t^n})^2 + \lambda_\tau \tau_t^2 + \beta V(\tilde{b}_t, \xi_t)\} \tag{47}$$

s.t. (8),(21),(23),(26),(41)-(43).

Definition 6 *An optimal discretion solution under cooperation is a Private Sector Equilibrium, private sector strategy functions (41)-(43), government strategy functions (44)-(45) and a* *value function $V(\tilde{b}_t, \xi_t)$ that satisfy: (i) Government minimization. The government strategy functions solve the minimization problem (46) and the value function $V(\tilde{b}_t, \xi_t)$ satisfies the Bellman equation (47) given an exogenous process $\{\xi_t\}$. (ii) Private sector maximization. The strategy functions of the private sector solve the household and firm maximization problems taking the government strategy functions and the exogenous process $\{\xi_t\}$ as given. (iii) Rational expectations and initial conditions.*

To characterize the strategy functions of the households and the government we can write a Lagrangian for the minimization problem in the Bellman equation:

$$
\begin{aligned}
L_t \quad = \quad & \frac{1}{2}\pi_t^2 + \frac{1}{2}\lambda_x(\frac{Y_t - Y_t^n}{Y_t^n})^2 + \frac{1}{2}\lambda_\tau \tau_t^2 + \beta V(\tilde{b}_t, \xi_t) \qquad (48) \\
& + \mu_t(\frac{\tilde{b}_t}{u_c(Y_t - G, \xi_t)} - \frac{1}{\beta f^e(\tilde{b}_{t-1}, \xi_{t-1})}\frac{\tilde{b}_{t-1}}{1+\pi_t} + G - \tau_t) + \eta_t(Y_t - S(\pi_t, \tilde{b}_{t-1}, \xi_{t-1})) \\
& + \psi_t(1 - \frac{u_c(Y_t, \xi_t)}{\beta f^e(\tilde{b}_t, \xi_t)}) + \gamma_t(\tilde{b}_t - \bar{b})
\end{aligned}
$$

There are six first-order conditions for this problem including 2 complementary slackness conditions. They are listed in Appendix A. The first-order conditions involve the derivative of the value function i.e. $V_{\tilde{b}}(\tilde{b}_t, \xi_t)$. It is not necessary to find an explicit form of the value function or its derivatives. The envelope condition for the Bellman equation gives an expression for $V_{\tilde{b}}(\tilde{b}_t, \xi_t)$ allowing us to substitute it out of the first-order conditions.

4.2 Solutions Paths for an Approximate Solution

In this subsection we illustrate an approximate solution to the equilibria defined in Definition 6. Again we approximate the constraints and the first-order conditions by a first-order Taylor expansion. The point we expand around is a constant solution defined in Appendix A. A nontrivial complication is raised by the presence of unknown private sector strategy functions and their derivatives in the approximate solution. In Appendix A we illustrate a solution method to find the value of these functions and their derivatives. Since the resulting system involves an inequality due to the zero bound on the nominal interest rate, we cannot apply standard methods for linear rational expectation models. Again we will use the simple assumptions about the stochastic process for g_t we showed in last section in Case 1 and 2.

It is useful to consider first the solution in the absence of a shock for a given initial value for real debt. In Appendix A we show that it takes the form:

$$b_t = \rho b_{t-1} \qquad (49)$$

$$\pi_t = \Pi b_{t-1} \qquad (50)$$

where we prove in Appendix A that ρ is a real number between 0 and 1. The coefficient Π is shown to be a positive number and is given by

$$\Pi = \frac{\lambda_\tau G}{\beta} + \rho\kappa^{-1}\sigma^{-1}\lambda_\tau G$$

This solution illustrates that debt can work as a device to effectively commit the government to inflation even if it is discretionary. The presence of debt creates inflation through two channels in our model: 1) If the government has outstanding *nominal* debt it has incentives to create inflation to reduce the real value of the debt. This effect is captured by $\frac{\lambda_\tau G}{\beta}$ in our expression for Π. 2) If the government issues debt at time t it has incentives to lower the real rate of return its pays on the debt it rolls over to time $t+1$. This incentive also translates into higher inflation and is captured by the term $\rho\kappa^{-1}\sigma^{-1}\lambda_\tau G$ in our expression for Π.[34]

Proposition 5 ***Committing to being Irresponsible.*** *If the Central Bank and the Treasury cooperate a discretionary government can commit to future inflation in a liquidity trap by cutting taxes and issuing nominal debt. Inflation is highest at first when out of the trap. It declines with public debt over the infinite horizon and converges to zero in absence of other shocks.*

Proof: see equations (49) and (50)

Proposition 5 does not establish whether or not it is optimal to cut taxes and issue debt in a liquidity trap (although our solution from the last section indicates that optimal monetary policy of an independent Central Bank should involve expected inflation). We now illustrate the optimal policy under discretion if monetary and fiscal policy are coordinated. Using (76),(77),(88)-(94) under C1A we can solve for the critical value of the shock g_0 that makes the zero bound binding. We call this value g_L^{CD} and it is given by:

$$g_L^{CD} = -\sigma\frac{(\kappa^2+\lambda_x)(1+\beta(1-\rho)+\beta G\Pi)-\sigma^{-2}G^2\lambda_\tau}{(\kappa^2+(1-\theta)\lambda_x)(1+\beta(1-\rho))}(1-\beta) \tag{51}$$

where the superscript CD stands for coordinated discretion. Note that this critical value differs from the value derived when the Central Bank is independent. It is easy to show that $g_L > g_L^{CD}$ when $(\kappa^2+\lambda_x)(1+\rho\beta\sigma^{-1}\kappa^{-1}) > \sigma^{-2}$ which is the case for a broad range of values. The zero bound is thus binding for a smaller range of values for g_0 under this condition . Let us assume C1B so that $g_0 < g_L^D$. Then solving the equations (76),(77),(88)-(94) in Appendix A yields (with several algebraic manipulations):

$$b_0 = -\hat{\tau}_0 = -\frac{\frac{\Pi}{\lambda_\tau}\beta\sigma(\kappa^2+(1-\theta)\lambda_x)}{1+\beta(1-\rho)+\frac{\Pi^2}{\lambda_\tau}\beta\sigma^2(\kappa^2+\lambda_x)-G\beta\Pi}(g_0-g_L^{CD})+b_L^{CD}>0 \tag{52}$$

where b_L^{CD} is a constant given by $b_L^{CD} \equiv \frac{G\beta}{1+\beta(1-\rho)}(1-\beta) > 0$. Equation (52) illustrates that deficit spending is optimal in a liquidity trap. The government cuts taxes and issues debt to effectively commit to future inflation. The solution for inflation in period 1 onwards is by (49)-(52):

$$\pi_t = \Pi\rho^t b_0 > 0 \text{ for } t \geq 1 \tag{53}$$

[34] Obstfeld (1991,1997) analyses a flexible price model with real debt (as opposed to nominal as in our model) but seignorage revenues due to money creation. He obtains a solution similar to (49) and (50) (i.e. debt in his model creates inflation but is paid down over time). Calvo and Guindotti (1990) similarly illustrate a flexible price model that has a similar solution. The influence of debt on inflation these authors illustrate is closely related to the first channel we discuss above. The second channel we show, however, is not present in these papers since they assume flexible prices.

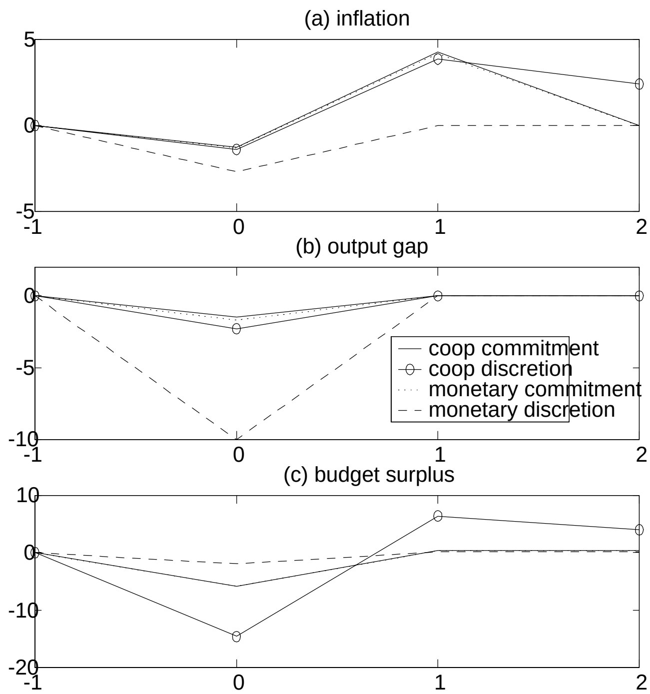

Figure 5: Committing to being irresponsible.

By committing to inflation the government curbs deflation and reduces output losses in period zero relative to the discretionary solution. This can be shown by solving (76),(77),(88)-(94) in Appendix A for the cooperative and non-cooperative cases:

$$x_0^D - x_0^{CD} = \sigma\Pi[\frac{\frac{\Pi}{\lambda_\tau}\beta\sigma(\kappa^2 + (1-\theta)\lambda_x)}{1+\beta(1-\rho)+\frac{\Pi^2}{\lambda_\tau}\beta\sigma^2(\kappa^2+\lambda_x) - G\beta\Pi}(g_0 - g_L^{CD}) - b_L^{CD}] < 0 \qquad (54)$$

$$\pi_0^D - \pi_0^{CD} = \sigma k\Pi[\frac{\frac{\Pi}{\lambda_\tau}\beta\sigma(\kappa^2 + (1-\theta)\lambda_x)}{1+\beta(1-\rho)+\frac{\Pi^2}{\lambda_\tau}\beta\sigma^2(\kappa^2+\lambda_x) - G\beta\Pi}(g_0 - g_L^{CD}) - b_L^{CD}] < 0 \qquad (55)$$

Proposition 6 ***The Optimality of Committing to being Irresponsible.*** *Suppose C1B. If the Central Bank and the Treasury cooperate, the government cuts taxes and issues positive amount of public debt in a liquidity trap. In doing so it will reduce the output gap and commits to inflation.*

Proof: See equations (52)-(55).

The evolution of each of the endogenous variables is shown in figure (5) and are labelled "cooperative discretion".[35] This figure shows the evolution under discretionary cooperation (described

[35] Again as the analytical results indicate our results do not in any way depend on the numerical values assumed. To calibrate the value of λ_τ we assume that the value of tax distortions associated with $G = 1/3$ is equivalent to social losses associated with 10% inflation. In Appendix C we show a table which illustrates that a discretionary maximizer would effectively commit to future inflation by issuing debt for a large range of values for λ_τ under C1B.

above) and contrasts it to the solution paths when the Central Bank is independent under discretion and commitment which we illustrated in the last section (we label these cases "monetary discretion" and "monetary commitment" in the figure above).[36] By cutting taxes and issuing debt in a liquidity trap the government curbs deflation and increases output almost to the same level as obtained under commitment by an independent Central Bank. The discretion solution is still inferior to the commitment solution since the price that has to be paid for this is higher inflation in period 2 onwards (when no inflation is desirable) and higher future taxes. Figure (6) illustrates that the same result holds true in the simple stochastic example we considered in the last section. There we contrasted the discretion solution under cooperation with the discretion solution that applies when the Central Bank is independent. Again a discretionary government can effectively commit to an inflation target in a liquidity trap by deficit spending, thereby curbing deflation and increasing the output gap.[37]

Proposition 5 and 6 summarize the central results of this paper. Even if the government is discretionary, it can regain the control of the price level that an independent Central Bank loses due to the zero bound. The government regains this control by coordinating fiscal and monetary policy, thus enabling it to increase output and prices in a liquidity trap. The principal policy tool we explore in this paper is deficit spending and debt accumulation. The channel is simple. Budget deficits generate nominal debt. Nominal debt in turn makes a higher inflation target in the future credible. Higher inflation expectations lower the real rate of interest and thus stimulate aggregate demand. This channel can be critical when there are large deflationary shocks since under these circumstances monetary policy can be frustrated by the zero bound on the short term nominal interest rate. Note that this policy involves direct actions by the government as opposed to only announcements about future policies. The government can announce an inflation target and then increase budget deficits until the target is reached. It is important to note that this channel of fiscal policy only works if fiscal and monetary policies are coordinated. If the Central Bank is independent, deficit spending has no effect on output and prices. To summarize:

Proposition 7 ***Deficit Spending in a Liquidity Trap.*** *If fiscal and monetary policies are coordinated, deficit spending will increase output and curb deflation in a liquidity trap by raising inflation expectations and lower the real rate of return. If the Central Bank is independent deficit spending has no effect on output and prices due to Ricardian equivalence.*

The effect of fiscal policy when coordinated with monetary policy is thus fundamentally different from its effects if the Central Bank is independent. This can be of potential importance in practice. Thus Krugman (2001) raises the question why deficit spending in Japan has failed to lift Japan out of its current depression while some economists believe that deficit spending helped Japan avoiding the Great Depression and that WWII jolted the US economy out of the

[36] The figure also shows the solution when the Central Bank and the Treasury cooperate and can commit. We describe that solution in the next section.

[37] It is worth noting that in both the numerical examples discussed the social welfare is higher under cooperation than non-cooperation when the government is discretionary as one would expect.

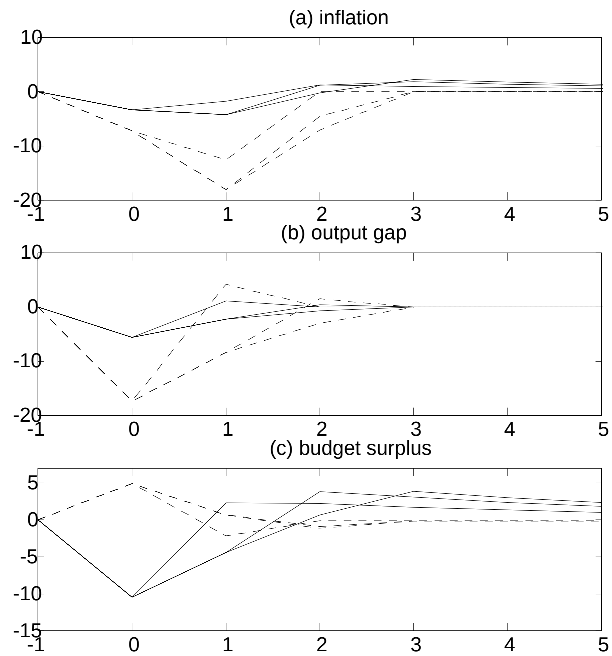

Figure 6: Committing to being irresponsible in a stochastic setting. The figure shows optimal cooperation under discretion (solid line) and discretion when the Central Bank is independent (dashed lines).

Great Depression. We argue in Section 7 that a critical difference between these two episodes of deficit spending is that the Bank of Japan is independent today whereas this was not the case in the Great Depression (and in the US the FED and the Treasury cooperated by establishing an interest rate peg in the 40's). This paper thus points towards an important channel of fiscal and monetary policy that may have been at work in Japan in the Great Depression and the US in WWII but is not present in Japan today. When monetary and fiscal policies are coordinated, deficit spending increases inflation expectations, which in turn lowers the real rate of return and stimulates aggregate demand.

4.3 Equilibria under Commitment when there is Cooperation

We finally define the optimal commitment solution under cooperation.

Definition 6 *The optimal commitment solution under cooperation is the Private Sector Equilibrium that maximizes social welfare. In this equilibrium the set of sequences* $\{\pi_t, \tau_t\}$ *minimizes (22) subject to (8), (21),(23), (24) and initial conditions given an exogenous process* $\{\xi_t\}$.

The Lagrangian problem can be written as:

$$
\begin{aligned}
L_s &= E_s \sum_{t=s}^{\infty} \beta^t [\frac{1}{2}\pi_t^2 + \frac{1}{2}\lambda_x (\frac{Y_t - Y_t^n}{Y_t^n})^2 + \frac{1}{2}\lambda_\tau \tau_t^2 \\
&+\mu_t (b_t - \frac{u_c(Y_{t-1}, \xi_{t-1})}{\beta f_{t-1}^e} \frac{b_{t-1}}{1+\pi_t} - G + \tau_t) + \eta_t (Y_t - S(\pi_t, u_{t-1}^e, v_{t-1}^e)) \\
&+\psi_t (1 - \frac{u_c(Y_t, \xi_t)}{\beta f_t^e}) + \gamma_t (u_c(Y_t, \xi_t) b_t - \bar{b}) \\
&+\phi_t^1 (u_t^e - u_c(Y_{t+1}, \xi_{t+1}) \frac{F^{-1}(Y_{t+1})}{1+\pi_{t+1}}) + \phi_t^2 (v_t^e - \tilde{v}_y(Y_{t+1}, \xi_{t+1}) F^{-1}(Y_{t+1})) + \phi_t^3 (f_t^e - \frac{u_c(Y_{t+1}, \xi_{t+1})}{1+\pi_{t+1}}]
\end{aligned} \tag{56}
$$

There are nine nonlinear first-order conditions that result from this minimization problem, including two complementary slackness conditions. They are relegated to Appendix A. The commitment solution is approximated by taking a first-order Taylor expansion of the first order conditions and the constraints around a constant solution. The constant solution and the necessary conditions for an approximate solution are shown in Appendix A.

4.4 Solutions Paths for an Approximate Solution

We will finally solve for an equilibrium where the Central Bank and the Treasury cooperate and the government can commit to the fully optimal solution. Again the value of g_0 that causes the zero bound to be binding is different from the solution when the Central Bank is independent. The value of g_0 that will make the zero bound binding is:

$$
g_L^{CC} = -\sigma \frac{\kappa^2 + \lambda_x - \sigma^{-2}\beta\lambda_\tau G^2}{\kappa^2 + (1-\theta)\lambda_x}(1-\beta) \tag{57}
$$

where the superscript CC stands for coordinated commitment. Note that $g_L < g_L^{CC}$ so that the zero bound will be binding for a larger set of values of g_0 in the cooperative commitment solution than if the Central Bank is independent.

The difference between optimal commitment under cooperation and optimal commitment when the Central Bank is independent is negligible. This can be seen by solving (69)-(77) for inflation and output under for these two cases and taking the difference of the result:

$$
\pi_1^{CC} - \pi_1^{C} = -\frac{\beta\lambda_\tau G^2 \sigma^{-2}[((1-\theta)\lambda_x + \kappa^2)g_0 - \sigma^{-2}\beta(1-\beta)]}{(\lambda_x + \kappa^2 + \beta\sigma^{-2} - \beta\lambda_\tau G^2\sigma^{-2})(\lambda_x + \kappa^2 + \sigma^{-2}\beta - \beta\lambda_\tau G^2 \sigma^{-2})} > 0
$$

The solution differs only slightly from the solution derived under optimal commitment by an independent Central Bank. The difference depends on the term $\beta\lambda_\tau G^2\sigma^{-2}$. For realistic values of G (e.g. in the range $0.2 - 0.4$) this term has negligible quantitative impact on the result since it involves the square of real government spending. Similarly it can be verified that the difference between the output gap and inflation in period zero is negligible. That this difference is negligible is illustrated by figure (5). There we show the solution paths for full government commitment and commitment when the Central Bank is independent, assuming the numerical values discussed in Section 5. The commitment solution under cooperation and non-cooperation are almost identical.

5 Extensions: Non-Standard Open-Market Operations

The model can be extended to analyze non-standard open market operations such as the purchasing of long-term bonds and foreign exchange or, even more exotically, dropping money from helicopters. Here we make a preliminary assessment of how these extensions could enrich our results.

5.1 Dropping Money from Helicopters

Friedman suggests that the government can always control the price level by increasing money supply, even in a liquidity trap. According to Friedman's famous *reductio ad absurdum* argument, if the government wants to increase the price level it can simply "drop money from helicopters." Eventually this should increase the price level – liquidity trap or not. Bernanke (2000) revisits this proposal and suggests that Japanese government should make "money-financed transfers to domestic households–the real-life equivalent of that hoary thought experiment, the "helicopter drop" of newly printed money." Our analysis supports Friedman's and Bernanke's suggestions if the Central Bank and the Treasury cooperate. Our analysis suggests, however, that it is not the increase in the money supply, as such, that has this effect, rather it is the increase in government liabilities (money +bonds).

Increasing money supply by buying government bonds will not have any effect when nominal interest rates are zero. At zero interest rates bonds and money are perfect substitutes. If the Central Bank buys bonds from the public all that happens is that people replace the bonds in their vaults with paper money. What happens if the government increases money supply without buying bonds, e.g. by dropping money from helicopters as Friedman's suggests? Since money and bonds are equivalent in a liquidity trap this is in fact exactly equivalent to issuing nominal bonds. When out of the trap and the nominal interest rate is positive again the Central Bank will simply replace the money in circulation with bonds by open market operation with out any cost.[38] So if we only consider equilibria where the government issues bonds that have one period maturity it is of no relevance whether the government increases its nominal debt or money supply by dropping money from helicopters. If the Treasury and the Central Bank cooperate the effects of a helicopter drop of money will thus be exactly the same as the effects of deficit spending that we have discussed in this paper. Thus the model of this paper can be interpreted as establishing a "fiscal theory" of dropping money from helicopters.

5.2 Open-Market Operations in Long-Term Bonds

It is often suggested that if long-term bonds have yields above zero, purchases of such bonds by the Central Bank should lower long-term interest rate and therefore increase spending.[39] As stressed

[38] This of course is critical since if large amounts of money is dropped from helicopters and not swapped back for bonds once out of the trap there will be hyperinflation.

[39] See e.g. Lebow (1993), Bernanke (1999), Blanchard (2000) and Clause, James, Dale Henderson, Athanasios Orphanides, David Small and Peter Tinsley (2000).

by Woodford (1999), the expectation theory of the term structure implies that this should not be possible, unless such actions are taken to signal a change in the bank's commitments regarding future monetary policy. Debt and taxes have no effect on the loss function of the Central Bank in our model. Therefore, open market operations in long-term debt have no effect if the Central Bank is independent. When the Central Bank and the Treasury cooperate on the other hand this fails to hold true. If the Central Bank buys long-term bonds with money in a liquidity trap under cooperation, it is in effect *changing the maturity structure* of outstanding government debt (if we consider the monetary base as government liability). Since money and short-term bonds are perfect substitutes in a liquidity trap, replacing long-term bonds with money is equivalent to replacing long-term bonds with short-term bonds. Thus the question of whether open market operations in long-term bonds is effective in a liquidity trap can be rephrased: Does changing the maturity structure of government debt increase inflation expectations? Preliminary results from work in progress by the author suggest that the answer is yes.[40] The logic behind this is straight forward. If the government holds long-term bonds it reduces its incentives to lower the short-term real rate of return as those returns will not apply to debt already issued. One of the two inflation incentives we discussed in Section 6 (for the case when all debt is short term) is thus reduced with higher maturity.[41] Since open market operations in long-term bonds shortens the maturity of outstanding debt, our preliminary results suggest that it may be effective to increase inflation expectations. An important caveat is that this channel will only be effective if the Central Bank is not independent.

5.3 Open Market Operations in Foreign Exchange

The model can also be extended to consider the effects of the government buying assets that have some real value or fixed rate of return such as foreign exchange. It is often suggested that purchases of foreign assets should be able to depreciate the exchange rate, and stimulate spending that way.[42] As pointed out by Christiano (1999) and Woodford (1999), however, the interest rate parity implies that such a policy should have no effect upon the exchange rate, except in so far as it changes expectations about future policy. Suppose it is the Treasury that buys the foreign exchange. For Japan this is a good assumption because by law it is the role of the Treasury to operate in the foreign exchange markets. If the Treasury buys large amounts of real assets (such as foreign exchange) it has no effect on the incentives of the Central Bank to create inflation if it is independent. Thus if the Central Bank is independent, foreign exchange market operations have no effect. If the Treasury and the Central Bank cooperate this result changes. Since open market operation in real assets by the Treasury would lead to a corresponding increase in public debt, this gives the government an incentives to create inflation through the same channels as we have

[40] These preliminary results are available from the author upon request.

[41] There will also be some effect on the government incentive to inflate away the real value of the debt (i.e. the first channel that debt affects the inflation incentives of the government discussed in section 4.2) but those effects go in the same direction and it can be shown that they are only of second order.

[42] See e.g. authors cited in footnote 5.

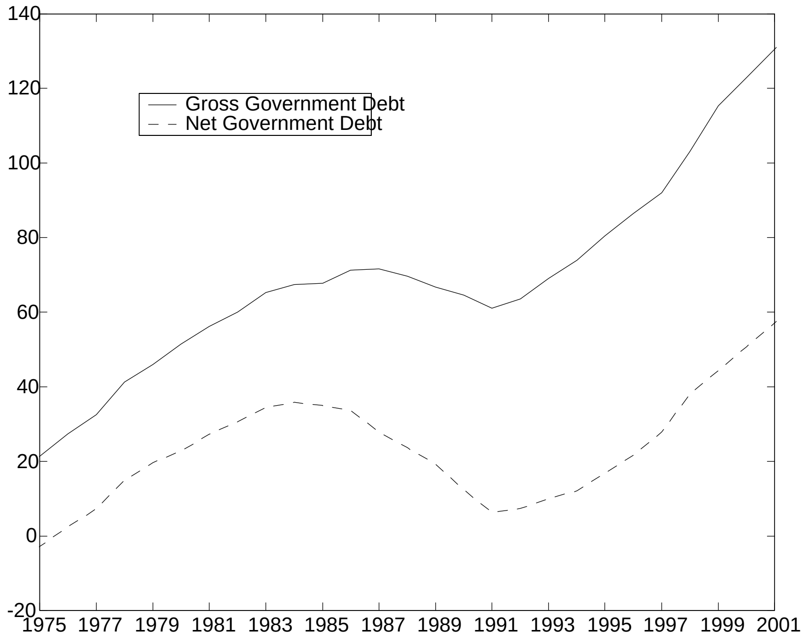

Figure 7: Ratio of net and gross government debt over GDP.

explored in this paper. Our conclusions thus once again depend on Central Bank independence.

6 Japan's Problems Today and how it Escaped from the Great Depression

It has been suggested by several commentators that fiscal policy has "reached its limit" in Japan today. Similarly it is suggested that monetary policy is ineffective due to the zero bound. Many have concluded that traditional tools of macroeconomics cannot get Japan out of its depression and that Japan needs "structural reforms." The figures presented on deflation and sluggish growth in the introduction, however, are easier to interpret as evidence of weak aggregate demand. For example Bernanke (2000) points out that "if Japan's slow growth were due entirely to structural problems on the supply side, inflation rather than deflation would presumably be in evidence." There is no doubt that there are various inefficiencies in the Japanese economy and that several government institutions and the banking sector could benefit from structural reforms. Regardless of that our analysis suggests that there is every reason to believe that the right combination of fiscal and monetary policies will stabilize inflation at a positive level and generate inflation expectations. This in turn lowers the real rate of interest and stimulates aggregate demand. Presumably this makes structural reforms easier to implement.

The reason it is commonly claimed that fiscal policy has reached its limit in Japan is the large public debt accumulated in recent years. Total government liabilities (money + bonds) have more than doubled, from roughly 64.5% in 1990 to 130% in 2000, largely due to deficit spending. This is the highest level of gross government debt in the G7 countries as is illustrated in Table 2 that shows the data for 2000. Figure (7) shows the evolution of government debt over GDP in Japan. The inability of deficit spending to jolt the Japanese economy out of its current depression is not inconsistent with our model. Our model suggests that if the Central Bank is independent, deficit spending has no effect on output or prices. The problem in Japan is not necessarily that fiscal policy is not expansive enough, but rather that fiscal and monetary policies are not coordinated; that the BOJ is "too independent". The Bank of Japan obtained legal independence in 1997. BOJ staff Fujiki, Okina, and Shiratsuka (2000) write: "The primary objective of monetary policy conducted by the Bank of Japan is to maintain price stability."

Although our model implies that the Central Bank's independence is a key obstacle to recovery, in practice the institutional changes needed are probably not as radical as one might think. Our "cooperative" solution is consistent with allowing the BOJ to maintain its *operational* independence, i.e. that it has the ability to determine day-to-day open market operations and interest rate movements. What our solution implies is that it should not have independence to determine its own *goals*. One simple solution that is consistent with our model involves the Japanese Parliament or the Cabinet setting an inflation target on regular basis and giving the BOJ the freedom to use its instruments to achieve it. That, however, is not enough. If the zero bound is binding a situation might arise where the Central Bank is unable to take any *actions* to achieve this target. This is particularly problematic in a liquidity trap because a higher future inflation target is more desirable than under normal circumstances (in order to make the real rate of return negative and stimulate demand). If the zero bound is binding, our solution indicates that fiscal policy can come to the aid of monetary policy and make an arbitrarily high future inflation target credible. The Ministry of Finance simply cuts taxes and increases debt to a level that is consisted with the desired level of inflation expectations.

To have the Cabinet or the Parliament of Japan decide an inflation target should not be considered a radical assault on the BOJ's independence. It is common practice in many countries to allow democratic bodies to determine the goals of monetary policy, even in the medium and short-run. In countries that have adapted a policy framework of inflation targeting, the target itself is often decided outside the Central Bank on a year-to-year basis. The Chancellor of Exchequer, for example, determines the inflation target of the Bank of England on an annual basis.[43] The important element that our analysis adds to the standard inflation targeting framework is that if the zero bound prohibits the Central Bank from reaching its current target or setting a higher future inflation target is not deemed as credible, direct *actions* can be taken by fiscal policy to solve the credibility problem.

Suppose the law of BOJ is changed, and the Japanese Cabinet or the Parliament is permitted to set an inflation target. Our analysis indicates that this is not enough in itself to guarantee a credible inflation target. In the absence of any debt issued the private sector has every reason expect the legislator to change the target once out of the trap.[44] In Japan, however, there is

[43] see e.g. Clementi (2000).

[44] In fact, given that the country is out of the trap, it would be in the interest of voters (if they have the social loss

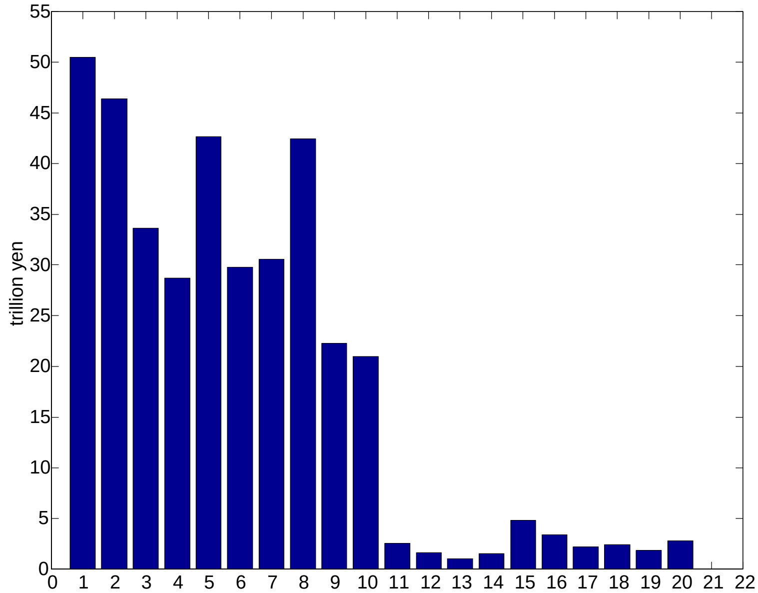

Figure 8: Time-to-maturity Structure of Outstanding Japanese Government Debt in years.

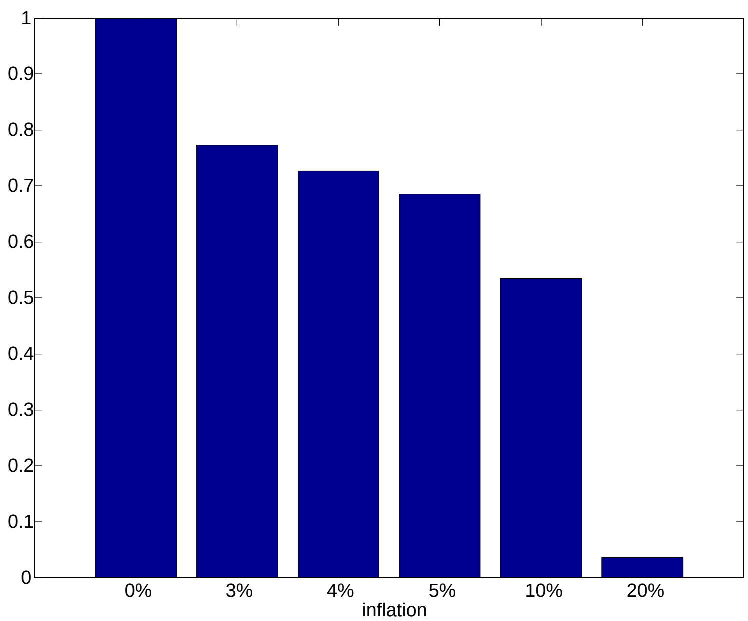

Figure 9: Reduction in the real value of Public debt if rolled over to 2021.

already substantial government debt outstanding. Would any further action be needed in Japan today apart from an inflation announcement by the Cabinet or the Parliament?

To analyze this question in the context of our model we need to study data on public debt. Figure (8) shows the maturity structure of outstanding debt in Japan, i.e. it shows the nominal value of debt due to be paid from 2001-2021. It is simple to calculate the government gains of inflation from this data if we make some simple assumption about the evolution of the natural rate of interest. Figure (9) illustrates how much the real debt would be reduced under different inflation rates. The underlying assumption is that the natural real rate is negative for 5 years at -3% and then returns to a positive rate. The figure shows the real value of the debt in 2021 if it is rolled over from 2001 onwards. We express the value of the debt as a ratio of real debt for a given inflation target over the real value of the debt if there would be zero inflation. We illustrate this ratio for 3,4,5,10 and 20 percent inflation. As illustrated in the figure there would be a substantial reduction in the real value of the debt with a credible inflation target, e.g. it is reduced by more than a quarter for 4 percent inflation. Given the high level of public debt today there seems to be substantial incentives for the government to adhere to the target.

	Net Government Debt	*Gross Government Debt*
Italy	98.66650022	110.8493514
Canada	66.02916459	104.8622681
Japan	50.70693774	122.8593272
France	42.54902586	64.44905255
United States	42.95781391	58.81071036
Germany	41.82264039	59.72264039
United Kingdom	33.49835674	54.4020255
Average in G7	53.74720563	82.27933934

Table 2: Gross and net government debt in the G7 countries.

There is, however, an important caveat to this argument. Although gross nominal debt over GDP is 130 percent in Japan today, that does not reflect the true inflation incentives of the government. Government institutions such as Social Security, Postal Savings, Postal Life Insurance and the Trust Fund Bureau hold a large part of this nominal debt. If the part of the public debt that is held by these institutions is subtracted from the total value of gross government debt it turns out that the "net" government debt over output is only 51 percent. The important thing to notice is that most of the government institutions that hold the government nominal debt have *real liabilities.* For example, Social Security (that holds roughly 25% of the nominal debt held by the government itself) pays Japanese pensions and medical expenses. Those pensions are *indexed to the CPI.* If inflation increases, the real value of Social Security assets will decrease but the real value of most its liabilities remain unchanged. Thus the Ministry of Finance would

function specified in earlier sections) to vote a government that promised to bring down inflation! Thus a rational expectations equilibrium with "credible" legislation cannot be established.

eventually have to step in to make up for any loss in the value of Social Security assets if the government is to keep its pension program unchanged. Therefore, the gains of reducing the real value of outstanding debt is partly offset by a decrease in the real value of the assets of government institutions such as Social Security. Thus the ratio of gross national debt to GDP *overestimates* the inflation incentives of the government, whereas the ratio of net value of government debt and GDP is perhaps a more realistic measure. Table 2 contrasts net government debt in Japan to net debt in the other G7 countries. As the table illustrates net government debt in Japan is below the average in the G7 countries. Thus trying to evaluate the inflation incentives of the government in Japan from data on gross debt can be highly misleading. In order for an inflation target determined outside the BOJ to be credible, further deficit spending or other actions that raise inflation expectations might be required (such as buying long-term bonds or foreign exchange).

	Change in GNP deflator	*Change in CPI*	*Change in WPI*	*Change in GNP*	*Government surplus over GNP*
1929	-	-2.3%	-2.8%	0.5%	-1.0%
1930	-	-10.2%	-17.7%	1.1%	2.0%
1931	-12.6%	-11.5%	-15.5%	0.4%	0.4%
1932	3.3%	1.1%	11.0%	4.4%	-3.5%
1933	5.4%	3.1%	14.6%	10.1%	-3.0%
1934	-1.0%	1.4%	2.0%	8.7%	-3.5%
1935	4.1%	2.5%	2.5%	5.4%	-3.3%
1936	3.0%	2.3%	4.2%	2.2%	-2.0%

Table 3: Coordination of Fiscal and Monetary Policy in the Great Depression in Japan.

An inflation-targeting regime of course is not the only possible interpretation of the cooperation solution in our model. Another possibility would be a complete abolition of Central Bank independence and the vesting of all monetary and fiscal policies in a single authority. There is an interesting historical precedent from Japan for this type of cooperative solution. During the late 1920's Japan was slipping into a depression. Growth had slowed down considerably, GNP rose by only 0.5 percent in 1929, 1.1 in 1930 and 0.4 percent in 1931. At the same time deflation was crippling the economy. This was registered by several macroeconomic indicators as is illustrated in Table (2). In December 1931 Korekiyo Takahasi was appointed the Finance Minister of Japan. Takahasi took three immediate actions. First, he abolished the gold standard. Secondly, he subordinated monetary policy to fiscal policy by having the BOJ underwrite government bonds. Third, he ran large budget deficits. These actions had dramatic effects as can be seen in Table 3. All the macroeconomic indicators changed in the direction predicted by our model. As the budget deficit increased, GNP rose and deflation was halted. During the same period, interest rates were at a historical low. Short-term nominal interest rates on commercial bills stood at 5.48% at the end of 1929. They went down to 4,38% by the end of 1932, 3,65% in 1933 and 3.29% in 1934 where they stayed throughout the 30's. That was the lowest level this measure of short-term nominal interest rates had ever reached to that date (data reaches back to 1883 and averages 7% up to the 30's). Since this data covers commercial bill rates it includes a risk premium and includes commercial paper of various maturity. It is thus unclear how much additional scope there was for lowering the real rate of return through the nominal interest rate. Our model indicates that the other actions taken, i.e. aggressive deficit spending that was financed by underwriting of government bounds, could have had considerable effects on the real rate of return through increasing *expected inflation*. This channel can be important in explaining the success of these policy measures in Japan. In 1936 Takahasi was assassinated and the government finances subjugated to military objectives. The following military expansion eventually led to excessive government debt and hyperinflation. Up til Takahasi's assassination, however, the economic policies in Japan during the 1930's were remarkably successful.

The result in Japan stands in sharp contrast with the experience in the US during the same period. Although the nominal interest rate (measured in terms of yields on short term government bonds that should thus have a small risk premium and only 3 month maturity) in the US went down close to zero during the 30's, this failed to generate a sustained increase in output and inflation.[45]

7 Conclusions

There are several lessons that can be drawn from our simple model. First, our model can answer puzzles such as: Why has deficit spending failed to get Japan out of its current depression while many economists maintain that it helped to get the Japanese economy out of the Great Depression in the 30's? Our model provides the stark conclusion that deficit spending will only increase output and prices if the Central Bank and the Treasury cooperate. Under cooperation, deficit spending increases inflation expectations, thereby lowering the real rate of return and stimulating aggregate demand. If the Central Bank is independent, deficit spending has no effect due to Ricardian equivalence. Second, our model casts some light on the costs of separating monetary policy from fiscal policy. Our central conclusion is that it can be beneficial in a liquidity trap to coordinate fiscal and monetary policy. It is worth stressing that our model does not indicate that Central Bank's independence is an inconvenient institutional framework under normal circumstances. What we have shown, however, is that under the unusual circumstances that create a liquidity trap, Central Bank independence can limit the government's ability to use non-standard methods to control the price level. The challenge for future research is to create an institutional

[45] It was not until 1942 that the Treasury and the FED implemented a similar arrangement of "cooperation" as in Japan. In 1942 an "interest rate peg" was established. The FED guaranteed a yield on Treasury bills of 0.33%. What followed was massive deficit spending due to WWII. The "cooperation" between the FED and the Treasury was not established as a response to deflationary pressures, as it was in Japan in the 30's. Rather it was a response to the financial needs of the Treasury during the war. The results, however, were similar to those seen in Japan a decade earlier. During the 40's, there was as sustained increase in output and inflation. Needless to say the reasons for the US recovery are too complicated to be captured by our simple model. It is possible, however, that some part of the explanation lies in the decrease in the real rate of interest that resulted from higher inflation expectations. Our model indicates that the cooperation between the FED and the Treasury in 1942 could have rationalized a substantial increase in inflation expectations.

framework that provides the price stability supplied by an independent Central Bank under normal circumstances and still allows fiscal and monetary policy to be coordinated when the zero bound prevents an independent Central Bank from fighting deflation effectively. Third our model casts light on several different policy options. By modeling the interaction of debt and inflation we argue that one can analyze the effects of several different policies in a single framework. We argue that foreign exchange interventions or open market operations in long-term bonds can be analyzed in terms of their effect on the structure of outstanding debt. This in turn influences the inflation incentives of the government. Working out the details of these extensions is a topic for further research.

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A First-Order Conditions and Approximate Solutions

A.1 Commitment

Below we show the first order condition for the minimization problem under commitment defined by the Lagrangians in (25) and (56). For compactness we only write the first order condition for (56). Below we explain how the first order conditions for (25) can be seen as a special case of these conditions:

$$\begin{aligned}\frac{\delta L_s}{\delta \pi_t} &= \pi_t + \mu_t \frac{u_c(Y_{t-1}, \xi_{t-1})}{\beta f^e_{t-1}} \frac{b_{t-1}}{(1+\pi_t)^2} - \eta_t S_\pi(\pi_t, u^e_{t-1}, v^e_{t-1}) \\ &+ \frac{1}{\beta}\phi^1_{t-1} \frac{u_c(Y_t, \xi_t) F^{-1}(Y_t)}{(1+\pi_t)^2} + \frac{1}{\beta}\phi^3_{t-1} \frac{u_c(Y_t, \xi_t)}{(1+\pi_t)^2} = 0\end{aligned} \tag{58}$$

$$\begin{aligned}\frac{\delta L_s}{\delta Y_t} &= \lambda_x\left(\frac{Y_t - Y^n_t}{Y^{n2}_t}\right) - \frac{\psi_t}{\beta} \frac{u_{cc}(Y_t, \xi_t)}{f^e_t} + \eta_t - \frac{u_{cc}(Y_t, \xi_t) b_t}{f^e_t} E_t \frac{\mu_{t+1}}{1+\pi_{t+1}} \\ &- \frac{\phi^1_{t-1}}{\beta}\left(\frac{u_{cc}(Y_t, \xi_t) F^{-1}(Y_t)}{1+\pi_t} + \frac{u_c(Y_t, \xi_t)}{F'(F^{-1}(Y_t))(1+\pi_t)}\right) \\ &- \frac{\phi^2_{t-1}}{\beta}\left(\tilde{v}_{yy}(Y_t, \xi_t) F^{-1}(Y_t) + \frac{\tilde{v}_y(Y_t, \xi_t)}{F'(F^{-1}(Y_t))}\right) - \frac{\phi^3_{t-1}}{\beta} \frac{u_{cc}(Y_t, \xi_t)}{1+\pi_t} + u_{cc}(Y_t,\xi_t)\gamma_t\end{aligned} \tag{59}$$

$$\frac{\delta L_s}{\delta \tau_t} = \lambda_\tau \tau_t + \mu_t = 0 \tag{60}$$

$$\frac{\delta L_s}{\delta b_t} = \mu_t - \frac{u_c(Y_t, \xi_t)}{f^e_t} E_t \frac{\mu_{t+1}}{1+\pi_{t+1}} + u_c(Y_t, \xi_t)\gamma_t = 0 \tag{61}$$

$$\frac{\delta L_s}{\delta u^e_t} = -\beta E_t \eta_{t+1} S_{u^e}(\pi_{t+1}, u^e_t, v^e_t) + \phi^1_t = 0 \tag{62}$$

$$\frac{\delta L_s}{\delta v^e_t} = -\beta E_t \eta_{t+1} S_{v^e}(\pi_{t+1}, u^e_t, v^e_t) + \phi^2_t = 0 \tag{63}$$

$$\frac{\delta L_s}{\delta f^e_t} = \frac{u_c(Y_t, \xi_t)}{\beta f^{e2}_t}\psi_t + \frac{u_c(Y_t, \xi_t)}{f^{e2}_t} b_t E_t \frac{\mu_{t+1}}{1+\pi_{t+1}} + \phi^3_t = 0 \tag{64}$$

Complementary slackness condition:

$$\frac{u_c(Y_t, \xi_t)}{\beta f^e_t} - 1 \geq 0, \psi_t \geq 0, \left(\frac{u_c(Y_t, \xi_t)}{\beta f^e_t} - 1\right),\ \psi_t = 0\ \forall\ t \tag{65}$$

$$\gamma_t \geq 0, \quad \bar{b} - u_c(Y_t, \xi_t) b_t \geq 0, \quad \gamma_t(\bar{b} - u_c(Y_t, \xi_t) b_t) = 0 \tag{66}$$

Note that a solution for the commitment program has to be accompanied by initial conditions for $\phi^1_t, \phi^2_t, \phi^3_t$ and b_{t-1}. We assume the initial conditions:

$$\phi^1_{-1} = \phi^2_{-1} = \phi^3_{-1} = \psi_{-1} = 0 \tag{67}$$

$$b_{-1} = 0 \tag{68}$$

An optimal commitment solution under cooperation is characterized by the processes $\{\pi_t, Y_t, \tau_t, b_t, \mu_t, \psi_t, \eta_t, \phi_t^1, \phi_t^2, \phi_t^3, \xi_t\}$ satisfying (8),(21),(24),(58)-(68) and the rational expectations constraints.

If the there is no cooperation the Central Bank will set the inflation and output gap independently of fiscal variables i.e. it will only minimize the period loss function $\pi_t^2 + \lambda_\tau x_t^2$. It can be verified that exactly the same solution is obtained as in the optimal commitment solution under cooperation above if we set $\lambda_\tau = 0$. The problem of the Treasury is to minimize (19) subject to (24) taking the evolution of $\{\pi_t, Y_t, f_t^e, v_t^e, u_t^e, \xi_t\}$ as given. It can be verified that this minimization problem gives the same first order condition as (60)-(61).

An optimal non-cooperative solution for the Central Bank under commitment is characterized by the processes $\{\pi_t, Y_t, \tau_t, f_t^e, v_t^e, u_t^e, \psi_t, \eta_t, \phi_t^1, \phi_t^2, \phi_t^3, \xi_t\}$ satisfying for $\lambda_\tau = 0$ (8),(24),(58)-(59), (62)-(65), (67) and the rational expectation constraints. The optimal noncooperative solution for the Treasury under commitment is characterized by the processes $\{b_t, \tau_t\}$ satisfying (21),(24),(60)-(61) and (68) where the set of processes $\{\pi_t, Y_t, f_t^e, v_t^e, u_t^e, \xi_t\}$ is exogenously given.

A.1.1 Approximate Solution

To obtain an approximate solution we do a first order Taylor expansion around a constant solution. The constant solution we expand around and solves the equation above is $\pi = b = \phi = \eta = \phi = 0$, $\mu = -\lambda_\tau \tau = -\lambda_\tau G$ and $\bar{\imath} = \frac{1}{\beta} - 1$. This constant solution would result for the initial conditions $\phi_{-1}^1 = \phi_{-1}^2 = \phi_{-1}^3 = \psi_{-1} = b_{-1} = 0$ at all times if there are no shocks to the economy so that $\xi_t = 0$ at all times. If the shocks are small enough the approximate equations will be arbitrarily close to the exact equations.

To simplify we can write the first order conditions and the constraints in the terms of the output gap $\hat{x}_t$ and the two shocks $\hat{r}_t^n$ and ϵ_t. These two shocks summarize as a linear combination all the possible shocks in the vector ξ_t. If we substitute out ϕ_t^1, ϕ_t^2 and ϕ_t^3 we can write the linearized first order conditions as (with several tedious algebraic manipulations):[46]

$$\pi_t - \kappa^{-1}\eta_t + \kappa^{-1}E_{t-1}\eta_t - \frac{1}{\beta^2}\psi_{t-1} = 0 \tag{69}$$

$$\lambda_x \hat{x}_t + \frac{\sigma^{-1}}{\beta}\psi_t - \frac{\sigma^{-1}}{\beta^2}\psi_{t-1} + \eta_t + \kappa^{-1}(\omega + \sigma^{-1})E_{t-1}\eta_t - \lambda_\tau G\sigma^{-1}b_t + \frac{\lambda_\tau G\sigma^{-1}}{\beta}b_{t-1} = 0 \tag{70}$$

$$\hat{\tau}_t - \hat{\mu}_t = 0 \tag{71}$$

$$\hat{\mu}_t - E_t\hat{\mu}_{t+1} - \sigma^{-1}(E_t\hat{x}_{t+1} - \hat{x}_t) - \hat{r}_t^n - \lambda_\tau^{-1}G^{-1}u_c\gamma_t = 0 \tag{72}$$

The complementary slackness conditions are:

$$E_t\hat{x}_{t+1} - \hat{x}_t + \sigma E_t\pi_{t+1} + \sigma\hat{r}_t^n + \beta\sigma\bar{\imath} \geq 0,\ \psi_t \geq 0,\ \psi_t(E_t\hat{x}_{t+1} - \hat{x}_t + \sigma E_t\pi_{t+1} + \sigma\hat{r}_t^n + \beta\sigma\bar{\imath}) = 0 \tag{73}$$

[46] It is useful to note that in the constant solution $S_{u^e} = \frac{\kappa^{-1}}{u^e}$, $S_{v^e} = -\frac{\kappa^{-1}}{v^e}$ and $S_\pi = \kappa^{-1}$.

$$\gamma_t \geq 0, \quad \bar{b} - u_c b_t \geq 0, \quad \gamma_t(\bar{b} - u_c b_t) = 0 \tag{74}$$

The initial conditions are:

$$b_{-1} = \psi_{-1} = 0 \tag{75}$$

The linearized AS equation and the budget constraint can be written as:

$$\pi_t = \kappa \hat{x}_t + E_{t-1}\pi_t + \epsilon_t \tag{76}$$

$$b_t = \frac{1}{\beta} b_{t-1} - G\hat{\tau}_t \tag{77}$$

All the hatted variables above are defined as a percentage deviation from the constant solution. An approximate solution for the cooperation case is found using the linearized equation if we make assumptions about the stochastic process of $\hat{r}_t^n$ and ϵ_t. Similarly an approximate solution for the Central Bank's problem in the noncooperative case is found by setting $\lambda_\tau = 0$ in the system of equations above. Finally an approximate solution for the Treasury's problem under noncooperation can be found by considering equation (71)-(72) and (74)-(75) for $\lambda_\tau > 0$. Note that the process for $\hat{r}_t^n$ and ϵ_t can be found if we make some assumption about the process for g_t since $\hat{r}_t^n = \sigma^{-1}E_t[(g_t - \hat{Y}_t^n) - E_t(g_{t+1} - \hat{Y}_{t+1}^n)] = \sigma^{-1}(1-\theta)E_t(g_t - g_{t+1})$ and $\epsilon_t = \kappa\theta(g_t - E_{t-1}g_t)$.

A.2 Discretion

Below we show the first order condition for the minimization problem under discretion defined by the Lagrangian in (48) and (32) and (34). For compactness we only write the first order condition for (48). Below we explain how the first order conditions for (32) and (34) can be seen as a special case of the condition written below:

$$\frac{\delta L_s}{\delta \pi_t} = \pi_t + \mu_t \frac{1}{\beta} f^e(\tilde{b}_{t-1}, \xi_{t-1})^{-1} \frac{\tilde{b}_{t-1}}{(1+\pi_t)^2} - \eta_t S_\pi(\tilde{b}_{t-1}, \pi_t) = 0 \tag{78}$$

$$\frac{\delta L_s}{\delta Y_t} = \lambda_x \frac{Y_t - Y_t^n}{Y_t^{n2}} - \mu_t \frac{\tilde{b}_t u_{cc}(Y_t, \xi_t)}{u_c(Y_t, \xi_t)^2} + \eta_t - \psi_t \frac{u_{cc}(Y_t, \xi_t)}{\beta f^e(\tilde{b}_t, \xi_t)} = 0 \tag{79}$$

$$\frac{\delta L_s}{\delta \tau_t} = \lambda_\tau \tau_t + \mu_t = 0 \tag{80}$$

$$\frac{\delta L_s}{\delta \tilde{b}_t} = \beta V_{\tilde{b}}(\tilde{b}_t, \xi_t) + \frac{\mu_t}{u_c(Y_t, \xi_t)} + \psi_t \frac{u_c(Y_t, \xi_t) f_{\tilde{b}}^e(\tilde{b}_t, \xi_t)}{\beta f^e(\tilde{b}_t, \xi_t)^2} + \gamma_t = 0 \tag{81}$$

Complementary slackness conditions:

$$\psi_t \geq 0, \ \frac{u_c(Y_t, \xi_t)}{\beta f_t^e} - 1 \geq 0, \quad \psi_t(\frac{u_c(Y_t, \xi_t)}{\beta f_t^e} - 1) = 0 \tag{82}$$

$$\gamma_t \geq 0, \quad \bar{b} - \tilde{b}_t \geq 0, \quad \gamma_t(\ \bar{b} - \tilde{b}_t) = 0 \tag{83}$$

The optimal plan under discretion also satisfies an envelope condition:

$$V_{\tilde{b}}(\tilde{b}_{t-1},\xi_{t-1}) = E_{t-1}[-\frac{\mu_t}{\beta(1+\pi_t)}(f^e(\tilde{b}_{t-1},\xi_{t-1})^{-1} - f^e(\tilde{b}_{t-1},\xi_{t-1})^{-2}f^e_{\tilde{b}}(\tilde{b}_{t-1},\xi_{t-1})\tilde{b}_{t-1}) - \eta_t S_{\tilde{b}}(\pi_t,\tilde{b}_{t-1},\xi_{t-1})] \tag{84}$$

Using the envelope condition we can substitute the derivative of the value function into (81) eliminating the value function from the first order conditions:

$$\frac{\mu_t}{u_c(Y_t,\xi_t)} - (1-\frac{f^e_{\tilde{b}}(\tilde{b}_t,\xi_t)\tilde{b}_t}{f^e(\tilde{b}_t,\xi_t)})(E_t\frac{u_c(Y_{t+1},\xi_{t+1})}{1+\pi_{t+1}})^{-1}E_t\frac{\mu_{t+1}}{1+\pi_{t+1}}$$

$$+\psi_t\frac{u_c(Y_t,\xi_t)f^e_{\tilde{b}}(\tilde{b}_t,\xi_t)}{\beta f^e(\tilde{b}_t,\xi_t)^2} - \beta E_t S_{\tilde{b}}(\pi_{t+1},\tilde{b}_t)\eta_{t+1} + \gamma_t = 0 \tag{85}$$

To obtain a solution to the system we must specify initial condition for both state variables:

$$\tilde{b}_{-1} = 0 \tag{86}$$

$$\xi_{-1} = 0 \tag{87}$$

An optimal discretion solution under cooperation is characterized by the processes $\{\pi_t, Y_t, \tau_t, b_t, \mu_t, \psi_t, \eta_t, \gamma_t, \xi_t\}$ satisfying (8),(21),(26),(78)-(80),(82),(85),(86) and (87).

If there is no cooperation the Central Bank will set inflation and output gap independently of fiscal variables. It can be verified that the solution for the Central Banks problem is the same as the optimal solution under cooperation shown above if $\lambda_\tau = 0$. Similarly it can be verified that the Treasury's problem will give the same first order conditions as shown in (80), (83) and (85) if we set $f_b = \psi_t = \eta_t = 0$.

An optimal non-cooperative solution for the Central Bank under discretion is characterized by the processes $\{\pi_t, Y_t, \tau_t, b_t, \mu_t, \psi_t, \eta_t, \gamma_t\}$ satisfying (8),(26), (78)-(80), (82), (85) and (87) when $\lambda_\tau = 0$. The optimal noncooperative solution for the Treasury under commitment is characterized by the processes $\{b_t, \mu_t\}$ that satisfy (21),(80)-(85), (86) and (87) when $f_b = \psi_t = \eta_t = 0$.

A.2.1 Approximate Solution

To obtain an approximate solution we once again do a first order Taylor expansion around a constant solution. The constant solution we expand around and solves the equation above is $\pi = b = \eta = \gamma = 0$, $\mu = -\lambda_\tau \tau = -\lambda_\tau G$ and $\bar{\imath} = \frac{1}{\beta} - 1$. This solution would result for the initial conditions $\phi^1_{-1} = \phi^2_{-1} = \phi^3_{-1} = \psi_{-1} = b_{-1} = 0$ if there are no shocks to the economy so that $\xi_t = 0$ at all times. If the shocks are small enough the approximate equations will be arbitrarily close to the exact equations.

As in the commitment case we will write the first order conditions in the terms of the output gap $\hat{x}_t$ and the two shocks $\hat{r}^n_t$ and ϵ_t. The linearized first order conditions can be written as (with several tedious algebraic manipulations):[47]

$$\pi_t - \frac{\lambda_\tau G}{\beta}b_{t-1} - \kappa^{-1}\eta_t = 0 \tag{88}$$

[47] Note that $\mathrm{S}_b = 0$ in the constant solution which elimintes the term η_{t+1} in (91).

$$\lambda_x \hat{x}_t - \sigma^{-1}\lambda_\tau G b_t + \eta_t + \frac{\sigma^{-1}}{\beta}\psi_t = 0 \tag{89}$$

$$\hat{\tau}_t - \hat{\mu}_t = 0 \tag{90}$$

$$\hat{\mu}_t - E_t\hat{\mu}_{t+1} - \sigma^{-1}E_t(\hat{x}_{t+1} - \hat{x}_t + \sigma\hat{r}_t^n) + f_b^e b_t + \frac{f_b^e}{\beta\lambda_\tau G}\psi_t + \lambda_\tau^{-1}G^{-1}u_c\gamma_t = 0 \tag{91}$$

The complementary slackness condition is:

$$E_t\hat{x}_{t+1} - \hat{x}_t + \sigma E_t\pi_{t+1} + \sigma\hat{r}_t^n + \beta\sigma\bar{\imath} \geq 0,\ \ \psi_t \geq 0,\ \ \psi_t(E_t\hat{x}_{t+1} - \hat{x}_t + \sigma E_t\pi_{t+1} + \sigma\hat{r}_t^n + \beta\sigma\bar{\imath}) = 0 \tag{92}$$

$$\gamma_t \geq 0, \quad \bar{b} - u_c b_t \geq 0, \quad \gamma_t(\bar{b} - u_c b_t) = 0 \tag{93}$$

The initial condition is:

$$b_{-1} = 0 \tag{94}$$

A solution can now be found to the linearized equation above in addition to linearized AS equation (76) and the budget constraint (77). This solution involves the derivative of an unknown function $f^e(\tilde{b}_t, \xi_t)$ in the constant solution i.e the term f_b^e. As we seek to find the derivative of f it is not enough to solve for the value of f in the constant solution. Rather we need to solve for the value of f away from the constant solution. It will not be necessary to solve for the transition dynamics of f for all type of shocks since we are only interested in evaluating the derivative of f with respect to $\tilde{b}$. This derivative can be found by assuming initial condition for $\tilde{b}$ that are different from the constant solution. In the absence of shocks $\tilde{b}_t$ is the only state variable of the game. Up to a first order approximation we can write $\tilde{b}_t = u_c b_t$. Then in the absence of shocks a first order solution can be found of the form:

$$\pi_t = \Pi b_{t-1} \tag{95}$$

$$b_t = \rho b_{t-1} \tag{96}$$

$$\hat{\tau}_t = \delta b_{t-1} \tag{97}$$

Consider the value of the function $f^e(\tilde{b}_t, \xi_t)$ at time t when there are no shocks to the economy and perfect foresight (given some initial value of $\tilde{b}_t$):

$$u_c f^e(\tilde{b}_t)^{-1} = 1 + \pi_{t+1} \tag{98}$$

A first order approximation to this equation yields

$$-f_{\tilde{b}}^e b_t = \pi_{t+1} \tag{99}$$

Equation (95) and (99) imply:

$$f_b^e = -\Pi$$

Equation (77) implies:

$$\delta = \frac{1 - \rho\beta}{\beta}G^{-1}$$

Equation (88)-(90) and (96) imply:

$$\Pi = \frac{\lambda_\tau G}{\beta} + \kappa^{-1}\sigma^{-1}\lambda_\tau G\rho$$

The value of ρ can be found considering (77),(91), (90) and our solution for f_b^e. In the absence of shocks those equations combined can be written as:

$$\beta E_t b_{t+1} - (1+\beta+\lambda_\tau G^2 + \beta\kappa^{-1}\sigma^{-1}\lambda_\tau G^2\rho)b_t + b_{t-1} + u_c\lambda_\tau^{-1}\gamma_t = 0 \tag{100}$$

Suppose the debt limit is not binding so that $\gamma_t = 0$. Then ρ solves the characteristic equation:

$$\Gamma(\rho) = \beta(1-\kappa^{-1}\sigma^{-1}\lambda_\tau \mathrm{G}^2)\rho^2 - (1+\beta+\lambda_\tau \mathrm{G}^2)\rho + 1 = 0 \tag{101}$$

Then the solution for b_t when the debt ceiling is not binding is of the form

$$b_t = c_1\rho_1^t + c_2\rho_2^t \tag{102}$$

Proposition 8 *The characteristic equation (101) has one root* $0 < \rho_1 < 1$ *and one root* $\rho_2 > \left|\frac{1}{\beta}\right|$

Proof. Suppose $1 > \kappa^{-1}\sigma^{-1}\lambda_\tau G^2$ Then $\Gamma(\rho = 0) = 1 > 0$. $\Gamma(\rho = 1) = \beta - \beta\kappa^{-1}\sigma^{-1}\lambda_\tau G^2 - 1 - \beta - \lambda_\tau G^2 + 1 = -(\beta\kappa^{-1}\sigma^{-1}+1)\lambda_\tau G^2 < 0$. Thus one root is less than 1. $\Gamma(\rho = \frac{1}{\beta}) = \frac{1}{\beta} - \frac{\kappa^{-1}\sigma^{-1}\lambda_\tau G^2}{\beta} - \frac{1}{\beta} - 1 - \frac{\lambda_\tau G^2}{\beta} + 1 = -\frac{\kappa^{-1}\sigma^{-1}\lambda_\tau G^2}{\beta} - \frac{\lambda_\tau G^2}{\beta} < 0$ so that one root must be bigger than $\frac{1}{\beta}$. Suppose $1 < \kappa^{-1}\sigma^{-1}\lambda_\tau G^2$ (*). $\Gamma(\rho = 0) = 1 > 0$ Then one root is positive and one is negative. $\Gamma(\rho = 1) = \beta - \beta\kappa^{-1}\sigma^{-1}\lambda_\tau G^2 - 1 - \beta - \lambda_\tau G^2 + 1 = -(\beta\kappa^{-1}\sigma^{-1}+1)\lambda_\tau G^2 < 0$. The positive root is less than one $\Gamma(\rho = -\frac{1}{\beta}) = \frac{1}{\beta} - \frac{\kappa^{-1}\sigma^{-1}\lambda_\tau G^2}{\beta}) + \frac{1}{\beta} + 1 + \frac{\lambda_\tau G^2}{\beta} + 1 = 2(1+\frac{1}{\beta}) + \frac{\lambda_\tau G^2}{\beta}(1-\kappa^{-1}\sigma^{-1}\lambda_\tau G^2) > 0$ if $2(1+\frac{1}{\beta}) > \frac{\lambda_\tau G^2}{\beta}(1-\kappa^{-1}\sigma^{-1}\lambda_\tau G^2)$. This must be the case since $\frac{\lambda_\tau G^2}{\beta}(1-\kappa^{-1}\sigma^{-1}\lambda_\tau G^2) < 0$ by (*). Thus the negative root must be greater in absolute value than $\frac{1}{\beta}$ ■

The borrowing limit imposes that $b_t \leq \bar{b}$. If the debt dynamic involved an explosive root the debt limit would be reached in finite time. This however cannot be an equilibrium because if the debt limit is reached in the absence of any shocks then (91) is violated. To see this consider the period in which the debt limit is reached.[48] Then $b_t = \bar{b} \geq b_{t+1}$ and $b_t > b_{t-1}$. Then by (100) $\gamma_t = \beta E_t b_{t+1} - (1+\beta+\lambda_\tau G^2 + \beta\kappa^{-1}\sigma^{-1}\lambda_\tau G^2\rho_1)b_t + b_{t-1} < 0$ that violates (93). Thus $c_2 = 0$ in (102) and

$$b_t = \rho b_{t-1} \qquad 0 < \rho < 1$$

Thus ρ solves:

$$\rho = \min\left\{\left|\frac{1+\beta+\lambda_\tau G^2 + \sqrt{(1+\beta+\lambda_\tau G^2)^2 - 4\beta(1-\kappa^{-1}\sigma^{-1}\lambda_\tau G^2)}}{2\beta(1-\kappa^{-1}\sigma^{-1}\lambda_\tau G^2)}\right|, \left|\frac{1+\beta+\lambda_\tau G^2 - \sqrt{(1+\beta+\lambda_\tau G^2)^2 - 4\beta(1-\kappa^{-1}\sigma^{-1}\lambda_\tau G^2)}}{2\beta(1-\kappa^{-1}\sigma^{-1}\lambda_\tau G^2)}\right|\right\}$$

[48] The argument only considers the case when $\rho_2 > 1/\beta$ which is the case of economic interest.

B Recursive Solution Method

B.1 Commitment

Here we outline a recursive solution method for discretion and commitment for Case 2. The solution for Case 1 can be seen as a special case of the solution illustrated. In Case 2 we assume that there is an unexpected shock to g_0 and that in every period $t > 0$ there is a probability $\alpha_t > 0$ that it reverses back to 0. Once it reverses back to zero it stay's there forever after. We call the stochastic date it reverses back to zero T. We assume that there is some date S (that can be arbitrarily far into the future) at which g_t must have returned back to zero with probability 1 (in the Case 1 $S = 1$).

The solution has to satisfy the following equations:

$$\pi_t - \kappa^{-1}\eta_t + \kappa^{-1}E_{t-1}\eta_t - \frac{1}{\beta^2}\psi_{t-1} = 0 \tag{103}$$

$$\lambda_x \hat{x}_t + \frac{\sigma^{-1}}{\beta}\psi_t - \frac{\sigma^{-1}}{\beta^2}\psi_{t-1} + \eta_t + \kappa^{-1}(\omega + \sigma^{-1})E_{t-1}\eta_t - \lambda_\tau G\sigma^{-1}b_t + \frac{\lambda_\tau G\sigma^{-1}}{\beta}b_{t-1} = 0 \tag{104}$$

$$-b_t + \frac{1}{\beta}b_{t-1} + E_t b_{t+1} - \frac{1}{\beta}b_t - \sigma^{-1}G(E_t\hat{x}_{t+1} - \hat{x}_t) - G\hat{r}_t^n = 0 \tag{105}$$

$$\hat{x}_t - E_t\hat{x}_{t+1} - \sigma E_t\pi_{t+1} - \sigma\hat{r}_t^n - \sigma\beta\bar{\imath} \geq 0, \psi_t \geq 0, \psi_t(\hat{x}_t - E_t\hat{x}_{t+1} - \sigma E_t\pi_{t+1} - \sigma\hat{r}_t^n - \sigma\beta\bar{\imath}) = 0 \tag{106}$$

$$\pi_t = \kappa\hat{x}_t + E_{t-1}\pi_t + \epsilon_t \tag{107}$$

Let us denote the any variable $\hat{q}_t^T$ as the value of the variable conditional on that the economy is in the trap and $\hat{q}_t^N$ as the value of the variable conditional on that the economy is out of the trap.

t>T

then $\psi_t^N = \hat{x}_t^N = \hat{r}_t^{nN} = 0$ Then by (103)

$$\pi_t^N = 0$$

and by (105)

$$b_t^N = b_T^N$$

t=T

$$\pi_T^N - \kappa^{-1}\eta_T^N + \kappa^{-1}\alpha_T\eta_T^N + +\kappa^{-1}(1-\alpha_T)\eta_T^T - \frac{1}{\beta^2}\psi_{T-1}^T = 0 \tag{108}$$

$$\begin{aligned} & \lambda_x\hat{x}_T^N - \frac{\sigma^{-1}}{\beta}\psi_T^N + \frac{\sigma^{-1}}{\beta^2}\psi_{T-1}^T + \eta_T^N + \kappa^{-1}(\omega+\sigma^{-1})\alpha_T\eta_T^N \\ & +\kappa^{-1}(\omega+\sigma^{-1})(1-\alpha_T)\eta_T^T - \lambda_\tau G\sigma^{-1}b_T^N + \frac{\lambda_\tau G\sigma^{-1}}{\beta}b_{T-1}^T \\ = \; & 0 \end{aligned} \tag{109}$$

$$\frac{1}{\beta}b_{T-1}^T - \frac{1}{\beta}b_T^N + \sigma^{-1}G\hat{x}_T^N = 0 \tag{110}$$

$$\psi_T^N = 0 \tag{111}$$

$$\pi_t^N = \kappa \hat{x}_t^N + \alpha_T \pi_t^N + (1-\alpha_T)\pi_T^T + \epsilon_T^N \tag{112}$$

$$Z_t = \begin{bmatrix} \pi_t^N \\ \hat{x}_t^N \\ \eta_t^N \\ b_t^N \end{bmatrix}, \; P_t = \begin{bmatrix} \psi_t^N \\ b_t^N \end{bmatrix}$$

We can write (108)-(112) on the form

$$Z_T^N = K_T Z_T^T + F_T P_{T-1}^T + W_T \tag{113}$$

t<T

$$\pi_t^T - \kappa^{-1}\eta_t^T + \kappa^{-1}\alpha_T \eta_t^N + \kappa^{-1}(1-\alpha_T)\eta_t^T - \frac{1}{\beta^2}\psi_{t-1}^T = 0 \tag{114}$$

$$\begin{aligned} & \lambda_x \hat{x}_t^T + \frac{\sigma^{-1}}{\beta}\psi_t^T - \frac{\sigma^{-1}}{\beta^2}\psi_{t-1}^T + \eta_t^T + \kappa^{-1}(\omega+\sigma^{-1})\alpha_t \eta_t^N \\ & +\kappa^{-1}(\omega+\sigma^{-1})(1-\alpha_t)\eta_t^T - \lambda_\tau G\sigma^{-1} b_t^T + \frac{\lambda_\tau G \sigma^{-1}}{\beta} b_{t-1}^T \\ = \; & 0 \end{aligned} \tag{115}$$

$$\begin{aligned} & -b_t^T + \frac{1}{\beta} b_{t-1}^T + \alpha_{t+1} b_{t+1}^N + (1-\alpha_{t+1}) b_{t+1}^T - \frac{1}{\beta} b_t^T \\ & -\sigma^{-1} G \alpha_{t+1} \hat{x}_{t+1}^N - \sigma^{-1} G (1-\alpha_{t+1}) \hat{x}_{t+1}^T + \sigma^{-1} G \hat{x}_t^T - G \hat{r}_t^{nT} \\ = \; & 0 \end{aligned} \tag{116}$$

$$\hat{x}_t^T - \alpha_{t+1}\hat{x}_{t+1}^N - (1-\alpha_{t+1})\hat{x}_{t+1}^T - \sigma\alpha_{t+1}\pi_{t+1}^N - \sigma(1-\alpha_{t+1})\pi_{t+1}^T - \sigma \hat{r}_t^{nT} - \sigma\beta\bar{\imath} = 0 \tag{117}$$

$$\pi_t^T = \kappa \hat{x}_t^T + \alpha_t \pi_t^N + (1-\alpha_t)\pi_t^T + \epsilon_t^T \tag{118}$$

We can write (114)-(118) on the form (where a_t^i are of dimension (6×2) and b_t^i of dimension (6×4)):

$$\begin{bmatrix} a_t^1 & b_t^1 \end{bmatrix} \begin{bmatrix} P_t^T \\ Z_t^T \end{bmatrix} = \begin{bmatrix} a_t^2 & b_t^2 \end{bmatrix} \begin{bmatrix} P_{t-1}^T \\ Z_{t+1}^T \end{bmatrix} + m_t + \begin{bmatrix} a_t^3 & b_t^3 \end{bmatrix} \begin{bmatrix} Z_t^N \\ Z_{t+1}^N \end{bmatrix} \tag{119}$$

Substitute (113) into (119) and we get

$$\begin{bmatrix} a_t^1 - b_t^3 F_{t+1} & b_t^1 - a_t^3 K_t \end{bmatrix} \begin{bmatrix} P_t^T \\ Z_t^T \end{bmatrix} = \begin{bmatrix} a_t^2 + a_t^3 F_t & b_t^2 + b_t^3 K_{t+1} \end{bmatrix} \begin{bmatrix} P_{t-1}^T \\ Z_{t+1}^T \end{bmatrix} + \begin{bmatrix} m_t + a_t^3 W_t + b_t^3 W_{t+1} \end{bmatrix}$$

Inverting the matrix on the left hand side we can write the system as:

$$\begin{bmatrix} P_t^T \\ Z_t^T \end{bmatrix} = \begin{bmatrix} A_t & B_t \\ C_t & D_t \end{bmatrix} \begin{bmatrix} P_{t-1}^T \\ Z_{t+1}^T \end{bmatrix} + \begin{bmatrix} M_t \\ V_t \end{bmatrix} \tag{120}$$

We know the matrixes A_t, B_t, C_t and D_t. They only vary with time because of α_t . We also know M_t and V_t that are only a function of the exogenous shocks and the parameters of the model. Note that

$$B_{S-1} = D_{S-1} = 0 \tag{121}$$

By recursive substitution we can find a solution of the form:

$$P_t^T = \Omega_t P_{t-1}^T + \Phi_t \tag{122}$$

$$Z_t^T = \Lambda_t P_{t-1}^T + \Theta_t \tag{123}$$

To find the solution for the coefficients $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t consider the solution of the system in period $S-1$ when $B_{S-1} = D_{S-1} = 0$. Consider the solution of the system in period $S-1$. By (120)

$$P_{S-1}^T = A_{S-1} P_{S-1}^T + M_{S-1}$$

$$Z_{S-1}^T = C_{S-1} P_{S-1}^T + V_{S-1}$$

Then we have:

$$\Omega_{S-1} = A_{S-1}$$

$$\Phi_{S-1} = M_{S-1}$$

$$\Lambda_{S-1} = C_{S-1}$$

$$\Theta_{S-1} = V_{S-1}$$

Consider the solution of the system in period $S-2$ for P_{S-2}^T:

$$\begin{aligned} P_{S-2}^T &= A_{S-2} P_{S-3}^T + B_{S-2} Z_{S-1}^T + M_{S-2} \\ &= A_{S-2} P_{S-3}^T + B_{S-2} (\Lambda_{S-1} P_{S-2}^T + \Theta_{S-1}) + M_{S-2} \end{aligned}$$

Then

$$P_{S-2}^T = (I - B_{S-2}\Lambda_{S-1})^{-1} A_{S-2} P_{S-3}^T + (I - B_{S-2}\Lambda_{S-1})^{-1} (B_{S-2}\Theta_{S-1} + M_{S-2})$$

Similarly the solution for S_{S-2} is:

$$\begin{aligned} Z_{S-2}^T &= C_{S-2} P_{S-3}^T + D_{S-2} Z_{S-1}^T + V_{S-2} \\ &= C_{S-2} P_{S-3}^T + D_{S-2} (\Lambda_{S-1} P_{S-2}^T + \Theta_{S-1}) + V_{S-2} \\ &= C_{S-2} P_{S-3}^T + D_{S-2} \Lambda_{S-1} P_{S-2}^T + D_{S-2} \Theta_{S-1} + V_{S-2} \\ &= C_{S-2} P_{S-3}^T + D_{S-2} \Lambda_{S-1} (\Omega_{S-2} P_{S-3}^T + \Phi_{S-2}) + D_{S-2} \Theta_{S-1} + V_{S-2} \\ &= (C_{S-2} + D_{S-2} \Lambda_{S-1} \Omega_{S-2}) P_{S-3}^T + D_{S-2} \Lambda_{S-1} \Phi_{S-2} + D_{S-2} \Theta_{S-1} + V_{S-2} \end{aligned}$$

Thus we can find of numbers $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t for period 0 to $S-2$ by solving

$$\Omega_t = [I - B_t \Lambda_{t+1}]^{-1} A_t \tag{124}$$

$$\Lambda_t = C_t + D_t \Lambda_{t+1} \Omega_t \tag{125}$$

$$\Phi_t = (I - B_t \Lambda_{t+1})^{-1} [B_t \Theta_{t+1} + M_t] \tag{126}$$

$$\Theta_t = D_t \Lambda_{t+1} \Phi_t + D_t \Theta_{t+1} + V_t$$

Given the solution for $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t we can find the solution for each of the endogenous variables in (122) and (123) using the initial condition for $P_{-1}^T = 0$. This defines the solution under the contingency that the economy stays in the trap for the maximum period times given by S (which can be made arbitrarily large). The solution for all of the other contingencies is then given by (113) where we now know the values for Z_t^T and P_{t-1}^T from the solution derived above.

B.2 Discretion

The solution has to satisfy the following equations:

$$\pi_t - \frac{\lambda_\tau G}{\beta} b_{t-1} - \kappa^{-1} \eta_t = 0 \tag{127}$$

$$\lambda_x \hat{x}_t - \sigma^{-1} \lambda_\tau G b_t + \eta_t + \frac{\sigma^{-1}}{\beta} \psi_t = 0 \tag{128}$$

$$\frac{1}{\beta} b_{t-1} - b_t - \frac{1}{\beta} b_t + E_t b_{t+1} - \sigma^{-1} G (E_t \hat{x}_{t+1} - \hat{x}_t + \sigma \beta \hat{r}_t^n) - \Pi G b_t - \frac{\Pi}{\beta \lambda_\tau} \psi_t = 0 \tag{129}$$

$$\hat{x}_t - E_t \hat{x}_{t+1} - \sigma E_t \pi_{t+1} - \sigma \hat{r}_t^n - \sigma \beta \bar{\imath} \geq 0, \ \psi_t \geq 0, \ \psi_t (\hat{x}_t - E_t \hat{x}_{t+1} - \sigma E_t \pi_{t+1} - \sigma \hat{r}_t^n - \sigma \beta \bar{\imath}) = 0 \tag{130}$$

$$\pi_t = \kappa \hat{x}_t + E_{t-1} \pi_t + \epsilon_t \tag{131}$$

t>T

then $\psi_t^N = \hat{x}_t^N = \hat{r}_t^{nN} = 0$

$$\pi_t^N = \Pi b_{t-1}^N \tag{132}$$

$$b_t^N = \rho_1 b_{t-1}^N \tag{133}$$

t=T

$$\pi_T^N - \frac{\lambda_\tau G}{\beta} b_{T-1}^T - \kappa^{-1} \eta_T^N = 0 \tag{134}$$

$$\lambda_x \hat{x}_T^N - \sigma^{-1} \lambda_\tau G b_T^N + \eta_T^N = 0 \tag{135}$$

$$\frac{1}{\beta} b_{T-1}^T - (1 - \rho_1 + \frac{1}{\beta}) b_T^N + \sigma^{-1} G \hat{x}_T^N - \Pi G b_T^N = 0 \tag{136}$$

$$\pi_t^N = \kappa \hat{x}_t^N + \alpha_T \pi_t^N + (1-\alpha_T)\pi_T^T + \epsilon_T^T \tag{137}$$

Again define $Z_t \equiv \begin{bmatrix} \pi_t \\ \hat{x}_t \\ \eta_t \\ b_t \end{bmatrix}$ and $P_t = \begin{bmatrix} \psi_t \\ b_t \end{bmatrix}$We can write:

$$Z_T^N = K_T Z_T^T + F_T P_{T-1}^T + W_T \tag{138}$$

t<T

$$\pi_t^T - \frac{\lambda_\tau G}{\beta} b_{t-1}^T - \kappa^{-1}\eta_t^T = 0 \tag{139}$$

$$\lambda_x \hat{x}_t^T - \sigma^{-1}\lambda_\tau G b_t^T + \eta_t^T + \frac{\sigma^{-1}}{\beta}\psi_t^T = 0 \tag{140}$$

$$\begin{aligned} & -\lambda_\tau(1+\frac{1}{\beta}+\Pi G)b_t^T + \frac{\lambda_\tau}{\beta}b_{t-1}^T + \lambda_\tau \alpha_{t+1} b_{t+1}^N + \lambda_\tau(1-\alpha_{t+1})b_{t+1}^T \\ & -\lambda_\tau \sigma^{-1} G \alpha_{t+1}\hat{x}_{t+1}^N - \lambda_\tau \sigma^{-1} G(1-\alpha_{t+1})\hat{x}_{t+1}^T + \lambda_\tau \sigma^{-1} G \hat{x}_t^T - \lambda_\tau G \hat{r}_t^{nT} - \frac{\Pi}{\beta}\psi_t^T \\ = \quad & 0 \end{aligned} \tag{141}$$

$$\hat{x}_t^T - \alpha_{t+1}\hat{x}_{t+1}^N - (1-\alpha_{t+1})\hat{x}_{t+1}^T - \sigma\alpha_{t+1}\pi_{t+1}^N - \sigma(1-\alpha_{t+1})\pi_{t+1}^T - \sigma\hat{r}_t^{nT} - \sigma\beta\bar{\imath} = 0 \tag{142}$$

$$\pi_t^T = \kappa \hat{x}_t^T + \alpha_t \pi_t^N + (1-\alpha_t)\pi_t^T + \epsilon_t^T \tag{143}$$

We can solve this in exactly the same fashion as was illustrated for the commitment solution.

C Independent Treasury

In this case the Treasury takes the processes for π_t, x_t and i_t as exogenously given. The program satisfies

$$\hat{\tau}_t = E_t \hat{\tau}_{t+1} + \hat{\imath}_t - E_t \pi_{t+1} \tag{144}$$

$$b_t = \frac{1}{\beta} b_{t-1} - G\hat{\tau}_t \tag{145}$$

t>T

Then

$$\hat{\tau}_t^N = \hat{\tau}_{t+1}^N \tag{146}$$

and

$$b_t^N = b_T^N \tag{147}$$

Then

$$\hat{\tau}_t^N = \frac{1-\beta}{\beta} G^{-1} b_T^N \tag{148}$$

t=T

$$\hat{\tau}_T^N = \hat{\tau}_{T+1}^N - \sigma^{-1}\hat{x}_T^N = \frac{1-\beta}{\beta}G^{-1}b_T^N - \sigma^{-1}\hat{x}_T^N = \frac{1-\beta}{\beta^2}G^{-1}b_{T-1}^T - \frac{1-\beta}{\beta}\hat{\tau}_T^N - \sigma^{-1}\hat{x}_T^N \tag{149}$$

$$\hat{\tau}_T^N = \frac{1-\beta}{\beta}G^{-1}b_{T-1}^T - \sigma^{-1}\beta\hat{x}_T^N \tag{150}$$

$$b_T^N = \frac{1}{\beta}b_{T-1}^T - \hat{\tau}_T^N \tag{151}$$

t<T

$$\hat{\tau}_t^T = (1-\alpha_{t+1})\hat{\tau}_{t+1}^T + \alpha_{t+1}\hat{\tau}_{t+1}^N - \beta\bar{\imath} - \alpha_{t+1}\pi_{t+1}^N - (1-\alpha_{t+1})\pi_{t+1}^T \tag{152}$$
$$= (1-\alpha_{t+1})\hat{\tau}_{t+1}^T + \alpha_{t+1}\frac{1-\beta}{\beta}G^{-1}b_t^T - \alpha_{t+1}\sigma^{-1}\beta\hat{x}_{t+1}^N - \beta\bar{\imath} - \alpha_{t+1}\pi_{t+1}^N - (1-\alpha_{t+1})\pi_{t+1}^T$$

$$b_t^T = \frac{1}{\beta}b_{t-1}^T - G\hat{\tau}_t^T \tag{153}$$

We can now use exactly the same solution method as before where $P_t = b_t$ and $Z_t = \tau_t$. Then only thing that changes is the dimensions of Ω, Φ, Λ and Θ.

D Calibration

	Full Commitment			Monetary Commitment		
Level of tax distortions	π_1	x_0	t_0	π_1	x_0	t_0
1%	4.2%	-1.7%	-5.8%	4.2%	-1.7%	-5.8%
3%	4.2%	-1.7%	-5.8%	4.2%	-1.7%	-5.8%
5%	4.2%	-1.6%	-5.8%	4.2%	-1.7%	-5.8%
10%	4.3%	-1.5%	-5.9%	4.2%	-1.7%	-5.8%
15%	4.4%	-1.2%	-6.0%	4.2%	-1.7%	-5.8%
20%	4.6%	-0.8%	-6.2%	4.2%	-1.7%	-5.8%

	Full Discretion			Monetary Discretion		
Level of tax distortions	π_1	x_0	t_0	π_1	x_0	t_0
1%	0.3%	-9.5%	-73.9%	0.0%	-10.0%	-1.8%
3%	1.5%	-7.1%	-49.0%	0.0%	-10.0%	-1.8%
5%	2.5%	-5.0%	-32.3%	0.0%	-10.0%	-1.8%
10%	3.9%	-2.3%	-14.6%	0.0%	-10.0%	-1.8%
15%	4.5%	-1.0%	-8.5%	0.0%	-10.0%	-1.8%
20%	4.8%	-0.3%	-5.7%	0.0%	-10.0%	-1.8%

Table 4: Solutions under different assumptions about the level of tax distortions.

Table 4 shows the value of inflation in period 1, output gap in period 0 and tax cuts in period 0 under different values for λ_τ. In the numerical example in the text we assume a value of λ_τ so that the losses due to 10% annual inflation would corresponds to tax distortions associated with $\tau_t = G = 1/3$. The table shows how the result would change by varying the degree of tax distortions in the economy.

E Data

Data on Debt

Maturity Structure, source: Japanese Government Bonds, Quarterly Newsletter of the Ministry of Finance of Japan, October 2001. Available at http://www.mof.go.jp/english/

Net and Gross Debt: OECD databank available at http://www.sourceoecd.com.

Data from Japan 1990-2000 in Table 1

From Datastream

Historical Data from Japan in Table 3

From Estimates of Long Term Economic Statistics of Japan since 1868:

Volume 1

GNP series, Table 23 p. 225.

GNP deflator, Table 30A p. 233.

Volume 8

CPI series, Table 2, p. 136

From Hundred Year Statistics of the Japanese Economy:

WPI series, Table 18, p. 76.

From Dilemmas of Growth in Prewar Japan, ed. Morley, James W.

Central Government Surpluses, Table 7, p. 250-251.