Dynamic Debt Deleveraging and Optimal Monetary Policy*

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Abstract

This paper proposes a post-crisis New Keynesian model that incorporates agent heterogeneity in borrowing and lending with a minimum set of assumptions and complexity. Unlike the standard framework, this model makes the natural rate of interest endogenous, and dependent on macroeconomic policy. We establish microfoundations for debt deleveraging based both on the accumulation of excessive debt by households and leverage constraints on banks, showing that they are isomorphic and thus integrating two popular narratives for the crisis of 2008. The main application is to study optimal monetary policy at the zero lower bound (ZLB). Such policy succeeds in raising the natural rate of interest by creating an environment that speeds up deleveraging and thus endogenously shortens the crisis and the duration of binding ZLB. Inflation should be front-loaded and overshoot its long term target during the ZLB episode.

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1 Introduction

This paper proposes a tractable post-crisis version of the canonical New Keynesian model. By "post-crisis" we mean the period after 2008, when several central banks had to cut their short-term interest rate to zero. While all models have shortcomings, the standard New Keynesian model came under special criticism for a few key abstractions that proved to be important omissions with respect to understanding some elements of the crisis. First, in its most basic form it posits only one risk-free short-term interest rate. Second, it makes no explicit provision for banking and it has only a single representative agent, with no distinction between savers and borrowers. Finally, shocks are somewhat reduced form, which makes it difficult to pinpoint the precise trigger of the crisis. The elements we intend to integrate into the standard New Keynesian post-crisis model are designed to make up for these omissions. We propose to do so with only a minor increment in complexity.

Our model builds on a rich literature developed before and after the crisis. The zero lower bound on short-term nominal interest was certainly not an unfamiliar concept to economists even before the crisis and right after it the literature has expanded on that basis very quickly. There is now a quite large literature on monetary and fiscal policy subject to ZLB.[1] Generally, however, these papers treat the shock that drives the economy to the zero bound, i.e. the shock to the "natural rate of interest", as exogenous. This means that under some basic policy specifications the duration of the trap is purely exogenous and the forces that perturb the economy – usually preference shocks – do not interact with the policy chosen. Here, instead, we model the origin of the crisis more explicitly making the duration of the negative natural rate of interest – and therefore the crisis – endogenous and a function of policy.

A recent strand of the literature seeks to model in greater detail the factors that may lead the economy against the zero bound, the very origin of the global economic crisis of 2008. We see this literature as offering two main narratives. One powerful account holds that the source was a deleveraging cycle on the household side. For recent theoretical contributions inspired see e.g. Eggertsson and Krugman (2012), Hall (2011), Guerrieri and Lorenzoni (2017) and Rognlie, Shleifer and Simsek (2014); Mian and Sufi (2011) provide extensive empirical evidence for this mechanism.[2] Another powerful narrative traces the origin of the crisis to banking turbulence (see e.g. Curdia and Woodford, 2010, Gertler and Kiyotaki, 2010).[3]

Consider first the household debt-deleveraging story. First, we have a period of excessive optimism, in which debtors borrow and spend aggressively via a process of leveraging (piling up debt). Since one person's debt is another's asset, creditors have to be induced to spend less by high real interest rates. Then comes a "Minsky moment" (Eggertsson and Krugman, 2012) when households realize that their new debt may in fact not all be sustainable, and

[1] See for example Eggertsson and Woodford (2003), Adam and Billi (2006), Eggertsson (2008) as pre-crisis and Eggertsson (2011), Christiano, Eichenbaum and Rebelo (2011) and Werning (2011) for post-crisis examples.

[2] See also Geanakoplos (2010) and references therein, although he, and the literature he cites, does not emphasize the connection of the leverage cycle with the interest rate channel as we do here and as the literature above does. Thus he does not focus as clearly on the interaction of the leverage cycle with ZLB, which is the central theme here.

[3] See also Andrès et al. (2013) and De Fiore and Tristani (2012, 2013) for alternative approaches.

we shift from leveraging to deleveraging. But the zero lower bound prevents the central bank from cutting the interest rate enough to induce sufficient spending by low-debt agents. Hence, one interpretation of a drop in the natural rate of interest is that debtors – as a group – are trying to deleverage very fast, so that the real interest rate needs to go negative in order to get the savers to spend enough to sustain full employment. A negative natural rate of interest can make the ZLB binding, which creates problems for macroeconomic policy. In earlier work on deleveraging, such as Eggertsson and Krugman (2012), the deleveraging shock coincides with a sudden drop in borrowing capacity to which the borrower must adapt. However, they posit that this adjustment takes place in only one period, "the short run". Here, instead, we relax this assumption so that deleveraging takes place smoothly over several periods – determined endogenously – as a result of household's optimal deleveraging decisions.[4]

Now let us consider the banking turbulence interpretation. There is a crisis in the interbank market that increases banks' cost of funding. This might be due to shocks to banks' capital or a need to deleverage. The banks' capital constraint tightens during a period of stress making them less willing to lend and triggering a downturn. In the end, however, the mechanism through which this affects the macro economy turns out to be largely analogous to the household deleveraging thesis. Indeed we show that as regards such aggregate variables as output, inflation and interest rate the two stories are isomorphic. From a basic New Keynesian perspective, therefore, there is no special reason for choosing one over the other, and we will refer to both as "dynamic deleveraging". Our prior is that both played an important role in the crisis.

Within our framework we generalize the standard New Keynesian (NK) prototype model as one that involves exactly the same pair of equations, familiar to many readers, namely the IS and the AS equations, typically summarized as follows (denoting output in log deviation from steady state with, $\hat{Y}_t$, inflation with π_t, the nominal interest rate with i_t and steady state inflation by π)

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(i_t - E_t(\pi_{t+1} - \pi) - r_t^n)$$

$$\pi_t - \pi = \kappa\hat{Y}_t + \beta(\pi_{t+1} - \pi)$$

where $\beta, \sigma, \kappa > 0$ are coefficients.[5] The only difference between the present model and the benchmark is that r_t^n (which is interpreted as the natural rate of interest) is now an endogenous variable that depends on the level of private debt. In this paper we show how this variable is determined in equilibrium by a system of equations that depends, among other things, on households' indebtedness and on the spread between the risk-free interest rate and the risky, hence higher rate charged to borrowers. In the case of households' debt-deleveraging, this corresponds to a "shock" to the "safe level" of debt, giving households

[4]Here we do expound one suggested extension discussed in the Web Appendix of Eggertsson and Krugman (2012), but this delivers a less compact model owing to a different specification of preferences and production. Moreover, they do not provide explicit microfoundations for the banking sector, which in our case allows us to nest both the household deleveraging story and the banking story in the same framework. Finally, they are silent on the welfare implications of alternative policies. Another closely related paper is Curdia and Woodford (2010), which we also build upon. Their focus, however, is mostly on shocks to the aggregate banking system not on sub-optimal monetary policy at the ZLB.

[5]These equations are illustrated in Clarida, Galì and Gertler (1999), Woodford (2003) and Galì (2008) among others.

the incentive to pay down their debt to a new steady state. In case of banking turbulence, it corresponds to a shock to the required leverage ratio or the cost of equity financing curtailing banks' lending to a new steady state. We show that the natural rate of interest can be transitorially negative.

One relatively minor difference from the standard one is that ours is written in terms of inflation in deviation from steady-state inflation, π, which may be positive: a reasonable number, for example, would be 2% in the US. What this implies is that the recession at the zero lower bound does not necessarily require actual deflation, only inflation below the target of the central bank. Some authors contend that the lack of deflation during the post-2008 crisis represents a major failure of the canonical New Keynesian model. Our proposed model fixes this problem.

A more important advantage of our framework is that the explicit introduction of borrowing and lending allows a more disciplined calibration of the shock that triggered the Great Recession. In much of the earlier literature (e.g. Eggertsson, 2011) the driving force is assumed to be an unobserved preference shock, calibrated so as to generate the Great Recession. Here, instead, we have two more observables: first, the endogenous level of debt held by the households and, second, a borrowing rate that differs from the risk-free interest rate. These two variables allow relatively straightforward calibration of the shocks, as we will see. We can then ask if the shocks calibrated to match these new observables can generate the Great Recession. The short answer is yes.

The paper's main contribution of the paper, in our view, consists in this parsimonious framework, which generalizes the canonical New Keynesian model but at the same time speaks more directly to the crisis of 2008. This framework can be useful for a number of applications (see, for instance, the effect of negative interest rates on reserves in Eggertsson, Juelsrud and Wold, 2017). The principal application, however, is our revisitation of the classical question of optimal monetary policy when the zero lower bound is binding – a situation faced by much of the industrial world after 2008.

The first conclusion is that the duration of a negative natural rate of interest and of the ZLB is now endogenous, rather than depending only on exogenous preference shocks or an implicitly specified "short run". Further, the duration depends on the policy stance. Under a monetary policy regime that targets inflation high enough to avoid the ZLB, for example, the economy natural rate of interest will last less than if the policy stance is insufficiently stimulating. The intuition is simple. In a recession there is a drop in overall income undercutting borrowers' ability to pay down their debt, so deleveraging will be slower than if recession is avoided by aggressive monetary and fiscal stimulus. Since it is the deleveraging process that drives the decrease in the natural rate of interest, this affects how long remains below its steady-state.

The second key result is a partial corollary of the first. Endogenous deleveraging will in general amplify the effect of policy at the zero bound. This is because now policy will not only mitigate the crisis today, as the literature emphasizes but also shorten its duration by impinging directly on the natural rate of interest. Consider the nominal interest rate path under a policy to stabilize inflation and the output gap under either dynamic deleveraging or exogenous preference shocks. We find that optimal policy prescribes shorter duration at the zero bound than under dynamic deleveraging under exogenous preference shocks, precisely because it has a direct effect on the natural rate of interest. Optimal policy is powerful enough to "jump start" the economy and thus leads to a more rapid normalization of the nominal rate.

The third result follows from the explicit derivation of a social welfare function in our heterogenous agent model. While the standard New Keynesian model considers only output and inflation. Our social welfare function involves an additional term because we have different types of agents, i.e. borrowers and savers, with incomplete insurance between them. By comparison with the standard objective, this additional term gives the policymaker an even stronger reason to aggressive countercyclical policy at the ZLB. For one thing, borrowers tend to suffer more in a debt-deleveraging recession and thus to have higher marginal utility of income. Meanwhile borrowers have more to gain from inflationary policy than savers, as inflation reduces their real debt, lowers the real interest rate on it moving forward, and increases their labor income when the marginal value of extra income is especially high for borrowers.

A fourth result is that in a liquidity trap under dynamic deleveraging optimal monetary policy prescribes excess inflation, and possibly output above potential, well above the inflation target, even while the zero bound is binding and the natural rate of interest negative. In part, this is because social welfare now takes account of the social benefit of redistribution in response to the shock, but also to some extent because an endogenous natural rate of interest prescribes even more aggressive policy action than in the standard model so as to speed up the recovery.

The paper is organized as follows. Section 2 describes dynamic deleveraging in a simple endowment economy to clarify some key assumptions and then discusses the general model and its log-linear approximation. Section 3 illustrates the calibration of the model and shows that when it is fed with debt deleveraging shocks it can capture the movements of key macrovariables in the US during the Great Recession. Section 4 describes the application that we run here. Subsection 4.1 studies the positive implications of dynamic debt deleveraging and contrasts this with the standard NK model; Subsection 4.2 characterizes normative aspects of the model, analyzing optimal monetary policy under commitment. Section 5 concludes.

2 The model

2.1 Dynamic deleveraging in an endowment economy

First, let us consider a simple endowment economy in order to clarify some key assumptions of the general model. A representative borrower (b) and saver (s) maximize utility

$$E_t \sum_{T=t}^{\infty} (\beta^j)^{T-t} \log C_t^j \qquad \text{with } j = b \text{ or } s$$

where C_t^j is consumption and β^j a discount factor (with $0 < \beta^b < \beta^s \leq 1$). The optimization problem is subject to the budget constraint:

$$\frac{b_t^j}{1 + r_t^j} = b_{t-1}^j + C_t^j - \frac{1}{2} Y + T_t^j \tag{1}$$

where b_t^j is a one-period risk-free real debt contracted in period t (inclusive of interest payment), r_t^j is the real interest rate, Y is the endowment and T_t^j is a lump-sum tax.

The gross risk-free real interest rate is $1 + r_t$ and each agent j faces the interest rate function

$$1 + r_t^j = \begin{cases} 1 + r_t & \text{if } b_t^j \leq \bar{b}_t \\ (1 + r_t)(1 + \phi(b_t^j - \bar{b}_t)) & \text{if } b_t^j > \bar{b}_t \end{cases}. \tag{2}$$

Figure 1 plots function (2): if the borrower's debt is lower than $\bar{b}_t$ then he faces the risk-free rate $1 + r_t$, if greater he pays a premium $1 + \phi(b_t^b - \bar{b}_t)$. This is a generalization of the strict borrowing constraint of Eggertsson and Krugman (2012) obtained in the limit when $\phi \to \infty$ so that $b_t^j \leq \bar{b}_t$ at all times.

Equilibrium is a collection of stochastic processes $\{C_t^b, C_t^s, r_t^b, r_t^s, b_t^b\}$ satisfying

$$\frac{1}{C_t^s} = \beta^s(1 + r_t^s)E_t\frac{1}{C_{t+1}^s} \tag{3}$$

$$\frac{1}{C_t^b} = \beta^b(1 + r_t^b)E_t\frac{1}{C_{t+1}^b} \tag{4}$$

$$1 + r_t^b = \begin{cases} 1 + r_t^s & \text{if } b_t^b \leq \bar{b}_t \\ (1 + r_t^s)(1 + \phi(b_t^b - \bar{b}_t)) & \text{if } b_t^b > \bar{b}_t \end{cases} \tag{5}$$

$$C_t^s + C_t^b = Y \tag{6}$$

$$b_t^b = (1 + r_t^b)[b_{t-1}^b + C_t^b - \frac{1}{2}Y] \tag{7}$$

where (3) and (4) are the consumption Euler equations of the saver and of the borrower, respectively, (5) determines the spread between the rate charged to the borrower and that paid to the saver, (6) is the resource constraint and (7) the budget constraint of the borrower.[6]

The steady state is apparent from the first two equations yielding $1 + r^s = (\beta^s)^{-1}$ and $1+r^b = (\beta^b)^{-1}$. This is enough to pin down the steady-state equilibrium debt via (5) implying

$$b^b = \bar{b} + \phi^{-1}\left(\frac{\beta^s}{\beta^b} - 1\right)$$

which is shown as point A in Figure **1.** The household borrows above the threshold $\bar{b}$ to a level such that $1 + r^b$ equals the inverse of the borrower's discount rate. This is in contrast to Eggertsson and Krugman (2012), where $\phi \to \infty$, the debt limit is binding, $b^b = \bar{b}$, and the borrower is at a corner solution.

[6]The first-order conditions are derived by writing up a Lagrangian. Here we make the simplifying assumption that the borrower takes b_t^b in the interest-rate premium function as exogenous (corresponding to aggregate debt in the economy). In the general model we allow the spread function to depend on both individual and aggregate debt. We also make the assumption that the spread between the two interest rates is rebated lump sum to the saver which is why no lump sum transfer appears in (7) and assume that the banks are owned by the savers.

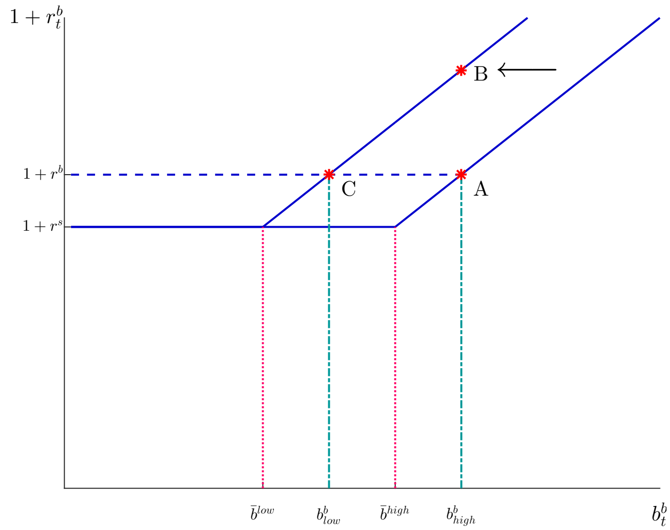

Figure 1: Plot of the function characterizing the cost of borrowing: equation (2) when $\bar{b} = \bar{b}^{high}$ and when $\bar{b} = \bar{b}^{low}$ with $\bar{b}^{high} > \bar{b}^{low}$. The initial steady state is A when $\bar{b} = \bar{b}^{high}$. As $\bar{b}$ moves to $\bar{b}^{low}$, the equilibrium moves to B and then to the final steady state C along the shifted line. Note that b^b_{high} (b^b_{low}) is the steady-state level of debt when the threshold is $\bar{b}^{high}$ ($\bar{b}^{low}$).

The key thought experiment in the paper is that the debt limit goes from some "high" level to a "low" level, i.e. $\bar{b}^{high} \to \bar{b}^{low}$, an experiment sometimes referred to as a Minsky moment. At this point, the borrower can no longer maintain his outstanding debt unless he pays a higher interest rate premium on it. This is shown in Figure 1, where in response to the shock the interest rate increases as is shown by point B. In the previous literature, such as Eggertsson and Krugman (2012), by assumption, the household pays down its debt in one period. Here, instead, the borrower repays over a period of time that is endogenously determined. As Figure 1 shows, the shock triggers a rise in the interest rate faced by the borrower if he leaves his debt unchanged. This gives the borrower an incentive to pay down debt and the optimal repayment path is determined by (3)-(7), the solution of which we turn to next. The dynamic deleveraging is what moves the borrower from point B down to point C in Figure 1, restoring an interest rate that is equal to the inverse of his discount factor, $(\beta^b)^{-1}$.

Figure 2 shows the path of each of the endogenous variables for a finite ϕ.[7] The deleveraging is shown in the last panel, where private debt falls from 108% to 88% of output.[8] The borrower cuts consumption immediately and gradually pays down the debt. As all output is consumed, the fall in the borrower's consumption needs to be offset by an increase in the saver's. This is achieved by a reduction in the interest rate. The interest rate paid to the saver may even turn negative if the shock to $\bar{b}$ is large enough. Since the saver's rate is the risk-free short-term interest rate, which in a more general setting corresponds to the nominal interest rate set by the central bank, this has major implications for monetary policy, as we

[7] Illustrative parameters assumed: $\phi = 0.0078$, $Y = 1$, $\beta^b = 0.9796$, $\beta^s = 0.9852$, $\bar{b}^{high} = .9773$, $\bar{b}^{low} = .78$. The model is log-linearized around the steady state to generate the figures.

[8] It should be noted that the shock $\bar{b}$ expressed as a ratio of output –the variable $\bar{b}^{gdp}$– moves from 97.73% to 78%.

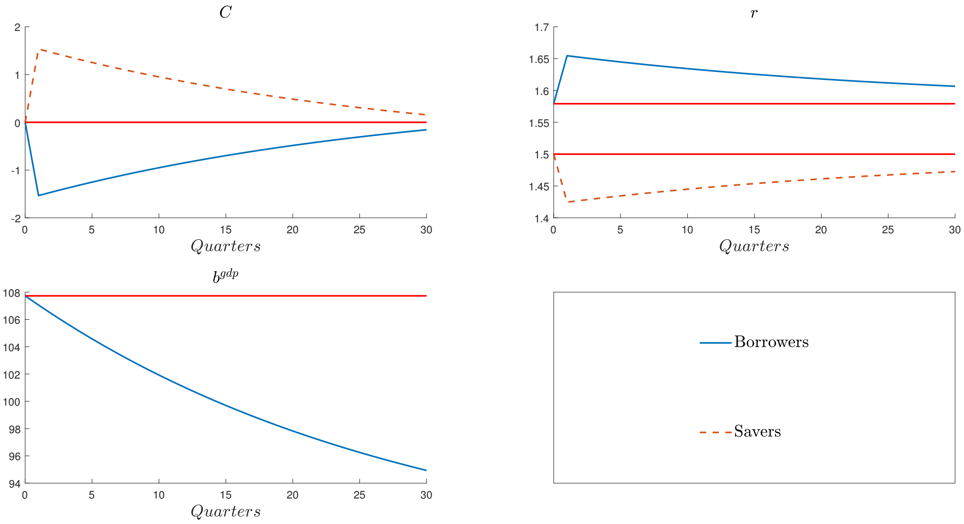

Figure 2: Responses following a deleveraging shock: $\bar{b}$ moves from $\bar{b}^{high}$ to $\bar{b}^{low}$, in the endowment-economy model. Variables are: consumption of borrowers (C^b), consumption of savers (C^s), real interest rate on borrowing (r^b), real interest rate on saving (r^s), debt of borrowers with respect to output (b^{gdp}), the risk-free borrowing threshold with respect to output ($\bar{b}^{gdp}$). C^b, C^s, are percentage deviations with respect to the initial steady state; r^b, r^s, b^{gdp} and $\bar{b}^{gdp}$ are percent and at annual rates.

will soon see.

A few comments are in order here. First, since the speed of deleveraging – as determined by how long the agents take to reach their new level of steady-state debt – is optimally determined, it is not hard to imagine that this speed may be affected by macroeconomic policy, an insight soon confirmed once production and macroeconomic policy are made endogenous. This implies that the duration of the negative real interest rate is endogenous, and this will be critical. Second, nothing in the experiment depends on the gap between β^b and β^s being large, as shown in Figure 1. Even if the gap is small, as long as $\bar{b}^{high}$ falls to $\bar{b}^{low}$, a spread opens to the same extent and the borrower deleverages. In other words, the dynamics of deleveraging are independent of the difference between β^b and β^s; only the steady state depends on this difference. Even if $\beta^b \to \beta^s$ the same thought experiment can be performed. This observation is useful because it is convenient to assume that $\beta^b \to \beta^s$ to derive social welfare.[9] When $\beta^b \to \beta^s$ borrowing and lending are no longer motivated by differences in discount factors but instead by the initial asset distribution (some agents are born with debt, others with assets).[10]

[9] Indeed when $\beta^b < \beta^s$ aggregate welfare cannot be written in a recursive way.

[10] In our example we can assume that debtors start from a level of debt $b^b = \bar{b}^{high}$. Observe that while there are initial conditions for debt consistent with lower values of the debt, it can be no higher than this value in steady state. Taking this initial value as given, then, and assuming a debt deleveraging shock, the new steady state will be uniquely defined as $b^b = \bar{b}^{low}$, precisely as in our exercise above.

2.2 General environment

We now turn to a more general environment. On a continuum between 0 and 1, a fraction χ of borrowers (b) and a fraction $1-\chi$ of savers (s) maximize expected utility

$$E_t \sum_{T=t}^{\infty} (\beta^j)^{T-t} \left[1 - \exp(-z C_T^j) - \frac{(L_T^j)^{1+\eta}}{1+\eta} \right] \text{ with } j = s \text{ or } b \tag{8}$$

where E_t is the expectation operator, z a positive parameter, β^j the discount factor with $0 < \beta^b \leq \beta^s < 1$ and C_t^j is aggregate consumption

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

where $C_t(i)$ is the consumption of good of variety i, $\theta > 1$ is the intratemporal elasticity of substitution between goods and L^j is hours worked subject to

$$\frac{B_t^j}{1+i_t^j} = B_{t-1}^j + P_t C_t^j - W_t^j L_t^j - \Psi_t^j - \Gamma_t^j + T_t^j \tag{9}$$

where B^j is nominal debt, P_t the price index, W_t^j the wage for type j labor, Ψ_t^j is firms' profits, Γ_t^j profits of financial intermediation and T_t^j lump-sum taxes.

Because debt is nominal, monetary policy impacts on the real value of debt through both inflation and the nominal interest rate. Utility is exponential in consumption, as is common in applied finance (see e.g. Calvet, 2001) to simplify aggregation.

The nominal interest rate i_t^j is specific to households. All savers deposit their savings in banks at the riskfree rate i_t while borrower j raises funds from banks at an interest rate given by

$$1 + i_t^j = (1+i_t)\phi\left(\frac{b_t^j}{\bar{b}_t}, \frac{b_t}{\bar{b}_t}, \zeta_t \right) \tag{10}$$

which is proportional to the saving rate through a premium determined by the spread function $\phi(\cdot,\cdot,\cdot)$.[11] In (10), this function is written in a general form (examples are provided in Appendix A using different banking models). The results do not depend on those details.

The spread function involves three terms which we discuss in turn. The spread, or borrowing premium, depends on agent j's real debt, defined as $b_t^j \equiv B_t^j/P_t$, in reference to $\bar{b}_t$, the maximum risk-free debt. The idea is that each agent has a debt capacity above which the bank needs to be compensated for default risk. According to the second term the spread is also a function of the aggregate debt (per borrower), $b_t = \left(\int_\chi b_t^j dj \right) / \chi$, which is also in reference to the debt capacity, so that there is risk associated with bank lending independent of the idiosyncratic risk of individual borrower. If aggregate borrowing is high, say, then every borrower will default at a higher rate for a given level of debt. One can read this as meaning that when aggregate borrowing is high banks have more limited resources

[11] A spread function is usually assumed in open-economy models to obtain stationarity of external debt, see among others Schmitt-Grohe and Uribe (2003), Benigno (2009) and Mehrotra (2018).

to monitor loans. Relative to the first term, the key difference is that in making their choices the agents take the evolution of aggregate debt as exogenous. Finally, there is an exogenous shifter, ζ_t, capturing other features of a banking model such as leverage ratio or cost of equity financing of banks (see Appendix A).

The consumption Euler equation of savers is

$$U_c(C_t^s) = \beta^s(1+i_t)E_t\left\{U_c(C_{t+1}^s)\frac{P_t}{P_{t+1}}\right\}, \tag{11}$$

which is standard. Borrowers, instead, face the borrowing premium and internalize the effects of their borrowing decisions on spreads. Borrower j satisfies the consumption Euler equation:

$$U_c(C_t^j) = \beta^b\frac{(1+i_t^j)}{1-\epsilon\left(\frac{b_t^j}{\bar{b}_t},\frac{b_t}{\bar{b}_t},\zeta_t\right)}E_t\left\{U_c(C_{t+1}^j)\frac{P_t}{P_{t+1}}\right\}, \tag{12}$$

where $\epsilon(\cdot;\cdot,\cdot)$ is the elasticity of the premium with respect to individual real debt

$$\epsilon\left(\frac{b_t^j}{\bar{b}_t},\frac{b_t}{\bar{b}_t},\zeta_t\right) \equiv \frac{b_t^j}{\bar{b}_t}\frac{\phi_{b^j}\left(\frac{b_t^j}{\bar{b}_t},\frac{b_t}{\bar{b}_t},\zeta_t\right)}{\phi\left(\frac{b_t^j}{\bar{b}_t},\frac{b_t}{\bar{b}_t},\zeta_t\right)}.$$

If $\phi(\cdot,\cdot,\cdot)$ is independent of the agent's specific debt level, then $\epsilon(\cdot;\cdot,\cdot)=0$ and the borrowers' Euler equation is of the same form as in Curdia and Woodford (2010, 2011) where the spread depends only on aggregate debt. In equilibrium, borrowers are identical so that $b_t^j = b_t$ and (12) simplifies to

$$U_c(C_t^b) = \beta^b\frac{(1+i_t^b)}{1-\epsilon\left(\frac{b_t}{\bar{b}_t},\frac{b_t}{\bar{b}_t},\zeta_t\right)}E_t\left\{U_c(C_{t+1}^b)\frac{P_t}{P_{t+1}}\right\}, \tag{13}$$

and (10) simplifies to

$$(1+i_t^b) = (1+i_t)\cdot\phi\left(\frac{b_t}{\bar{b}_t},\frac{b_t}{\bar{b}_t},\zeta_t\right) \tag{14}$$

where $(1+i_t^j) = (1+i_t^b)$.[12]

A positive spread implies profits from financial intermediation denoted by

$$\Gamma_t = \int_\chi\left(\frac{1}{1+i_t}-\frac{1}{1+i_t^j}\right)b_t^j.$$

[12]We can combine equations (11), (13) and (14) in the steady state to obtain some restrictions on the spread function $\beta^s/\beta^b = \phi\left(b/\bar{b},b/\bar{b},\zeta\right)/\left\{1-b//\bar{b}\cdot\phi_{b^j}\left(b/\bar{b},b/\bar{b},\zeta\right)/\phi\left(b/\bar{b},b/\bar{b},\zeta\right)\right\}$. In general, and out of the steady state, we assume borrowers can never borrow at a rate lower than the risk-free interest rate, i.e., $\phi(\cdot,\cdot,\cdot)\geq 1$. We assume that the borrowing premium is non-decreasing in the amount of borrowing of agent j, i.e. we assume that the derivative of the function with respect to the first argument is non-negative, $\phi_{b^j}(\cdot,\cdot,\cdot)\geq 0$. Moreover, the borrowing premium is also non-decreasing in aggregate borrowing, meaning that the derivative of the function with respect to the second argument is non-negative $\phi_b(\cdot,\cdot,\cdot)\geq 0$. Since savers are more patient than borrowers, i.e. $\beta^s > \beta^b$, then $\phi(\frac{b^*}{\bar{b}},\frac{b^*}{\bar{b}},\zeta) > 1$ and $\phi_{b^j}(\frac{b^*}{\bar{b}},\frac{b^*}{\bar{b}}\zeta) > 0$ where b^* is the steady-state equilibrium debt and ζ is the steady state of ζ_t. Therefore, in a steady state in which $\beta^s > \beta^b$, borrowers face a higher interest rate than savers, and the premium at which they borrow is increasing in their individual debt position. Finally, we assume that when ζ_t is at its steady-state level ζ and $b\leq\bar{b}$ the borrowing premium and its derivative are such that $\phi(1,1,\zeta)=1$ and $\phi_{b^j}(1,1,\zeta)=0$.

Banks raise funds from the savers at the risk-free nominal interest rate i_t to lend to borrowers at the higher interest rate i_t^j (see Appendix A). The cost function (10) arises as a technological constraint on the financial intermediation of the banks. Bank profits are rebated to the owners, who are the savers.[13]

The optimal supply of labor implies

$$\frac{(L_t^j)^\eta}{z\exp(-zC_t^j)} = \frac{W_t^j}{P_t} \quad j = s \text{ or } b. \tag{15}$$

Turning to the production side, we assume a continuum of firms of measure one, each producing a single good. The production function is linear in labor, $Y(i) = L(i)$. A key simplification of the model is that each agent receives a constant fraction of total income, in contrast to existing work on heterogeneity in New Keynesian models where income shares vary according to their labor response (see e.g. Eggertsson and Krugman, 2012, Curdia and Woodford, 2010, 2011). The assumption that generates constant income shares is that the aggregate labor input of the firm is a Cobb-Douglas function of the individual agents' labor $L(i) = (L^s(i))^{1-\chi}(L^b(i))^\chi$.

Given this condition, it can be shown that each agent is paid the same aggregate wage $W_t^s L_t^s = W_t^b L_t^b = W_t L_t$ where $W_t \equiv (W_t^s)^{1-\chi}(W_t^b)^\chi$.[14] This, together with the assumption of exponential utility, implies that for $j = s, b$ (15) can be aggregated with weights $1-\chi$ and χ to yield aggregate labor supply

$$\frac{(L_t)^\eta}{z\exp(-zC_t)} = \frac{W_t}{P_t}, \tag{16}$$

where C_t is aggregate consumption given by $C_t = (1-\chi)C_t^s + \chi C_t^b$. Aggregate output is

$$Y_t = (1-\chi)C_t^s + \chi C_t^b. \tag{17}$$

We can now formulate the firm's problem as equivalent to hiring the labor composite L_t from a common labor market at wage W_t so that the firms' pricing problem can be formulated as in the standard New Keynesian model.[15] Firms are subject to price rigidities, as in the Calvo model. A fraction $(1-\alpha)$ of firms with $0 < \alpha < 1$ change their price, which remains in effect at time T with a probability α^{T-t}. This price is indexed to the inflation target over

[13] The quantitative results can change with alternative assumptions as explained later, but not the quantitative results. We could also add a general cost of financial intermediation that absorbs real resources. Under the assumption that this cost is second-order, the log-linear approximation of the model equilibrium conditions is not going to be affected at all. However, the second-order approximation of the welfare could contain additional terms where the intermediation cost is a function of aggregate debt. See also Curdia and Woodford (2010).

[14] This can be shown by solving the static cost minimization problem for firm i, i.e. $\min_{L_t^s, L_t^b}\{W_t^s L_t^s(i) + W_t^b L_t^b(i)\}$ s.t. $(L^s(i))^{1-\chi}(L_t^b(i))^\chi = \bar{L}$ and noting that market clearing requires $\int_0^1 L^s(i)di = \int_\chi^1 L^s(j)dj = (1-\chi)L_t^s$ where i denotes the index of firms, of measure 1 and j denotes the index of savers, who are of measure $1-\chi$. The problem above implies a labor demand for each type of labor of the form $L_t^s/L_t = W_t/W_t^s$ and $L_t^b/L_t = W_t/W_t^b$.

[15] To see this, use footnote 14 to observe that we can write $W^s \int L^s(i) + W^b \int L^b(i) = (1-\chi)W_t^s L_t^s + \chi W_t^b L_t^b$ and therefore that $W^s(i)\int L^s(i) + W^b(i)\int L^b(i) = W_t L_t$ using $L_t^s/L_t = W_t/W_t^s$ and $L_t^b/L_t = W_t/W_t^b$.

the period given by Π^{T-t}. Adjusting firms choose prices to maximize the present discounted value of the profits assuming the prices chosen remain constant. The discounting of profits depends on firms' ownership. The simplest assumption is that firms are under the control of the saver, but an alternative interpretation of what follows is the special case in which $\beta^s \to \beta^b = \beta$. It should be noted that this is the first instance in which this restriction is imposed. Later it will be also useful to derive a recursive representation of utility to perform welfare analysis.[16]

The present discounted value of profits is:

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left[(1+\tau)\Pi^{T-t} \frac{P_t(i)}{P_T} Y_T(i) - \frac{W_T}{P_T} Y_T(i) \right]$$

where λ_t is a linear combination of the marginal utilities of the real income of the two agents, $\lambda_t = [(1-\varpi)U_c(C_t^s) + \varpi U_c(C_t^b)]$, where $1-\varpi$ and ϖ are the profit shares of savers and borrowers, respectively, with $0 \leq \varpi \leq 1$ and τ is a constant subsidy on firms' revenues. The first-order condition of the optimal pricing problem implies

$$\frac{P_t^*}{P_t} = \mu \frac{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t} \frac{1}{\Pi^{T-t}} \right)^{\theta} \frac{W_T}{P_T} Y_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t} \frac{1}{\Pi^{T-t}} \right)^{\theta-1} Y_T \right\}}, \tag{18}$$

where $\mu \equiv \theta/[(\theta-1)(1+\tau)]$ and in equilibrium $P_t(i) = P_t^*$, since all firms adjusting their price fix it at the same level. The remaining fraction α of firms indexes to the inflation target $\bar{\Pi}$, so the law of motion of the aggregate price index is given by

$$P_t^{1-\theta} = (1-\alpha) P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta} \Pi^{1-\theta}. \tag{19}$$

Our assumptions concerning preferences and production simplify the aggregate-supply equation by netting out any implications of the heterogenous-agent model. The firms' real marginal cost is given by the real wage W/P which, using equations (16) and (17), can be written as function of output

$$\frac{W_t}{P_t} = \frac{(Y_t \Delta_t)^{\eta}}{z \exp(-z Y_t)}, \tag{20}$$

using (17) and $L_t = Y_t \Delta_t$ where $\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\theta} di$ follows the law of motion

$$\Delta_t = \alpha \left(\frac{\Pi_t}{\Pi} \right)^{\theta} \Delta_{t-1} + (1-\alpha) \left(\frac{1 - \alpha \left(\frac{\Pi_t}{\Pi} \right)^{\theta-1}}{1-\alpha} \right)^{\frac{\theta}{\theta-1}}. \tag{21}$$

[16] Assuming that $\beta^s \to \beta^b = \beta$ has implications for the values taken by the function $\phi(\cdot,\cdot,\cdot)$ in the steady state: in particular $\phi(1,1,\zeta) = 1$ and $\phi_{b^j}(1,1,\zeta) = 0$, implying that in the steady state there is no spread between the borrowers and the risk-free interest rate and that $\epsilon(1,1,\zeta) = 0$. We provide an expression for the more general case in Appendix C, which leads to identical linearized equations, as long as the firm is using the savers' discount factor in its decision.

Combine (18), (19), (20) and $Y_t \Delta_t = L_t$ to yield aggregate supply

$$\left(\frac{1 - \alpha \left(\frac{\Pi_t}{\Pi} \right)^{\theta - 1}}{1 - \alpha} \right)^{\frac{1}{\theta - 1}} = \frac{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t} \frac{1}{\Pi^{T-t}} \right)^{\theta - 1} Y_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t} \frac{1}{\Pi^{T-t}} \right)^{\theta} \mu \frac{\Delta_T^{\eta} Y_T^{1+\eta}}{z \exp(-zY_t)} \right\}}. \tag{22}$$

To complete the characterization of the model we specify the distributive details of dividends and fiscal policy. The way wages and profits are distributed is essential to the policy transmission mechanism. Real dividends are

$$div_t = (1 + \tau) Y_t - \frac{W_t}{P_t} L_t$$

which using (20) and $Y_t \Delta_t = L_t$ can be expressed as

$$div_t = Y_t (1 + \tau) - \frac{Y_t^{1+\eta} \Delta_t^{1+\eta} \exp(z Y_t)}{z}. \tag{23}$$

The profits distributed to borrowers and savers are $\Psi_t^b = (\varpi/\chi) \cdot div_t$ and $\Psi_t^s = (1 - \varpi/\chi) \cdot div_t$, respectively, reflecting ownership according to ϖ. The government raises taxes τY_t to cover the firms' subsidies levied in the proportion $T_t^b = (\varpi/\chi)(\tau Y_t)$ and $T_t^s = (1 - \varpi/\chi)(\tau Y_t)$.

In what follows, we assume that $\varpi = \chi$ so that firms' profits and taxes are proportional to the size of each group in the population. The relevance of this assumption is discussed in the conclusions. Intermediation profits are all rebated to the savers, i.e. $\Gamma_t^b = 0$.

Using (23), the flow budget constraint of borrowers (9) can now be expressed as

$$\frac{b_t}{1 + i_t^b} = \frac{b_{t-1}}{\Pi_t} + C_t^b - \left(1 - \frac{\varpi}{\chi} \right) \frac{Y_t^{1+\eta} \Delta_t^{1+\eta} \exp(z Y_t)}{z} - \frac{\varpi}{\chi} Y_t. \tag{24}$$

An equilibrium is given by the set of stochastic processes $\{C_t^b, C_t^s, i_t, i_t^b, b_t, Y_t, \Pi_t, \lambda_t, \Delta_t\}_{t=t_0}^{\infty}$ that solves the equilibrium conditions (11), (13), (14), (17), (21), (22), (24) together with a policy rule, the definition $\lambda_t = [(1 - \varpi) U_c(C_t^s) + \varpi U_c(C_t^b)]$ and for given exogenous sequences $\left\{ \bar{b}_t, \zeta_t \right\}_{t=t_0}^{\infty}$, considering the zero lower bound on the nominal interest rate $i_t \geq 0$. See Appendix B for further details on the recursive formulation of the aggregate-supply equation.

2.3 Log-linear approximation of the equilibrium conditions and a parallel to the standard New Keynesian model

The non-linear equilibrium conditions can be log-linearized into two blocks: (i) an *aggregate demand block* of five equations, namely the consumption Euler Equation for the saver (25) and the borrower (26), a spread function relating the borrowing and saving rates (27), the borrower's budget constraint (28) and the aggregate resource constraint (29); and (ii) an *aggregate supply block* consisting of the New-Keynesian Phillips curve (30):

$$E_t \hat{C}_{t+1}^s - \hat{C}_t^s = \sigma [\hat{\imath}_t - E_t (\pi_{t+1} - \pi)] \tag{25}$$

$$E_t\hat{C}^b_{t+1} - \hat{C}^b_t = \sigma\left[\hat{i}^b_t + \upsilon\left(\hat{b}_t - \hat{d}_t\right) - E_t(\pi_{t+1} - \pi)\right] \tag{26}$$

$$\hat{i}^b_t = \hat{\imath}_t + \varphi(\hat{b}_t - \hat{d}_t) \tag{27}$$

$$\hat{b}_t = \frac{1}{\beta}\hat{b}_{t-1} + \frac{\tilde{b}}{\beta}(\beta\hat{i}^b_t - (\pi_t - \pi)) + (1+i)\hat{C}^b_t - (1+i)\left(1 + \left(1 - \frac{\varpi}{\chi}\right)(\eta + \sigma^{-1})\right)\hat{Y}_t \tag{28}$$

$$\hat{Y}_t = \chi\hat{C}^b_t + (1-\chi)\hat{C}^s_t \tag{29}$$

$$\pi_t - \pi = \kappa\hat{Y}_t + \beta E_t(\pi_{t+1} - \pi) \tag{30}$$

where we have defined the variables $\hat{\imath}_t \equiv \ln(1+i_t)/(1+i)$, $\hat{i}^b_t \equiv \ln(1+i^b_t)/(1+i)$, $\pi_t \equiv \ln \Pi_t$, $\hat{C}^j_t \equiv (C^j_t - C^j)/Y$ for each $j = s, b$, $\hat{b}_t \equiv (b_t - \bar{b}^{high})/Y$, $\hat{d}_t \equiv (\bar{b}_t - \bar{b}^{high})/Y + (\zeta_t - \zeta)/\zeta$ and $\hat{Y}_t \equiv (Y_t - Y)/Y$. The coefficients of the model are defined as $\sigma \equiv 1/(zY)$, $\upsilon \equiv Y/\bar{b}^{high} \cdot (\phi_{b^i,b}(1,1,\zeta) + \phi_{b^i,b^i}(1,1,\zeta)) > 0$, $\varphi \equiv Y/\bar{b}^{high} \cdot \phi_b(1,1,\zeta) > 0$, $\tilde{b} \equiv \bar{b}^{high}/Y$ and $\kappa \equiv (1-\alpha)(1-\alpha\beta)(\eta+\sigma^{-1})/\alpha$.[17] These coefficients are written under the assumption that $\beta^b \to \beta^s$. We report the more general case in Appendix D.

Equations (25)-(30) determine the equilibrium allocation for $\{\pi_t, \hat{C}^b_t, \hat{C}^s_t, \hat{Y}_t, \hat{i}^b_t, \hat{\imath}_t, \hat{b}_t\}_{t=t_0}^{\infty}$ given the specification of monetary policy, an exogenous process $\hat{d}_t$ and initial condition $\hat{b}_{t_0-1}$. Movements in $\hat{d}_t$ isomorphically capture movements in $\bar{b}_t$ and ζ_t, i.e.

$$\hat{d}_t \equiv (\bar{b}_t - \bar{b}^{high})/Y + (\zeta_t - \zeta)/\zeta$$

highlighting the result we emphasized in the introduction: that is, that a household debt deleveraging shock and a banking shock have the same effect on aggregate dynamics. We will be considering a permanent debt deleveraging shock $d^{high} -> d^{low}$, which can then be explained either by an abrupt change in the perceived safe level of households' debt $\bar{b}^{high} -> \bar{b}^{low}$ or by an increase in the leverage requirement of banks or its equity cost $\zeta^{high} -> \zeta^{low}$ (see Appendix A for details of the microfoundation of the banking model).

To obtain a parallel to the standard New Keynesian model combine equations (25), (26), (27) and (29) to yield

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(\hat{\imath}_t - E_t(\pi_{t+1} - \pi) - r^n_t) \tag{31}$$

where r^n_t is

$$r^n_t \equiv -\chi(\upsilon + \varphi)\left(\hat{b}_t - \hat{d}_t\right). \tag{32}$$

Equation (31) is the same as the IS equation of the standard model, except that in that model r^n_t is exogenous. Here, instead, it is endogenously determined by (32), which in turn is specified by the aggregate-demand block (25)-(29). Aggregate demand is still determined by the real interest rate and expected future income, but the level of private indebtedness shifts this relationship.

The AS equation is exactly the same as in the standard model shown in (30), in contrast to other recent extensions of the NK models (see e.g. Curdia and Woodford, 2010). This

[17] In the approximation we have further normalized $\zeta\phi_{b^i,\zeta}(1,1,\zeta) = -Y/\bar{b}^{high} \cdot (\phi_{b^i,b}(1,1,\zeta) + \phi_{b^i,b^i}(1,1,\zeta))$ and $Y/\bar{b}^{high} \cdot \phi_b(1,1,\zeta) = -\zeta\phi_\zeta(1,1,\zeta)$.

Table 1: Parameters under Benchmark Model

Parameter	Value	Source or Target
Intertemporal elasticity of substitution in consumption	$\sigma = 0.66$	Smets and Wouters (2007)
Inverse of the Frisch elasticity of labor supply	$\eta = 1$	Justiniano et al. (2015)
Slope of the AS equation	$\kappa = 0.02$	Eggertsson and Woodford (2003)
Share of borrowers	$\chi = 0.61$	Justiniano et al. (2015)
Steady-state inflation rate	$\Pi = 1.005$	Corresponding to 2% at annual rates: Justiniano et al. (2015)
Intertemporal discount factor	$\beta = .9963$	Match real interest rate of 1.5%: Domeij and Ellingsen (2015)
Elasticity of substitution among varieties of goods	$\theta = 7.88$	Rotemberg and Woodford (1997)
Parameter of spread function	$\varphi = 0.0078$	Match the data
Parameter of the borrowers' Euler equation	$\upsilon = 0.0225$	Match the data.
Initial real debt	$\bar{b}^{high} = 4.0869$	Initial debt over GDP at 107.73%
Final real debt	$\bar{b}^{low} = 3.3384$	Final debt over GDP at 88%
Profit shares of borrowers	$\varpi = \chi$	

result depends essentially on our assumption on preferences and production, which implies that the wealth of each agent varies proportionally with output.

The natural rate of interest, defined in (32), is a useful concept here as in the standard New Keynesian model. Conditional on the level of debt $\hat{b}_{t-1}$ at time t, it corresponds to the real interest rate with prices flexible from period t onwards. As in the standard model, it can be interpreted as the real interest rate that the central bank would like to achieve at time t to stabilize output going forward.

3 Calibration: matching the Great Recession

We now show how our model generates movements in key variables of approximately the same size as in the U.S. during the Great Recession in response to a debt deleveraging shock. Where possible, our parameters are drawn from the existing literature as in Table 1. What remains to be chosen is the size of the shock (d^{high} to d^{low}), as well as φ and υ. To determine these parameters, we compare the model variables for household debt, b_t, and the borrowing rate, i_t^b, to their respective counterparts in the data. An important choice in the calibration is the choice of d^{high} and d^{low}, which reflects either a reversion in the household debt limit or a banking shock as trigger of the crisis. We choose d^{high} so that equilibrium debt corresponds to the level of household debt accumulation prior to the crisis of 2008. We pick d^{low} so that it corresponds to the value of private debt the quarter before the Federal Reserve started raising rates in 2015.[18] We choose (φ, ν) so as to minimize the mean squared errors of the model with respect to the data with results reported in Table 1 [19] For monetary policy, we assume that the central bank successfully targets 2 percent inflation unless it is precluded from doing so by the ZLB (in which case $i_t = 0$ and $\pi_t \leq 2\%$).

The top charts in Figure 3 show time series data on household debt and borrowing rate (dashed line) in comparison with the model output (solid line). As noted, the values of

[18] For a justification of this strategy see the Appendix E.
[19] See Appendix E for details.

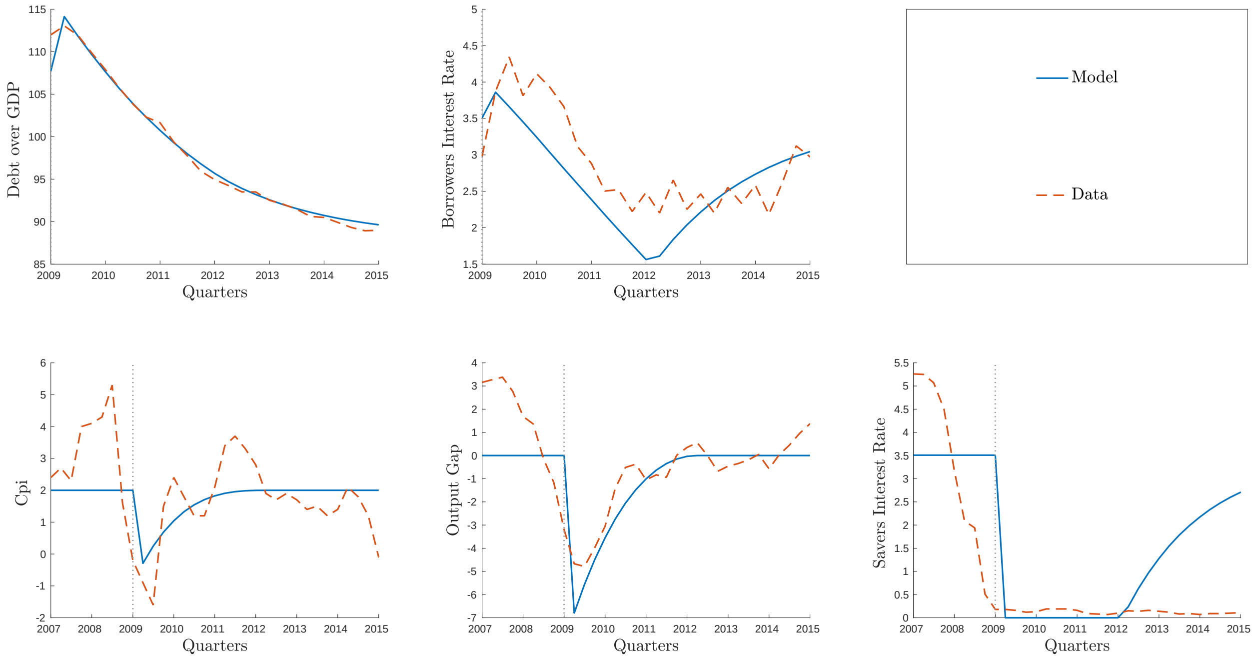

Figure 3: The top charts show the dynamic of nominal debt as a percentage of GDP and the dynamic of the borrowers' interest rate implied by the model. The bottom charts plot the model impulse responses of inflation, output gap, and savers' interest rate following the deleveraging shock. The continuous blue line displays the impulse response function implied by the model and the dashed red line displays the empirical counterparts. All variables are in %.

$(d^{high}, d^{low}, \varphi, \nu)$ have been chosen to match the empirical data.

The lower charts in Figure 3 compare data and model for output, inflation and interest rate, which we have not tried to match directly, denoting the time at which the reversion in the debt limit occurs with a dashed line. First, we see that the debt deleveraging shock generates enough downward pressure on the nominal interest rate to make ZLB binding. Second, the resulting recession in output is of roughly the same severity as in the data. Third, the model generates a drop in inflation; actual deflation appears only in a single quarter, but the rate then remains below target until the ZLB ceases to be binding. In short, the model generates a benchmark that broadly matches the US data following the crisis of 2008. It now becomes interesting to explicitly uncover the underlying dynamics that generate this outcome and explore the role for policy.

Before proceeding, it is worth highlighting some features of the data that the model misses. The short-term nominal interest rate starts to rise at the end of 2012 in the model so the recession lasts roughly 3 years. Although the actual recession (measured as deviation of output from trend) did not last much longer than this, the upper charts show that the short-term nominal interest rate remained at zero until the end of 2015, three full years longer. One reason for this failure of the model is that the spread between the borrowing and lending rate was fairly low and constant from 2012 on, which the model interprets as meaning that there is no longer need for negative real rates to achieve the inflation target. Adding more internal propagation to the model may resolve this issue, as it corresponds to the most stripped-down version of the New Keynesian model, which tends not to generate

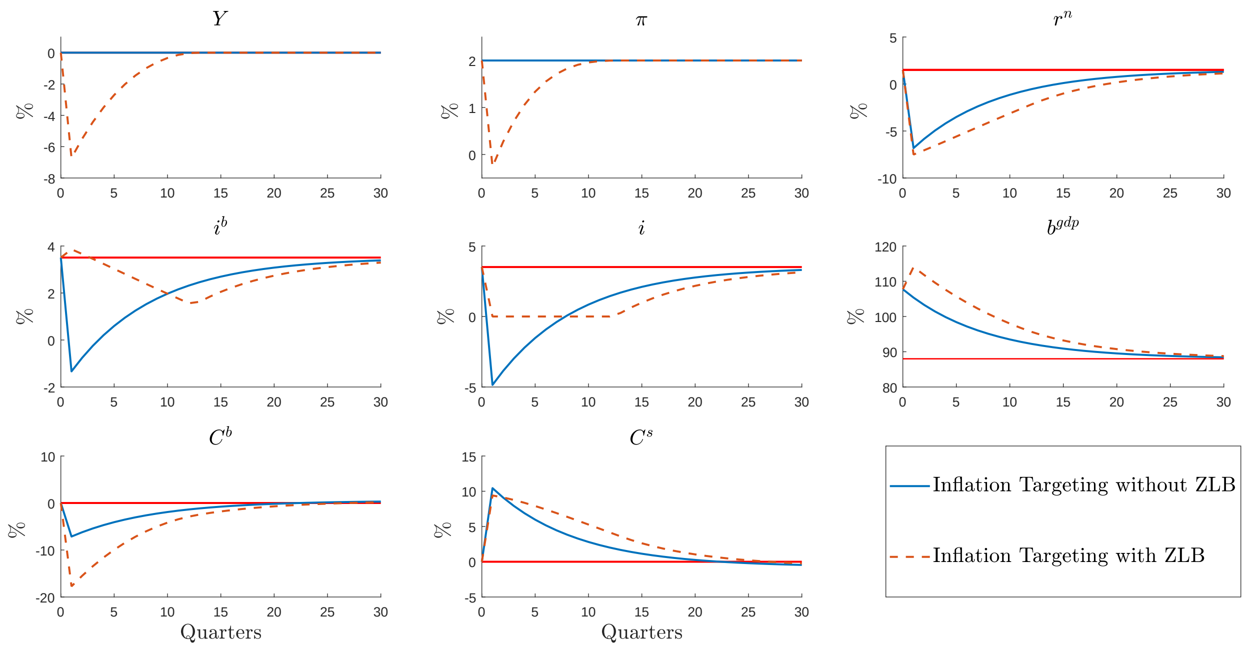

Figure 4: Responses following a deleveraging shock when the central bank can target constant inflation without (line "Inflation Targeting without ZLB") and with (line "Inflation Targeting with ZLB") taking into consideration the ZLB. Variables are: output (Y), inflation rate (π), natural rate of interest defined as in (32) (r^n), nominal interest rate on borrowing (i^b), nominal interest rate on saving (i), aggregate debt over GDP (b^{gdp}), consumption of borrowers (C^b), and consumption of savers (C^s). Y , C^b and C^s are in percentage deviation with respect to the steady state; π, r^n, i^b, i and b^{gdp} are in percent and at annual rates.

persistent responses to shocks. Another way to explain why the lift-off comes earlier than in the data is that our model disregards some secular factors that lengthen the duration of the ZLB, a theme of the recent literature on secular stagnation.[20] Under that interpretation, the dynamics can be interpreted as specific to the financial crisis, which could be layered on top of the slower-moving forces considered in the secular stagnation literature.

4 Main results

In the previous Section we saw that the model can generate a recession comparable to the U.S. Great Recession using data on household debt and borrowing rates to calibrate the trigger shock. The present Section is divided into two parts. Section 4.1 presents the positive analysis on the role of endogenous debt deleveraging in determining the depth and the duration of the recession. Section 4.2 offers a normative analysis of optimal monetary policy, highlighting that endogenous deleveraging calls for more aggressive policy than would be suggested by the standard New Keynesian model.

4.1 Positive analysis: dynamic debt deleveraging and the ZLB

In the standard New Keynesian account (see e.g. Eggertsson and Woodford, 2003) the duration of the Great Recession is purely exogenous; that is, it is attributed to the exogenous preference shocks that drive the natural rate of interest temporarily into negative territory. In our framework, by contrast, the initial impulse is given by the extent to which private debt exceeds the level considered as safe, or alternatively by how overleveraged the banking sector is. We assume that the realization of this shock is immediate, so the duration of the crisis is endogenously determined as a function of households' deleveraging choices. Unlike the standard account, then our version comprises the natural rate of interest and the depth and duration of the recession.

To understand how the deleveraging process propagates the recession, it is helpful to solve the model without imposing the ZLB. In this case, targeting inflation at 2% replicates flexible price allocation, which is analogous to the simple endowment example posited at the beginning of the paper. Given the preference and technology specification, (potential) output remains at the steady state as shown in the top panel in Figure 4. However, other variables move. There is an increase in the spread between the borrowing and the saving rate, i_t^b and i_t (middle charts in Figure 4), triggered by the exogenous shock d_t. In response the borrowers find it optimal to start deleveraging so we see a decline in their outstanding debt b_t in the third chart in the middle row. How do the borrowers deleverage? They can cut consumption, C_t^b, and increase hours worked, L_t^b. This is perfectly offset by a decrease in savers' hours worked and an increase in their consumption. It is clear why the borrowers decide to deleverage: They are facing higher borrowing costs. But why should the savers decide to consume more and work less? The reason is that the risk-free interest rate i_t declines, making current consumption relatively costly. It is also in the savers' interest to reduce working hours: higher consumption reduces the marginal utility of consumption, and with it the incentive to work.[21]

The continuous line in the last column and top row of Figure 4 shows how much the real interest rate needs to drop for output to remain unchanged: about 6 percentage points. The nominal interest rates that are consistent with this equilibrium, however, are negative (middle row, first column). For a central bank that targets inflation at 2%, as we assume here, this means that if the natural rate of interest is below -2% then the zero bound becomes binding and the equilibrium adjustment we have just explored is not feasible.

When the ZLB is imposed (the dashed red lines of the Figure) output and inflation drop since the central bank cannot accommodate the shock, as shown in the top row of Figure 4.[22] The third chart in the middle row clarifies that the ZLB introduces an endogenous

[20] See e.g. Eggertsson and Mehrotra (2014), Benigno and Fornaro (2018) and Garga and Singh (2017). The latter two papers show how hysteresis can arise due to the R&D mechanism.

[21] Real wages of savers increase following the shock partly offsetting the wealth effect on their labor supply. Without the increase in real wages, labor supply would fall twice as far. However, the labor responses of the savers and the borrowers are not central to the dynamics of the model. Wage rigidity, for example, would greatly reduce the asymmetric labor supply response of agents. Another simple way of doing away with the large asymmetric response of labor supply of the two agents is to use GHH preferences. We omit this extension because it does not nest the NK benchmark model.

[22] In Figure 4, and in what follows, the inflation-targeting policy considering the ZLB is defined as $\pi_t = \pi$ whenever $i_t > 0$, otherwise $i_t = 0$.

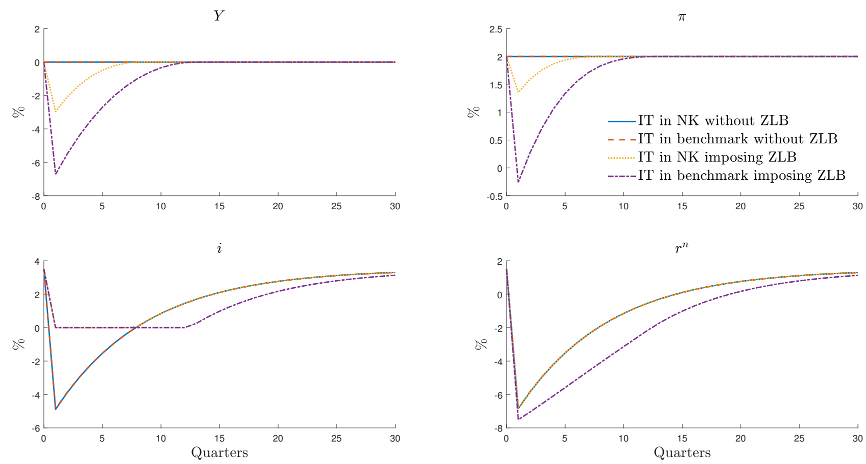

Figure 5: Comparison between the responses to a deleveraging shock in the deleveraging model, under inflation targeting without and with the ZLB (lines "IT in benchmark without ZLB" and "IT in benchmark imposing ZLB"), with those of the benchmark New-Keynesian model, under inflation targeting without and with the ZLB (lines "IT in NK without ZLB" and "IT in NK imposing ZLB"). (The lines "IT in benchmark without ZLB" and "IT in NK without ZLB" coincide by construction). Variables are: output (Y), inflation rate (π), nominal interest rate on savings (i), natural rate of interest defined as in (32) (r^n). Y is in percentage deviation with respect to the steady state; π, i and r^n are in percent and at annual rates.

component to the dynamic deleveraging. As the ZLB is now binding, this reduces output and thus borrowers' income. This in turn implies a slowdown in deleveraging. A simple way of seeing this is to compute the natural rate of interest given by (32). This process is now endogenous and, as we can see in the third column of the top row recovers more slowly with a binding ZLB (red dashed line) than otherwise. This means that endogenous deleveraging lengthens the recession by creating a vicious circle of falling income and a slower deleveraging, which is not found when the shocks are purely exogenous. This has important policy implications, as we shall soon see.

Direct comparison of this model with the standard NK model helps to highlight the role of debt deleveraging. Consider the solution without imposing the ZLB (Figure 4), where both inflation and output are perfectly stabilized via interest rate cuts. Equations (31) and (32) show that when inflation and output are stabilized, the real interest rate is equal to the natural rate. In this case, the model behaves exactly like the standard New Keynesian model, in which the real interest rate in equation (31) is now exogenously given by say, preference shocks, as long as we set the parameters of (31) and (32) in the same way in both models. Taking the real interest rate as representing the natural rate in the NK model, but keeping track of the fundamental shock in the original model, what are the implications of imposing the ZLB in the NK model vis-à-vis the more general model? Comparing the two outcomes we analyze how making the natural rate endogenous affects the solution. That is, we can determine to what extent it matters that in our new model the decline in output leads to endogenous propagation as the recession makes it harder for borrowers to pay down their debt, thus delaying the recovery of the natural rate of interest to its steady state and prolonging the recession.

Figure 5 shows the evolution of output, inflation and the natural and nominal interest rates in the standard NK model compared with our dynamic deleveraging model. As the red and blue lines show the two solutions coincide when the ZLB is not imposed. This is by construction, since we parameterize the NK model exactly like our benchmark model using equation (31). What is interesting is how the solutions differ when the ZLB is imposed, (purple and yellow lines). We can see that with endogenous deleveraging the effects on output and inflation are both larger than in the standard case. The reason is that under dynamic deleveraging the output slack drives the natural rate of interest further down and makes it more persistent, so the ZLB becomes more binding. Since aggregate demand depends on the current and expected future nominal interest rate as well as expected inflation and expected output, this interrelation feeds into lower current demand, hence lower output and inflation, and so on. We see further that this effect is quite large: the inflation and output drops are more than twice as great without dynamic debt deleveraging. Furthermore, the ZLB is binding for several more quarters when persistence of the natural rate of interest is endogenous.

We conclude then that adding dynamic deleveraging can have significant effects on the actual dynamics at the zero bound, both in terms of persistence of the recession (for a given shock as measured by the natural rate of interest) and its severity. What are the policy implications? We now turn to this.

4.2 Normative analysis: optimal policy under dynamic deleveraging

In the previous section we have shown that dynamic deleveraging makes the natural rate of interest endogenous, with it the duration of the ZLB. We will now show that this means policy should be even more aggressive than is implied by the standard model. To determine the optimal policy, consider a policymaker who maximizes social welfare[23]

$$W_t = E_t \left\{ \sum_{t=t_0}^{\infty} \beta^{T-t} \left[(1-\tilde{\chi}) \left(U(C_t^s) - V(L_t^s) \right) + \tilde{\chi} \left(U(C_t^b) - V(L_t^b) \right) \right] \right\} \tag{33}$$

for weight $\tilde{\chi} \in (0, 1)$. The heterogeneity in the model gives rise to special considerations. Deleveraging shifts the economy's distribution of wealth. Our strategy is to approximate the model to the efficient steady state under the assumption that this will be reached in the long run, once deleveraging is completed**.** The long-run steady state is efficient provided that weights $\tilde{\chi}$ are defined by the first-order conditions for the maximization (33) subject to

$$Y_t = (L_t^s)^{1-\chi}(L_t^b)^{\chi} = (1-\chi)C_t^s + \chi C_t^b, \tag{34}$$

which imply

$$\frac{U_c(C_t^s)}{U_c(C_t^b)} = \frac{\tilde{\chi}}{(1-\tilde{\chi})} \frac{(1-\chi)}{\chi}. \tag{35}$$

Using (35) and the steady-state levels of borrowers' and savers' consumption reached at the end of the deleveraging period (see (C.10) and (C.11) in Appendix C) gives a unique value for $\tilde{\chi}$.[24]

[23] As we noted in the context of the motivating example the assumption $\beta^s - > \beta^b$ is helpful here.

[24] If we choose an alternative weight $\tilde{\chi}$, the final steady state will be inefficient creating an incentive for policy to deviate from the inflation target Π in order to correct for the inefficient final distribution of wealth.

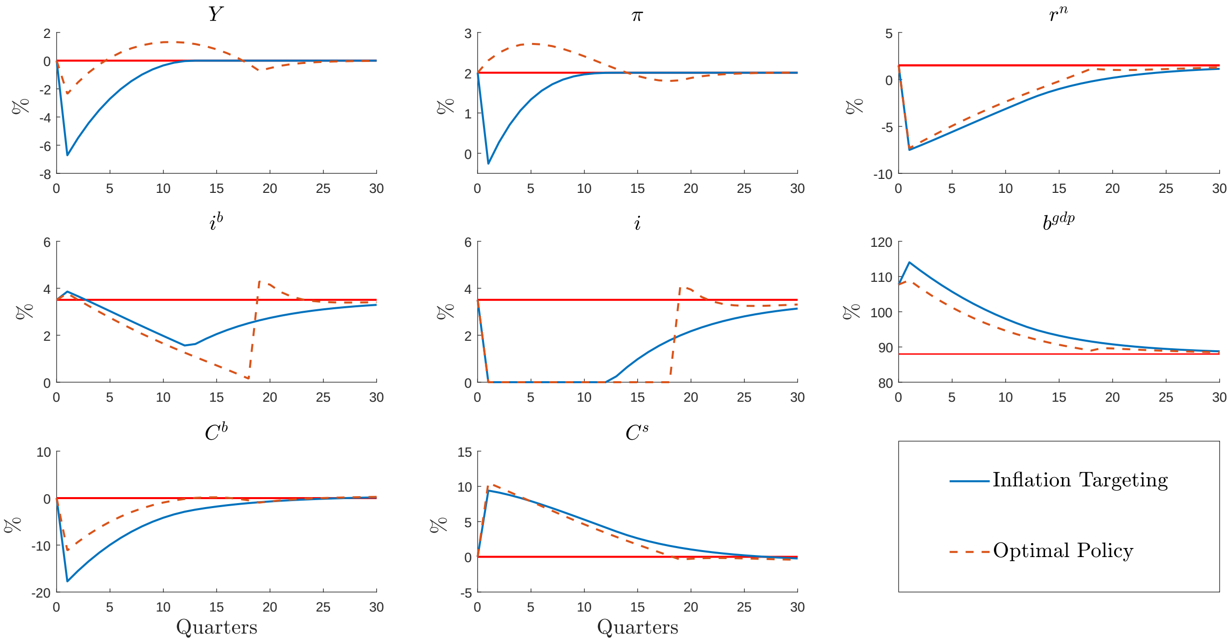

Figure 6: Responses following a deleveraging shock under optimal monetary policy with commitment (line "Optimal Policy") compared with inflation-targeting policy (line "IT") taking into consideration the ZLB. Variables are: output (Y), inflation rate (π), natural rate of interest defined as in (32) (r^n), nominal interest rate on borrowing (i^b), nominal interest rate on saving (i), aggregate debt over GDP (b^{gdp}), consumption of borrowers (C^b), and consumption of savers (C^s). Y, C^b and C^s are in percentage deviation with respect to the steady state; π, r^n, b^{gdp}, i^b and i are in percent and at annual rates.

As is shown in detail in Appendix F, a second-order approximation of (33) yields

$$L_{t_0} = \frac{1}{2} E_t \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\hat{Y}_t^2 + \chi(1-\chi)\lambda_c(\hat{C}_t^b - \hat{C}_t^s - c^R)^2 + \lambda_\pi(\pi_t - \pi)^2 \right] \right\} \tag{36}$$

where $\tilde{C}_t^j \equiv (C_t^j - \bar{C}^j)/Y$; $\bar{C}^j$ is the efficient steady-state level and $c^R \equiv [(C^b - \bar{C}^b) - (C^s - \bar{C}^s)]/Y$.[25]

As is shown in (36), the policymaker would like to keep inflation and output on target and achieve the efficient levels of consumption for the two agents. But, these three objectives can be reached simultaneously only in the long run. As Section 4.1 shows a deleveraging shock under an inflation-targeting policy produces short-run divergences between borrowers' and savers' consumption, which generate welfare losses according to objective (36), even without taking into account the ZLB. Adding the latter makes things worse, as output and inflation drop.

The presence of this "long-run" incentive is not convenient since it blurs understanding of the optimal adjustment following a deleveraging shock. Moreover, to deal with a distorted steady state, we have to adopt a more complex approximation procedure through second-order approximations of the equilibrium conditions implied by the optimization problem of private agents and by the resource constraints. This procedure has the cost of a more untidy analysis without the benefit of any substantial additional insight.

[25] A similar result is derived in Nisticó (2016), in which the consumption dispersion across agents is further related to financial wealth.

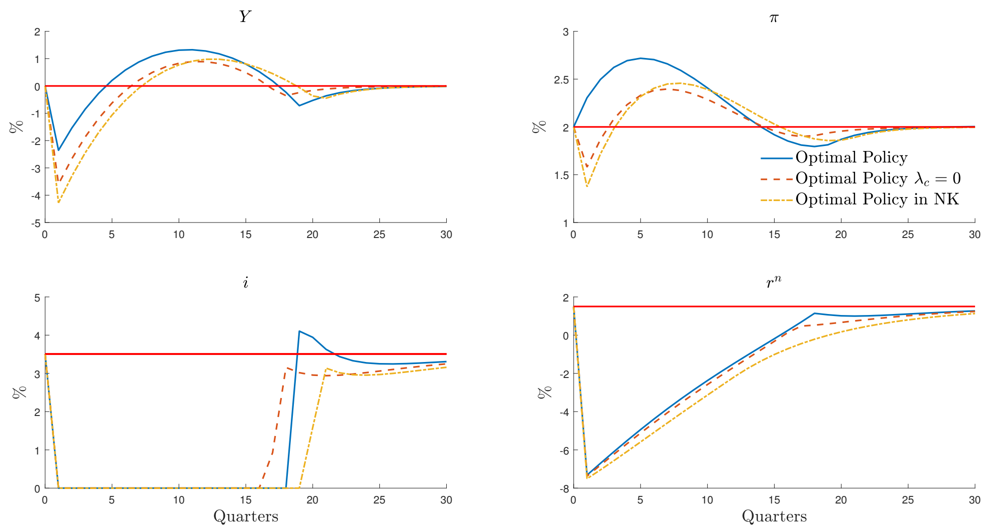

Figure 7: Comparison between the responses to a deleveraging shock. "Optimal Policy": optimal policy under commitment in the deleveraging model. "Optimal Policy $\lambda_c = 0$": optimal policy under commitment in the deleveraging model when $\lambda_c = 0$ in (36). "Optimal Policy in NK": optimal policy in the standard NK model. Variables are: output (Y), inflation rate (π), nominal interest rate on savings (i), natural rate of interest defined as in (32) (r^n). Y is in percentage deviation with respect to the steady state; π, i and r^n are in percent and at annual rates.

The benevolent planner minimizes the loss function (36) under commitment by choosing the sequence $\left\{\pi_t, \hat{C}_t^b, \hat{C}_t^s,\ \hat{Y}_t, \hat{\imath}_t^b, \hat{\imath}_t, \hat{b}_t\right\}_{t=t_0}^{\infty}$ given the constraints (25), (26), (27), (28), (29), (30), the exogenous process $\hat{d}_t$ and an initial condition on $\hat{b}_{t_0-1}$ and taking into account the ZLB constraint. For the details of the first-order conditions of the optimal policy problem see Appendix G.

The charts in Figures 6 show the responses of some key variables to a permanent shock to $\hat{d}_t$ under both optimal policy and inflation targeting considering the zero bound on nominal interest rates. Optimal policy has important elements in common with the standard NK model (see e.g. Eggertsson and Woodford, 2003). Figure 6 shows that optimal policy commits to keeping the nominal interest rates low for a considerably longer time than if the central bank targets inflation. The result of this commitment is an output boom and inflation higher-than-target during and after the trap. A key difference is that this commitment is stronger than in the standard model, because it implies an accommodation that is forceful enough for inflation to overshoot the 2% target for the entire duration of the ZLB whereas it never undershoots. This feature of optimal policy is new and different relative to the standard model.

Figure 7 shows the comparison with the case in which $\lambda_c = 0$. Including the consumption-risk-sharing argument in the loss function is the cause of the initial overshooting of inflation. At the exit from the liquidity trap, inflation and output undershoot their targets when $\lambda_c > 0$ relative to the case $\lambda_c = 0$. Taking the distributional impact of policy into account in a heterogeneous-agent model requires a more expansionary reaction to deleveraging, increasing output and mitigating the cost of deleveraging to borrowers.

An important conclusion then is that factoring in heterogeneity between borrowers and savers has important implications for optimal policy due to its welfare effects. Inflation policy becomes more attractive than in the standard model, because borrowers suffer more than savers in a debt deleveraging cycle so their marginal utility is greater, and because the benevolent planner cares about the distribution of the cost of the recession across agents. The utility of borrowers, in turn, improves with higher inflation than in the standard model. Even if it comes at the expense of savers, a policymaker maximizing welfare of form (36) will set a higher inflation target. The labor market is perfectly flexible, so that one way in which the borrower can react to the shock is by increasing labor supply. With more realistic frictions, the ability to deleverage by increasing labor supply decreases significantly making the case for inflation even stronger.

One interesting implication of our results is a theoretical rationale for inflation policy based on special consideration for borrowers, beyond the traditional case made in the modern ZLB literature. The improved welfare of borrowers as a consequence of inflation was a key explanation for the inflationary policy pursued by the US government during the Great Depression in 1933 (see e.g. Eggertsson, 2008).

At the end of Subsection 4.1 (see Figure 5), we compared the solution of our model to the NK model with parameters set for the two to yield identical outcomes for output, interest rates and inflation under an inflation targeting regime, as long as the ZLB is not imposed. We now analyze how optimal monetary policy differs across the two models when the ZLB is binding.

A first important difference is that the loss function in the benchmark NK model corresponds to (36) but with $\lambda_c = 0$. Second, in our setting – unlike the standard model – policy takes account of the fact it endogenously affects the deleveraging process (and the natural rate of interest). Figure 7 shows that the output and inflation implications of the two optimal policies are quite different. In our dynamic deleveraging model, optimal policy (line "Optimal Policy") is aggressive enough to bring about an immediate rise in inflation, thus overshooting the implicit inflation target by a significant amount. In the benchmark NK model (line "Optimal Policy in NK"), inflation overshoots the target less aggressively and with some delay, and recovery peaks later. The most interesting feature in the comparison is the behavior of the nominal interest rate. Optimal policy in our model implies an earlier lift-off than in the standard model, even though it is consistent with a smaller decline in output and inflation. How is it possible, then, that there is more expansion in output and inflation under our model than in the benchmark NK? This is the effect of making the natural rate endogenous. In the deleveraging model, the zero bound policy speeds up deleveraging and mitigates the fall in the natural rate of interest (as shown in the right-bottom panel) resulting in a more expansionary policy in terms of output and inflation. In the NK model no such feedback effect between policy and the natural rate is present.

The difference in optimal policy is partly explained by the endogenous feedback between policy and the natural rate of interest under debt deleveraging, and partly by the different objective functions of the government in the two models. Figure 7 shows the implications of optimal policy assuming $\lambda_c = 0$ in (36) (in which case the policymaker does not care about the distribution of income across the two agents). Optimal policy now yields dynamics more similar to the standard NK model, but three important differences remain. First, both inflation and output overshoot their long-term target earlier than in the standard case (and before the ZLB ceases to bind). Second, optimal policy raises the natural rate of interest above the level we feed exogenously into the NK model. Third, and relatedly, optimal policy now prescribes a substantially shorter duration of the zero interest rate than in the NK model while achieving a similar pattern of inflation and output. The reason for this last point is not that the policy is less aggressive. Rather, it is again because it is successful in endogenously raising the natural rate of interest and generating an output boom and inflation that the liftoff of rates comes earlier than in the absence of the policy easing.

Appendix G.1 explores some robustness analysis, analyzing the consequences of different degrees of nominal rigidity and of an alternative spread function with a debt-over-gdp argument in lieu of real debt. In the first extension, the greater the price flexibility, the less monetary policy needs to stay at the ZLB. By contrast, the different argument of the spread function does not make much quantitative difference.

5 Conclusions and future work

Explicitly deriving how the optimal monetary policy can be implemented, is beyond the scope of this paper. Here let us set forth few observations by way of conclusion.

One issue for policy implementation is that where a rich stochastic structure underlies the model, it could be difficult to communicate it in terms of state-contingent paths for the interest rate. One solution, suggested by Eggertsson and Woodford (2003), is a simple price-level targeting criterion; that is the central bank keeps the nominal interest rate at zero until it reaches a certain target that is the combination of price and output levels. If the target is not reached due to the ZLB, a formula is provided for how the target should be revised upward. Critically, however, the formula for the targeting criterion depends only on past deviations of policy from the suggested target.

The reason why the Eggertsson and Woodford (2003)'s targeting criterion depends only on lagged variables, however, is that the model is strictly forward-looking, for the reasons set out in Giannoni and Woodford (2017). In our setting, however, debt is a state variable. As Giannoni and Woodford (2017) show, this implies that the targeting criterion will include terms that are forward-looking. In the present context, this implies that the central bank's commitment to zero interest rate should depend not only on the realization of the current economic variables but also on forecasts which in turn depend on the economy's debt capacity and the speed of deleveraging. The problem of implementation is thus a rich and interesting avenue for further research.

In this paper we have extended the standard New Keynesian model include dynamic deleveraging. This yields a relatively general framework that we expect to be useful for further applications. We have kept the analysis as simple as possible to provide a workhorse post-crisis model.

We have focused chiefly on deleveraging shocks to households, but we have also remarked on the isomorphism with credit shocks that capture changes in intermediaries' leverage or other financial constraints. The analysis can be extended to study other sources of disturbances such as the more standard productivity and cost-push shocks. The way profits and taxes are distributed across agents significantly affects the response to shocks and accordingly warrants more investigation, as has been pointed out by the most recent literature on heterogenous-agent models.

One important extension would be to enlarge our framework into medium-scale DSGE model that can be estimated. Here we have elected not to do so in order to get a tractable model offering analytic predictions about optimal policy and clearly generalizing the existing literature on the zero lower bound. We hope future research can take this analysis a step further to produce a fully estimated model, possibly along the lines pursued by Justiniano et al. (2014).

Finally, one can imagine applications of the approach developed here to open economies or currency areas, so as to study the endogeneity of a country's deleveraging embedded in an international transmission mechanism. Benigno and Romei (2014), Bhattarai et al. (2015) and Fornaro (2014) are examples of such work. In particular, Fornaro (2014) analyzes a currency-area model that has some similarities with the framework set out here, with the demonstration that if it relinquishes control over the exchange rate, a country may suffer a severe contraction and a prolonged stay at the zero lower bound.

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A Banking and deleveraging

This Appendix provides a microfounded structure for borrowing and lending, with explicit banking technology, to help the reader interpret the general function presented in Section 2.2.

Broadly, we think of the spread function as stemming from two main sources. One is the existence of a certain level of debt that is "safe", i.e. above which lending to a particular group of individuals is at risk of default. The other is the cost of capital faced by the financial system as a whole. As we shall see, we do not need to take a position on which of these factors is the ultimate source. Let us start with the first interpretation.

A bank can be seen as a technology that converts the deposit contracts of some people into loan contracts of others. We posit that only banks can produce loan contracts (i.e. households cannot lend directly to one another). Consider the profits of a bank issuing loans to a group of individuals j (it can be a continuum of measure 1), financed via deposits by individual(s) i (the identity of the depositors is not important, they could be many or just one, for we assume all depositors receive the same risk-free deposit rate which is determined in equilibrium). Suppose that within the group of loans j, there is a probability $\gamma_t(l_t(j), \bar{b}_t^j, b_t^b)$ of any given loan not being repaid, so that in aggregate $\gamma_t(l_t(j), \bar{b}_t^j, b_t^b)l_t(j)$ equals the resources lost by the bank on its lending. This extra cost can already be predicted at time t. Intermediaries are unable to distinguish ex ante which type j borrowers will default. Assume now that this probability is higher, the further the loan is from what the bank deems "safe", i.e. $\bar{b}_t^j$. Similarly, the greater the aggregate debt in the economy b_t^b, the higher the probability of default. The terms of both loan and deposit contracts are determined in period t to be paid out in period $t+1$. At the end of period t some people abscond with the money, in a way we will specify below. In period $t+1$ the remaining loans are collected and deposits paid.

The profit of a bank offering loan contracts of type j and taking deposits i is

$$d_t(i) - l_t(j) - \gamma_t(l_t(j), \bar{b}_t^j, b_t^b)l_t(j) + E_t R_{t,t+1}\{(1+r_t^b)l_t(j) - (1+r_t^d)d_t(i)\} \tag{A.1}$$

where $R_{t,t+1}$ is a stochastic discount factor used to price the real value of next-period income flows.

The problem of the bank can be greatly simplified by the following assumption: Suppose that if there are profits on the loan contracts, the bank pays them to its owner (the representative saver) in period t and holds only enough assets at the end-of-period to pay off the depositors in period $t+1$. This implies that $(1+r_t^b)l_t(j) = (1+r_t^d)d_t(i)$ so that the last term of the profit function drops out. Furthermore, using this to substitute for $d_t(i)$, we can simplify (A.1) to

$$\left\{\frac{r_t^b - r_t^d}{1+r_t^d}\right\} l_t(j) - \gamma_t(l_t(j), \bar{b}_t^j, b_t^b)l_t(j)$$

in which case the bank's problem is simply to determine how much to lend to borrowers j (funded by taking deposits at the rate r_t^d). This yields the first order condition

$$\frac{r_t^b - r_t^d}{1+r_t^d} = \gamma_t^1 \tag{A.2}$$

where $\gamma_t^1 \equiv \frac{\partial \gamma_t(l_t(j),\bar{b}_t^j,b_t^b)}{\partial l_t(j)} l_t(j) + \gamma_t(l_t(j), \bar{b}_t^j, b_t^b)$. This problem looks exactly the same as in the previous section if we make one additional assumption: namely that "Fraud opportunities," and hence default, arrive exogenously to savers when they can "pose" as borrowers. In this case the proceeds of the fraud show up in the exogenous lump sum term in equation (1), while the borrowers' budget constraint remains unchanged. A "Minsky moment" can then be defined as a sudden reduction in $\bar{b}_t^j$, which is the perceived borrowing capacity of the group of borrowers of type j, which is also the borrowing capacity of the economy as a whole and shows up exactly in the same fashion as we have already analyzed.[26]

Consider now an alternative environment in which the spread reflects instead some cost of funding to banks. This was our second interpretation of the shock triggering the crisis. One example of such a cost is a capital requirement: that is, the bank needs to hold a certain capital, k_t^s, as a fraction ζ of its outstanding borrowing, b_t^b, i.e.

$$k_t^s \geq \zeta \frac{b_t^b}{1 + r_t^b}$$

in which ζ is the inverse of the leverage ratio. The bank raises this capital from savers, so we need to adjust the saver's budget constraint to reflect this while the borrower's constraint remains unchanged.[27] Suppose lending some capital to the bank is completely risk-free for savers, so that $r_t^k = r_t^s$. In writing the bank's problem, let us now imagine, as in Jermann and Quadrini (2012) or Justiniano et al (2014), that there is some cost of equity funding to the bank beyond r_t^k. In particular, assume a function $f(\cdot)$ that is weakly convex and captures the cost of equity funding above a certain threshold $\bar{k}$, with the property that $f(1) = 0$. The profit of the bank can now be written as

$$\Psi_{t+1} = (1 + r_t^b)\tilde{b}_t^b + (1 + r_t^s)\tilde{b}_t^s - (1 + r_t^s)k_t^s \left[1 + \frac{1}{\zeta} f\left(\frac{(1 + r_t^s)k_t^s}{\bar{k}}\right)\right], \tag{A.3}$$

where we have appropriately re-scaled the function $f(\cdot)$ by ζ and defined $\tilde{b}_t^b \equiv b_t^b/(1+r_t^b)$ and $\tilde{b}_t^s \equiv b_t^s/(1 + r_t^s)$. Considering that the capital-requirement constraint binds in equilibrium and that $\tilde{b}_t^b + \tilde{b}_t^s = k_t^s$, it is easy to show that the first-order condition of the optimization problem implies

$$(1 + r_t^b) = (1 + r_t^s) \left[1 + F\left(\frac{(1 + r_t^s)k_t^s}{\bar{k}}\right)\right],$$

in which

$$F\left(\frac{(1 + r_t)k_t^s}{\bar{k}}\right) \equiv f\left(\frac{(1 + r_t^s)k_t^s}{\bar{k}}\right) + \frac{(1 + r_t^s)k_t^s}{\bar{k}} f'\left(\frac{(1 + r_t^s)k_t^s}{\bar{k}}\right).$$

Using again the fact that the capital requirement binds in equilibrium $(1 + r_t^s)k_t^s = \zeta b_t^b$, we can further write the above first-order condition as

$$(1 + r_t^b) = (1 + r_t^s) \left[1 + F\left(\frac{\zeta b_t^b}{\bar{k}}\right)\right].$$

[26] The first-order condition (A.2) is in fact equivalent to the type of friction we assumed in equation (2). To see this, rewrite (A.2) as $1 + r_t^b = (1 + r_t^d)(1 + \gamma_t^1)$ which reduces to (5) if we assume that $\gamma_t = b_t^b - \bar{b}_t$.

[27] In particular the saver's budget constraint (1) should be written as $b_t^s/(1 + r_t^s) - k_t^s = b_{t-1}^s - (1 + r_{t-1}^k)k_{t-1}^s + C_t^s - (1/2)Y + T_t^s$.

This, once again, gives a spread between the lending and the deposit rate that is a function of the aggregate debt in the economy, as in (2). Here, however, it is due to capital constraints of the banks. In particular we can see that this spread may increase either because of an abrupt change in the required leverage ratio of the banks (an increase in ζ), or because of an increase in the cost of the bank's equity funding (a fall in $\bar{k}$). Any of the foregoing interpretations is valid for the general spread function that we choose in the general model of Section 2.2.

B Recursive formulation of the AS equation

Here, we describe the recursive formulation of the AS equation. This is given by

$$\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{1}{\theta-1}} = \frac{F_t}{K_t}, \tag{B.4}$$

where F_t and K_t satisfy:

$$F_t = \lambda_t Y_t + \alpha\beta E_t \left\{ F_{t+1} \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta-1} \right\}, \tag{B.5}$$

$$K_t = \mu \frac{\lambda_t \Delta_t^{\eta} Y_t^{1+\eta}}{z\exp(-z(Y_t - \Xi_t))} + \alpha\beta E_t \left\{ K_{t+1} \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta} \right\}. \tag{B.6}$$

C Steady State

The steady state implied by the equilibrium conditions is of particular importance, since we are taking log-linear approximations of the model. We consider an initial steady state in which $\bar{b}_t = \bar{b}^{high}$, $\zeta_t = \zeta$ and monetary policy sets the inflation rate to the target $\Pi_t = \Pi$. It clearly follows from (21) that $\Delta_t = 1$. In this steady state, the Euler equations of the savers, (11), and borrowers, (13), imply, respectively, that

$$(1+i) = \beta^{-1}\Pi, \tag{C.7}$$

and

$$(1+i^b) = \beta^{-1}\Pi\left(1-\epsilon\left(\frac{b}{\bar{b}^{high}}, \frac{b}{\bar{b}^{high}}, \zeta\right)\right) \tag{C.8}$$

while the borrowing premium is given by

$$\frac{(1+i^b)}{(1+i)} = \phi\left(\frac{b}{\bar{b}^{high}}, \frac{b}{\bar{b}^{high}}, \zeta\right), \tag{C.9}$$

following equation (14).

Combining (C.7), (C.8) and (C.9) we get

$$\frac{\phi\left(\frac{b}{\bar{b}^{high}}, \frac{b}{\bar{b}^{high}}, \zeta\right)}{1-\epsilon\left(\frac{b}{\bar{b}^{high}}, \frac{b}{\bar{b}^{high}}, \zeta\right)} = 1$$

which implicitly defines the level of debt b for each borrower with respect to the risk-free threshold $\bar{b}^{high}$. In particular, under minor restrictions on the functions $\phi(\cdot)$ and $\epsilon(\cdot)$, b can be set equal to $\bar{b}^{high}$ implying that $\phi(\cdot) = 1$ so that the borrowing and saving rates are equal in the steady state, $i^b = i$, while $\epsilon(\cdot) = 0$, implying that $\phi_{b^j}(1, 1, \zeta) = 0$.

Having determined the steady-state level of debt, we obtain the consumption of each borrower from (24)

$$C^b = Y - \frac{(1-\beta)}{\Pi}\bar{b}^{high},$$

under the assumption that $\varpi = \chi$, while from the aggregate resource constraint (17), we obtain the consumption of savers

$$C^s = Y + \frac{(1-\beta)}{\Pi}\frac{\chi}{1-\chi}\bar{b}^{high}.$$

Given the policy rule $\Pi_t = \Pi$, the aggregate-supply block of the model, characterized by equations (B.4)–(B.6), implies that steady-state output is determined by

$$\frac{Y^\eta}{z\exp(-zY)} = 1,$$

where we have also assumed a subsidy on firms' revenues equal to $\tau = 1/(\theta - 1)$ such that $\mu = 1$.

An important implication of our preference specification is that steady-state output is independent of the distribution of wealth, and therefore of the debt deleveraging process. In particular, we are interested in studying the effects of a permanent reduction in $\bar{b}$ from $\bar{b}^{high}$ to $\bar{b}^{low}$. Following this shock, the consumption of savers and of borrowers converges to new levels defined by

$$\bar{C}^b = Y - \frac{(1-\beta)}{\Pi}\bar{b}^{low}, \tag{C.10}$$

$$\bar{C}^s = Y + \frac{(1-\beta)}{\Pi}\frac{\chi}{1-\chi}\bar{b}^{low}. \tag{C.11}$$

D Derivation of the log-linearized Euler equation and spread function when $\beta^s > \beta^b$

This section derives the log-linearized Euler equation for borrowers and the spread function when savers are more patient than borrowers, i.e. when $\beta^s > \beta^b$. First we focus on the Euler equation and then we analyze the spread function.

For clarity we rewrite equation (13):

$$U_c(C_t^b) = \beta^b \frac{(1+i_t^b)}{1-\epsilon\left(\frac{b_t}{\bar{b}_t}, \frac{b_t}{\bar{b}_t}, \zeta_t\right)} E_t\left\{U_c(C_{t+1}^b)\frac{P_t}{P_{t+1}}\right\}, \tag{D.12}$$

where the elasticity of the premium with respect to the individual real debt is defined as

$$\epsilon\left(\frac{b_t}{\bar{b}_t}, \frac{b_t}{\bar{b}_t}, \zeta_t\right) \equiv \frac{b_t}{\bar{b}_t} \frac{\phi_{b^j}\left(\frac{b_t}{\bar{b}_t}, \frac{b_t}{\bar{b}_t}, \zeta_t\right)}{\phi\left(\frac{b_t}{\bar{b}_t}, \frac{b_t}{\bar{b}_t}, \zeta_t\right)}.$$

Before log-linearizing the Euler equation (D.12) we must define some coefficients:

$$\nu \equiv \frac{Y}{\bar{b}} \frac{1}{\phi\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)} \frac{1}{\left(1-\epsilon\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)\right)} \left(\phi_{b^j}\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right) + \frac{b^*}{\bar{b}}\left(\phi_{b^j,b^j}\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right) + \phi_{b^j,b}\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)\right)\right.$$
$$\left. - \frac{b^*}{\bar{b}} \frac{\phi_{b^j}\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)}{\phi\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)} \left(\phi_b\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right) + \phi_{b^j}\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)\right)\right),$$

and

$$\nu_\zeta \equiv \frac{b^*}{\bar{b}} \frac{\zeta}{\phi\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)} \frac{1}{\left(1-\epsilon\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)\right)} \left(\phi_{b^j,\zeta}\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right) - \frac{\phi_{b^j}\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)\phi_\zeta\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)}{\phi\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)}\right),$$

where we define b^* as the individual borrower's steady-state debt and $\phi_b(.,.,.)$ and $\phi_\zeta(.,.,.)$ as the derivatives of the function $\phi(.,.,.)$ with respect to the second and the third argument, respectively. Finally, we define $\phi_{b^j,b^j}(.,.,.)$ as the second derivative with respect to the first argument and $\phi_{b^j,b}(.,.,.)$ and $\phi_{b^j,\zeta}(.,.,.)$ as the cross derivatives with respect to the first and the second arguments and to the first and third arguments, respectively. In addition, we assume that

$$\phi_{b^j}\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right) + \frac{b^*}{\bar{b}}\left(\phi_{b^j,b^j}\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right) + \phi_{b^j,b}\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)\right) \geq$$
$$\frac{b^*}{\bar{b}} \frac{\phi_{b^j}\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)}{\phi\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)} \left(\phi_b\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right) + \phi_{b^j}\left(\frac{b^*}{\bar{b}}, b\right.\right.$$

and

$$\phi_{b^j,\zeta}\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right) \leq \frac{\phi_{b^j}\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)\phi_\zeta\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)}{\phi\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)}.$$

Therefore, we log-linearize the borrowers Euler equation (D.12) around the steady-state allocation obtaining:

$$E_t\hat{C}^b_{t+1} - \hat{C}^b_t = \sigma\left[\hat{\imath}^b_t + \upsilon\left(\hat{b}_t - \hat{d}_{1,t}\right) - E_t(\pi_{t+1} - \pi)\right], \tag{D.13}$$

where $\hat{d}_{1,t} \equiv \left(\bar{b}_t - \bar{b}^{high}\right)/Y - \frac{\nu_\zeta}{\nu}(\zeta_t - \zeta)/Y$.[28] Equation (D.13) is almost identical to equation (26) except for the definition of the shock $\hat{d}_{1,t}$ in place of $\hat{d}_t$.

[28]The other variables correspond to those defined in Section 2.3.

Second, we consider equation (14):

$$(1 + i_t^b) = (1 + i_t) \cdot \phi\left(\frac{b_t}{\bar{b}_t}, \frac{b_t}{\bar{b}_t}, \zeta_t\right). \tag{D.14}$$

We define

$$\varphi \equiv \frac{Y}{\bar{b}} \frac{1}{\phi\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)} \left(\phi_{b^j}\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right) + \phi_b\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)\right) \quad \varphi_\zeta \equiv \frac{\zeta}{\bar{b}} \frac{\phi_\zeta\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)}{\phi\left(\frac{b^*}{\bar{b}}, \frac{b^*}{\bar{b}}, \zeta\right)},$$

and assume that $\phi_{b^j}(.,.,.) \geq 0$, $\phi_b(.,.,.) \geq 0$ and $\phi_\zeta(.,.,.) \leq 0$ in line with Section 2.2. Thus, we log-linearize equation (D.14) around the steady state, obtaining:

$$\hat{\imath}_t^b = \hat{\imath}_t + \varphi(\hat{b}_t - \hat{d}_{2,t}), \tag{D.15}$$

where $\hat{d}_{2,t} \equiv \left(\bar{b}_t - \bar{b}^{high}\right)/Y - \frac{\varphi_\zeta}{\varphi}(\zeta_t - \zeta)/Y$. Again, equation (D.15) is almost identical to (27), except for the definition of the shock, $\hat{d}_{2,t}$. In addition, the shock to the Euler equation of the borrowers $\hat{d}_{1,t}$ may differ from the shock in the spread function $\hat{d}_{2,t}$.

E Calibration

To calibrate the shock, we rely on our interpretation of the model as driven by debt deleveraging on the household side. A possible alternative, however, would be some measure of disturbances in the banking system.

First, as an empirical proxy for household debt, we use the series of U.S. nominal debt for Households and Nonprofit Organizations taken from the Board of Governors of the Federal Reserve System.[29] This series is shown in Figure 8.[30] Second, we use the Commercial Bank Credit Card Interest Rate as a proxy for the borrowers' interest rate.[31] We show this series, appropriately adjusted, on the top row and second column of Figure 3.[32]

The model and the data are quarterly. The calibrated parameters, shown in Table 1, are largely standard and taken directly from the literature cited in the Table.[33] Particular to our model are the parameters φ and υ which govern the spread function. The main new element of the calibration is the choice of shock, which is a one-time reduction in d from d^{high} to d^{low}, with its implications for the new observables we have introduced. We use the data on debt to discipline the choice of d^{high} to d^{low}. We set $d^{high} = 4.0869$ to match the value

[29] Following Eggertsson, Ferrero and Raffo (2014), we approximate GDP as the sum of Consumption and Gross Investment from the NIPA tables.

[30] The series shows different trends over time, possibly as a result of some structural break.

[31] The source of the Commercial Bank Credit Card Interest Rate is the Board of Governors of the Federal Reserve System.

[32] We took the series from the Account Interest Assessed. This measure of spread was fluctuating around 10%. Since in our model the steady-state spread is zero, we de-meaned the data spread using the historical mean from 1995 to the first quarter of 2009. To compute the borrowers' interest rate we add this de-meaned spread to the Federal Funds rate.

[33] Except for the fraction of borrowing and lending, where we rely on Justiniano, Primiceri and Tambalotti (2015).

Figure 8: US Private Debt over GDP in percent, with dashed trend lines for different subperiods.

of the debt to GDP ratio in the second quarter of 2008.[34] We set $d^{low} = 3.3384$ so that the final debt matches the debt to GDP ratio of 88%, observed in third quarter of 2015. This is also an interesting benchmark for a reason shown in Figure 8, which draws a trend line for the increase in the debt/output ratio for the period 1987-2000. The year 2001 marks the break point at which we see a very rapid increase in debt, a period many have associated with the real estate "bubble". The value 88% corresponds, as seen in the figure, to the 2015 value of this trend projected on 1987-2000 data which happens to coincide exactly with the observed value of real debt over GDP in that year.[35] While this is only one illustrative case, we experiment with other values as further discussed below. The two key parameters left to be determined are φ and υ. These parameters capture the characteristics of function (10), which determines the difference between borrowing and lending rates and the extent to which households internalize this in their optimizing decisions (which in turn determine the speed of debt deleveraging). Our strategy is to set these two parameters to match the

[34] Debt over GDP was equal to 107.73%

[35] We consider private debt over GDP at quarterly frequency from the first quarter of 1952 to the third quarter of 2015. We divide this series into four subsamples: Q1-1952–Q2-1964, Q3-1964–Q2-1984, Q3-1984–Q4-1986 and Q1-1987–Q4-2000. We choose these subsamples since they have different linear trends. Excluding the third subsample, we compute the linear trend of all the subsamples. We consider the third subsample as a discontinuity jump. Finally, we project the linear trend of the fourth subsample up to the third quarter of 2015.

data on the top row of Figure 3 as closely as possible using as a criterion Minimum Mean Square Error of the data relative to the model. This procedure results in $\varphi = .0078$ and $\upsilon = .0225$.[36]

By construction, the model matches the data in Figure 3 relatively well, as we have chosen φ and υ to do precisely that. Let us now see what happens to the variables that we have not tried explicitly to match, feeding the shock into the model, i.e. d^{high} falls to d^{low}. Figure 3 shows the model and the data. As our empirical measures we take the annual percentage change in CPI for inflation and detrended GDP, through HP filter, for the deviation of output from potential. The short-term nominal interest rate – i.e. the risk-free rate paid by the saver – is the Federal Funds rate. The duration of the output contraction is about three years about the same as for our measure of the output gap according to the HP filter, which shows output back up to a trend around 2012.[37] As noted in the main text, the key discrepancy between the model and the data is the Federal Funds rate. We have also experimented with lenghtening the recession by choosing a lower d^{low}. Another natural benchmark, relative to the one we choose, is the household debt over GDP in 2001, or 76.5%. Accordingly, we have re-estimated the values of υ and φ. This adjustment does increase the duration of the ZLB by about three quarters. While the changes in output and inflation are of similar order, this parametrization is a bit worse at matching the spreads and the debt deleveraging, which is why we focus on the numerical example considered in the text.

F Derivation of the loss function (36)

In this section we show the derivations of the second-order approximation of the welfare function (33). The approximation is with respect to an efficient steady state, which maximizes (33) under the resource constraint (34).

At the efficient steady state, the following conditions hold

$$\begin{aligned}
(1-\tilde{\chi})\bar{U}_c^s &= (1-\chi)\bar{\lambda}; \\
\tilde{\chi}\bar{U}_c^b &= \chi\bar{\lambda}; \\
(1-\tilde{\chi})\bar{V}_l^s &= (1-\chi)\bar{\lambda}\frac{\bar{Y}}{\bar{L}^s}; \\
\tilde{\chi}\bar{V}_l^b &= \chi\bar{\lambda}\frac{\bar{Y}}{\bar{L}^b}
\end{aligned}$$

where all upper bars denote steady-state values and $\bar{\lambda}$ is the steady-state value of the Lagrange multiplier associated with constraint (34). Note that these conditions imply

[36] We create a grid of φ and υ. For every couple we simulate our model, assuming that the central bank targets inflation and that the nominal interest rate cannot go below zero. We compute the square difference of the deleveraging in our model and in the actual data as well as the square difference of the borrowers' interest rate in the data and in our model. We pick the couple that minimizes the sum of these square differences, that is $\phi = .0078$ and $\upsilon = .0225$.

[37] To be clear, we do not think this is the most reasonable estimate of the output gap, but we use it here since it is very transparent and widely used, and thus helpful for illustrative purposes. We took the series of GDP as previously defined from 1990 till the last data available, divided by the CPI and de-trended by using the HP filter. Since this series is at quarterly frequency, we set the multiplier λ_{HP} equal to 1600.

$\bar{U}_c^s/\bar{U}_c^b = (1-\chi)\tilde{\chi}/[\chi(1-\tilde{\chi})]$ so that an appropriately chosen $\tilde{\chi}$ determines the efficient distribution of wealth.

By taking a second-order expansion of the utility flow around the efficient steady state, we obtain

$$\begin{aligned} U_t &= \bar{U} + (1-\tilde{\chi})\left[\bar{U}_c^s(C_t^s - \bar{C}^s) + \frac{1}{2}\bar{U}_{cc}^s(C_t^s - \bar{C}^s)^2\right] + \\ &+\tilde{\chi}\left[\bar{U}_c^b(C_t^b - \bar{C}^b) + \frac{1}{2}\bar{U}_{cc}^b(C_t^b - \bar{C}^b)^2\right] + \\ &-(1-\tilde{\chi})\left[\bar{V}_l^s(L_t^s - \bar{L}^s) + \frac{1}{2}\bar{V}_{ll}^s(L_t^s - \bar{L}^s)^2\right] - \\ &-\tilde{\chi}\left[\bar{V}_l^b(L_t^b - \bar{L}^b) + \frac{1}{2}\bar{V}_{ll}^b(L_t^b - \bar{L}^b)^2\right] + \mathcal{O}(||\xi||^3) \end{aligned}$$

where the upper-bar variable denotes the efficient steady-state variable and $\mathcal{O}(||\xi||^3)$ collects terms in the expansion that are of an order higher than the second. We can use the steady-state conditions to write the above equation as

$$\begin{aligned} U_t &= \bar{U} + (1-\chi)\bar{\lambda}\left[(C_t^s - \bar{C}^s) + \frac{1}{2}\frac{\bar{U}_{cc}^s}{\bar{U}_c^s}(C_t^s - \bar{C}^s)^2\right] + \\ &+\chi\bar{\lambda}\left[(C_t^b - \bar{C}^b) + \frac{1}{2}\frac{\bar{U}_{cc}^b}{\bar{U}_c^b}(C_t^b - \bar{C}^b)^2\right] + \\ &-(1-\chi)\bar{\lambda}\frac{\bar{Y}}{\bar{L}^s}\left[(L_t^s - \bar{L}^s) + \frac{1}{2}\frac{\bar{V}_{ll}^s}{\bar{V}_l^s}(L_t^s - \bar{L}^s)^2\right] - \\ &-\chi\bar{\lambda}\frac{\bar{Y}}{\bar{L}^b}\left[(L_t^b - \bar{L}^b) + \frac{1}{2}\frac{\bar{V}_{ll}^b}{\bar{V}_l^b}(L_t^b - \bar{L}^b)^2\right] + \mathcal{O}(||\xi||^3). \end{aligned}$$

Note that for a generic variable X, we have

$$X_t = \bar{X}\left(1 + \tilde{X}_t + \frac{1}{2}\tilde{X}_t^2\right) + \mathcal{O}(||\xi||^3)$$

where $\tilde{X}_t \equiv \ln X_t/\bar{X}$, and further recall that

$$Y_t = \chi C_t^s + (1-\chi)C_t^b.$$

We can write the above approximation as

$$\begin{aligned} U_t &= \bar{U} + \bar{\lambda}\bar{Y}\left[\tilde{Y}_t + \frac{1}{2}\tilde{Y}_t^2\right] - \frac{1}{2}\bar{\lambda}z\left[(1-\chi)(C_t^s - \bar{C}^s)^2 + \chi(C_t^b - \bar{C}^b)^2\right] + \\ &-\chi\bar{\lambda}\bar{Y}\left[\tilde{L}_t^s + \frac{1}{2}(1+\eta)(\tilde{L}_t^s)^2\right] \\ &-(1-\chi)\bar{\lambda}\bar{Y}\left[\tilde{L}_t^b + \frac{1}{2}(1+\eta)(\tilde{L}_t^b)^2\right] + \mathcal{O}(||\xi||^3), \end{aligned} \quad \text{(F.16)}$$

where we have also used the fact that with the preference specification used $\bar{U}^s_{cc}/\bar{U}^s_c = \bar{U}^b_{cc}/\bar{U}^b_c = -z$ and $\bar{V}^s_{ll}\bar{L}^s/\bar{V}^s_l = \bar{V}^b_{ll}\bar{L}^b/\bar{V}^b_l = \eta$. Note that the efficient steady state of output is equal also to its initial steady state. Therefore, in what follows we can use the fact that $\bar{Y} = Y$ and also clearly $\tilde{Y}_t = \hat{Y}_t$.

Notice the following equivalences

$$\frac{(L^s_t)^{1+\eta}}{z\exp(-zC^s_t)} = \frac{(L^b_t)^{1+\eta}}{z\exp(-zC^b_t)} = \frac{(\Delta_t Y_t)^{1+\eta}}{z\exp(-zY_t)}$$

where we have used $W_t L_t = W^s_t L^s_t = W^b_t L^b_t$ and $L_t = \Delta_t Y_t$. The above equations imply exactly that

$$\tilde{L}^s_t = \tilde{\Delta}_t + \hat{Y}_t - \frac{z}{1+\eta}[(C^s_t - \bar{C}^s) - (Y_t - Y)],$$

$$\tilde{L}^b_t = \tilde{\Delta}_t + \hat{Y}_t - \frac{z}{1+\eta}[(C^b_t - \bar{C}^b) - (Y_t - Y)].$$

and therefore that

$$\tilde{L}^s_t = \tilde{\Delta}_t + \hat{Y}_t - \frac{\sigma^{-1}}{1+\eta}(\tilde{C}^s_t - \hat{Y}_t),$$

$$\tilde{L}^b_t = \tilde{\Delta}_t + \hat{Y}_t - \frac{\sigma^{-1}}{1+\eta}(\tilde{C}^b_t - \hat{Y}_t).$$

where $\tilde{C}^b_t \equiv (C^b_t - \bar{C}^b)/Y$ and $\tilde{C}^s_t \equiv (C^s_t - \bar{C}^s)/Y$. Moreover,

$$\tilde{C}^s_t - \hat{Y}_t = -\chi(\tilde{C}^b_t - \tilde{C}^s_t)$$

$$\tilde{C}^b_t - \hat{Y}_t = (1-\chi)(\tilde{C}^b_t - \tilde{C}^s_t)$$

which can be substituted into (F.16) to obtain

$$\begin{aligned} U_t &= \bar{U} - \frac{1}{2}\bar{\lambda}Y\left\{(\eta+\sigma^{-1})\cdot\hat{Y}^2_t + \chi(1-\chi)\sigma^{-1}(\tilde{C}^b_t - \tilde{C}^s_t)^2 + \right. \\ &\quad \left. +\chi(1-\chi)\frac{\sigma^{-2}}{(1+\eta)}(\tilde{C}^b_t - \tilde{C}^s_t)^2\right\} - \bar{\lambda}Y\cdot\hat{\Delta}_t + \mathcal{O}(||\xi||^3). \end{aligned}$$

Note that

$$\Delta_t = \alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta}\Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta}{\theta-1}}.$$

By taking a second-order approximation of $\hat{\Delta}_t$, as is standard in the literature, and integrating appropriately across time, we obtain that

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\hat{\Delta}_t = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\theta\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{(\pi_t-\pi)^2}{2} + \text{t.i.p.} + \mathcal{O}(||\xi||^3).$$

We can therefore write

$$W_{t_0} = -\bar{\lambda}(\eta+\sigma^{-1})Y\cdot\frac{1}{2}E_t\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}L_t\right\} + \text{t.i.p.} + \mathcal{O}(||\xi||^3)$$

where

$$L_t = \hat{Y}_t^2 + \chi(1-\chi)\lambda_c(\tilde{C}_t^b - \tilde{C}_t^s)^2 + \lambda_\pi(\pi_t - \pi)^2$$

where we have defined

$$\lambda_c \equiv \frac{\sigma^{-1}(1+\eta) + \sigma^{-2}}{(1+\eta)(\eta + \sigma^{-1})}$$

$$\lambda_\pi \equiv \frac{\theta}{\kappa}.$$

G First-order conditions of optimal policy under commitment

In this section, we characterize the optimal policy problem in detail.

Optimal monetary policy under commitment minimizes the loss function

$$L_{t_0} = \frac{1}{2} E_t \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\hat{Y}_t^2 + \chi(1-\chi)\lambda_c(\hat{C}_t^b - \hat{C}_t^s - c^R)^2 + \lambda_\pi(\pi_t - \pi)^2 \right] \right\} \tag{G.17}$$

where c^R captures the relative difference between the initial and the final steady-state consumptions of borrowers and savers defined as $c^R \equiv [(C^b - \bar{C}^b) - (C^s - \bar{C}^s)]/Y$. The minimization is constrained by the following set of structural equations of the model:

$$\hat{Y}_t = \chi \hat{C}_t^b + (1-\chi)\hat{C}_t^s \quad (\lambda_1) \tag{G.18}$$

$$E_t \hat{C}_{t+1}^b - \hat{C}_t^b = \sigma[\hat{\imath}_t^b - E_t(\pi_{t+1} - \pi) + \upsilon(\hat{b}_t - \hat{d}_t)] \quad (\lambda_2) \tag{G.19}$$

$$E_t \hat{C}_{t+1}^s - \hat{C}_t^s = \sigma[\hat{\imath}_t^s - E_t(\pi_{t+1} - \pi)] \quad (\lambda_3) \tag{G.20}$$

$$\hat{C}_t^b = \frac{\bar{b}}{(1+i)}(\hat{b}_t - (\hat{\imath}_t^b)) - \frac{\bar{b}}{\beta(1+i)}(\hat{b}_{t-1} - (\pi_t - \pi)) + \hat{Y}_t \quad (\lambda_4) \tag{G.21}$$

$$\hat{\imath}_t^b = \hat{\imath}_t^s + \varphi\left(\hat{b}_t - \hat{d}_t\right) \quad (\lambda_5) \tag{G.22}$$

$$\pi_t - \pi = \kappa \hat{Y}_t + \beta E_t(\pi_{t+1} - \pi) \quad (\lambda_6) \tag{G.23}$$

$$-\hat{\imath}_t^s + \hat{\imath}_{ss,t} \leq 0. \quad (\lambda_7) \tag{G.24}$$

Note that for each of these equations we have written on the right-hand side its Lagrange multiplier.

The first-order conditions of the optimal policy problem are:

$$\hat{Y}_t : \qquad \hat{Y}_t + \lambda_{1,t} - \lambda_{4,t} - k\lambda_{6,t} = 0 \tag{G.25}$$

$$\hat{C}_t^s : \qquad -(\chi(1-\chi)\lambda_c)\left(\hat{C}_t^b - \hat{C}_t^s - \hat{C}_t^R\right) - (1-\chi)\lambda_{1,t} - \lambda_{3,t} + \frac{\lambda_{3,t-1}}{\beta} \tag{G.26}$$

$$\hat{C}_t^b: \qquad (\chi(1-\chi)\lambda_c)\left(\hat{C}_t^b - \hat{C}_t^s - \hat{C}_t^R\right) - (\chi)\lambda_{1,t} - \lambda_{2,t} + \frac{\lambda_{2,t-1}}{\beta} + \lambda_{4,t} = 0 \tag{G.27}$$

$$\hat{\pi}_t: \qquad \lambda_\pi(\pi_t - \pi) + \sigma\frac{\lambda_{2,t-1}}{\beta} + \sigma\frac{\lambda_{3,t-1}}{\beta} - \frac{\bar{b}}{(1+i)\beta}\lambda_{4,t} + \lambda_{6,t} - \lambda_{6,t-1} = 0 \tag{G.28}$$

$$\hat{i}_t^s: \qquad -\lambda_{3,t}\sigma - \lambda_{5,t} - \lambda_{7,t} = 0 \tag{G.29}$$

$$\hat{i}_t^b: \qquad -\lambda_{2,t}\sigma + \frac{\bar{b}}{(1+i)}\lambda_{4,t} + \lambda_{5,t} = 0 \tag{G.30}$$

$$\hat{b}_t: \qquad -\frac{\bar{b}}{(1+i)}\lambda_{4,t} + \frac{\bar{b}}{(1+i)}E_t\lambda_{4,t+1} - \phi\lambda_{5,t} - \sigma\upsilon\lambda_{2,t} = 0. \tag{G.31}$$

$$\lambda_{7,t}(-\hat{i}_t^s + \hat{i}_{ss,t}) = 0. \tag{G.32}$$

The set of first-order conditions together with the equilibrium constraints is solved using a solution method that takes into account the zero lower bound (see also Eggertsson and Woodford, 2003).

G.1 Robustness analysis

In this section, we explore the properties of our model as regards robustness to different assumptions. First, we analyze how optimal policy changes for different degrees of price stickiness. Under our benchmark model, the calibration of the slope of the AS equation is $\kappa = 0.02$. We conduct experiments with higher values of κ, implying greater degree of price flexibility, $\kappa = 0.1$ and $\kappa = 0.5$. Figure 9 shows the comparison. The greater the price flexibility (the higher κ), the less time monetary policy needs to stay at the zero lower bound. Inflation surges more, even on impact, when prices are more flexible, and output stabilization improves accordingly. Indeed, the costs of inflation variability in the loss function are smaller, the greater the degree of price flexibility. We also investigated whether it makes a difference to assume that the spread function in (27) is a function of debt/GDP rather than real debt. This distinction produces only minor differences in the computation of the optimal policy.

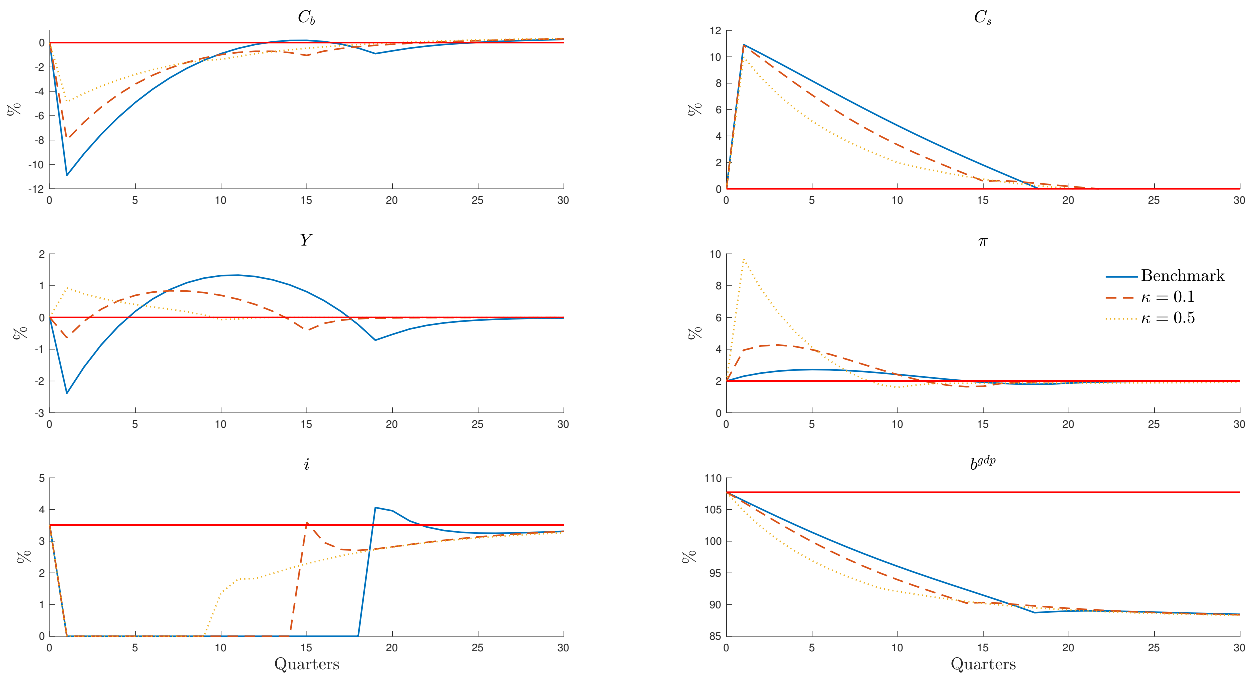

Figure 9: Comparison between the optimal responses to a deleveraging shock under different degrees of price stickiness. "Benchmark": optimal policy under commitment. "$\kappa = 0.1$": optimal policy under commitment when κ in equation (30) is equal to 0.1. "$\kappa = 0.5$": optimal policy under commitment when κ in equation (30) is equal to 0.5. Variables are: consumption of borrowers and savers, (C^b) and (C^s), output (Y), inflation rate (π), nominal interest rate on savings (i), and debt to steady-state GDP ratio (b^{gdp}). Y, C^b and C^s are in percentage deviation with respect to the steady state; π, i and b^{gdp} are in percent and at annual rates.