Is Increased Price Flexibility Stabilizing? Redux[☆]

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Abstract

What are the implications of increased price flexibility on output volatility? In a simple DSGE model, we show analytically that more flexible prices always amplify output volatility for supply shocks and also amplify output volatility for demand shocks if monetary policy does not respond strongly to inflation. More flexible prices often reduce welfare, even under optimal monetary policy if full efficiency cannot be attained. Our results extend to a model with both sticky information and/or wages. We estimate a quantitative DSGE model using Bayesian methods and using counterfactual experiments show that our results from the simple model continue to apply.

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1. Introduction

In this paper we explore the consequences for output volatility of an exogenous increase in price flexibility. Our main thought experiment is a comparative static. Taking the price flexibility parameter as exogenously given (for example, the expected price duration of a Calvo (1983) pricing firm), our experiment asks what the implications are of making this parameter move towards greater price flexibility. Does output volatility increase or decrease? There are many reasons why one might be interested in the answer to a question of this kind. For one, the surge of online retailing may make prices move more frequently and thus correspond more closely to the flexible price benchmark. Does this imply that price rigidities will become irrelevant in the foreseeable future? A thought experiment in similar spirit can be found in a classic paper by Woodford (1998), contemplating a "cashless limit." In this work, he explores the consequences of progressively less cash being required for transaction services. Similarly, here, we compare the behavior of a series of economies with staggered price setting and explore the effect of exogenously changing the frequency of price changes.

Our main result is that the answer to the question – whether an increase in the flexibility of prices stabilizes or destabilizes output – critically depends on two considerations. The first is the way in which monetary policy rule is formulated – in particular, how strongly the nominal interest rate reacts to inflation. The second is what shocks are driving the business cycle – in particular, whether they are in broad terms "demand" or "supply" shocks.

If demand shocks drive the business cycle, then whether an increase in price flexibility increases or decreases output volatility will depend upon how responsive monetary policy is to inflation. If monetary policy is not responsive to inflation, one can show in a simple model analytically that higher flexibility is destabilizing, because it makes the real interest rate more volatile.[1] As aggregate demand depends upon the real interest rate, the higher

[1] The simple model we consider for our baseline results is the well-known sticky price model with a feedback rule for monetary policy, whose equilibrium can be summarized by three equations.

volatility of the real rate translates into higher output volatility. Meanwhile, if monetary policy is responsive enough to inflation, then higher price flexibility will always decrease output volatility in response to demand shocks. In contrast, in response to supply shocks, higher price flexibility will always increase output volatility, regardless of the monetary policy response. We show this analytically in a simple model. The economic logic behind the result underlying why supply shocks are always destabilizing is somewhat subtle, even paradoxical, especially, when monetary policy is not very responsive to inflation.[2]

These analytical results suggest that ultimately, whether higher price flexibility is destabilizing or not is an empirical question that depends upon the relative weight of demand shocks to supply shocks and the estimated monetary policy rule. To address this empirical question, our paper estimates an empirical DSGE model of US economy. According to our estimates, monetary policy reacts strongly to inflation, and the degree to which higher price flexibility would have been destabilizing then depends upon if supply shocks were sufficiently important. In our estimated model, supply shocks play a relatively large role, and thus, higher price flexibility would have been destabilizing. With several counterfactual experiments, we verify that our analytical results continue to apply in this quantitative model.

Our main thought experiment is rather special, and subject to a number of qualifications and criticism. Accordingly, we consider various tractable extensions to the simple analytical model. One important issue concerns implications for welfare, especially as some supply shock driven business cycles are efficient in standard models. The simple model used to derive analytical results features a well-known approximation to household welfare and so we address this question analytically in that set-up. Results show that welfare can decline with increased price flexibility even for efficient supply shocks (productivity shocks, for instance), as the welfare loss associated with the increase in inflation volatility can dominate the gains from the stabilization of the welfare relevant output gap volatility.[3] Similarly, an important

[2] We therefore, devote considerable space in discussing the economic logic in the paper.

[3] Obviously, the large weight that inflation receives in the welfare approximation in this class of model

issue is the specification of monetary policy. In particular, our analysis considers a classic Lucas critique of our baseline experiment: Since in our baseline model, the policy rule is kept fixed as we change the frequency of prices one might ask: What happens, if instead, the government sets optimal policy under discretion and changes its policy in response to the higher frequency of price changes? We find that the same basic results apply. In a similar spirit, our analysis considers a particular type of non-responsive monetary policy: the central bank is constrained by a binding zero lower bound on nominal interest rates but otherwise acts optimally under discretion. Results show the same basic result in this case as the general case when monetary policy is unresponsive to inflation, which is consistent for instance, with the results in Eggertsson (2010).

Next, our analysis considers models with other nominal frictions as it is important to establish that our results are not too specific to a model with sticky price frictions only. First, our analysis uses an extended version of the model with both sticky nominal prices and wages. In this set-up as well, and regardless of whether our analysis considers only output volatility or welfare and whether our analysis considers a simple monetary policy rule or optimal policy, our results continue to apply.[4] Next, our analysis incorporates sticky information as in the classic paper by Mankiw and Reis (2002). In particular, our analysis considers a model variant that features both sticky information and sticky prices. Similar results as our baseline case continue to apply for aggregate volatility when we make either information or prices more flexible.[5]

Our paper is related to several strands of the literature. In recent years, for example, there has been an explosion in empirical research addressing the empirical question of how frequently prices adjust (see for example, Bils and Klenow (2004), Klenow and Kryvtsov

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has been criticized to be not very realistic. We agree with the spirit of this criticism and for this reason do not do a micro-founded welfare evaluation in the empirical model. We simply want to make a point with this exercise in simple models that are analytically tractable.

[4] In fact this case, as we emphasize in detail in the paper, is particularly interesting as efficient productivity shocks lead to a trade-off for optimal policy.

[5] In the model with just sticky information, we also present some results on welfare.

(2008), and Nakamura and Steinsson (2008)). This literature has found that prices adjust between once every five months on average to above one year.[6] For a casual reader of this literature the question posed by this paper may then strike as odd. It might seem obvious that more flexible prices make monetary frictions as given by rigid prices – and more specifically monetary policy – play little or no role in stabilizing or destabilizing the business cycle. The most basic point of this paper is that this conclusion is far from obvious and one can easily make the opposite case.

Moreover, the question – whose answer is sometimes taken as being self-evident – is in fact an old and classic question in macroeconomics and one that remains unsettled. To make this clear we have stolen the title from De Long and Summers (1986), a paper published 25 years ago. They use a dynamic IS-LM model with rational expectations and Taylor-type wage contracts to show that an increase in flexibility can increase output volatility when demand shocks are perturbing the economy. But this argument goes even farther back as these authors point out. A similar observation is made, for example, by Tobin (1975) using a more old-style Keynesian model. Similarly, Keynes (1936) declared that "it would be much better that wages should be rigidly fixed and deemed incapable of material changes, than the depression should be accompanied by a gradual downward tendency of money-wages" using more informal arguments. In fact, the question about the relationship between price flexibility and output volatility even pre-dates Keynes. As early as 1923, Fisher (1923, 1925) saw the business cycle as "largely a dance of the dollar:" According to Fisher, expected deflation leads to high anticipated real interest rates that suppresses investment and output.

That increase in price flexibility can be destabilizing has already been noted in the literature on the zero bound on short term nominal interest rates (see e.g. Eggertsson (2010)). Galí (2012), focusing on wage flexibility, also shows that an increase in wage flexibility may reduce welfare in a series of numerical examples in a model with both price and wages rigidi-

[6]For a survey of this literature, see Klenow and Malin (2010).

ties under optimal monetary policy while Galí and Monacelli (2016) extend the results to an open economy. Relative to this literature, our main contribution is to show via a series of analytical propositions when exactly price and/or wage flexibility is destabilizing for either output and/or welfare under general specifications for policy, both in terms of policy rules and optimal monetary policy. Moreover, our paper shows all these results at positive interest rates, which makes it clear that they are not being driven by a specific thought experiment of a binding zero lower bound that renders monetary policy non-responsive. Importantly, we show how the results depend upon different shocks together with the policy reaction function of the central bank.[7] Finally, unlike the previous literature, we also estimate a medium-scale DSGE model, which allows us to verify the analytical predictions of our model as we conduct empirically relevant counterfactual experiments.

2. A Simple Sticky Price Model

This section presents the textbook three-equation sticky price model and addresses the main questions of this paper analytically. Since this model has become standard by now, we do not write up the micro-foundations, which can be found in textbooks such as Woodford (2003). To fix notation the following contains the main elements of the model as needed.

2.1. Does an increase in price flexibility increase output volatility?

Consider the standard New Keynesian model with time-dependent pricing as in Calvo (1983). From the optimization problem of the firm, which chooses its price anticipating that it only gets to revisit this choice with an exogenous probability $1 - \alpha$ every period, one can derive the optimal pricing equation. A log-linear approximation of the model and firms' pricing decisions implies the New Keynesian Phillips curve, or the "AS" equation

$$\pi_t = \kappa \hat{Y}_t - \kappa \hat{Y}_t^n + \beta E_t \pi_{t+1} \tag{1}$$

[7] Galí (2012), for example, only considers productivity shocks in his analysis.

where π_t is inflation, $\hat{Y}_t$ is output in log-deviation from steady state, and $\hat{Y}_t^n$ is a disturbance term, often called the "natural level" of output. The parameter $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\phi+\sigma^{-1}}{1+\phi\theta} > 0$ measures the slope of the Phillips curve, where β is the discount rate, ϕ^{-1} is the Frisch elasticity of labor supply, σ is the intertemporal elasticity of substitution, and θ is the elasticity of substitution among different varieties of goods. Our focus lies in what happens as we increase "price flexibility." We interpret this as increasing the exogenous probability of adjusting prices, that is $1-\alpha$, which results in a higher κ. $\hat{Y}_t^n$ is given by

$$\hat{Y}_t^n = \frac{1+\phi}{\sigma^{-1}+\phi}A_t - \frac{1}{\sigma^{-1}+\phi}\mu_t \tag{2}$$

where A_t denotes productivity shocks and μ_t markup shocks of firms.[8] These shocks appear in the AS equation in exactly the same way, hence for the moment we will simply refer to shocks to the natural level of output as productivity shocks, A_t, or "supply shocks". The distinction between these different sources of variation in the natural level of output will become relevant when our analysis considers welfare.

From the households maximization problem, one obtains as a log-linear approximation to the Euler equation, the "IS" relationship

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(\hat{\imath}_t - E_t\pi_{t+1}) + \psi_t - E_t\psi_{t+1} \tag{3}$$

where $\hat{\imath}_t$ is the nominal interest rate and ψ_t is an exogenous preference shock. We refer to this as "demand shock" since it only affects the IS equation.[9] Monetary policy is given by a

[8]We can also include time-varying labor taxes $\hat{\tau}_t^w$ in the model, which will show up as $\frac{1}{\sigma^{-1}+\phi}\hat{\tau}_t^w$ in this expression. It will imply the same results later in the paper as markup shocks.

[9]This specification cleanly separates the main effects we are interested in – that is, exogenous forces that perturb the IS equation on the one hand – "demand shocks" and exogenous forces that perturb the AS equation on the other – "supply shocks". The preference shock and productivity shock are clear examples of shocks that affect only one of these margins, but there are other shocks that may affect both, such as exogenous variations in government spending (which both has a direct demand effect and a "wealth effect" via labor supply which changes the natural rate of output).

reaction function of the standard "Taylor rule" type

$$\hat{\imath}_t = \phi_\pi \pi_t + \phi_y \hat{Y}_t + \eta_t \tag{4}$$

where $\phi_\pi,\ \phi_y > 0$ are the feedback parameters and η_t is a monetary policy shock. Our equilibrium selection device is that we look for a determinate bounded solution.[10] To ensure a determinate equilibrium, assume that the following condition is satisfied

$$\phi_\pi + \frac{1-\beta}{\kappa}\phi_y > 1. \tag{5}$$

Finally, our analysis assumes that each of the exogenous processes $A_t, \psi_t,$ and η_t follow a first-order AR process with persistence ρ^i and i.i.d. component ϵ_t^i where i indexes $A, \psi,$ or η. One can now state the following lemma.

Lemma 1. *Suppose each of the following shocks are independent of one another* (ψ_t, A_t, η_t), *and follow an AR(1) with persistence* ρ_i, $i = \psi, \eta, A$. *Then the variance of output that can be attributed to each shock is given by*

$$VAR(\hat{Y}_t/\psi_t) = \left(\frac{\sigma(1-\beta\rho_\psi)(1-\rho_\psi)}{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)+\sigma\kappa[\phi_\pi-\rho_\psi]}\right)^2 VAR(\psi_t),$$

$$VAR(\hat{Y}_t/A_t) = \left(\frac{\kappa\sigma[\phi_\pi-\rho_A]}{[(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma[\phi_\pi-\rho_A]]}\gamma_A\right)^2 VAR(A_t),$$

$$VAR(\hat{Y}_t/\eta_t) = \left(\frac{\sigma\,(1-\beta\rho_\eta)}{(1-\rho_\eta+\sigma\phi_y)(1-\beta\rho_\eta)+\sigma\kappa[\phi_\pi-\rho_\eta]}\right)^2 VAR(\eta_t)$$

where $\gamma_A = \frac{1+\phi}{\sigma^{-1}+\phi}$.

Proof. *In Appendix A.* ■

Following this lemma, one can now state the first key proposition.

[10] Determinacy is required for the comparative static we study to be well defined. If there is indeterminacy it is not clear which equilibria to pick so whether increasing flexibility is stabilizing or not may depend on equilibrium selection in an arbitrary way.

Proposition 1. *The effect of higher price flexibility on output variance is given by*

$$\text{If } \phi_\pi - \rho_\psi > 0, \text{ then } \frac{\partial VAR(\hat{Y}_t/\psi_t)}{\partial \kappa} < 0; \text{If } \phi_\pi - \rho_\psi < 0, \text{ then } \frac{\partial VAR(\hat{Y}_t/\psi_t)}{\partial \kappa} > 0,$$

$$\frac{\partial VAR(\hat{Y}_t/A_t)}{\partial \kappa} > 0 \text{ for any values of } \phi_\pi, \rho_A,$$

and

$$\text{If } \phi_\pi - \rho_\eta > 0, \text{ then } \frac{\partial VAR(\hat{Y}_t/\eta_t)}{\partial \kappa} < 0; \text{ If } \phi_\pi - \rho_\eta < 0, \text{ then } \frac{\partial VAR(\hat{Y}_t/\eta_t)}{\partial \kappa} > 0.$$

Proof. *In Appendix A.* ■

The proofs of both the lemma and the proposition are straight-forward. To provide the intuition, it is useful to write out explicitly the unique bounded solution and graph it up.

2.1.1. Discussion: Demand shocks

Let us assume first that ψ_t is the only source of economic fluctuations. Under this assumption, since the model is linear and ψ_t is the only state variable, eqn.(5) guarantees a solution of the form $\hat{Y}_t = Y_\psi \psi_t$, $\pi_t = \pi_\psi \psi_t$, and $\psi_t = \rho_\psi \psi_{t-1} + \epsilon_t$ where Y_ψ and π_ψ are coefficients to be determined. This implies that $E_t \hat{Y}_{t+j} = \rho_\psi^j Y_\psi \psi_t$, $E_t \pi_{t+j} = \rho_\psi^j \pi_\psi \psi_t$. Consider now the solution in period t, which we subscript with S (for short run): $\hat{Y}_S = \hat{Y}_t = Y_\psi \psi_t$ (once the economy has been perturbed by a shock $\psi_t = \psi_S \neq 0$). The IS equation can be combined with the policy rule to yield an aggregate demand, "AD" equation

$$(1 - \rho_\psi + \phi_y \sigma)\hat{Y}_S = -\sigma(\phi_\pi - \rho_\psi)\pi_S + (1 - \rho_\psi)\psi_S \tag{6}$$

where we have substituted $E_t \hat{Y}_{t+1} = \rho_\psi \hat{Y}_S$ and $E_t \pi_{t+1} = \rho_\psi \pi_S$. The AS equation is similarly

$$(1 - \rho_\psi \beta)\pi_S = \kappa \hat{Y}_S. \tag{7}$$

For later purposes, note here that the slope of the AD equation given by eqn.(6) depends on whether $(\phi_\pi - \rho_\psi) \gtreqless 0$.

The AD and AS relationships as given by eqns. (6) and (7) are plotted in Figure 1 Panel (a) for the case in which $\phi_\pi > \rho_\psi$. The figure shows the effect of a negative demand shock, from AD_1 to AD_2 under two assumptions, that is, when prices are rigid or more flexible (shown via a steeper AS curve). Under rigid prices, a given drop in demand results in a steeper contraction (point A) compared to the case when prices are more flexible (point B). The reason for this is relatively simple. In this economy, production is demand-determined, that is, the firms produce as many goods as are demanded by the customers that show up in front of their doors. This demand, however, depends not on any measure of price rigidity, but instead only on expectations about future output and the difference between the real interest rate and the demand shock $(\psi_t - E_t\psi_{t+1})$. To clarify things further, let us for a moment assume that $\rho_\psi = 0$. Then the expectation terms drop out since the economy is in steady state the next period. The central bank responds to a negative demand shock in the short run by cutting the nominal interest rate (since $\phi_\pi > 0$). This cut, however, will be bigger the greater is the drop in inflation associated with the demand shock. As prices become more flexible, then, the central bank cuts the nominal interest rate by more, and thus has a bigger effect on demand.

Consider now the case when $\rho_\psi > 0$, that is when the shock is more persistent. The logic described above still applies but some additional effects come into play. A persistent shock influences aggregate demand in two ways, as can be seen in eqn.(3): a more persistent shock changes both expected inflation and expected output. In particular, a persistent negative shock can potentially reduce future inflation expectations to such an extent that it actually destabilizes demand. To see this, consider an increase in ρ_ψ for a given ϕ_π. As we see from eqn.(6), this means that the AD curve becomes steeper, suggesting that a given nominal interest rate cut (in response to a reduction in π_S) now leads to a smaller increase in demand because once the shock is persistent, it not only triggers a reduction in current nominal interest rate today, it also triggers expectations of lower inflation in the future. The lower expected inflation in the future, in turn, increases the real interest rate, thus offsetting some of the expansionary effect of the decline in the nominal interest rate today. If the shock is persistent enough, and the interest response weak enough, the effect given by lower expected inflation can be so strong that it dominates and AD becomes upward sloping. Figure 1 Panel (b) shows the effect of a demand shock in the short run when $\phi_\pi - \rho_\psi < 0$. In this case, an increase in price flexibility leads to a bigger output contraction, from point A to point B. Then, price flexibility is destabilizing in the face of demand shocks.

Let us now comment upon the intuition for the monetary policy shock η_t. The intuition is exactly the same as for ψ_t. To see this, just note that if we substituted for the monetary policy reaction function given by eqn.(4) into the IS equation given by eqn.(3), then the shock η_t appears exactly in the same way as $\psi_t - E_t\psi_{t+1}$.

2.1.2. Discussion: Supply shocks

The case of the supply shock leads to additional insights. Here, we find that regardless of the monetary policy reaction function, the variance of output always increases with higher price flexibility. The reason is that while output increases in response to a positive technology shock when policy reacts strongly to inflation it decreases when the interest rate does not respond strongly enough.[11] And both these reactions get exaggerated, the higher is the degree of price flexibility. This, then, increases output *variance* unambiguously.

The required derivations are very similar to the case above and so we present them in Appendix A and move directly to discussing a presentation based on Figures. Figure 1 Panel (c) shows the effect of a productivity shock when $\phi_\pi > \rho_A$ and policy responds strongly to inflation. A positive technology shocks shifts out the aggregate supply curve as it reduces the marginal costs of firms so that they can now produce more output with the same inputs. For

[11] This is thus a generalization of the so-called paradox of toil, see Eggertsson (2010)), to a case that applies even at positive interest rates.

this to show up in more aggregate output, however, the consumers will need to be induced to buy more. Under the assumption that $\phi_\pi > \rho_A$, the AD curve is downward sloping because the central bank will cut the nominal interest rate in response to a drop in inflation. Consider first the case when the supply shock is i.i.d. so that $\rho_A = 0$. The increase in output then happens via cuts in the nominal interest rate, and the greater the drop in the price level, the bigger is the drop in the nominal interest rates. This then leads to a more robust expansion. Consider now $\rho_A > 0$. The figure is unchanged for low enough values of ρ_A but there are now additional forces at work because the supply shock not only triggers a drop in the nominal interest rate. The fact that it is persistent may also affect expected inflation $E_t\pi_{t+1} = \rho_A \pi_A A_t$. Note that this effect is contractionary, because it leads to lower future expected inflation which increases the real interest rate. This means that the AD curve in Figure 1 Panel (c) is now steeper and the expansionary effect of the supply shock is smaller (but more price flexibility still leads to a bigger expansion).

If ρ_A is large enough, or alternatively ϕ_π low enough, so that $\phi_\pi < \rho_A$, then the contractionary effect of lower expected inflation is *dominating* and the AD curve becomes upward-sloping in the (Y_S, π_S) space as shown in Figure 1 Panel (d). As we can see here then technology shocks are contractionary as improvement in technology triggers deflationary expectations that increase the real interest rate. This effect is stronger with more flexible prices because then the increase in expected deflation is higher. Accordingly, as shown in Figure 1 Panel (d), the drop in output is bigger. Hence, more flexible prices lead to higher variance of output under both policy specifications.

2.2. Does an increase in price flexibility increase welfare?

Our focus so far has been on output volatility, partly because this has been a focus of the previous literature cited in the introduction, but also, and perhaps surprisingly, because considering the most popular welfare criterion will in fact generate even somewhat starker results in certain respects regarding what constitutes non-responsive monetary policy. In the context of our model, a natural criterion is the unconditional welfare of the representative household, which as shown in Woodford (2003), can be approximated via second-order approximation around an efficient steady state to yield[12]

$$W \propto -\left[\frac{\theta}{\kappa} VAR\left(\pi_t\right) + VAR\left(\hat{Y}_t - \hat{Y}_t^e\right) + t.i.p.\right]. \tag{8}$$

where $t.i.p.$ denotes "terms independent of policy."[13] The term $\hat{Y}_t^e$ corresponds to $\hat{Y}_t^e = \frac{1+\phi}{\sigma^{-1}+\phi} A_t$ and denotes variations in the efficient level of output.

We see that relative to the total output variability, this criterion is different as it involves (i) the appropriately weighted variability of inflation and (ii) the welfare-relevant output gap (output variation relative to efficient variation in output due to (only) one of the supply shocks A_t). The next lemma, which computes the contribution of inflation variability to social welfare as well as that of the output gap, greatly facilitates understanding why we can get a weaker condition for non-responsive monetary policy to lead to a decrease in welfare.

Lemma 2. *The effect of higher price flexibility on the welfare-weighted inflation volatility is given by*

$$\frac{\partial \frac{\theta}{\kappa} VAR\left(\pi_t / j_t\right)}{\partial \kappa} > 0 \textit{ if } \phi_\pi - \rho_j < \Gamma_j \textit{ for } j = A, \psi, \eta \textit{ and } \mu,$$
$$\frac{\partial \frac{\theta}{\kappa} VAR(\pi_t / j_t)}{\partial \kappa} < 0 \textit{ if } \phi_\pi - \rho_j > \Gamma_j \textit{ for } j = A, \psi, \eta \textit{ and } \mu,$$

where $\Gamma_j \equiv \frac{(1-\rho_j+\sigma\phi_y)(1-\beta\rho_j)}{\kappa\sigma} > 0$. *The effect of higher price flexibility on the welfare relevant output gap for technology shocks, is given by*

$$\textit{If } (\phi_\pi - \rho_A) > 0, \textit{ then } \frac{\partial VAR(\hat{Y}_t - \hat{Y}_t^e / A_t)}{\partial \kappa} < 0; \textit{If } (\phi_\pi - \rho_A) < 0, \textit{ then } \frac{\partial VAR(\hat{Y}_t - \hat{Y}_t^e / A_t)}{\partial \kappa} > 0.$$

[12] For simplicity we express the welfare function for the limiting case in which $\beta- > 1$ to facilitate comparison with our earlier result.

[13] Note here that we are only considering the welfare effect of the element of the loss function that interacts with policy decisions, we do not consider how an increase in price flexibility may affect the term t.i.p, that is, the "terms independent of policy."

Proof. *In Appendix A.* ■

Note first that for the demand shock, the welfare relevant weighted inflation volatility goes up when $\phi_\pi - \rho_\psi < \Gamma_\psi > 0$. We already know that output volatility, which for this shock means welfare relevant output gap volatility, goes up when $\phi_\pi - \rho_\psi < 0$. Thus for the demand shock, considering the welfare criterion in eqn. (8) actually leads to a weaker condition than the one we need just for output. Whenever output volatility goes up, welfare goes down unambiguously for demand shocks. Moreover, since $\Gamma_\psi > 0$, and the condition for price flexibility stabilizing output volatility was $\phi_\pi - \rho_\psi > 0$, we have a range of parameters $\Gamma_\psi > \phi_\pi - \rho_\psi > 0$ when higher price flexibility reduces output volatility, but still reduces aggregate welfare due to the negative effect it has on inflation volatility.

Next for markup shocks, output volatility, which means welfare relevant output gap volatility, goes up regardless of the monetary policy reaction function. Lemma 2 above shows that the welfare relevant inflation volatility goes up when $\phi_\pi - \rho_\mu < \Gamma_\mu > 0$. Then clearly $\phi_\pi - \rho_\mu < \Gamma_\mu > 0$ is a sufficient condition for welfare to go down. Thus, the condition is weaker than requiring a non-responsive monetary policy of the type: $\phi_\pi - \rho_\mu < 0$.

Finally, for the technology shock, note that taking into account the welfare relevant output gap implies that the variance goes up with price flexibility only when $\phi_\pi - \rho_A < 0$. But note that the welfare relevant inflation volatility goes up for a weaker condition, $\phi_\pi - \rho_A < \Gamma_A > 0$. Therefore, we will find that for technology shocks, a weaker condition than $\phi_\pi - \rho_A < 0$ will be needed to get welfare to decrease with increased price flexibility. Thus, when output gap variance goes up, it is sufficient to imply that welfare will go down. That is, the negative effect on welfare due to the increase in inflation volatility will dominate the welfare improvement due to the reduction in output gap variance for technology shocks under conditions that are weaker than $\phi_\pi - \rho_A < 0$. These results are then formally shown in the Proposition below.[14]

[14] For markup shocks, the necessary and sufficient condition is in the Appendix as it is not very clean.

Proposition 2. *(i) The effect of higher price flexibility on welfare for demand shocks is given by*

$$\frac{\partial W(./\psi_t)}{\partial \kappa} < 0 \ \textit{if} \ (\phi_\pi - \rho_\psi) < \Lambda_\psi; \frac{\partial W(./\psi_t)}{\partial \kappa} > 0 \ \ \textit{if} \ (\phi_\pi - \rho_\psi) > \Lambda_\psi$$

where $\Lambda_\psi \equiv \frac{\theta(1-\beta\rho_\psi)(1-\rho_\psi+\sigma\phi_Y)}{\sigma(2(1-\beta\rho_\psi)^2+\kappa\theta)} > 0$. *(ii) A sufficient condition for higher price flexibility to have a negative effect on welfare for markup shocks is given by*

$$\phi_\pi - \rho_\mu < \Gamma_\mu.$$

(iii) The effect of higher price flexibility on welfare for technology shocks is given by

$$\frac{\partial W(./A_t)}{\partial \kappa} < 0 \ \textit{if} \ (\phi_\pi - \rho_A) < \Lambda_A; \frac{\partial W(./A_t)}{\partial \kappa} > 0 \ \ \textit{if} \ (\phi_\pi - \rho_A) > \Lambda_A$$

where $\Lambda_A \equiv \frac{\theta(1-\beta\rho_A)(1-\rho_A+\sigma\phi_Y)}{\sigma(2(1-\beta\rho_A)^2+\kappa\theta)} < \Gamma_A$.

Proof. *In Appendix A.* ■

In sum, overall, a weaker condition than $\phi_\pi - \rho_j < 0$ is required to generate welfare loss from increased price flexibility regardless of the shock. In particular for demand and technology shocks, variance of output/output gap increasing is sufficient to imply that welfare will decrease with increased price flexibility. Our results focussing on welfare are then even starker than those focussing on output or output gap. A brief remark however, is in order. This result overall depends critically on two aspects. First, as price flexibility increases, the weight on inflation, that is $\frac{\theta}{\kappa}$, goes down as the level of price flexibility increases and accordingly the volatility of inflation. One alternative is to assume commonly used preferences for the policy maker in policy institutions, $\lambda_\pi \pi_t^2 + \lambda_y(\hat{Y}_t - \hat{Y}_t^e)^2$, where the weights do not depend on the degree of price flexibility. Second, the implied high weight on inflation in the micro-founded welfare criterion might be unrealistic, which is why in empirical work an equally-weighted objective, with $\lambda_\pi = \lambda_y = \frac{1}{2}$, is often used. Our analysis therefore considers this more stringent alternative in the quantitative model.

3. Quantitative Analysis

This section performs a quantitative evaluation of the effects of greater price flexibility on output volatility.[15] The results in the simple model suggested that the output effect of greater price flexibility depended on the identity of underlying shocks and the endogenous response of monetary policy, suggesting that the answer to the key question is empirical. To address the empirical question, our analysis fits the well-known Smets and Wouters (2007) model to U.S. data.[16] We estimate the structural parameters and the underlying historical shocks. Conditional on the estimated values for all other parameters of the model and the estimated historical shocks the analysis performs several counterfactual experiments.

3.1. Estimation

We refer the reader to the Smets and Wouters (2007) paper for a detailed description of the model, which by now is largely standard, and to our working paper for other estimated model alternatives.[17] The complete model and the log-linearized equations are in Appendix B. Our estimation of the model differs only in some instances from the exact exercise in Smets and Wouters (2007). The difference lies in that incorporated in the original estimation in Smets and Wouters (2007) are markup shocks that are scaled by the price flexibility parameters. Since our focus lies in changing the price flexibility parameter, rather than treating it as fixed in counterfactual experiments, we re-estimate the model so that the estimated markup shocks are independent of nominal flexibility parameter.[18] Overall, since our exercise is

[15] As we describe in detail later, we focus on variance of (model consistent) detrended output. We do not undertake a complete exercise on model implied welfare in this section, but instead just look at simple, empirically relevant, welfare functions that weigh inflation and output gap differentially.

[16] We use quarterly U.S. data from 1966:I -2004:IV on log difference of real GDP, real consumption, real investment, real wage, and the GDP deflator, log hours worked, and the federal funds rate.

[17] In a working paper version of the paper, we estimate two well known alternatives to the SW model and find largely comparable results. These alternatives addressed some short-coming on identification and interpretation of shocks in the original Smets and Wouters (2007) paper. Note also that an alternative interpretation of the wage/price mark-up shocks is that they are shocks to the Calvo probability.

[18] Otherwise, in making prices more or less flexible, this would artificially change the volatility of the "fundamental shocks" which we are taking as exogenous. Moreover, we use not just the posterior mean, but also the entire distribution of the estimates in our counterfactual experiments. Thus we cannot directly read

extremely close to that of Smets and Wouters (2007), the estimates are in line with their results. The only exceptions are the estimates pertaining to the price and wage markup shocks for reasons described above.

All the details of the estimation, including the prior distributions and the posterior estimates, are in Appendix C. The two most important aspects in light of the simple model are the monetary policy rule and the shocks considered in the model. One point worth emphasizing here then is that the policy rule is estimated with a Taylor rule coefficient on inflation of $\phi_\pi = 2.07$. This puts us in the parameter region in which price flexibility is stabilizing for demand shocks while destabilizing for supply shocks. Finally, the economy is driven by seven fundamental aggregate shocks: shocks to total factor productivity, investment-specific technology, risk premium, exogenous government spending, monetary policy shock, price markup, and wage markup.

3.2. Counterfactual Experiments

Given the structural estimation, we now describe our counterfactual exercises that complement our analytical results in the previous section.

3.2.1. Baseline counterfactual comparative static

Our first counterfactual experiment is a comparative static on price flexibility. Taking the (posterior mean) estimates of the model and policy rule parameters as given and considering all shocks, Panel (a) of Figure 2 shows for a whole range of price stickiness the implied (unconditional) volatility of (de-trended) output.[19] The Figure normalizes the variance of output at our posterior mean estimates to 1 and shows how the variance changes as the probability of price adjustment increases. For reference, the vertical lines show the posterior

off the estimates from Smets and Wouters (2007).

[19] This is model-consistent de-trended output. As the estimated model features deterministic growth, output is made stationary by de-trending it by the level of technology.

mean of probability of price adjustment as well as the 90% probability intervals.[20] As is clear, except at a small range very close to full price rigidity, making prices more flexible increases output volatility monotonically. This result reflects the fact that according to our estimates, supply shocks play a non-trivial role in explaining output variation.

Next, in light of the results from the simple model regarding welfare, we consider the same exercise, but now for inflation volatility, (welfare relevant) output gap volatility, and an often-used welfare function that considers an equally weighted variance of inflation and output gap.[21] Panels (b)-(d) show that increased price flexibility unambiguously increases inflation volatility, and again except for a small range close to full price rigidity, increases output gap volatility and decreases the equally weighted average of inflation and output gap. The result on increased inflation volatility is intuitive. Additionally, the result on increase in output gap volatility reflects the fact that according to our estimates, inefficient supply shocks play a non-trivial role in explaining output variation. Finally, welfare, according to our measure, declines as both inflation and output gap volatility increase with flexibility.

3.2.2. Marginal posterior distribution of derivatives

The baseline exercise is a natural counterfactual experiment. Our focus, however, has been a comparative static exercise where the price stickiness parameter can be very far from the estimated credibility set. We next implement an exercise that uses the entire posterior distribution of our parameter estimates. In particular, we present the marginal posterior distribution of the (numerical) derivative of output with respect to price stickiness, evaluated at the posterior mean. Panel (a) of Figure 3 presents the results on output.[22] According

[20] Since our baseline estimation does not feature measurement error, using smoothed series of shocks together with posterior mean of parameters would lead to the same volatility of output growth, which is what we use in the estimation exercise. Also, note that we plot $1-\xi_p$, where ξ_p is the probability of price adjustment in our notation in the model in the Appendix.

[21] This equally weighted function can be thought of as resembling the dual mandate of the Federal Reserve.

[22] In this Figure, the marginal posterior distribution is plotted with frequency on the y-axis and the numerical derivatives on the x-axis.

to the estimation this derivative is never negative.[23] Panels (b)-(d) show results from the same exercise but now for inflation, output gap, and an equally weighted average of inflation and output gap. Looking at the entire marginal distribution, the derivative is positive for inflation, positive for output gap, and negative for the simple measure of welfare. To sum up, given our estimates, this result shows that the marginal posterior distribution of the derivative of output with respect to price flexibility is always positive while the derivative of welfare with respect to price flexibility is always negative in the estimated model.

3.2.3. Shocks and monetary policy response

We next analyze in further detail two key aspects highlighted in the analytical results: output volatility and welfare increase or decrease with greater price flexibility depend on (i) the monetary policy response to inflation and (ii) the nature of shocks. For (ii) it matters whether supply shocks or demand shocks dominate in terms of explaining the variation in the data explained by the model.

Our analysis first shows how the derivative of output volatility with respect to price stickiness depends on ϕ_π. Panel (a) of Figure 4 show that in a large range of values such that the monetary policy rule satisfies the Taylor principle ($\phi_\pi > 1$), this derivative is positive. For reference, the vertical lines show the posterior mean of ϕ_π as well as the 90% probability intervals. We next do this same exercise, but for inflation, output gap, and an equally weighted average of inflation and output gap. Panels (b)-(d) of Figure 4 shows that the derivative is positive for inflation and output gap and negative for the weighted average in a large range of values such that $\phi_\pi > 1$. These results verify that the result derived in our analytical model in a richer structural one, but recall they suggested that when monetary policy is responsive (to inflation), output, inflation, and output gap volatility all

[23] Thus, the property we saw above where at close to full price rigidity, the variance of output can decrease with increased price flexibility is not empirically relevant for us, given the posterior distribution of our estimates. Moreover, note that all that is relevant for us here is whether this derivative is positive or negative.

are expected to increase in response to supply shocks and the estimation is suggesting that inefficient supply shocks explain a non-trivial variation of output in the data.

Our analysis next shows how the derivative of output volatility with respect to price stickiness depends on relative volatility of the shocks that are in the estimated model. This gives us a metric of how the results depend on how dominant each type of shocks are in the model. In particular, our measure of relative volatility is the ratio of standard deviation of a given shock to the sum of the standard deviation of all the seven shocks in the model. Figure 5 shows that the derivative is positive, and increasing in the relative volatility for the supply shocks (technology, price markup, and wage markup), while it is also positive but decreasing in the relative volatility for demand shocks (risk premium, government spending, monetary policy, and investment specific).[24] A vertical line shows the relative volatility of the shock according to our posterior mean estimates. The two dashed line show the 90% probability intervals. This is consistent with our analytical results that supply shocks are always destabilizing with higher price flexibility while higher flexibility is stabilizing in the case of demand shocks. Thus, in principle, if demand shocks were estimated to be of sufficiently larger importance, the overall derivative could be negative. The 90% probability intervals however, suggest that none of the demand shocks are significant enough for the output variance derivative to switch sign according to our estimation. The key reason, as emphasized earlier, is the large role of the estimated supply shocks which account for the bulk of fluctuations in the estimated model.[25] We finally show how the derivative of a weighted average of variance of inflation and output gap with respect to price stickiness depends on relative volatility of the shocks. To conserve space, our exposition shows results for the weighted average instead of inflation and output gap separately. Figure 6 shows that this

[24] Note that we are primarily only interested in whether these derivatives are positive or negative. To get a sense of magnitudes however, we can convert these derivatives to elasticities. As an example, for the wage markup shock, the elasticity of output variance with respect to price flexibility at the posterior mean of relative volatility would be 0.42.

[25] In a working paper version we consider two popular alternatives to this model and re-estimate the relative importance of supply and demand shocks and find this result to be robust.

derivative is always negative, i.e. increasing price flexibility reduces welfare.

3.2.4. An extreme example

For reference it is helpful to get a sense of an upper bound on the role nominal rigidities can play according to the estimated model. If we run a counterfactual experiment in which there is no nominal rigidity (in either prices or wages), the standard deviation of annualized output growth doubles. It increases from 2.2 percentage points to 4.4 percentage points.[26]

4. Extensions

This section considers several extensions, in particular those pertaining to the simple model for which there are analytical results.

4.1. Monetary policy rules and optimal policy

The simple model considered a simple Taylor-type rule, as given by eqn.(4), as a description of monetary policy. Our analysis now considers several variants of such rules that are popular in applied work, allowing for a welfare-relevant output gap, interest rate smoothing, and a response to the growth rate of the output gap (this is in particular motivated by our quantitative model's interest-rate rule specification). Overall, these extensions do not have a significant qualitative effect on the results. All the numerical results are in Appendix I. Figure I.10 shows results for both the variance of output and welfare given both demand and supply (markup and technology) shocks as a function of 1/(1-α), which is the expected duration of the price contract.[27] For this exercise, we use the parameters in Table H.6, where we point out that we use $\phi_\pi = 1.5$.[28] Note that this parametrization implies that monetary policy is "responsive." Then, in line with our analytical results, Figure I.10 shows that for all

[26] Note that we report here the effects on growth rate of output as that is the data we use in the estimation.
[27] Figure I.10 contains the exact specification of the policy rules we use.
[28] Our baseline parameterization in Table H.6 of $\kappa = 0.02$ corresponds to duration of 3.2 quarters. We only provide numerical results here since analytical results are tedious with interest rate smoothing.

of the Taylor rule specifications, for demand shocks, the variance of output decreases with increased price flexibility while for supply shocks, it increases. In the case of the unresponsive policy the opposite result for demand shocks (as in the analytical section) while the sign remains the same for supply shocks.[29]

Moving to welfare, one can also confirm our theoretical results across all policy rules. The figure shows clearly that for all the Taylor rule specifications and for all the shocks, welfare decreases with increased price flexibility for a high enough level of price stickiness. Once prices are flexible enough, however, then welfare improves with further increases in price flexibility, much as our theoretical results suggested. Interestingly, this inflection point occurs later for the alternative policy rules, suggesting that our result that increasing price flexibility can be welfare-reducing applies to an even broader parameter range for these alternative policy rules.[30]

So far, we have discussed the effects on output volatility and welfare of increased price flexibility while modeling monetary policy as following an interest-rate rule. Thus, the increase in price flexibility does not change the policy reaction function of the government at the same time. How sensitive is our conclusion to allowing for the feature that an increase in κ may simultaneously change government behavior, for example, ϕ_π or ϕ_y? In order to address this question one needs to have some theory of how the government behaves in response to changes in the level of stickiness in price setting. One approach is to assume that policy is determined to maximize social welfare, so that policy may then endogenously react if there is a change in κ. Hence the government maximizes eqn.(8) subject to eqns.(1) and (3).[31] Consider first optimal policy under discretion (Markov Perfect equilibrium), that is, the government maximizes welfare but is unable to commit to future policy. Then the first-order conditions of the government maximization problem can be combined to yield the

[29] Since it is not empirically the most relevant, we do not report the unresponsive case here.

[30] Notice that for preference and markup shocks, the results in the Figure are identical whether we have output or the welfare-relevant output gap in the Taylor rule.

[31] All the details of the derivation are contained in the appendix.

following relationship between output and inflation

$$\theta\pi_t + (\hat{Y}_t - \hat{Y}_t^e) = 0. \tag{9}$$

We see that this relationship, the so-called "targeting rule," is independent of the degree of price flexibility. In particular, observe that if the only shocks in the economy are A_t and ψ_t this relationship implies an equilibrium in which $\pi_t = \hat{Y}_t - \hat{Y}_t^e = 0$ at all times. This means that neither output nor inflation volatility depends upon the degree of price rigidity. Compared to our previous policy rule in eqn.(4), discretion thus corresponds to the special case when $\phi_\pi \to \infty$, that is, the central bank completely offsets any effect of preference and technology shocks on inflation.

It is easy to see that in this case, also, changes in price rigidities have no effect on welfare since the central bank is fully replicating the flexible price, and thereby, the efficient allocation. We show this result formally in the Appendix. This result, however, is relatively special and relies heavily on the "divine coincidence" feature demonstrated by Blanchard and Galí (2007) in the most simple variation of the New Keynesian model. However, this divine coincidence is absent in a more general setting (with wage frictions), as we shall shortly see. Moreover, for a more general specification of the shocks, and when the first best is no longer obtained, our previous results are unchanged as we now demonstrate.

Thus, let us consider a case when the first best is not achieved due to an inefficient supply shock μ_t. In this case, $\hat{Y}_t^e = 0$, and there is a trade-off between inflation and output. In this case one can prove the following proposition.

Proposition 3. *Suppose monetary policy is set optimally under discretion and the markup shock μ_t follows an AR(1) process. Then the variance of output is given by*

$$VAR(\hat{Y}_t) = \theta^2 \left(\frac{\kappa}{1 + \kappa\theta - \beta\rho_u} \right)^2 VAR(\mu_t)$$

which is always increasing in price flexibility so that

$$\frac{\partial VAR(\hat{Y}_t)}{\partial \kappa} = \frac{\theta^2(1-\beta\rho_u)2\kappa}{(1+\kappa\theta-\beta\rho_u)^3} VAR(\mu_t) > 0.$$

Similarly, welfare is decreasing in price flexibility

$$\frac{\partial W}{\partial \kappa} < 0$$

if $\frac{1+\kappa\theta}{1+2\kappa\theta} > \beta\rho_\mu$.

Proof. *In Appendix D.* ■

This is a close analogue to Proposition 1 which showed that output variability always increases under a Taylor rule (and the complementary Proposition 2 for welfare). Thus, with an inefficient shock μ_t and under optimal monetary policy, not only does the variance of output increase, but welfare can also be reduced with increased price flexibility as long as $\frac{1+\kappa\theta}{1+2\kappa\theta} > \beta\rho_\mu$. In particular, with i.i.d. shocks ($\rho_\mu = 0$), welfare always declines when inefficient supply shocks hit the economy and monetary policy is conducted optimally under discretion. Moreover, as before with a Taylor rule, the condition for welfare to decline with additional increased price flexibility becomes harder to fulfill as prices become more flexible:[32] $\lim_{\kappa\to\infty} = \frac{1+\kappa\theta}{1+2\kappa\theta} = \frac{1}{2}$. Let us also point out that the policy under the optimal commitment has a similar flavor as outlined above.[33]

4.2. Zero lower bound

So far our analysis has not incorporated the zero lower bound on the nominal interest rate. Given that this has been a relevant situation for many central banks recently, and especially because of our results that even at positive interest rates, the responsiveness of monetary

[32]In the Appendix, we also show that the persistence of the shock matters for our condition in the proposition as it affects strongly the variance of the welfare-weighted inflation term.

[33]One simply needs to modify eqn.(9) to include $(\hat{Y}_t - \hat{Y}_t^e) - (\hat{Y}_{t-1} - \hat{Y}_{t-1}^e)$ instead of $(\hat{Y}_t - \hat{Y}_t^e)$.

policy to inflation matters critically, we now analyze the case of the zero lower bound. For this we generalize the Taylor rule as $\hat{\imath}_t = \max(\beta - 1, \phi_\pi \pi_t + \phi_y \hat{Y}_t)$ where $\hat{\imath}_t \geq \beta - 1$ as we consider deviations from steady-state. Next, for tractability, consider a shock as in Eggertsson and Woodford (2003) in which $\psi_t = \psi_S < 0$ in period 0 and which reverts back to steady state $\psi_t = \bar{\psi}$ with a fixed probability $1 - \mu$ every period thereafter. Then it is easy to confirm (see for example, Eggertsson (2010)) that the solution for inflation and output is $\pi_S = \frac{\kappa}{(1-\mu)(1-\beta\mu)-\mu\sigma\kappa}\psi_S < 0, \hat{Y}_S = \frac{(1-\beta\mu)}{(1-\mu)(1-\beta\mu)-\mu\sigma\kappa}\psi_S < 0$. Thus, there is deflation and output contraction when the zero lower bound binds. Moreover, Appendix E shows that greater price flexibility leads to a bigger drop in output.

Hence, if the shock to ψ_t is large enough so that the zero bound is binding, an increase in price flexibility is no longer stabilizing, it instead is destabilizing regardless of the value of ϕ_π and ϕ_y. The logic of this proposition is in fact the same as we showed in Figure 1 Panel (b). The intuition for this relies heavily on the fact that the nominal interest rate does not respond strongly to the drop in inflation and output since it is stuck at zero. This is therefore a specific case of our general non-responsive monetary policy reaction function. Finally, let us briefly address the issue of optimal policy. Even if the government maximizes welfare as in the previous section, Eggertsson (2008) shows that one obtains exactly the same equilibrium under optimal monetary policy under discretion as analyzed and presented above. It follows that even if the government conducts policy under discretion, at the zero bound, the more flexible the prices, the greater is the drop in output. Moreover, in Appendix E welfare also declines with increased price flexibility in this case.

4.3. Sticky prices and wages

So far, the analytic results of the New Keynesian model only featured price stickiness. However, estimated models in the literature (including the one we analyze in the previous section) typically feature both price and wage stickiness. It is worthwhile to consider how our analytical results are affected if we allow for both price and wage stickiness. We undertake this exercise next. Our analysis is related to Galí (2012), who shows that an increase in wage flexibility may reduce welfare in a model with both price and wages rigidities under a Taylor rule and under optimal monetary policy with commitment. Galí (2012) shows numerical results where wage flexibility is increased while price stickiness is held constant under technology shocks. Relative to that work we study more shocks and make some useful simplifying assumptions regarding the extent of price and wage stickiness, which enables us to show several results in closed-form. In particular, under these simplifying assumptions, our analysis always considers a case where both price and wage flexibility change, and for some shocks, the insights from the simple model with only price stickiness can be shown to go through analytically. Finally, our analysis consider general specifications for policy and in particular, focuses on a case of no commitment under optimal policy, where again the analysis from the model with only price stickiness carries over in a transparent way.[34] All these results are in Appendix F.

Our analysis uses a standard model of price and wage stickiness as described in Woodford (2003). The private sector equilibrium conditions as well as the approximation of household welfare for this model are provided in Appendix F. As is well-known in this model both a price and a wage Phillips curve appear. Moreover, in the price Phillips curve, compared to the standard sticky price model used above for analytical results, a new term appears. This term is the real wage gap, that is the gap between real wages and its natural counterpart. This term is important as it leads to a trade-off for monetary policy even with technology shocks – so the divine coincidence no longer applies. Finally, in terms of welfare, both price and wage inflation variability matter, together with (welfare-relevant) output gap variability. One of our main contributions here is that by making an additional assumption that the slopes of the two Phillips curve are the same, we obtain several insights.

First, as shown in the Appendix, under this case the model simplifies the analysis of de-

[34]The case of commitment is also studied in the Appendix.

mand shocks to exactly the same as the simple model with only sticky prices. Our theoretical results, in particular those for welfare, then apply exactly. Second, for the technology shock, the divine coincidence no longer holds. Then, numerically we show in the Appendix that for both the responsive and non-responsive monetary policy case, the variance of output increases and welfare decreases as nominal flexibility increases. These results are thus stronger than those for the technology shock under the sticky-prices-only baseline model. Third, similar results hold when we consider optimal policy under discretion, in which case one can derive an extended version of the targeting rule eqn.(9). Now that the divine coincidence does not apply, the government cannot achieve the first best in response to even technology shocks. This in turn will lead to a greater degree of output volatility and a reduction in welfare for a reasonable parameterization of the model, as shown in the Appendix.

4.4. Sticky prices and information

So far our analysis has considered sticky price and wage models modeling nominal rigidities using the Calvo (1983) framework. A natural question is whether our main message also applies to other models of nominal rigidities. To address this question, we now consider variants of the highly influential sticky-information model of Mankiw and Reis (2002).

In particular, this section will use the welfare criterion in this class of model derived by Ball et al. (2005) and the variant of the model developed by Coibion and Gorodnichenko (2011) that features both sticky prices and information. This variant allows us to change both sticky information and sticky price parameters and also enables us to consider a case of not responsive monetary policy without having indeterminacy issues.[35] To preserve space, we refer the reader to the Ball et al. (2005) and Coibion and Gorodnichenko (2011) papers for details. All the relevant log-linearized conditions as well as the utility based welfare criterion

[35] In the standard sticky information model, as there is no long-run trade-off between inflation and output gap, if monetary policy does not satisfy the Taylor principle, the model does not have a determinate solution. But remarkably, we show later that for results to be interesting, we do not even need to violate the Taylor principle.

from Ball et al. (2005) are in Appendix G. The model in Coibion and Gorodnichenko (2011) features two sectors, one that has sticky prices firms and the other sticky information firms. Our calibration is based on the estimates in Coibion and Gorodnichenko (2011) and is also explained in more detail in Appendix G. For reference, Table 1 provides all the parameter values used in the exercise.

In the model with both sticky information and prices, the probability of updating information is given by $1 - \delta_{si}$ while the probability of price adjustment in given by $1 - \delta_{sp}$. Figure 7 shows comparative statics with respect to these parameters for output volatility. The share of sticky price firms, is given by $s = 0.75$ and we consider two main shocks: preference and technology.[36] The shocks follow a persistent process ($\rho = 0.99$). In the Figure, the level of volatility in our baseline calibration is normalized to be 1. Moreover, our analysis splits the results by the case where monetary policy is responsive ($\phi_\pi = 1.5,\ \phi_y = 0.25$) and non-responsive ($\phi_\pi = 0.25,\ \phi_y = 1.5$).[37] For the non-responsive case, the Taylor rule continues to lead to determinacy in this general model. Figures 7 shows that in this more general model, the insights of our analytical model with only sticky prices continue to hold. For demand shocks, with responsive monetary policy, either making information or prices more flexible leads to lower volatility in output. In contrast, with non-responsive monetary policy, making information or prices more flexible leads to increased volatility in output for demand shocks. For technology shocks, in either case of monetary policy response, making information or prices more flexible leads to increased volatility in output.

How about welfare implications in the sticky information model? For this, we turn to the set-up in Ball et al. (2005), which also derives the utility based welfare criterion. In this variant, there is only sticky information and so for the non-responsive monetary policy case, one has to satisfy the Taylor principle for determinacy ($\phi_\pi = 1.01,\ \phi_y = 0.25$). Shocks follow

[36] The initial values of δ_{si} and δ_{sp} and the value of s are from Coibion and Gorodnichenko (2011).

[37] We numerically solve this model using a truncated state space. We have checked extensively that the period we truncate at does not affect our results.

a persistent process ($\rho = 0.99$). We now only do comparative statics for the probability of updating information $1 - \delta_{si}$. Figures and 8 -9 show the results, where for completeness we show both output and welfare loss for three shocks: preference, technology, and markup. In the Figures, the level of volatility and welfare loss in our baseline calibration is normalized to be 1. A pattern that is consistent with our analytical results based on the sticky prices only model also appears here. When monetary policy responds weakly to inflation, even though it still responds more than one-for-one, there is a greater range over which welfare declines when information is more flexible for all three shocks. We emphasize that this is a general pattern. For instance, in the less responsive to inflation case, if there is a larger response to output, then the range over which welfare declines expands further.[38] Moreover, finally note here that for output, volatility can increase in the case of preference shocks when information is more flexible, under weak response of monetary policy.

5. Conclusion

Our analysis in this paper explores the consequences for output volatility of an exogenous increase in price flexibility and shows that the results depend critically on the nature of shock and the monetary policy reaction function. Similar results hold for welfare. If demand shocks hit the economy, output volatility goes up and welfare goes down if monetary policy is not responsive enough to inflation; if technology shocks hit the economy, output gap volatility goes up and welfare goes down if monetary policy is not responsive to inflation. Moreover, if efficiency cannot be attained, say due to inefficient supply shocks or due to even efficient shocks in a richer model, then similar results apply even under optimal policy. We show that our results extend to a model with both sticky prices and information. Finally, we use an estimated quantitative model to verify the analytical results as well as conduct counterfactual experiments.

[38] In this case, for comparison, we obviously use a higher response to output also in the responsive case. Thus, in this unreported extension we use for the two cases: $\phi_\pi = 1.5/1.01$; $\phi_y = 0.5$.

In the quantitative exercise using U.S. data, we find that conditional on the estimated values of the structural parameters and shocks, increased price flexibility would indeed have been destabilizing. The key reason for this conclusion is the important role played by supply shocks, both efficient and inefficient, in the estimated model. For this result to be overturned, we suspect the model would need to be amended in such a way as to give demand shocks a greater role. Finally, while the following two points are a bit speculative, the mechanism we have uncovered could explain two other empirical phenomena. First, our paper may shed some light on why the Great Recession triggered a far smaller drop in output than the Great Depression: the Great Recession was associated with a relatively modest decline in inflation, while the Great Depression was characterized by excessive deflation. This could in principle be explained by prices being more flexible during the Great Depression than during the Great Recession, a question we leave open for future research. Second, our model may shed light on cross-country variation in output volatility. One important factor may be that in some countries monetary policy is relatively unstable, which may make prices more flexible. The model suggests that if certain shocks are driving the business cycle, this may play a role in explaining cross-country variation in output volatility.

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6. Tables

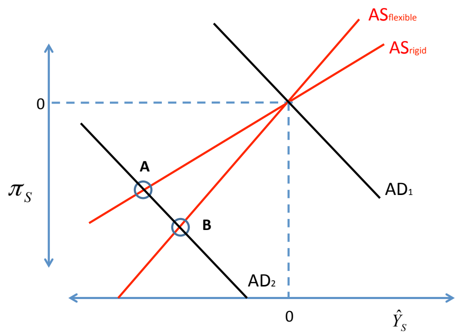
Table 1: Parameter Values of the Sticky Price and Information Model

Parameter	Value	Parameter	Value
β	0.99	σ	1
κ	0.01818	θ	10
ρ	0.99	ϕ_y	0.25/1.5
ϕ_π	1.5/0.25/1.01	η	1

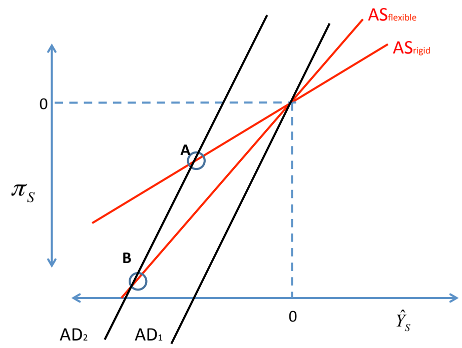
NOTE: The parameter β denotes the rate of time preference, σ the intertemporal elasticity of substitution, θ the elasticity of substitution among different varieties of goods, η the Frisch elasticity of labor supply, $\kappa = \frac{\sigma+\eta}{1+\eta\theta}$ the degree of strategic complementarity, ϕ_π the systematic response to inflation in the Taylor rule, and ϕ_y the systematic response to output in the Taylor rule. Other than the Taylor rule parameters, all other parameters are from Coibion and Gorodnichenko (2011).

7. Figures

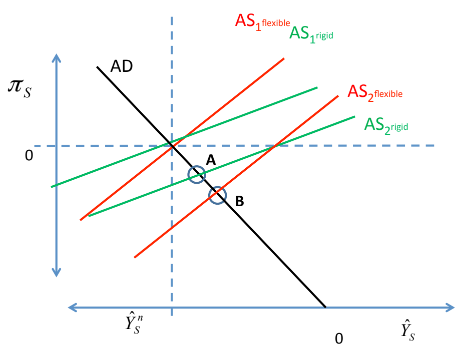
Figure 1: Theoretical Effect of Price Flexibility on Output



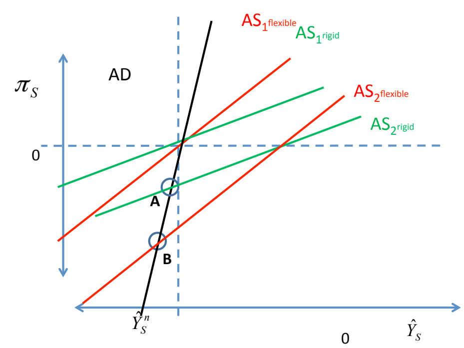
(a) Responsive Monetary Policy, Demand Shock



(b) Non-Responsive Monetary Policy, Demand Shock



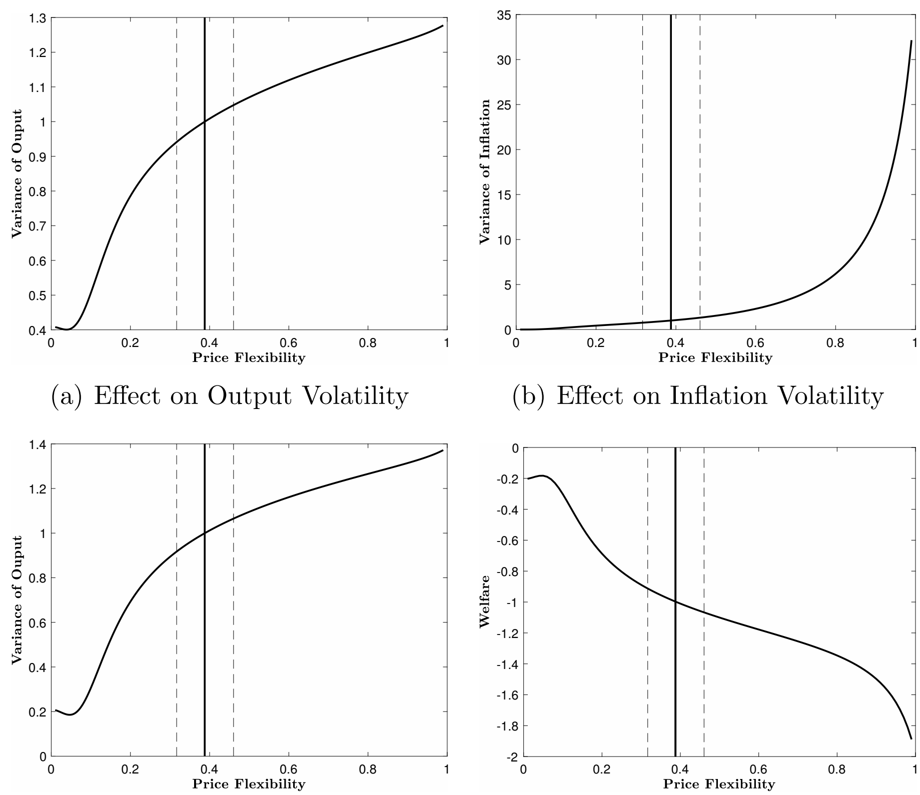
(c) Responsive Monetary Policy, Supply Shock



(d) Non-Responsive Monetary Policy, Supply Shock

NOTE: The figure illustrates the effect of demand and supply shocks under responsive and not responsive enough monetary policy, as defined for the simple sticky price model in Section 2. AD denotes the aggregate demand curve, AS the aggregate supply curve. Section 2 further clarifies notation.

Figure 2: Model Estimated Effect of Price Flexibility on Key Aggregates



(a) Effect on Output Volatility

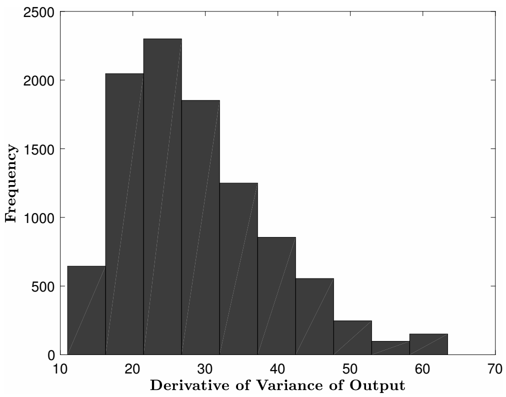
(b) Effect on Inflation Volatility

(c) Effect on Output Gap Volatility

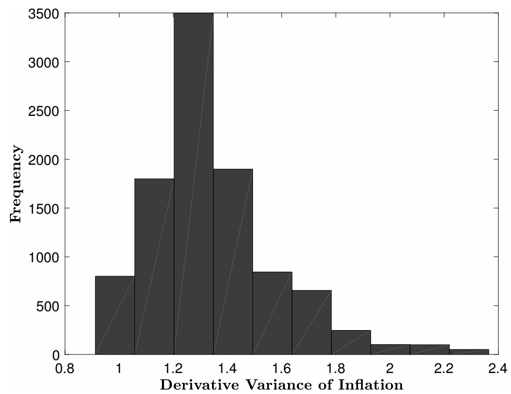
(d) Effect on Welfare

NOTE: The figure illustrates the effect of price flexibility on the volatility of output, inflation, the output gap and equal-weighted welfare. The effect is obtained from the estimated medium-scale model in Section 3 by varying the price flexibility parameter while holding all other parameters at their posterior means. Price flexibility is given by 1 less the Calvo parameter. The y-axis of all panels is normalized to 1 for the posterior means. Vertical lines display the posterior means (solid lines) and 90% error bands (dashed lines). Equal-weighted welfare shows an equal-weighted, negatively signed composite of inflation and the output gap.

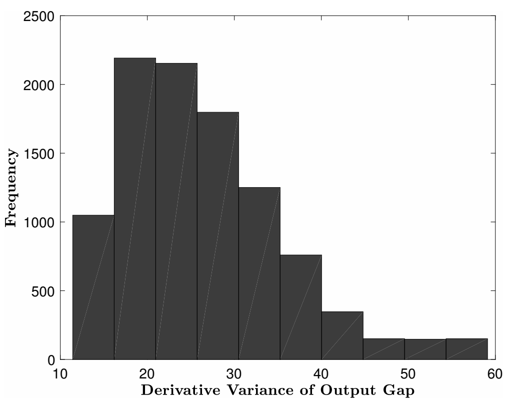
Figure 3: Distribution of Model Estimated Effect of Increased Price Flexibility on Derivatives of Key Aggregates



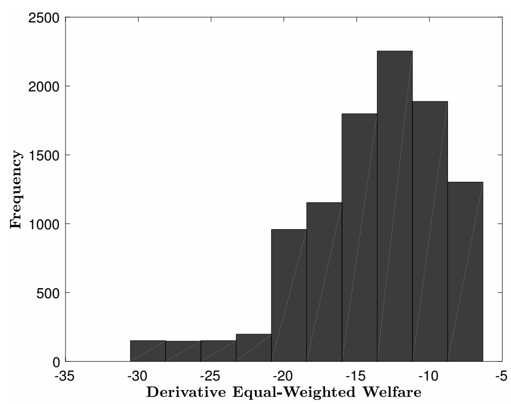
(a) Distribution of Derivative of Output Volatility



(b) Distribution of Derivative of Inflation Volatility



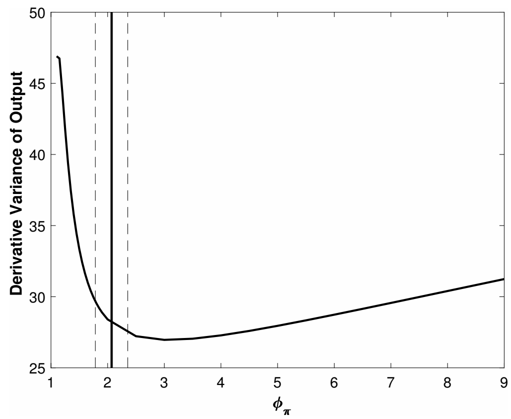
(c) Distribution of Derivative of Output Gap Volatility



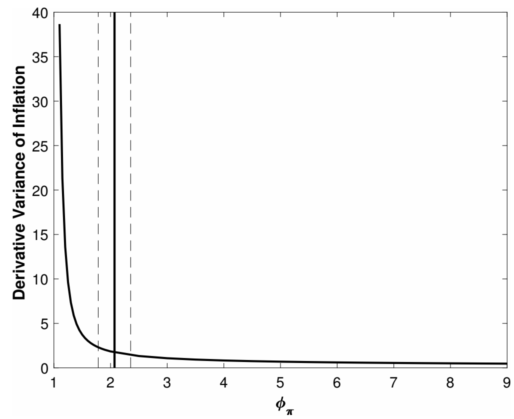
(d) Distribution of Derivative of Welfare

Note: The figure illustrates the distribution of the effect of price flexibility on the derivative of the volatility of output, inflation, the output gap and equal-weighted welfare. The distribution is obtained from 10,000 draws of the Bayesian estimation of the medium-scale model in Section 3 and by varying the price flexibility parameter while holding all other parameters at their draws. Equal-weighted welfare is computed as an equal-weighted, negatively signed composite of inflation and the output gap.

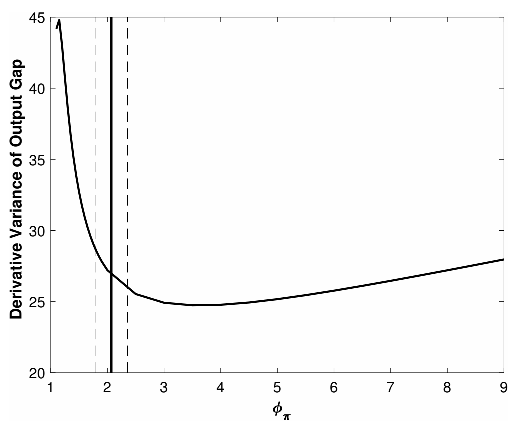
Figure 4: Model Estimated Effect of Systematic Response to Inflation on Derivatives of Key Aggregates with Respect to Price Flexibility



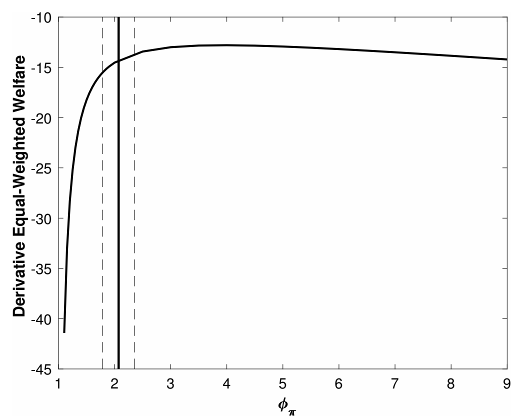
(a) Effect on Derivative of Output Volatility with Respect to Price Flexibility



(b) Effect on Derivative of Inflation Volatility with Respect to Price Flexibility



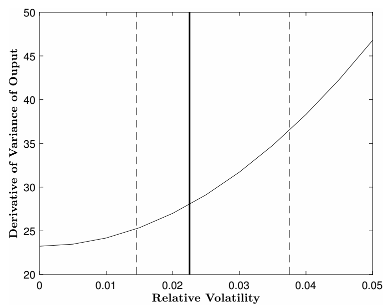
(c) Effect on Derivative of Output Gap Volatility with Respect to Price Flexibility



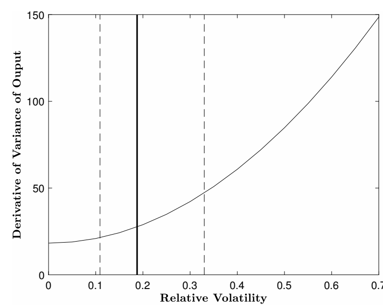
(d) Effect on Derivative of Welfare with Respect to Price Flexibility

NOTE: The figure illustrates the effect of the systematic response to inflation in the interest-rate rule on the volatility of output, inflation, the output gap and equal-weighted welfare. The effect is obtained from the estimated medium-scale model in Section 3 as the numerical derivative of output volatility with respect to price flexibility as we vary the parameter ϕ_π while holding all other parameters at their posterior means. Equal-weighted welfare is computed as an equal-weighted, negatively signed composite of inflation and the output gap.

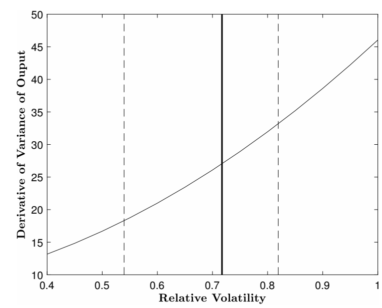
Figure 5: Effect of Importance of Shocks on Model-Estimated Derivative of Output Volatility with Respect to Price Flexibility



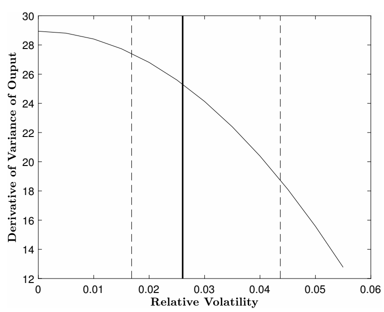
(a) Productivity Shock



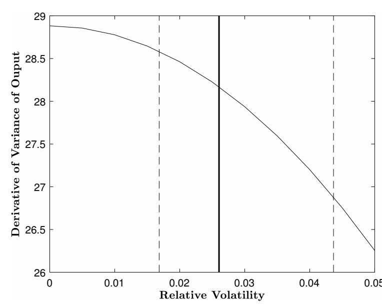
(b) Price Markup Shock



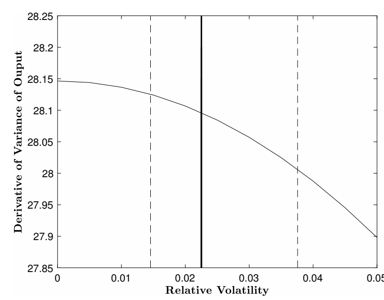
(c) Wage Markup Shock



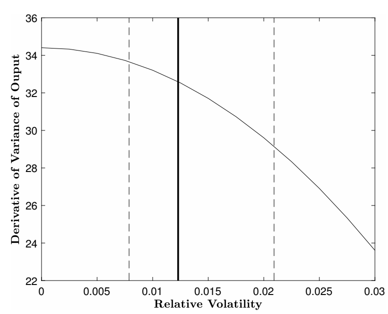
(d) Risk-Premium Shock



(e) Government-Spending Shock



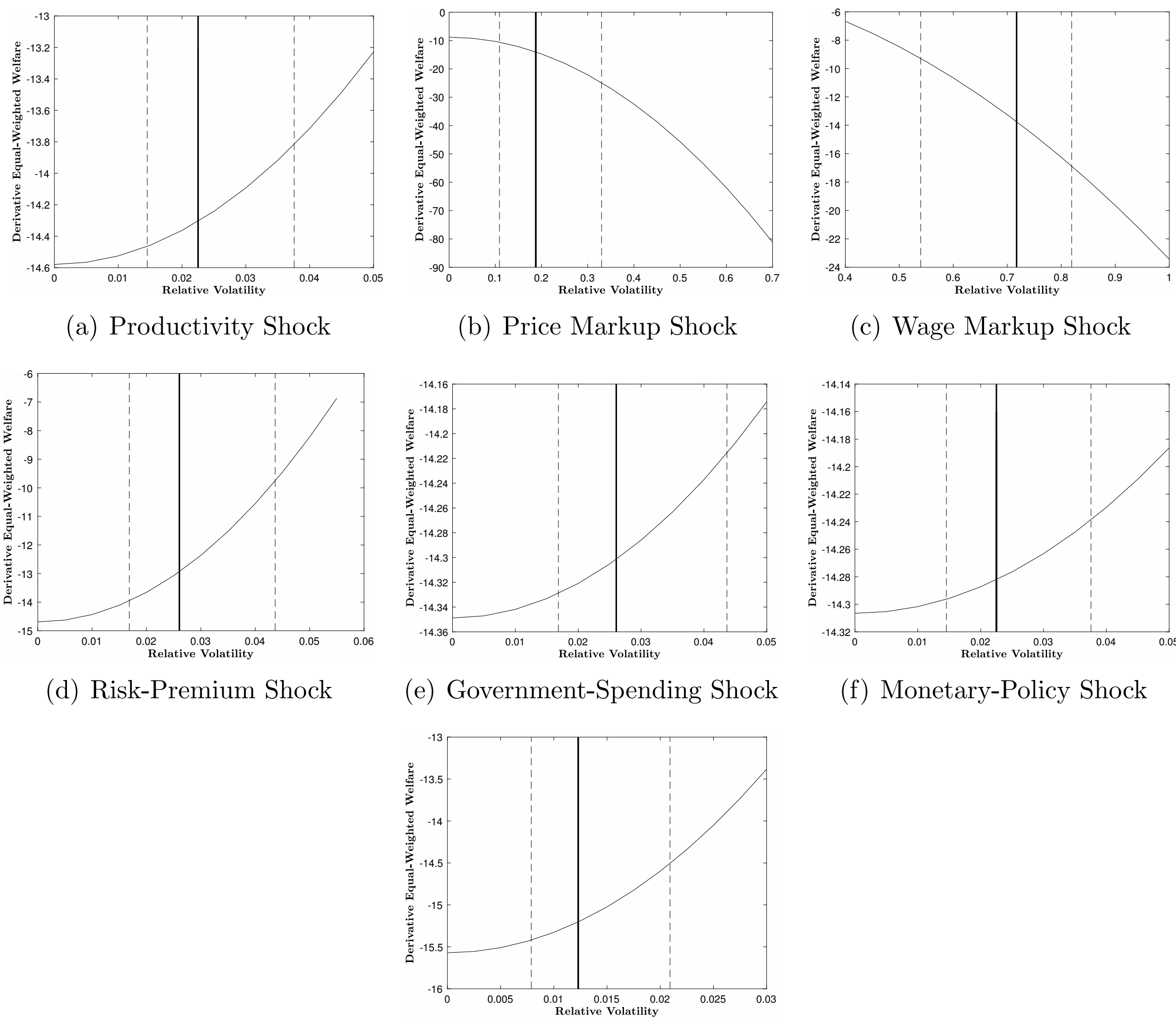
(f) Monetary-Policy Shock



(g) Investment-Specific Shock

Note: The figure illustrates the effect of increasing the importance of shocks on the model-estimated derivative of output volatility. The effect is obtained from the estimated medium-scale model in Section 3 as the numerical derivative of output volatility with respect to price flexibility as we increase the volatility of one shock at a time while holding all other parameters, in particular the volatility of all other shocks, at their posterior means. Vertical lines display the ratio of posterior means of the volatility of the shock considered and all shocks (solid lines), and 90% error bands (dashed lines).

Figure 6: Effect of Importance of Shocks on Model-Estimated Derivative of Equal-Weighted Welfare with Respect to Price Flexibility



(a) Productivity Shock

(b) Price Markup Shock

(c) Wage Markup Shock

(d) Risk-Premium Shock

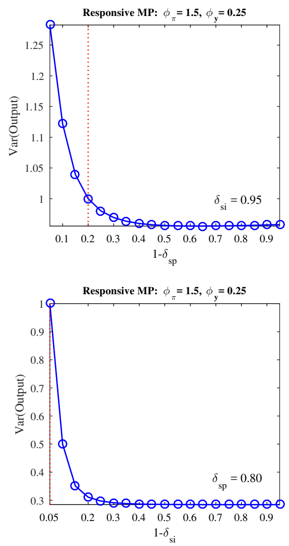
(e) Government-Spending Shock

(f) Monetary-Policy Shock

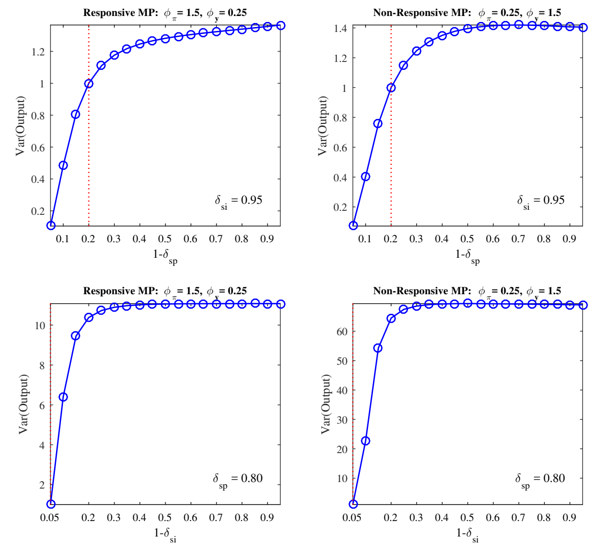
(g) Investment-Specific Shock

NOTE: The figure illustrates the effect of increasing the importance of shocks on the model-estimated derivative of equal-weighted welfare. The effect is obtained from the estimated medium-scale model in Section 3 as the numerical derivative of equal-weighted welfare with respect to price flexibility as we increase the volatility of one shock at a time while holding all other parameters, in particular the volatility of all other shocks, at their posterior means. Equal-weighted welfare shows an equal-weighted, negatively signed composite of inflation and the output gap. Vertical lines display the ratio of posterior means of the volatility of the shock considered and all shocks (solid lines), and 90% error bands (dashed lines).

Figure 7: Sticky Information/Sticky Price Model: Preference and Technology Shocks



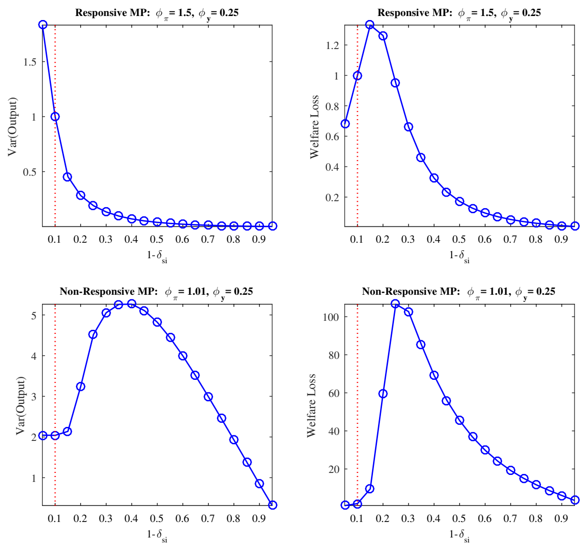
(a) Productivity Shock



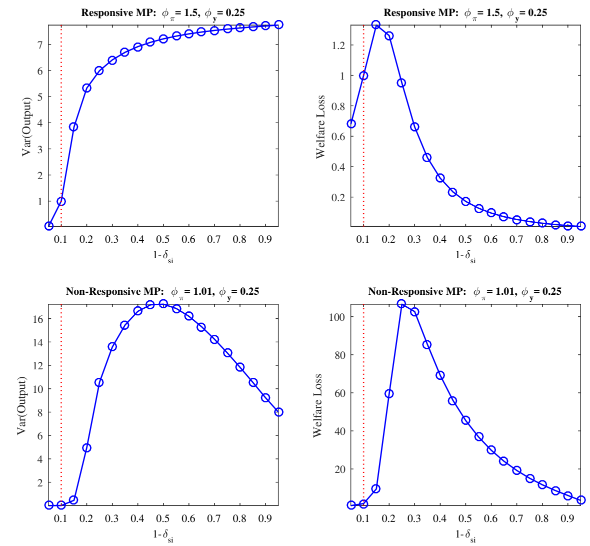
(b) Technology Shock

Note: The figure illustrates the effect of increasing either the flexibility of information or price flexibility on the volatility of output, both in a non-responsive or responsive case of monetary policy. The effect is obtained from the model in Section 4 for both preference and technology shocks.

Figure 8: Welfare in the Sticky Information Model: Preference and Technology Shocks



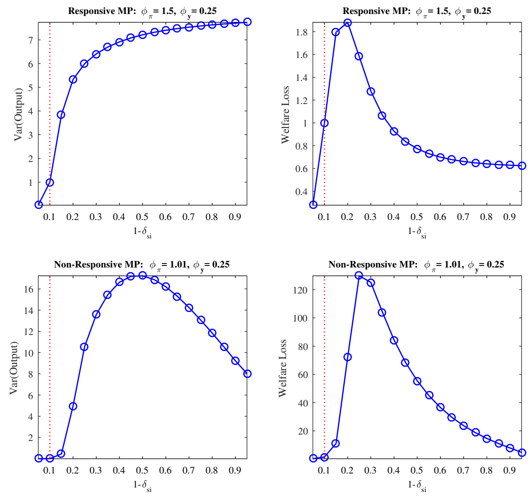
(a) Productivity Shock



(b) Technology Shock

Note: The figure illustrates the effect of increasing either the flexibility of information or price flexibility on the volatility of output, both in a non-responsive or responsive case of monetary policy. The effect is obtained from the model in Section 4 for both preference and technology shocks. Welfare represents the usual approximation to household utility.

Figure 9: Welfare in the Sticky Information Model: Markup Shock



Note: The figure illustrates the effect of increasing either the flexibility of information or price flexibility on the volatility of output, both in a non-responsive or responsive case of monetary policy. The effect is obtained from the model in Section 4 for both markup shocks. Welfare represents the usual approximation to household utility.

NOT FOR PUBLICATION

Appendix A. Proofs of Lemmas and Propositions in Section 2

In the following, to keep clutter to a minimum, we only keep track of A_t and ψ_t since η_t shows up in the same way as $\sigma^{-1}(\psi_t - E_t\psi_{t+1})$ and μ_t and τ_t^w as A_t. Then, under endogenous nominal demand and Taylor rule, the following equations hold:

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(\hat{\imath}_t - E_t\pi_{t+1}) + \psi_t - E_t\psi_{t+1}$$

$$\pi_t = \kappa\hat{Y}_t - \kappa\gamma_A A_t + \beta E_t\pi_{t+1}$$

$$\hat{i}_t = \phi_\pi\pi_t + \phi_y\hat{Y}_t + \eta_t$$

where demand and technology shocks evolve first-order auto-regressively as $\psi_t = \rho_\psi\psi_{t-1} + \epsilon_t^\psi$ and $A_t = \rho_A A_{t-1} + \epsilon_t^\psi$, and $\gamma_A = \frac{1+\phi}{\sigma^{-1}+\phi}$.

Appendix A.1. Welfare - Lemma 1 and Proposition 1

First, only consider demand shocks ψ_t. The system has a solution of the following form: $Y_t = Y_\psi\psi_t$, $\pi_t = \pi_\psi\psi_t$ and $i_t = i_\psi\psi_t$ which implies that $E_tY_{t+1} = Y_\psi\rho_\psi\psi_t$ and $E_t\pi_{t+1} = \pi_\psi\rho_\psi\psi_t$.

Matching coefficients yields the following expressions:

$$\begin{aligned} Y_\psi &= \left[\frac{\sigma(1-\beta\rho_\psi)\,(1-\rho_\psi)}{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)+\kappa\sigma\,(\phi_\pi-\rho_\psi)}\right] \\ \pi_\psi &= \left[\frac{\kappa\sigma\,(1-\rho_\psi)}{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)+\kappa\sigma\,(\phi_\pi-\rho_\psi)}\right] \end{aligned}$$

This implies the following expression for the variance of output and inflation:

$$\begin{aligned} var(\hat{Y}_t/\psi_t) &= \left(\frac{\sigma(1-\beta\rho_\psi)(1-\rho_\psi)}{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)+\sigma\kappa[\phi_\pi-\rho_\psi]}\right)^2 var(\psi_t) \\ var(\pi_t/\psi_t) &= \left(\frac{\kappa\sigma\,(1-\rho_\psi)}{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)+\kappa\sigma\,(\phi_\pi-\rho_\psi)}\right)^2 var(\psi_t) \end{aligned}$$

Then, the derivative of the variance of output with respect to κ is:

$$\frac{\partial VAR(\hat{Y}_t/\psi_t)}{\partial k} = -2\sigma^2 \frac{\sigma(\phi_\pi - \rho_\psi)(1-\beta\rho_\psi)^2(1-\rho_\psi)^2}{((1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)+\sigma\kappa[\phi_\pi-\rho_\psi])^3} var(\psi_t)$$

If $(\phi_\pi - \rho_\psi) > 0$, then this derivative is always negative. The sign of the derivative flips iff $(\phi_\pi - \rho_\psi) < 0$. Note that the denominator is always positive which follows from the bounds implied by the determinacy condition.

Now, only consider technology shocks A_t. The system has a solution of the following form: $Y_t = Y_A A_t$, $\pi_t = \pi_A A_t$ and $i_t = i_A A_t$ which implies that $E_t Y_{t+1} = Y_A \rho_A A_t$ and $E_t \pi_{t+1} = \pi_A \rho_A A_t$.Matching coefficients yields the following expressions:

$$Y_A = \frac{\kappa\sigma[\phi_\pi - \rho_A]}{[(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma[\phi_\pi-\rho_A]]}\gamma_A$$

$$\pi_A = \frac{\kappa\gamma_A(1-\rho_A+\sigma\phi_y)}{[(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma(\phi_\pi-\rho_A]]}\gamma_A$$

This implies the following expression for the variance of output and inflation:

$$var(\hat{Y}_t/A_t) = \left(\frac{\kappa\sigma\gamma_A[\phi_\pi - \rho_A]}{[(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma[\phi_\pi-\rho_A]]}\gamma_A\right)^2 var(A_t)$$
$$var(\pi_t/A_t) = \left(\frac{-\kappa\sigma\gamma_A(1-\rho_A+\sigma\phi_y)}{[(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma(\phi_\pi-\rho_A]]}\gamma_A\right)^2 var(A_t)$$

Then, the derivative of the variance of output with respect to κ is:

$$\begin{aligned}\frac{\partial var(Y_t)}{\partial \kappa} &= 2Y_A \frac{\partial Y_A}{\partial \kappa} \\ &= 2\gamma_A \left[\frac{\kappa\sigma\left(\phi_\pi - \rho_A\right)}{(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma\left(\phi_\pi-\rho_A\right)}\right] \\ &\gamma_A \left[\frac{\sigma\left(\phi_\pi-\rho_A\right)(1-\beta\rho_A)(1-\rho_A+\sigma\phi_y)}{(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma\left(\phi_\pi-\rho_A\right)^2}\right] \\ &> 0\end{aligned}$$

since the denominator is always positive which follows from the bounds implied by the determinacy condition.

Next, notice that the shock η_t appears exactly in the same way as $\psi_t - E_t\psi_{t+1}$. Hence, the derivative of output has the same sign with respect to κ, depending on $\phi_\eta - \rho_\eta$: $\frac{\partial var(Y_t/\eta_t)}{\partial \kappa} < 0$

if $\phi_\eta - \rho_\eta < 0$ and $\frac{\partial var(Y_t/\eta_t)}{\partial \kappa} > 0$ if $\phi_\eta - \rho_\eta > 0$. The coefficients in the case of an idiosyncratic monetary policy shock η_t are:

$$
\begin{aligned}
Y_\eta &= \frac{-\sigma(1-\beta\rho_\eta)}{1-\rho_\eta+\sigma\phi_y+\sigma\kappa(\phi_\pi-\rho_\eta)} \\
\pi_\eta &= \frac{\kappa}{1-\beta\rho_\eta}\frac{-\sigma(1-\beta\rho_\eta)}{1-\rho_\eta+\sigma\phi_y+\sigma\kappa(\phi_\pi-\rho_\eta)}
\end{aligned}
$$

Finally, consider a markup shock $\hat{\mu}_t$ – shocks to labor taxes $\hat{\tau}_t^w$ have isomorphic derivations. First, note that we have $Y_t^n = Y_t^n - Y_t^e = -\frac{1}{\sigma^{-1}+\phi}\hat{\mu}_t$. Then, applying the method of undetermined coefficients in a setup analogous to the above yields the following coefficients:

$$
\begin{aligned}
Y_\mu &= -\left(\frac{1}{\sigma^{-1}+\phi}\right)\left[\frac{\kappa\sigma\left(\phi_\pi-\rho_\mu\right)}{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma\left(\phi_\pi-\rho_\mu\right)}\right] \\
\pi_\mu &= -\frac{\kappa}{(1-\beta\rho_\mu)}\left(\frac{1}{\sigma^{-1}+\phi}\right)\left[\frac{-\left(1-\beta\rho_\mu\right)\left(1-\rho_\mu+\sigma\phi_y\right)}{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma\left(\phi_\pi-\rho_\mu\right)}\right]
\end{aligned}
$$

This directly implies that the variance of output is

$$
var\left(Y_t\right) = \left[\left(\frac{1}{\sigma^{-1}+\phi}\right)\left[\frac{\kappa\sigma\left(\phi_\pi-\rho_\mu\right)}{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma\left(\phi_\pi-\rho_\mu\right)}\right]\right]^2 var(\mu_t)
$$

Taking derivatives of the variance of output with respect to kappa yields:

$$
\begin{aligned}
\frac{\partial var(Y_t)}{\partial \kappa} &= 2Y_\mu\frac{\partial Y_\mu}{\partial \kappa} \\
&= 2\left(\frac{1}{\sigma^{-1}+\phi}\right)\left[\frac{\kappa\sigma\left(\phi_\pi-\rho_\mu\right)}{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma\left(\phi_\pi-\rho_\mu\right)}\right] \\
&\left(\frac{1}{\sigma^{-1}+\phi}\right)\frac{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)\sigma(\phi_\pi-\rho_\mu)}{\left((1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma\left(\phi_\pi-\rho_\mu\right)\right)^2} \\
&= 2\frac{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)}{\kappa\left((1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma\left(\phi_\pi-\rho_\mu\right)\right)}var\left(Y_t\right) \\
&> 0
\end{aligned}
$$

Note that the denominator is always positive which follows from the bounds implied by the determinacy condition.

Appendix A.2. Welfare - Lemma 2 and Proposition 2

For the technology shock, first note the for $Y_t^e = \frac{1+\phi}{\sigma^{-1}+\phi}\hat{A}_t$, we have that

$$var\left(Y_t - Y_t^e\right) = \left[\left(\frac{1+\phi}{\sigma^{-1}+\phi}\right)\left[\frac{-\left(1-\beta\rho_A\right)\left(1-\rho_A+\sigma\phi_y\right)}{(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma\left(\phi_\pi-\rho_A\right)}\right]\right]^2$$

Second, we take derivatives of the weighted variance term:

$$\begin{aligned}
\frac{\partial\frac{\theta}{\kappa}var(\pi_t)}{\partial\kappa} &= -\theta\frac{1}{\kappa^2}var(\pi) + \theta\frac{1}{\kappa}\frac{\partial var(\pi_t)}{\partial\kappa} \\
&= \theta\left[\frac{1}{(1-\beta\rho_A)}\right]^2 var\left(Y_t - Y_t^e\right)\left(\frac{(1-\rho_A+\sigma\phi_Y)(1-\beta\rho_A)-\kappa\sigma\left(\phi_\pi-\rho_A\right)}{(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma\left(\phi_\pi-\rho_A\right)}\right) \\
&> 0 \text{ if } \phi_\pi - \rho_A < \Gamma_A = \frac{(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)}{\kappa\sigma} \\
&< 0 \text{ if } \phi_\pi - \rho_A > \Gamma_A = \frac{(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)}{\kappa\sigma}
\end{aligned}$$

For the demand shock, $Y_t^e = Y_t$. Since $var(\pi_t) = \frac{\kappa^2}{(1-\beta\rho_\psi)^2}var(Y_t)$, some algebra directly implies that

$$\begin{aligned}
\frac{\partial\frac{\theta}{\kappa}var(\pi_t)}{\partial\kappa} &= -\theta\frac{1}{\kappa^2}var(\pi_t) + \theta\frac{1}{\kappa}\frac{\partial var(\pi_t)}{\partial\kappa} \\
&= \frac{\theta}{\kappa^2}var\left(Y_t\right)\left[\frac{\kappa}{(1-\beta\rho_\psi)}\right]^2\left[-1+2\frac{\left(1-\rho_\psi+\sigma\phi_y\right)\left(1-\beta\rho_\psi\right)}{\left(1-\rho_\psi+\sigma\phi_y\right)\left(1-\beta\rho_\psi\right)+\sigma\kappa\left(\phi_\pi-\rho_\psi\right)}\right] \\
&= \frac{\theta}{\kappa^2}var\left(Y_t\right)\left[\frac{\kappa}{(1-\beta\rho_\psi)}\right]^2\left(\frac{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)-\kappa\sigma(\phi_\pi-\rho_\psi)}{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)+\kappa\sigma(\phi_\pi-\rho_\psi)}\right) \\
&> 0 \text{ iff } \phi_\pi - \rho_\psi < \Gamma_\psi = \frac{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)}{\kappa\sigma} \\
&< 0 \text{ iff } \phi_\pi - \rho_\psi > \Gamma_\psi = \frac{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)}{\kappa\sigma}
\end{aligned}$$

For the markup shock, some algebra directly implies that

$$\begin{aligned}
\frac{\partial\frac{\theta}{\kappa}var(\pi_t)}{\partial\kappa} &= \frac{\theta}{\kappa^2}\left[\frac{(1-\rho_\mu+\sigma\phi_y)}{\sigma\left(\phi_\pi-\rho_\mu\right)}\right]^2 var(Y_t)\left(\frac{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)-\kappa\sigma(\phi_\pi-\rho_\mu)}{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma(\phi_\pi-\rho_\mu)}\right) \\
&< 0 \text{ iff } \phi_\pi - \rho_\mu > \Gamma_\mu = \frac{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)}{\kappa\sigma} \\
&> 0 \text{ if } \phi_\pi - \rho_\mu < \Gamma_\mu = \frac{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)}{\kappa\sigma}
\end{aligned}$$

Noting that for the demand shock ψ_t it holds true that $Y_t^e = Y_t$, we take derivatives of

W with respect to κ :

$$
\begin{aligned}
\frac{\partial W}{\partial \kappa} &= -\left(\phi+\sigma^{-1}\right)\left[\frac{\partial \frac{\theta}{\kappa} var\left(\pi_t\right)}{\partial \kappa}+\frac{\partial var\left(Y_t\right)}{\partial \kappa}\right] \\
&= \frac{\left(\phi+\sigma^{-1}\right) var(Y_t)}{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)+\kappa\sigma(\phi_\pi-\rho_\psi)} \\
&\left(-\frac{\theta}{\left(1-\beta\rho_\psi\right)^2}\left((1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)-\kappa\sigma(\phi_\pi-\rho_\psi)\right)+2\sigma(\phi_\pi-\rho_\psi)\right) \\
&< 0 \text{ iff } \left(\phi_\pi-\rho_\psi\right) < \Lambda_\psi = \frac{\theta(1-\beta\rho_\psi)(1-\rho_\psi+\sigma\phi_y)}{\sigma(2(1-\beta\rho_\psi)^2+\kappa\theta)} \\
&> 0 \text{ iff } \left(\phi_\pi-\rho_\psi\right) > \Lambda_\psi = \frac{\theta(1-\beta\rho_\psi)(1-\rho_\psi+\sigma\phi_y)}{\sigma(2(1-\beta\rho_\psi)^2+\kappa\theta)}
\end{aligned}
$$

For the markup shock, first consider the derivative of the weighted inflation term with respect to κ:

$$
\begin{aligned}
\frac{\partial \frac{\theta}{\kappa} var(\pi_t)}{\partial \kappa} &= -\theta\frac{1}{\kappa^2}var(\pi)+\theta\frac{1}{\kappa}\frac{\partial var(\pi_t)}{\partial \kappa} \\
&= \frac{\theta}{\kappa^2}\left[\frac{(1-\rho_\mu+\sigma\phi_y)}{\sigma\left(\phi_\pi-\rho_\mu\right)}\right]^2 var(Y_t)\left(\frac{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)-\kappa\sigma(\phi_\pi-\rho_\mu)}{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma(\phi_\pi-\rho_\mu)}\right) \\
&< 0 \text{ iff } \phi_\pi-\rho_\mu > \Gamma_\mu = \frac{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)}{\kappa\sigma} \\
&> 0 \text{ iff } \phi_\pi-\rho_\mu < \Gamma_\mu = \frac{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)}{\kappa\sigma}
\end{aligned}
$$

Second, since $Y_t^e=0$, taking derivatives of welfare with respect to κ yields after some algebra:

$$
\begin{aligned}
\frac{\partial W}{\partial \kappa} &= -\left(\phi+\sigma^{-1}\right)\frac{1}{\kappa\left((1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma\left(\phi_\pi-\rho_\mu\right)\right)}var\left(Y_t\right) \\
&\left(\frac{\theta}{\kappa}\left[\frac{(1-\rho_\mu+\sigma\phi_y)}{\sigma\left(\phi_\pi-\rho_\mu\right)}\right]^2(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)-\kappa\sigma(\phi_\pi-\rho_\mu)+2(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)\right) \\
&< 0 \text{ if } \frac{\theta}{\kappa}\left[\frac{(1-\rho_\mu+\sigma\phi_y)}{\sigma\left(\phi_\pi-\rho_\mu\right)}\right]^2(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu) \\
&-\kappa\sigma(\phi_\pi-\rho_\mu)+2(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu) > 0 \\
&> 0 \text{ if } \frac{\theta}{\kappa}\left[\frac{(1-\rho_\mu+\sigma\phi_y)}{\sigma\left(\phi_\pi-\rho_\mu\right)}\right]^2(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu) \\
&-\kappa\sigma(\phi_\pi-\rho_\mu)+2(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu) < 0
\end{aligned}
$$

A sufficient but not necessary condition for $\frac{\partial W}{\partial \kappa} < 0$ is $(\phi_\pi - \rho_\mu) < 0$.

For technology shocks, $Y_t^e = \frac{1+\phi}{\sigma^{-1}+\phi}\hat{A}_t$. This implies that

$$var\left(Y_t - Y_t^e\right) = \left[\left(\frac{1+\phi}{\sigma^{-1}+\phi}\right)\left[\frac{-\left(1-\beta\rho_A\right)\left(1-\rho_A+\sigma\phi_y\right)}{(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma\left(\phi_\pi-\rho_A\right)}\right]\right]^2$$

so that

$$\begin{aligned}
\frac{\partial var(Y_t - Y_t^e)}{\partial \kappa} &= -2\left(\frac{1+\phi}{\sigma^{-1}+\phi}\right)^2 \frac{\sigma(\phi_\pi-\rho_A)((1-\beta\rho_A)\left(1-\rho_A+\sigma\phi_y\right))^2}{((1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma\left(\phi_\pi-\rho_A\right))^3} \\
&= -2\frac{\sigma(\phi_\pi-\rho_A)}{((1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma\left(\phi_\pi-\rho_A\right))} var(Y_t - Y_t^e) \\
&> 0 \text{ iff } \left(\phi_\pi-\rho_A\right) < 0 \\
&< 0 \text{ iff } \left(\phi_\pi-\rho_A\right) > 0
\end{aligned}$$

We combine results from above, which directly yields after some algebra:

$$\begin{aligned}
\frac{\partial W}{\partial \kappa} &= -\left(\phi+\sigma^{-1}\right)\left[\frac{\partial \frac{\theta}{\kappa} var\left(\pi_t\right)}{\partial \kappa} + \frac{\partial var\left(Y_t - Y_t^e\right)}{\partial \kappa}\right] \\
&(-\frac{\theta}{\left(1-\beta\rho_A\right)^2}\left((1-\rho_A+\sigma\phi_y)(1-\beta\rho_A) - \kappa\sigma(\phi_\pi-\rho_A)\right) + 2\ \sigma(\phi_\pi-\rho_A)) \\
&< 0 \text{ if } \left(\phi_\pi-\rho_A\right) < \Lambda_A = \frac{\theta(1-\beta\rho_A)(1-\rho_A+\sigma\phi_Y)}{\sigma(2(1-\beta\rho_A)^2+\kappa\theta)} \\
&> 0 \text{ if } \left(\phi_\pi-\rho_A\right) > \Lambda_A = \frac{\theta(1-\beta\rho_A)(1-\rho_A+\sigma\phi_Y)}{\sigma(2(1-\beta\rho_A)^2+\kappa\theta)}
\end{aligned}$$

Appendix B. Smets-Wouters Model

We refer the reader to the original Smets and Wouters (2007) paper for a detailed description of the model. Here we present the log-linearized equilibrium conditions in line with the notation in their paper (for the expressions for the reduced-form parameters below as a function of the structural parameters, please see Smets and Wouters (2007) and its appendix).

$$\hat{c}_t = c_1\hat{c}_{t-1} + (1-c_1)E_t\hat{c}_{t+1} - c_2\{\hat{r}_t - E_t\hat{\pi}_{t+1} + \varepsilon_t^b\} - c_3\left(E_t\hat{n}_{t+1} - \hat{n}_t\right)$$

$$\hat{\imath}_t = i_1 \hat{\imath}_{t-1} + (1 - i_1) E_t \hat{\imath}_{t+1} + i_2 \hat{q}_t + \varepsilon_t^q$$

$$\hat{q}_t = -\left(\hat{r}_t - E_t \hat{\pi}_{t+1} + \varepsilon_t^b\right) + q_1 E_t \hat{r}_{t+1}^k + (1 - q_1) E_t \hat{q}_{t+1}$$

$$\hat{y}_t = c_y \hat{c}_t + i_y \hat{\imath}_t + \hat{g}_t + v_y \hat{v}_t$$

$$\hat{\pi}_t - \iota_p \hat{\pi}_{t-1} = \bar{\beta}\bar{\gamma}\left(E_t \hat{\pi}_{t+1} - \iota_p \hat{\pi}_t\right) - \pi_1 \left(-\left(\alpha \hat{r}_t^k + (1-\alpha)\hat{w}_t - a_t\right) - \hat{\mu}_t^p\right)$$

$$\hat{\pi}_t^w - \iota_w \hat{\pi}_{t-1} = \bar{\beta}\bar{\gamma}\left(E_t \hat{\pi}_{t+1}^w - \iota_w \hat{\pi}_t\right) - \lambda_w \left(\hat{w}_t - \left(\left(\frac{1}{1-h/\bar{\gamma}}\right)\hat{c}_t - \left(\frac{h/\bar{\gamma}}{1-h/\bar{\gamma}}\right)\hat{c}_{t-1} + \sigma_l \hat{n}_t\right) - \hat{\mu}_t^w\right)$$

$$\widehat{\bar{k}}_t = k_1 \widehat{\bar{k}}_{t-1} + (1 - k_1)\,\hat{\imath}_t + k_2 \varepsilon_t^q$$

$$\hat{k}_t = \hat{v}_t + \widehat{\bar{k}}_{t-1}$$

$$\hat{v}_t = \left(\frac{1}{(\psi/1-\psi)}\right)\hat{r}_t^k$$
$$\hat{k}_t = \hat{w}_t - \hat{r}_t^k + \hat{n}_t$$

$$r_t = \rho r_{t-1} + (1-\rho)\left(r_\pi \pi_t + r_y \widehat{ygap}_t\right) + r_{\Delta y}\widehat{\Delta ygap}_t + \varepsilon_t^r$$

640 Appendix C. Solution and Estimation Method

We use a Bayesian framework for estimation. The first-order approximation to the equilibrium conditions of the model can be written as

$$\Gamma_0(\theta)\, s_t = \Gamma_1(\theta)\, s_{t-1} + \Gamma_\varepsilon(\theta)\varepsilon_t + \Gamma_\eta(\theta)\pi_t$$

where s_t is a vector of model variables and ε_t is a vector of shocks to the exogenous processes. π_t is a vector of rational expectations forecast errors, which implies $E_{t-1}\pi_t = 0$ for all $t,$ and θ contains the structural model parameters. The solution to this system is given by

$$s_t = \Omega_1(\theta)s_{t-1} + \Omega_\varepsilon(\theta)\varepsilon_t.$$

which can be obtained using standard methods in the literature. Finally, the model variables are related to the observables by the measurement equation

$$y_t = Bs_t$$

where y_t is the vector of observables.

Let $Y = \{y\}_{t=1}^{T}$ be the data. In a Bayesian framework, the likelihood function $L(Y \mid \theta)$ is combined with a prior density $p(\theta)$ to yield the posterior density

$$p(\theta \mid Y) \quad \propto \quad p(\theta)L(Y \mid \theta).$$

Assuming Gaussian shocks, it is straightforward to evaluate the likelihood function using the Kalman filter. A numerical optimization routine is used to maximize $p(\theta \mid Y)$ and find the posterior mode. Then, we can generate draws from $p(\theta \mid Y)$ using the Metropolis-Hastings algorithm where we use a Gaussian proposal density in the algorithm, using an inverse of a scaled Hessian computed at the posterior mode as the covariance matrix.

The Metropolis-Hastings algorithm works as follows. Let the posterior mode computed from the numerical optimization routine be $\tilde{\theta}$. Let the inverse of the Hessian computed at $\tilde{\theta}$ be $\tilde{\Sigma}$.

(a) Choose a starting value θ^0. Then use a loop over the following steps (b)-(d).

(b) For $d = 1, ..., D,$ draw a θ^* from the proposal distribution $N(\theta^{d-1},\ c\tilde{\Sigma})$.

(c) Accept $\theta^*,$ that is $\theta^d = \theta^*,$ with probability $\min\{1, r(\theta^{d-1},\ \theta^*)\}$. Reject θ^*, that is

$\theta^d = \theta^{d-1}$, otherwise.

(d) $r(\theta^{d-1},\ \theta^*)$ is given by:

$$r(\theta^{d-1},\ \theta^*) = \frac{p(\theta^*)L(Y \mid \theta^*)}{p(\theta^{d-1})L(Y \mid \theta^{d-1})}$$

The scale parameter c is chosen to lead to acceptance rates of around 30%.

To settle on a model specification, we do Bayesian model comparison using the marginal data densities of the models. In comparing models A and B we are interested in the relative posterior probabilities of the models given the data. That is, $\frac{p(A|Y)}{p(B|Y)} = \frac{p(A)}{p(B)} \frac{p(Y|A)}{p(Y|B)}$ where $p(A)$ and $p(B)$ are the prior probabilities of the models A and B. Since we do not specifying different prior probabilities over the models, we just compare the marginal data densities given by $p(Y \mid A)$ and $p(Y \mid B)$. The marginal data density of a model is given by

$$p(Y) = \int p(\theta)L(Y \mid \theta)\ d\theta.$$

Note that this measure penalizes overparameterized models.

The marginal data density is approximated by the Geweke (1998) modified harmonic-mean estimator. First note that we can write

$$\frac{1}{p(Y)} = \int \frac{f(\theta)d\theta}{p(\theta)L(Y \mid \theta)} d\theta$$

where f is a probability density function such that $\int f(\theta)d\theta = 1$. Then, we can use the following estimator

$$\hat{p}(Y) = \left[\frac{1}{D}\sum_{d=1}^{D} \frac{f(\theta^d)}{p(\theta^d)L(Y \mid \theta^d)}\right]^{-1}$$

where d denotes the posterior draws obtained using the Metropolis-Hastings algorithm. For f , Geweke (1998) proposed a truncated multivariate normal distribution.

Table C.2: Prior Distribution of Structural Parameters

Parameters	Domain	Density	Prior Mean	Prior Stdev
φ	$\mathbb{R}$	Normal	4.00	1.50
σ_c	$\mathbb{R}$	Normal	1.50	0.37
h	[0,1)	Beta	0.70	0.10
ξ_w	[0,1)	Beta	0.50	0.10
σ_l	$\mathbb{R}$	Normal	2.00	0.75
ξ_p	[0,1)	Beta	0.50	0.10
ι_w	[0,1)	Beta	0.50	0.15
ι_p	[0,1)	Beta	0.50	0.15
ψ	[0,1)	Beta	0.50	0.15
Φ	$\mathbb{R}$	Normal	1.25	0.12
r_π	$\mathbb{R}$	Normal	1.50	0.25
ρ	[0,1)	Beta	0.75	0.10
r_y	$\mathbb{R}$	Normal	0.12	0.05
$r_{\Delta y}$	$\mathbb{R}$	Normal	0.12	0.05
$\overline{\pi}$	$\mathbb{R}^+$	Gamma	0.62	0.10
$100(\beta^{-1}-1)$	$\mathbb{R}^+$	Gamma	0.25	0.10
$\bar{l}$	$\mathbb{R}$	Normal	0.00	2.00
$\overline{\gamma}$	$\mathbb{R}$	Normal	0.40	0.10
α	$\mathbb{R}$	Normal	0.30	0.05

Table C.3: Prior Distribution of Shock Processes

Parameters	Domain	Density	Prior Mean	Prior Stdev
ρ_a	[0,1)	Beta	0.5	0.2
ρ_b	[0,1)	Beta	0.5	0.2
ρ_g	[0,1)	Beta	0.5	0.2
ρ_I	[0,1)	Beta	0.5	0.2
ρ_r	[0,1)	Beta	0.5	0.2
ρ_p	[0,1)	Beta	0.5	0.2
ρ_w	[0,1)	Beta	0.5	0.2
ρ_{ga}	[0,1)	Beta	0.5	0.2
μ_p	[0,1)	Beta	0.5	0.2
μ_w	[0,1)	Beta	0.5	0.2
σ_a	$\mathbb{R}^+$	InvG	0.10	0.5
σ_b	$\mathbb{R}^+$	InvG	0.10	2
σ_g	$\mathbb{R}^+$	InvG	0.10	2
σ_I	$\mathbb{R}^+$	InvG	0.10	2
σ_r	$\mathbb{R}^+$	InvG	0.10	2
σ_p	$\mathbb{R}^+$	InvG	4.00	4
σ_w	$\mathbb{R}^+$	InvG	5.00	5

Table C.4: Posterior Estimates of Structural Parameters

Parameters	Prior Mean	Posterior Mean	Probability Interval 90%
φ	4.00	5.4878	[3.8117 7.1473]
σ_c	1.50	1.3444	[1.1319 1.5553]
h	0.70	0.7173	[0.6469 0.7892]
ξ_w	0.50	0.6184	[0.5182 0.7220]
σ_l	2.00	1.4825	[0.6539 2.2722]
ξ_p	0.50	0.6124	[0.5396 0.6833]
ι_w	0.50	0.6143	[0.4158 0.8182]
ψ	0.50	0.5712	[0.3912 0.7492]
Φ	1.25	1.6149	[1.4869 1.7426]
r_π	1.50	2.0701	[1.7830 2.3554]
ρ	0.75	0.7936	[0.7516 0.8367]
r_y	.125	0.0843	[0.0474 0.1209]
$r_{\Delta y}$	.125	0.2169	[0.1699 0.2638]
$\overline{\pi}$	.625	0.8310	[0.6115 0.9660]
$100(\beta^{-1}-1)$	0.25	0.1690	[0.6580 1.0013]
$\bar{l}$	0.00	-0.0723	[-1.8855 1.7663]
$\overline{\gamma}$	0.40	0.4276	[0.4033 0.4520]
α	0.30	0.1914	[0.1620 0.2203]

Table C.5: Posterior Estimates of Shock Processes

Parameters	Prior Mean	Posterior Mean	Probability Interval 90%
ρ_a	0.5	0.9563	[0.9376 0.9754]
ρ_b	0.5	0.2041	[0.0651 0.3349]
ρ_g	0.5	0.9751	[0.9614 0.9890]
ρ_I	0.5	0.7149	[0.6202 0.8130]
ρ_r	0.5	0.1730	[0.0592 0.2817]
ρ_p	0.5	0.9050	[0.8441 0.9705]
ρ_w	0.5	0.9733	[0.9570 0.9899]
ρ_{ga}	0.5	0.5196	[0.3714 0.6666]
μ_p	0.5	0.5587	[0.3699 0.7464]
μ_w	0.5	0.8030	[0.7021 0.9055]
‘σ_a	0.10	0.4568	[0.4107 0.5025]
σ_b	0.10	0.2434	[0.2045 0.2821]
σ_g	0.10	0.5285	[0.4773 0.5782]
σ_I	0.10	0.4520	[0.3704 0.5322]
σ_r	0.10	0.2494	[0.2233 0.2746]
σ_p	4.00	3.8069	[2.0399 5.5395]
σ_w	5.00	14.5539	[5.6942 23.5754]

Appendix D. Extensions: Monetary policy rules and optimal policy

Optimal policy under discretion can be characterized easily here since there are no state variables. The problem is just a static one of minimizing

$$L_t = \left(\phi + \sigma^{-1}\right)\left[\frac{\theta}{\kappa}\pi_t^2 + \left(Y_t - Y_t^e\right)^2\right]$$

subject to

$$\pi_t = \kappa Y_t - \kappa Y_t^n + \beta E_t \pi_{t+1}.$$

Lets reformulate it as minimizing

$$L_t = \left[\frac{\theta}{\kappa}\pi_t^2 + \left(Y_t - Y_t^e\right)^2\right]$$

subject to

$$\pi_t = \kappa\left(Y_t - Y_t^e\right) + \kappa\left(Y_t^e - Y_t^n\right) + \beta E_t \pi_{t+1}.$$

The FOC of this problem leads to the simple, well-known targeting rule

$$\theta\pi_t + \left(Y_t - Y_t^e\right) = 0.$$

Now we have to work with two equations only to pin down the solution of the model

$$\theta\pi_t + \left(Y_t - Y_t^e\right) = 0$$

$$\pi_t = \kappa\left(Y_t - Y_t^e\right) + \kappa\left(Y_t^e - Y_t^n\right) + \beta E_t \pi_{t+1}$$

Replace the first into the second

$$\pi_t = -\kappa\theta\pi_t + \kappa\left(Y_t^e - Y_t^n\right) + \beta E_t \pi_{t+1}$$

Now replace for

$$Y_t^e - Y_t^n = \frac{1}{\sigma^{-1} + \phi}\hat{\mu}_t$$

Then get

$$\pi_t = -\kappa\theta\pi_t + \kappa\frac{1}{\sigma^{-1} + \phi}\hat{\mu}_t + \beta E_t \pi_{t+1}$$

This gives the following first-order forward looking difference equation in π_t

$$(1+\kappa\theta)\,\pi_t = \beta E_t \pi_{t+1} + \frac{\kappa}{\sigma^{-1}+\phi}\hat{\mu}_t$$

Guess

$$\pi_t = \pi_\mu \hat{\mu}_t$$

which gives

$$E_t \pi_{t+1} = \pi_\mu \rho_\mu \hat{\mu}_t$$

Replace above and match coefficients to get

$$\pi_\mu = \frac{\kappa}{(\sigma^{-1}+\phi)\,(1+\kappa\theta-\beta\rho_\mu)}$$

Thus,

$$\pi_t = \frac{1}{(\sigma^{-1}+\phi)}\left(\frac{\kappa}{(1+\kappa\theta-\beta\rho_\mu)}\right)\hat{\mu}_t.$$

This implies that

$$(Y_t - Y_t^e) = -\theta\pi_t = -\frac{\theta}{(\sigma^{-1}+\phi)}\left(\frac{\kappa}{(1+\kappa\theta-\beta\rho_\mu)}\right)\hat{\mu}_t.$$

We first start with establishing what happens to the variance of output when prices become more flexible. First note that two cases are particularly easy. For demand shocks, output does not respond at all as long as the ZLB does not bind. That is, in that case, we have

$$Y_t = 0.$$

So variance of output does not depend on price stickiness.

For technology shocks, output responds one-to-one since we have

$$\pi_t = (Y_t - Y_t^e) = 0.$$

This means

$$Y_t = Y_t^e = \frac{1+\phi}{\sigma^{-1}+\phi}\hat{A}_t.$$

Again, variance of output does not depend on price stickiness.

For markup shocks, we have as the solution for output (since $Y_t^e = 0$)

$$Y_t = -\frac{\theta}{(\sigma^{-1} + \phi)} \left(\frac{\kappa}{(1 + \kappa\theta - \beta\rho_\mu)} \right) \hat{\mu}_t.$$

$$var\,(Y_t) = \theta^2 \left(\frac{\kappa}{(1 + \kappa\theta - \beta\rho_\mu)} \right)^2.$$

Then

$$\frac{\partial var\,(Y_t)}{\partial \kappa} = \frac{\theta^2 (1 - \beta\rho_\mu)\, 2\kappa}{(1 + \kappa\theta - \beta\rho_\mu)^3} > 0.$$

Now, lets look at the effects of increased price flexibility on welfare. As is well-known with technology shocks only, both π_t and $(Y_t - Y_t^e)$ can be put to zero and one gets to first-best. Thus, there is no interesting relationship between price flexibility and welfare. With mark-up shocks, there is a trade-off as can be seen above.

For mark-up shocks, we want to evaluate

$$W = -\left(\phi + \sigma^{-1}\right) \left[\frac{\theta}{\kappa} var\,(\pi_t) + var\,(Y_t - Y_t^e) \right]$$

We have as the targeting rule

$$\theta\pi_t + (Y_t - Y_t^e) = 0$$

which gives

$$\theta^2 var\,(\pi_t) = var\,(Y_t - Y_t^e)\,.$$

Then, welfare is given by

$$W = -\left(\phi + \sigma^{-1}\right) \theta \left[\left(\frac{1}{\kappa} + \theta \right) var\,(\pi_t) \right].$$

We have as the solution of the model

$$\pi_t = \frac{1}{(\sigma^{-1} + \phi)} \left(\frac{\kappa}{(1 + \kappa\theta - \beta\rho_\mu)} \right) \hat{\mu}_t$$

or

$$var\,(\pi_t) = \left[\frac{1}{(\sigma^{-1} + \phi)} \left(\frac{\kappa}{(1 + \kappa\theta - \beta\rho_\mu)} \right) \right]^2.$$

We can then establish how variance of inflation depends on price flexibility.

$$\frac{\partial var(\pi_t)}{\partial \kappa} = \frac{(1-\beta\rho_\mu)\,2\kappa}{(1+\kappa\theta-\beta\rho_\mu)^3} > 0.$$

Then, we can establish how the welfare relevant variance of inflation depends on price flexibility

$$\frac{\partial \frac{\theta}{\kappa} var\,(\pi_t)}{\partial \kappa} = \frac{1-\kappa\theta-\beta\rho_\mu}{(1+\kappa\theta-\beta\rho_\mu)^3} > 0 \text{ if } 1-\kappa\theta > \beta\rho_\mu.$$

Thus, while with a low ρ_μ this variance of welfare relevant inflation term is increasing with greater price flexibility, it can decrease for a high enough ρ_μ. Third, we can consider how the variance of welfare relevant output gap depends on increased price flexibility (this is basically the same as the variance of output since $Y_t^e = 0$)

$$\frac{\partial var\,(Y_t - Y_t^e)}{\partial \kappa} = \theta^2 \frac{(1-\beta\rho_\mu)\,2\kappa}{(1+\kappa\theta-\beta\rho_\mu)^3} > 0.$$

Now, lets finally move to welfare. Replace the expression for inflation above, along with the relationship between to get π_t and $(Y_t - Y_t^e)$ to get

$$W = -\frac{1}{(\sigma^{-1}+\phi)}\theta\left[\frac{\kappa\,(1+\theta\kappa)}{(1+\kappa\theta-\beta\rho_\mu)^2}\right]$$

For simplicity, first consider $\rho_\mu = 0$. Then, we have

$$W = -\frac{1}{(\sigma^{-1}+\phi)}\theta\left[\frac{\kappa}{(1+\kappa\theta)}\right].$$

It is easy to see that in such a case

$$\frac{\partial W}{\partial \kappa} < 0.$$

When we consider a general ρ_μ however, note that this is not always the case. In particular, for a high enough ρ_μ, it can be the case that increased price flexibility leads to higher welfare. Generally,

$$\frac{\partial W}{\partial \kappa} = -\frac{1+\kappa\theta-\beta\rho_\mu\,(1+2\kappa\theta)}{(1+\kappa\theta-\beta\rho_\mu)^3}$$

The denominator is always positive, but the numerator can take either positive or negative value. Thus,

$$\frac{\partial W}{\partial \kappa} < 0 \text{ if } \frac{1+\kappa\theta}{1+2\kappa\theta} > \beta\rho_\mu.$$

There are two forces at work: the variance of the welfare relevant output gap is always increasing in price flexibility, but the variance of the welfare relevant inflation can decrease with higher flexibility if ρ_μ is big enough.

We can also study optimal monetary policy under commitment, which means specifying a fully state-contingent path at $t = 0$ for the endogenous variables to minimize the loss-function subject to

$$\pi_t = \kappa Y_t - \kappa Y_t^n + \beta E_t \pi_{t+1}.$$

Lets then define the Lagrangian

$$\begin{aligned}\mathcal{L} =& \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\theta}{\kappa} \pi_t^2 + (Y_t - Y_t^e)^2 \right] \\ &+ E_0 \sum_{t=0}^{\infty} \beta^t q_{1,t} \left\{ \pi_t - \kappa \left(Y_t - Y_t^e \right) - \kappa \left(Y_t^e - Y_t^n \right) - \beta E_t \pi_{t+1}. \right\}\end{aligned}$$

where $\{q_{1,t}\}$ is the sequence of Lagrange multiplier.

First order conditions are given as:

$$\partial \pi_t : 0 = \frac{\theta}{\kappa} \pi_t + q_{1,t} - q_{1,t-1}$$

$$\partial \left(Y_t - Y_t^e \right) : 0 = \left(Y_t - Y_t^e \right) - \kappa q_{1,t}$$

Consequently, the equilibrium time path of

$$\left\{ \hat{Y}_t, \pi_t, q_{1,t} \right\}_{t=0}^{\infty}$$

is characterized by the following 3 equations

$$\begin{aligned}\pi_t &= \kappa \left(Y_t - Y_t^e \right) - \kappa \left(Y_t^e - Y_t^n \right) - \beta E_t \pi_{t+1} \\ 0 &= \frac{\theta}{\kappa} \pi_t + q_{1,t} - q_{1,t-1} \\ 0 &= \left(Y_t - Y_t^e \right) - \kappa q_{1,t}\end{aligned}$$

given exogenous processes and initial conditions. We assume that all the variables are in the steady state initially: $q_{-1} = 0$.

So assuming the "time-less perspective" we have as the "targeting rule"

$$\theta\pi_t + (Y_t - Y_t^e) - \left(Y_{t-1} - Y_{t-1}^e\right) = 0.$$

Appendix E. Extensions: Zero lower bound

When ψ_t becomes negative enough, the ZLB binds. We assume, like in Eggertsson and Woodford (2003) and Eggertsson (2008) that the shock to $\psi_t = \psi_S < 0$ in period 0 and which reverts back to steady state $\psi_S = \bar{\psi} > 0$ with a fixed probability $1-\mu$ every period thereafter. Under discretion, out of the trap, optimal policy is able to achieve $Y_t - Y_t^e, \pi_t = 0$. At the ZLB, we have $i_t = \beta - 1$.

First, consider the Phillips curve (no shock to Y_t^n now) where we denote by S the time in the trap:

$$\pi_S = \kappa Y_S + \beta\mu\pi_S.$$

Next, consider the IS equation (no shock to Y_t^n now)

$$Y_S = \mu Y_S + \sigma\mu\pi_S + (1-\mu)\psi_S$$

Some algebra directly implies that

$$Y_S = \frac{(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - \kappa\sigma\mu}\psi_S$$

and

$$\pi_S = \frac{\kappa}{(1-\mu)(1-\beta\mu) - \kappa\sigma\mu}\psi_S$$

Note that here $Y_t^e = 0$. Consider each derivative of the welfare function with respect to κ. First, some algebra directly implies that

$$\frac{\partial var(Y_S)}{\partial\kappa} = \frac{2\sigma\mu}{(1-\mu)(1-\beta\mu) - \kappa\sigma\mu} var(Y_S) > 0$$

Next, since $\pi_S = \frac{\kappa}{1-\beta\mu} Y_S$, we have that

$$\begin{aligned}
\frac{\partial var(\pi_S)}{\partial\kappa} &= 2\left(\frac{\kappa}{1-\beta\mu}\right)\frac{1}{1-\beta\mu} var(Y_S) + \left(\frac{\kappa}{1-\beta\mu}\right)^2 \frac{\partial var(Y_S)}{\partial\kappa} \\
&= \left[\frac{(1-\mu)(1-\beta\mu)}{[(1-\mu)(1-\beta\mu) - \kappa\sigma\mu]}\right]\frac{2\kappa}{(1-\beta\mu)^2} var(Y_S) > 0
\end{aligned}$$

This implies that the derivative of the weighted variance of inflation is

$$\begin{aligned}
\frac{\partial \frac{\theta}{\kappa} var\left(\pi_S\right)}{\partial \kappa} &= -var\left(\pi_S\right)\frac{\theta}{\kappa^2} + \frac{\theta}{\kappa}\frac{\partial var(\pi_S)}{\partial \kappa} \\
&= \left[\frac{\left(1-\mu\right)\left(1-\beta\mu\right)+\kappa\sigma\mu}{\left(1-\mu\right)\left(1-\beta\mu\right)-\kappa\sigma\mu}\right]\frac{\theta}{\left(1-\beta\mu\right)^2} var\left(Y_S\right) > 0
\end{aligned}$$

Therefore, the derivative of welfare with respect to κ is negative:

$$\frac{\partial W}{\partial \kappa} < 0.$$

since all loss components have a positive derivative with respect to κ and are multiplied by -1.

Finally, when studying optimal monetary policy, we take into account the zero lower bound on interest rates explicitly. This happens when r_t^n becomes negative enough so that the ZLB binds. We assume, like in Eggertsson and Woodford (2003) and Eggertsson (2008) that the shock to $r_t^n = r_S^n < 0$ in period 0 and which reverts back to steady state $r_S = \bar{r} > 0$ with a fixed probability $1-\mu$ every period thereafter.

Under discretion, out of the trap, optimal policy is able to achieve $Y_t - Y_t^e$, $\pi_t \ = 0$. At ZLB, we have $i_t = \beta - 1$.We have to consider two cases: when the economy is in a ZLB situation and when it is out of it. Out of the trap, as discussed above, in this simple model under discretion, both $Y_t - Y_t^e$ and π_t are equal to zero when the shock that hits the economy is a shock to r_t^n such as a preference shock. In the trap, $i_t = \beta - 1$.

Now, consider the Phillips curve (no Y_t^n shock now)

$$\pi_t = \kappa Y_t + \beta E_t \pi_{t+1}$$

which we rewrite as

$$\pi_S = \kappa Y_S + \beta\mu\pi_S.$$

Next, consider the IS equation (no Y_t^n shock now)

$$Y_t = E_t Y_{t+1} - \sigma(i_t - E_t\pi_{t+1} - r_t^n)$$

which we rewrite as

$$Y_S = \mu Y_S + \sigma\mu\pi_S + \sigma r_S^n$$

Lets manipulate these two expressions

$$\pi_S = \left(\frac{\kappa}{1-\beta\mu}\right) Y_S$$

$$\pi_S = \frac{(1-\mu)\,Y_S - \sigma r_S^n}{\sigma\mu}$$

and combine them to get

$$\left(\frac{\kappa}{1-\beta\mu}\right) Y_S = \frac{(1-\mu)\,Y_S - \sigma r_S^n}{\sigma\mu}$$

or

$$\left[\left(\frac{1-\mu}{\sigma\mu}\right) - \left(\frac{\kappa}{1-\beta\mu}\right)\right] Y_S = \frac{1}{\mu} r_S^n$$

$$\left[\frac{(1-\mu)\,(1-\beta\mu) - \kappa\sigma\mu}{\sigma\mu\,(1-\beta\mu)}\right] Y_S = \frac{1}{\mu} r_S^n$$

$$Y_S = \frac{\sigma\,(1-\beta\mu)}{(1-\mu)\,(1-\beta\mu) - \kappa\sigma\mu} r_S^n$$

and then

$$\pi_S = \left(\frac{\kappa}{1-\beta\mu}\right) Y_S$$

$$\pi_S = \left(\frac{\kappa}{1-\beta\mu}\right) \frac{\sigma\,(1-\beta\mu)}{(1-\mu)\,(1-\beta\mu) - \kappa\sigma\mu} r_S^n$$

$$\pi_S = \frac{\sigma\kappa}{(1-\mu)\,(1-\beta\mu) - \kappa\sigma\mu} r_S^n.$$

For welfare, we want to evaluate

$$W = -\left(\phi + \sigma^{-1}\right)\left[\frac{\theta}{\kappa} var\left(\pi_t\right) + var\left(Y_t - Y_t^e\right)\right]$$

which here is

$$W = -\left(\phi + \sigma^{-1}\right)\left[\frac{\theta}{\kappa} var\left(\pi_t\right) + var\left(Y_t\right)\right]$$

First, we have

$$var\left(\pi_S\right) = \left(\frac{\kappa}{1-\beta\mu}\right)^2 var\left(Y_S\right)$$

and

$$var\left(Y_S\right) = \left[\frac{\sigma\left(1-\beta\mu\right)}{\left(1-\mu\right)\left(1-\beta\mu\right)-\kappa\sigma\mu}\right]^2$$

so

$$W = -\left(\phi + \sigma^{-1}\right)\left[\frac{\theta}{\kappa} var\left(\pi_S\right) + var\left(Y_S\right)\right]$$

$$W = -\left(\phi + \sigma^{-1}\right)\left[\frac{\theta}{\kappa}\left(\frac{\kappa}{1-\beta\mu}\right)^2 var\left(Y_S\right) + var\left(Y_S\right)\right]$$

$$W = -\left(\phi + \sigma^{-1}\right)\left[\left(\frac{\theta}{\kappa}\left(\frac{\kappa}{1-\beta\mu}\right)^2 + 1\right) var\left(Y_S\right)\right]$$

$$W = -\left(\phi + \sigma^{-1}\right)\left[\left(\frac{\theta\kappa}{\left(1-\beta\mu\right)^2} + 1\right) var\left(Y_S\right)\right].$$

Now

$$\frac{\partial var(Y_S)}{\partial\kappa} = 2\left[\frac{\sigma\left(1-\beta\mu\right)}{\left(1-\mu\right)\left(1-\beta\mu\right)-\kappa\sigma\mu}\right]\frac{\sigma\left(1-\beta\mu\right)}{\left(\left(1-\mu\right)\left(1-\beta\mu\right)-\kappa\sigma\mu\right)^2}\sigma\mu > 0$$

which also gives

$$\frac{\partial var(Y_S)}{\partial\kappa} = \frac{2\sigma\mu}{\left(1-\mu\right)\left(1-\beta\mu\right)-\kappa\sigma\mu} var\left(Y_S\right).$$

Next

$$\frac{\partial var(\pi_S)}{\partial\kappa} = 2\left(\frac{\kappa}{1-\beta\mu}\right)\frac{1}{1-\beta\mu} var\left(Y_S\right) + \left(\frac{\kappa}{1-\beta\mu}\right)^2\frac{\partial var(Y_S)}{\partial\kappa}$$

$$\frac{\partial var(\pi_S)}{\partial\kappa} = 2\left(\frac{\kappa}{1-\beta\mu}\right)\frac{1}{1-\beta\mu} var\left(Y_S\right) + \left(\frac{\kappa}{1-\beta\mu}\right)^2\frac{2\sigma\mu}{\left(1-\mu\right)\left(1-\beta\mu\right)-\kappa\sigma\mu} var\left(Y_S\right)$$

$$\frac{\partial var(\pi_S)}{\partial\kappa} = \left[\frac{1}{\kappa} + \frac{\sigma\mu}{\left(1-\mu\right)\left(1-\beta\mu\right)-\kappa\sigma\mu}\right] 2\left(\frac{\kappa}{1-\beta\mu}\right)^2 var\left(Y_S\right)$$

$$\frac{\partial var(\pi_S)}{\partial \kappa} = \left[\frac{(1-\mu)(1-\beta\mu)}{[(1-\mu)(1-\beta\mu)-\kappa\sigma\mu]}\right]\frac{2\kappa}{(1-\beta\mu)^2}var\left(Y_S\right) > 0$$

Finally, the weighted variance of inflation term

$$\frac{\partial \frac{\theta}{\kappa} var\left(\pi_S\right)}{\partial \kappa} = -var\left(\pi_S\right)\frac{\theta}{\kappa^2} + \frac{\theta}{\kappa}\frac{\partial var(\pi_S)}{\partial \kappa}$$

$$\frac{\partial \frac{\theta}{\kappa} var\left(\pi_S\right)}{\partial \kappa} = -\left(\frac{\kappa}{1-\beta\mu}\right)^2 var\left(Y_S\right)\frac{\theta}{\kappa^2} + \frac{\theta}{\kappa}\left[\frac{(1-\mu)(1-\beta\mu)}{[(1-\mu)(1-\beta\mu)-\kappa\sigma\mu]}\right]\frac{2\kappa}{(1-\beta\mu)^2}var\left(Y_S\right)$$

$$\frac{\partial \frac{\theta}{\kappa} var\left(\pi_S\right)}{\partial \kappa} = -\frac{\theta}{(1-\beta\mu)^2}var\left(Y_S\right) + \left[\frac{(1-\mu)(1-\beta\mu)}{[(1-\mu)(1-\beta\mu)-\kappa\sigma\mu]}\right]\frac{2\theta}{(1-\beta\mu)^2}var\left(Y_S\right)$$

$$\frac{\partial \frac{\theta}{\kappa} var\left(\pi_S\right)}{\partial \kappa} = \left[-1 + \left[\frac{2(1-\mu)(1-\beta\mu)}{[(1-\mu)(1-\beta\mu)-\kappa\sigma\mu]}\right]\right]\frac{\theta}{(1-\beta\mu)^2}var\left(Y_S\right)$$

$$\frac{\partial \frac{\theta}{\kappa} var\left(\pi_S\right)}{\partial \kappa} = \left[\frac{(1-\mu)(1-\beta\mu)+\kappa\sigma\mu}{(1-\mu)(1-\beta\mu)-\kappa\sigma\mu}\right]\frac{\theta}{(1-\beta\mu)^2}var\left(Y_S\right) > 0$$

So,

$$\frac{\partial W}{\partial \kappa} < 0.$$

Appendix F. Wage and Price Flexibility

Appendix F.1. General model

Woodford (2003) presents a simple model with wage and price stickiness that can be summarized under a Taylor rule as

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(\hat{\imath}_t - E_t\pi^p_{t+1} - r^e_t) \tag{F.1}$$

$$\pi^p_t = \kappa_p(Y_t - Y^n_t) + \xi_p(\hat{w}_t - \hat{w}^n_t) + \beta E_t\pi^p_{t+1} \tag{F.2}$$

$$\pi_t^w = \kappa_w(Y_t - Y_t^n) - \xi_w(\hat{w}_t - \hat{w}_t^n) + \beta E_t \pi_{t+1}^w \tag{F.3}$$

$$\hat{\imath}_t = \phi_\pi \pi_t^p + \phi_y \hat{Y}_t \tag{F.4}$$

$$\hat{w}_t = \hat{w}_{t-1} + \pi_t^w - \pi_t^p \tag{F.5}$$

where we have that $\hat{w}_t^n = (1+\omega_p)a_t - \omega_p \hat{Y}_t^n$ and $\hat{Y}_t^n = \frac{1+\omega}{\sigma^{-1}+\omega}a_t - \frac{1}{\sigma^{-1}+\omega}\hat{\mu}_t + \frac{1}{\sigma^{-1}+\omega}\hat{\tau}_t^w$. Also, the welfare- objective around the efficient steady state is given by

$$L_t = \lambda_p(\pi_t^p)^2 + \lambda_w(\pi_t^w)^2 + \lambda_x(\hat{Y}_t - \hat{Y}_t^e)^2.$$

Here, we have $\xi_w = \frac{(1-\alpha_w)(1-\alpha_w\beta)}{\alpha_w(1+\nu\theta_w)}$, $\xi_p = \frac{(1-\alpha_p)(1-\alpha_p\beta)}{\alpha_p(1+\omega_p\theta_p)}$, $\kappa_w = \xi_w\left(\omega_w + \sigma^{-1}\right)$, $\kappa_p = \xi_p\omega_p$, $\kappa_w = \frac{(1-\alpha_w)(1-\alpha_w\beta)}{\alpha_w}\frac{\left(\omega_w+\sigma^{-1}\right)}{(1+\nu\theta_w)}$, $\kappa_p = \frac{(1-\alpha_p)(1-\alpha_p\beta)}{\alpha_p}\frac{\omega_p}{(1+\omega_p\theta_p)}$, $\lambda_p = \frac{\theta_p\xi_p^{-1}}{\theta_p\xi_p^{-1}+\theta_w\phi_h^{-1}\xi_w^{-1}} > 0$, $\lambda_w = \frac{\theta_w\phi_h^{-1}\xi_w^{-1}}{\theta_p\xi_p^{-1}+\theta_w\phi_h^{-1}\xi_w^{-1}} > 0$, and $\lambda_x = \frac{\sigma^{-1}+\omega}{\theta_p\xi_p^{-1}+\theta_w\phi_h^{-1}\xi_w^{-1}} > 0$. Moreover, $\nu \equiv \frac{v_{hh}h}{v_h}$, $\phi_h \equiv \frac{f(h)}{hf'(h)}$, $\omega_w = \nu\phi_h$, and $\omega = \omega_w + \omega_p$.

Assume the production function $y_t(i) = A_t h_t(i)^\gamma$ to get $\phi_h = 1/\gamma$, $\omega_w = \nu/\gamma$, and $\omega_p = \frac{1-\gamma}{\gamma}$.

Appendix F.2. Simplified Approximate model

Next, we make the assumption that simplifies the model and leads to sharp insights. We assume that $\kappa_p = \kappa_w = \kappa$. After some manipulation and using that $\Delta\hat{w}_t = \pi_t^w - \pi_t^p$, we obtain

$$\Delta w_t = -(\xi_w + \xi_p)(w_t - w_t^n) + \beta E_t \Delta w_{t+1}$$

and the solution for w_t can then be written as

$$w_t = \Gamma_w w_{t-1} + \Gamma_n w_t^n$$

where Γ_w is the root less than 1 of the polynomial $\mu^2 - \frac{\left(\beta+\kappa\left(\frac{1}{\omega_w+\sigma^{-1}}+\frac{1}{\omega_p}\right)+1\right)}{\beta}\mu + \frac{1}{\beta} = 0$ and $\Gamma_n = \frac{\Gamma_w}{1-\beta\Gamma_w\rho_A}\kappa\left(\frac{1}{\omega_w+\sigma^{-1}} + \frac{1}{\omega_p}\right)$.

As our second result, in this simplified case, the rest of the model equations reduce to three equations as given by

$$\pi_t^p = \kappa(Y_t - Y_t^n) + \beta E_t \pi_{t+1}^p + \kappa \frac{1}{\omega_p}(w_t - w_t^n)$$

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \sigma(i_t - E_t \pi_{t+1}^p - r_t^e)$$

$$i_t = \phi_\pi \pi_t^p + \phi_y \hat{Y}_t$$

This then implies that our previous result on demand shocks will fully go through in this case.

For productivity shocks, it is tedious to analytically show how the variance of output varies with κ. The solution of the model however can be shown in closed-form. For simplicity, assume log-utility ($\sigma = 1$) and i.i.d. technology shocks ($\rho_A = 0$). Then,

$$\hat{Y}_t = Y_A a_t + Y_w w_t$$

where

$$Y_A = \frac{\kappa\left(1 + \frac{1}{\omega_p}\right)}{\kappa + \frac{(\phi_y + 1)}{\phi_\pi}}; Y_w = -\frac{\kappa\left(\phi_\pi - \Gamma_w\right)}{\omega_p\left[\kappa\left(\phi_\pi - \Gamma_w\right) - \left(\Gamma_w - 1 - \phi_y\right)\left(1 - \beta\Gamma_w\right)\right]}.$$

This together with

$$w_t = \Gamma_w w_{t-1} + \Gamma_n w_t^n$$

and

$$w_t^n = a_t$$

completes the solution.

Appendix F.3. Discretion

The objective function is given by the following:

$$L_t = \lambda_p(\pi_t^p)^2 + \lambda_w(\pi_t^w)^2 + \lambda_x(\hat{Y}_t - \hat{Y}_t^e)^2$$
$$\lambda_p = \frac{\theta_p \xi_p^{-1}}{\theta_p \xi_p^{-1} + \theta_w \phi_h^{-1} \xi_w^{-1}} > 0, \lambda_w = \frac{\theta_w \phi_h^{-1} \xi_w^{-1}}{\theta_p \xi_p^{-1} + \theta_w \phi_h^{-1} \xi_w^{-1}} > 0, \lambda_x = \frac{\sigma^{-1} + \omega}{\theta_p \xi_p^{-1} + \theta_w \phi_h^{-1} \xi_w^{-1}} > 0$$
$$Y_t^e = \frac{1+\omega}{\sigma^{-1} + \omega} a_t$$

Given our specific assumptions, w_t is an exogenous process and hence there are no endogenous state variables in the model. This greatly simplifies things as the discretion problem just reduces to a period by period minimization problem. Also, note that our assumptions $\xi_w = \frac{1}{\omega_w + \sigma^{-1}}\kappa$, $\xi_p = \frac{1}{\omega_p}\kappa$ and $\Delta\hat{w}_t = \pi_t^w - \pi_t^p$ and the assumption of log utility (for expository reasons only) allow us to write the following Lagrangian after some manipulation:

$$\begin{aligned}\mathcal{L}_t = &\frac{1}{2}\left[\lambda_p(\pi_t^p)^2 + \lambda_w(\Delta\hat{w}_t + \pi_t^p)^2 + \lambda_x(\hat{Y}_t - \hat{Y}_t^e)^2\right] \\ &+ q_{1,t}\left\{\pi_t^p - \kappa(\hat{Y}_t - \hat{Y}_t^e) - \beta E_t \pi_{t+1}^p + \kappa\frac{1}{\omega_p}a_t - \kappa\frac{1}{\omega_p}w_t\right\}\end{aligned}$$

where the central bank will take expectation functions as given since there are no endogenous state variables and we use the fact that the IS equation is not binding. This yields the following FOCs:

$$\frac{\partial \mathcal{L}}{\partial \pi_t^p} = \lambda_p \pi_t^p + \lambda_w\left(\Delta\hat{w}_t + \pi_t^p\right) + q_{1,t} = 0$$
$$\frac{\partial \mathcal{L}}{\partial\left(\hat{Y}_t - \hat{Y}_t^e\right)} = \lambda_x(\hat{Y}_t - \hat{Y}_t^e) - \kappa q_{1,t} = 0$$

with the exogenous processes

$$w_t = \Gamma_w w_{t-1} + \frac{\beta\Gamma_w}{1 - \beta\Gamma_w \rho_A}\kappa\left(\frac{1}{\omega_w + 1} + \frac{1}{\omega_p}\right)a_t$$
$$\hat{Y}_t^e = a_t$$

Now combine the two FOCs to get the targeting rule, which is our main result here

$$\left(\lambda_p + \lambda_w\right)\pi_t^p + \lambda_w \Delta\hat{w}_t + \frac{\lambda_x}{\kappa}(\hat{Y}_t - \hat{Y}_t^e) = 0.$$

Assuming i.i.d. shock for simplicity, one can derive the solution of the model in closed form:

$$\pi_t^p = \pi_{w1}\hat{w}_{t-1} + \pi_w \hat{w}_t + \pi_A a_t$$

$$\pi_{w1} = \frac{\frac{\kappa}{(1+\omega)}\theta_w \phi_h^{-1}(\omega_w + 1)}{\left(1 + \frac{\kappa\left(\theta_p\omega_p + \theta_w\phi_h^{-1}(\omega_w+1)\right)}{(1+\omega)}\right)}$$

$$\pi_w = \left[\frac{1}{\left(1 + \frac{\kappa\left(\theta_p\omega_p + \theta_w\phi_h^{-1}(\omega_w+1)\right)}{(1+\omega)}\right) - \beta\Gamma_w}\right]\left[\beta\frac{\frac{\kappa}{(1+\omega)}\theta_w \phi_h^{-1}(\omega_w + 1)}{\left(1 + \frac{\kappa\left(\theta_p\omega_p + \theta_w\phi_h^{-1}(\omega_w+1)\right)}{(1+\omega)}\right)} - \frac{\kappa}{(1+\omega)}\theta_w\phi_h^{-1}(\omega_w+1) + \kappa\frac{1}{\omega_p}\right]$$

$$\pi_A = \frac{1}{\left(1 + \frac{\kappa\left(\theta_p\omega_p + \theta_w\phi_h^{-1}(\omega_w+1)\right)}{(1+\omega)}\right)}\left[-\kappa\frac{1}{\omega_p}\right]$$

For the output gap, we have

$$\left(\theta_p\omega_p + \theta_w\phi_h^{-1}(\omega_w+1)\right)\pi_t^p + \theta_w\phi_h^{-1}(\omega_w+1)\Delta\hat{w}_t + (1+\omega)(\hat{Y}_t - \hat{Y}_t^e) = 0$$

which gives

$$(\hat{Y}_t - \hat{Y}_t^e) = -\frac{\left(\theta_p\omega_p + \theta_w\phi_h^{-1}(\omega_w+1)\right)}{(1+\omega)}\pi_t^p - \frac{\theta_w\phi_h^{-1}(\omega_w+1)}{(1+\omega)}(\hat{w}_t - \hat{w}_{t-1})$$

and for output, since $\hat{Y}_t^e = a_t$

$$\hat{Y}_t = -\frac{\left(\theta_p\omega_p + \theta_w\phi_h^{-1}(\omega_w+1)\right)}{(1+\omega)}\pi_t^p - \frac{\theta_w\phi_h^{-1}(\omega_w+1)}{(1+\omega)}(\hat{w}_t - \hat{w}_{t-1}) + a_t$$

Appendix F.4. Commitment

In the case of commitment, we have:

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left[\lambda_p(\pi_t^p)^2 + \lambda_w(\Delta\hat{w}_t + \pi_t^p)^2 + \lambda_x(\hat{Y}_t - \hat{Y}_t^e)^2\right]\\ &+ E_0\sum_{t=0}^{\infty}\beta^t q_{1,t}\left\{\pi_t^p - \kappa(\hat{Y}_t - \hat{Y}_t^e) - \beta E_t\pi_{t+1}^p + \kappa\frac{1}{\omega_p}a_t - \kappa\frac{1}{\omega_p}w_t\right\}\end{aligned}$$

where the central bank can commit and hence does not take $E_t \pi_{t+1}^p$ as given

$$\frac{\partial \mathcal{L}}{\partial \pi_t^p} = \lambda_p \pi_t^p + \lambda_w \left(\Delta \hat{w}_t + \pi_t^p \right) + q_{1,t} - q_{1,t-1} = 0$$
$$\frac{\partial \mathcal{L}}{\partial \left(\hat{Y}_t - \hat{Y}_t^e \right)} = \lambda_x (\hat{Y}_t - \hat{Y}_t^e) - \kappa q_{1,t} = 0$$

with the exogenous processes

$$w_t = \Gamma_w w_{t-1} + \frac{\beta \Gamma_w}{1 - \beta \Gamma_w \rho_A} \kappa \left(\frac{1}{\omega_w + 1} + \frac{1}{\omega_p} \right) a_t$$
$$\Delta \hat{w}_t = \left(\Gamma_w - 1 \right) w_{t-1} + \frac{\beta \Gamma_w}{1 - \beta \Gamma_w \rho_A} \kappa \left(\frac{1}{\omega_w + 1} + \frac{1}{\omega_p} \right) a_t$$
$$\hat{Y}_t^e = a_t$$

Now combine the two FOCs to get the targeting rule

$$\left(\lambda_p + \lambda_w \right) \pi_t^p + \lambda_w \Delta \hat{w}_t + \frac{\lambda_x}{\kappa} (\hat{Y}_t - \hat{Y}_t^e) - \frac{\lambda_x}{\kappa} (\hat{Y}_{t-1} - \hat{Y}_{t-1}^e) = 0.$$

The closed-form solution of the model is not very instructive, although possible.

Appendix G. Sticky Information Model

Appendix G.1. Pure Sticky Information Model

- Flow welfare-based loss function

$$\frac{\sigma+\eta}{\theta\left(1+\theta\eta\right)}x_t^2+\sum_{i=1}^{\infty}\zeta_i\left(p_t-E_{t-i}p_t\right)^2$$

where $x_t = y_t - y_t^e$ and $\zeta_i = \frac{\delta_{si}^i(1-\delta_{si})}{\left(1-\delta_{si}^i\right)\left(1-\delta_{si}^{i+1}\right)}$

- Efficient rate of output

$$y_t^e=\frac{1+\eta}{\sigma+\eta}a_t$$

- Natural level of output

$$y_t^n=\frac{1+\eta}{\sigma+\eta}a_t-\frac{1}{\sigma+\eta}\mu_t$$

where a_t is technology shocks and μ_t is markup shocks.

- IS equation

$$\lambda_t=E_t\lambda_{t+1}+i_t-E_t\pi_{t+1}$$

- Marginal utility of consumption

$$\lambda_t=\psi_t-\sigma y_t$$

- Instantaneous optimal (relative) desired price

$$\begin{aligned}p_t^{\#}&=p_t^*-p_t\\&=\frac{\sigma+\eta}{1+\theta\eta}\left(y_t-y_t^e\right)+\frac{1}{1+\theta\eta}\mu_t\end{aligned}$$

- Sticky-information sector price dynamics

$$\pi_t=\frac{1-\delta_{si}}{\delta_{si}}p_t^{\#}+\left(1-\delta_{si}\right)\sum_{k=0}^{\infty}\delta_{si}^kE_{t-1-k}\left(\pi_t+\kappa\Delta y_t\right)$$

- Policy rule and shock processes

$$i_t = \phi_\pi \pi_t + \phi_y y_t$$
$$\psi_t = \rho_\psi \psi_{t-1} + \varepsilon_t^\psi$$
$$a_t = \rho_\psi a_{t-1} + \varepsilon_t^a$$
$$\mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t^\mu$$

We set $\beta = 0.99$, $\sigma = 1$, $\eta = 1$(Frisch elasticity of labor supply), $\theta = 10$ (CES parameter), $\kappa = \frac{\sigma+\eta}{1+\eta\theta} = 0.1818$. Here, $1-\delta_{si}$ is the probability of updating information for firms in sticky-information world.

Appendix G.2. Sticky Information and Sticky Price Model

- Household preference

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \psi_t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\int N_t(j)^{1+\eta} dj}{1+\eta} \right) \right]$$

- (Linearized) Dynamic IS curve

$$\lambda_t = E_t[\lambda_{t+1}] + i_t - E_t[\pi_{t+1}]$$
$$\lambda_t = \hat{\psi}_t - \sigma y_t$$

- Instantaneous optimal desired price

$$p_t^* = p_t + \frac{\sigma+\eta}{1+\theta\eta} \left(y_t - \frac{1+\eta}{\sigma+\eta} a_t \right)$$

- Optimal reset price for firm with ability of change price

$$\begin{aligned} b_t &= (1-\beta\delta) \sum_{k=0}^{\infty} (\beta\delta)^k E_t p_{t+1}^* \\ &= (1-\beta\delta) p_t^* + \beta\delta E_t b_{t+1}^* \end{aligned}$$

- Sticky-price sector price dynamics

$$p_t^{sp} = (1-\delta) b_t + \delta p_{t-1}^{sp}$$

- Sticky-information sector price dynamics

$$p_t^{si} = (1-\lambda)\, p_{t-1}^{si} + \lambda p_t^* + \lambda (1-\lambda) \sum_{k=0}^{\infty} (1-\lambda)^k E_{t-1-k} (\pi_t + \kappa \Delta y_t)$$

- Aggregate price level

$$p_t = s p_t^{sp} + (1-s)\, p_t^{si}$$

(in steady-state $\frac{P^{sp}}{P} = \frac{P^{si}}{P} = 1$).

- Policy rule and shock processes

$$i_t = \phi_\pi \pi_t + \phi_y y_t$$

$$\hat{\psi}_t = \rho_\psi \hat{\psi}_{t-1} + \varepsilon_t^{\psi}$$

$$a_t = \rho_\psi a_{t-1} + \varepsilon_t^{a}$$

We set $\beta = 0.99$, $\sigma = 1$, $\eta = 1$ (Frisch elasticity of labor supply), $\theta = 10$ (CES parameter), $\kappa = \frac{\sigma+\eta}{1+\eta\theta} = 0.1818$. Also, $s = 0.75$ (fraction of firms under sticky-price constraint). Here, $1 - \delta_{sp}$ is the probability of getting chance to change price for firms in sticky-price world, $1 - \delta_{si}$ is the probability of updating information for firms in sticky-information world.

Appendix H. Tables

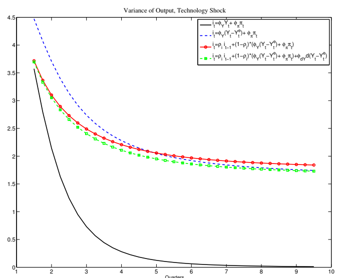
Table H.6: Illustrative Parameter Values

Parameter	Value	Parameter	Value
β	0.99	σ	1
κ	0.02	θ	10
ρ_j	0.9	$\theta_w = \theta_p$	10
γ	0.7	ϕ_y only	0.125
ν	1	$\phi_\pi; \phi_y$	$1.5, 0.2; 0.125, 20$

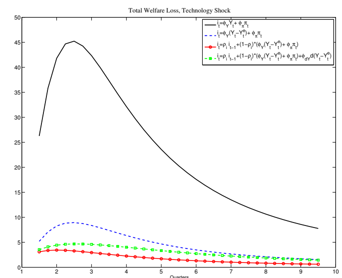
NOTE: The parameter β denotes the rate of time preference, σ the intertemporal elasticity of substitution, θ the elasticity of substitution among different varieties of goods, ν the Frisch elasticity of labor supply, $\kappa = \frac{\sigma+\nu}{1+\nu\theta}$ the degree of price flexibility, ϕ_π the systematic response to inflation in the Taylor rule, ϕ_y the systematic response to output in the Taylor rule, γ the returns to scale and θ_w and θ_p the elasticity of substitution across differentiated varieties.

Appendix I. Figures

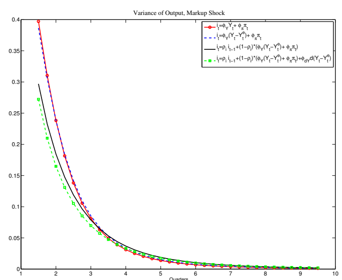
Figure I.10: Variance of Output and Welfare under Alternative Policy Rules



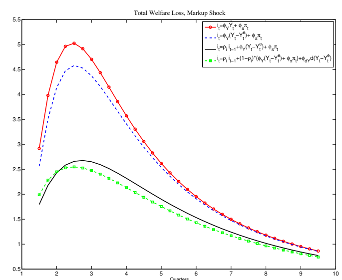
(a) Output Volatility and Technology Shock



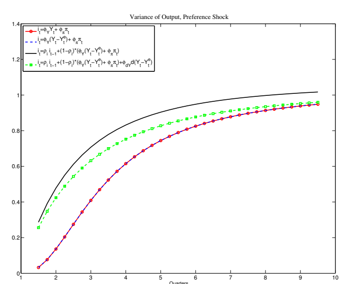
(b) Welfare and Technology Shock



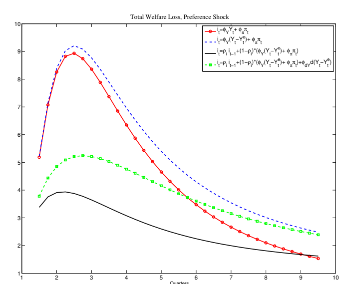
(c) Output Volatility and Markup Shock



(d) Welfare and Markup Shock



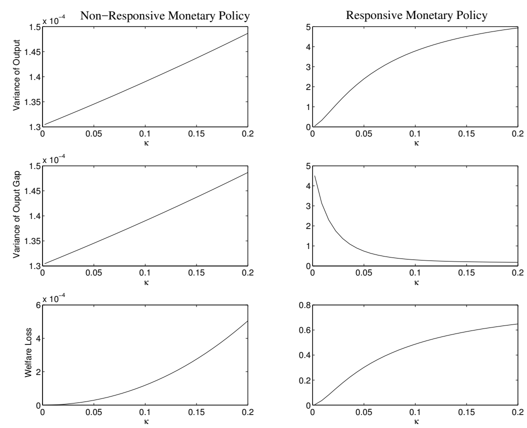
(e) Output Volatility and Preference Shock



(f) Welfare and Preference Shock

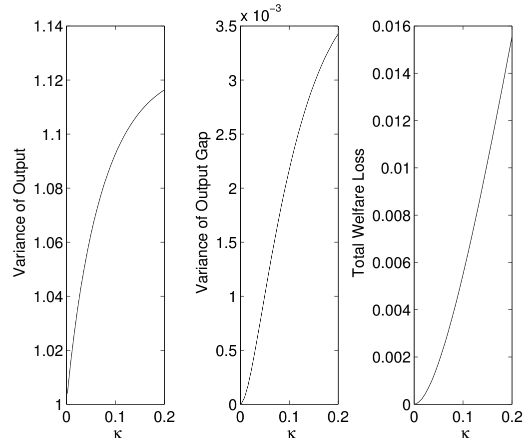
NOTE: This figure shows the volatility of output and welfare in our simple three-equation model under alternative policy rules as we vary the expected duration of price contracts, as discussed in section 4. The policy rules are shown in the legends of the panels. We consider technology, markup and preference shocks.

Figure I.11: Variance of Output, and Welfare Loss as a Function of Flexibility with Technology Shocks



NOTE: This figure shows variance of output, and the welfare loss as a function of price flexibility given technology shocks.

Figure I.12: Variance of Output and Welfare Loss under Discretion as a Function of Flexibility with Technology Shocks



NOTE: This figure shows variance of output, and the welfare loss under discretion as a function of price flexibility given technology shocks.