It's Baaack: The Surge in Inflation in the 2020s and the Return of the Non-Linear Phillips Curve*

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Abstract

This paper proposes a non-linear New Keynesian Phillips curve ⅃ (Inv-L NK Phillips Curve) to explain the surge of inflation in the 2020s. Economic slack is measured as firms' job vacancies over the number of unemployed workers. After showing empirical evidence of statistically significant non-linearities, we propose a New Keynesian model with search and matching frictions, complemented by a form of wage rigidity, in the spirit of Phillips (1958), that generates strong nonlinearities. Policy implications include the thesis that appropriate monetary policy can bring inflation down without a significant recession and that the recent inflationary surge was mostly generated by a " labor shortage" – i.e. an exceptionally tight labor market, which in addition to triggering inflation pressures by itself, amplifies the impact of other supply shortages.

—- Preliminary, currently undergoing substantial revision —

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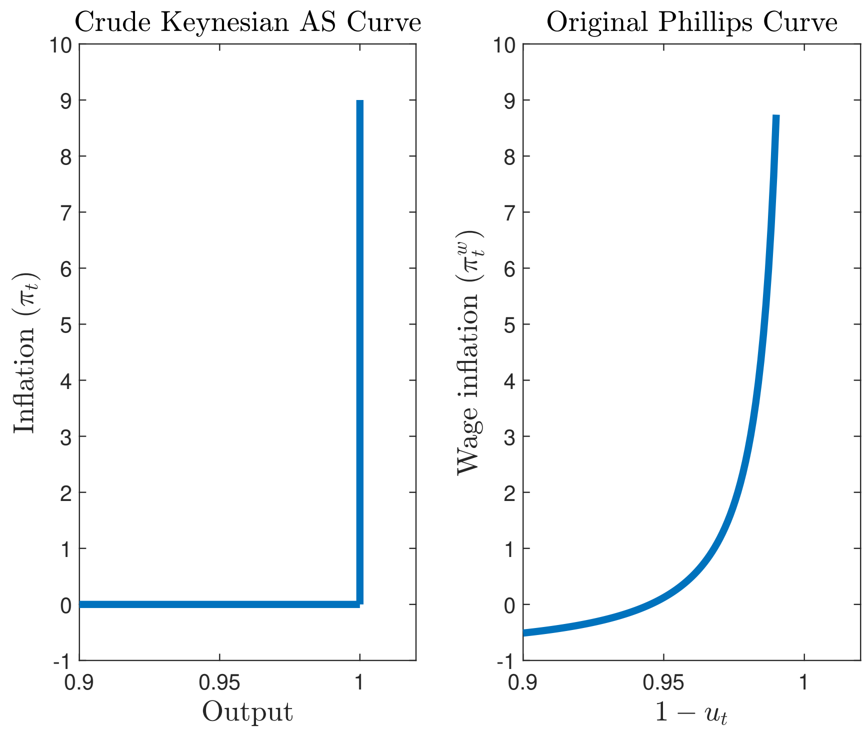

Figure 1: The crude Keynesian Phillips curve versus the original Phillips curve proposed and estimated by Phillips in 1958.

1 Introduction

This paper proposes to replace the canonical New Keynesian Phillips curve with an Inverted-L New Keynesian Phillips curve (Inv-L NK Phillips curve), which is nonlinear. The Inv-L NK Phillips curve can explain the sharp, unexpected rise in inflation in the U.S. in the early 2020s following the COVID-19 pandemic. Demand shocks are seen to have played a major role in explaining the surge, which carries policy implications. At the same time, if the economy enters the non-linear part of the Phillips curve, this supercharges the effect of supply shocks on inflation. According to our account supply shocks also played a major role in the inflation surge, especially early on. Another implication is that appropriate monetary policy can engineer a "soft landing."That is, for each percentage point decrease in inflation, the Federal Reserve can achieve a smaller increase in unemployment compared to the Volcker recession, which successfully curbed inflation following the 1970s' Great Inflation but at the expenses of a substantial increase in unemployment.

The U.S. time series data show statistically significant and quantitatively important support for a non-linear Phillips curve. Moreover, the data indicates increased amplification of supply shocks once the non-linearity kicks in. Below we set out the broader motivation for our approach, the historical background, and the main contribution of this paper.

The early Keynesian literature assumes a stark relationship between prices and output, namely the inverted (or backward) L in the left-hand panel of Figure 1. Blinder (2022) labels this "crude Keynesianism."When output is below potential, there are idle workers and empty factories, in which case prices are fixed. Once all workers are employed and factories are in full swing, output can go no higher, and aggregate supply becomes vertical. In this case the economy enters a neoclassical world of flexible prices. Keynesianism took a sharp turn with Phillips' seminal work (1958), one of those papers that are so well known that they are rarely cited even in the literature that builds on them. Instead, the author's name has simply become synonymous with the *Phillips curve*.[1]

The Phillips curve is the best known and most controversial equation in macroeconomics. Its most common version is:[2]

$$\underbrace{\pi_t = \kappa x_t}_{\substack{\text{Keynesian} \\ \text{Phillips Curve}}} \quad + \quad \underbrace{\varrho_t + \beta E_t \pi_{t+1}}_{\substack{\text{New Keynesian} \\ \text{Phillips Curve}}} \tag{1}$$

where π_t is inflation, κ is a coefficient, and x_t is some measure of economic activity such as the output gap, in which case $\kappa > 0$, or the unemployment gap, in which case $\kappa < 0$. The coefficient β is between 0 and 1, E_t is an expectation operator and ϱ_t is a supply shock.

The first part of equation (1), indicated by the first curly bracket, is the Keynesian Phillips curve popular in the 1960s. Suggested by Samuelson and Solow (1960), it implied a stable trade-off between inflation and the output gap. The second term, which includes a supply shock and expectations on future inflation, was emphasized by the literature that developed after the 1970s. The formulation in equation (1) is known as the New Keynesian Phillips curve, and it is currently the backbone of most macroeconomic models.

In the course of the 1970s the Keynesian Phillips curve collapsed as a stable statistical relationship (see Figure 13 in the Appendix). This empirical failure had a decisive impact on macroeconomics. Arguably, it gave birth to the rational expectations revolution and made microfoundations for macroeconomic models mainstream, partially because they offer an explicit account of how expectations are formed. What made the demise of the Keynesian Phillips curve dramatic – a watershed moment for macroeconomics – was that it had been prominently foreseen by Phelps (1967) and Friedman (1968). They predicted that a breakdown of this kind *should* occur – as a matter of theory — if the government tried to exploit the inflation-unemployment trade-off, because inflation expectations catch up and endogenously shift the relationship as in equation (1). With inflation rising in the late 1960s, it looked to many observers as if the government was indeed accepting excessive inflation in return for higher employment. That the relationship broke down in the 1970s, just as predicted, lent considerable credibility to Friedman and Phelps's prophecy.

The main explanation now put forward for the demise of the Keynesian Phillips curve is the combination of the unanchoring of inflation expectations in the 1970s and supply disruptions. The presence

[1] Like the original paper by John Nash, which is seldom cited when Nash equilibrium is applied in game theory.
[2] See e.g. Woodford (2003) and Galí (2015) for textbook treatments.

of expected inflation in (1), which is central to any modern Phillips curve, explains central banks' focus today on anchoring inflation expectations. As this relationship makes clear, higher inflation expectations have an effect similar to that of a negative supply shock.

Someone reading Phillips' classic paper today will realize immediately that modern research pays little or no attention to its central proposition. The Phillips of 1958 would hardly recognize the linear relationship (1) as his own construction, even apart from the terms that represent expectations.[3] First, the relationship Phillips suggests is between unemployment and wage inflation. Second, and more importantly, his point is that this relationship *is strongly non-linear*. The curve proposed in Phillips (1958) is

$$\pi_t^w = a + b\left(\frac{1}{u_t}\right)^c$$

where π_t^w is wage inflation, u_t is the unemployment rate and a, b, c are estimated coefficients.

The original Phillips curve is plotted in the right-hand panel of Figure 1 using the coefficient values Phillips estimated for the period 1861-1913, with $1 - u_t$ plotted on the x-axis representing a higher level of economic activity to relate it to the crude Keynesianism on the left.[4] As the figure reveals, this relationship is strongly non-linear and resembles the inverted L of "crude Keynesianism" much more closely than what became synonymous with his name, largely due to when the Phillips curve came to America via Samuelson and Solow (1960) who did not highlight the non-linearities.

While Phillips' paper is empirical, his theoretical argument for nonlinearity is straightforward. He writes that with "very few unemployed we should expect employers to bid up wages quite rapidly, each firm and each industry being continually tempted to offer a little above the prevailing wage." But why does the asymmetry arise when unemployment is high? Phillips suggests that "workers are reluctant to offer their services at less than the prevailing rate"so "wages fall only very slowly".[5] A very non-linear curve is Phillips's central proposition and the concluding line of his opening paragraph.

The main objective of this article is to resurrect Phillips' original idea in order to explain the surge in inflation in the 2020s. We first present suggestive time-series empirical evidence. We then propose a theoretical model for a non-linear Phillips curve and explore policy implications.

The recent increase in inflation took policy makers by surprise, at least if we go by the Summary of Economic Projections (SEP) of the Federal Reserve. It was also unexpected by private forecasters, as judged by Survey of Professional Forecasters (SPF), as shown in Figures 14 and 15 in the Appendix. Neither SEP nor SPF anticipated the surge in prices in 2021. And both consistently projected it to decline rapidly to the Fed's 2 percent target rate. Instead, inflation continued to increase in the following quarters.

[3] Phillips did, however, recognize that supply shocks could be an important factor in shifting his proposed empirical relationship.

[4] This is Figure 1 in Phillips (1958). The paper estimates b=9.636 and c=1.394 via least squares and sets a=-0.9 via "trial and error" based on data from the U.K. between 1861 and 1913.

[5] See also the discussion by Tobin (1972).

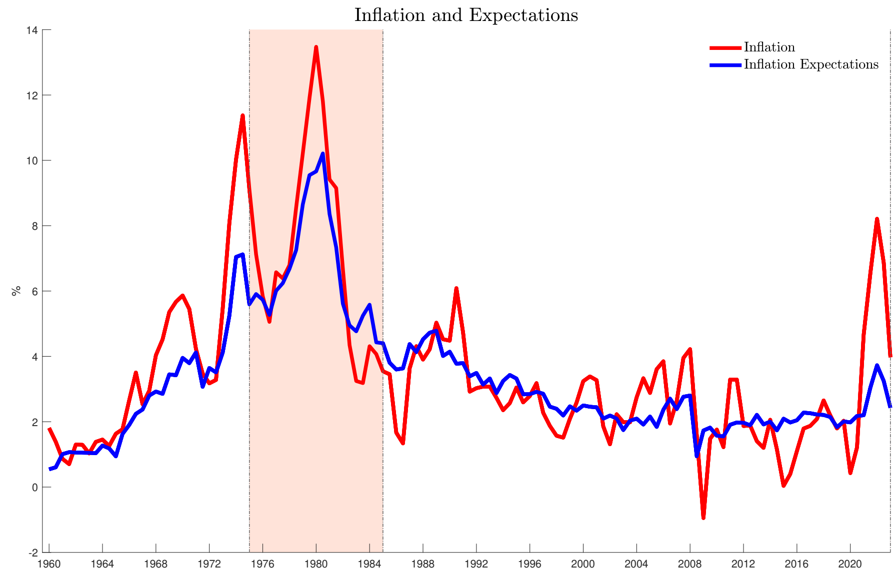

Figure 2: Inflation: CPI inflation rate at annual rates. 12-month Livingston inflation expectations.

The reason why both private forecasters and policy makers were caught flatfooted is largely explained by three broad observations, which motivate our approach.

First, the conventional wisdom, forged by the realization of Phelps and Friedman's prophecy in the 1970s, is that persistent increases in inflation are triggered when inflation expectations become unanchored. During the 2020s, however, there was no increase in inflation expectations comparable to that of the 1970s, and both policymakers and market participants accordingly thought that 2021 surge was transitory. Figure 2 shows inflation expectations according to the Livingstone survey, which asks participants what they expect inflation to be in a year. Whereas this measure had peaked at 10 percent in the late 1970s, in the 2020s it rose only modestly. Several alternative measures of expectation that proxy longer-run inflation expectations are even more striking. These measures suggest that the Great Inflation of the 1970s unanchored expectations so drastically that at their peak people were expecting inflation of 10 percent to persist even over a five-to-ten year horizon. Several such proxies, reported in the Appendix, are used for robustness checks of the empirical analysis in the next section.[6] In short, one of the key suspects for driving the Great Inflation of the 1970s was simply missing in the 2020s.

A second main reason why forecasters missed the surge is that most estimates of the slope of the Phillips curve leading up to the inflation surge, i.e. κ in equation (1), are very low. This view was reflected in virtually all the models used for inflation projections at policy institutions (see e.g. the

[6] In the Appendix we show the Cleveland Fed's five-year inflation expectations since 1982. The expectations were at 6 percent in 1982 even though current inflation was lower. As of this writing (December 2023), they stand at 2.36 percent and have never gone above 2.6 percent during the surge. Even more compelling is the evidence on the five-year five-year forward inflation rate, what markets expect inflation to be five years ahead. It has held remarkably stable throughout the 2020s surge. While a market-based measure such as this is only available from 1997, Groen and Middledorp (2013) identifies 108 dataseries to back-cast this measure to 1970 using partial least-squares. They find that it peaked at 10 percent during the Great Inflation of the 1970s indicating that market participants thought the high inflation was there to stay (see Figure 16 in the Appendix). The Blue Chip Economic indicators of ten-year inflation expectation (dating back to 1980), maintained by the Philadelphia Federal Reserve, show similar results. At the end of 1980, for example, ten-year inflation expectations were 8.4 percent.

discussion in Gopinath (2023) of the IMF model).[7] According to the widely cited recent estimate of Hazzell, Herreno, Nakamura and Steinsson (2022), for example, a 1-percentage-point reduction in unemployment generates only a 0.34-point increase in inflation. Their analysis uses a carefully designed identification strategy based on a cross-section of Metropolitan Statistical Areas in the United States for 1978-2018. To summarize, while a number of commentators suggested that demand was considerably above potential output, the mainstream macroeconomic models predicted that this should have a modest inflationary impact.[8]

Finally, a third culprit in the 1970s was supply disruptions. A key difficulty of explaining the inflation of the 2020s by supply distributions alone, is that conventional measures which researchers used *prior to the inflation surge*, such as the difference between headline and core inflation or the difference between the rates of change in the import-price and GDP deflators did not increase nearly as much in the 2020s as in the 1970s, and where generally not quantitatively much out of line with supply disruption observed since the 1970s, as we show in Figure 17 in the Appendix. On the surface, then, supply disruptions seem like a less plausible candidate for the inflation surge in the 2020s relative to the 1970s, especially as inflation expectations have remained anchored, and as long as one only considers supply measures, which were considered relevant prior to the surge, rather than measures researchers have used to rationalize the inflation surge *ex post*.[9] As we will see, however, even if we consider supply disturbances measured in the conventional way, they can have a major impact when interacted with a tight labor market so the economy enters the non-linear part of the Phillips curve.[10]

This brings us to the labor market. A key statistic both in our empirical analysis and in the modeling framework is *labor market tightness*, captured by $\theta_t = \frac{\text{Job Vacancies}}{\text{Unemployed workers}}$, which is a standard measure tightness in the search and matching literature (for a recent contribution in this literature see e.g. Michaillat and Saez, 2023). The numerator, i.e. the number of vacancies at firms, summarizes how many jobs firms are seeking to fill, while the denominator is the number of job seekers. This variable is plotted in the top panel of Figure 3 for the period 1960-2022, which gives the reader a hint to our main hypothesis, The average value of θ, conditional on its being lower than 1, is 0.56. This is relatively close to our empirical assessment of the state of the labor market as neither inflationary nor deflationary, even if the exact estimate is subject to uncertainty.

On two occasions, however, this metric is greater than θ^* which we define as the threshold identifying a labor market shortage. The exact value of θ^* is an empirical question we will address and find that $\theta^* = 1$ is a reasonable approximation based on an empirical criteria discussed in next section. This definition of *labor shortage*, means that there are more jobs firms are looking to fill, than there are

[7]Gopinath draws the conclusion that the failure of the models currently used by major institutions to predict the inflation surge are plausibly explained by the fact that there may be "important nonlinearities in the Phillips curve slope: price and wage pressures from falling unemployment become more acute when the economy is running hot than when it is below full employment." This is precisely the type of mechanism our model formalizes.

[8]Most prominently, in a Washington Post op-ed of 5 February 2021, Lawrence Summers (2021) issued an early warning that the fiscal stimulus planned was big enough to push demand substantially above potential output and "set off inflationary pressures of a kind we have not seen in a generation."

[9]Refer to Benigno et al. (2022) for a new index capturing global supply chain pressures.

[10]During the early part of the surge, for example, there were widespread reports of temporary bottlenecks in the supply chain, which led many see the inflation spike as transitory.

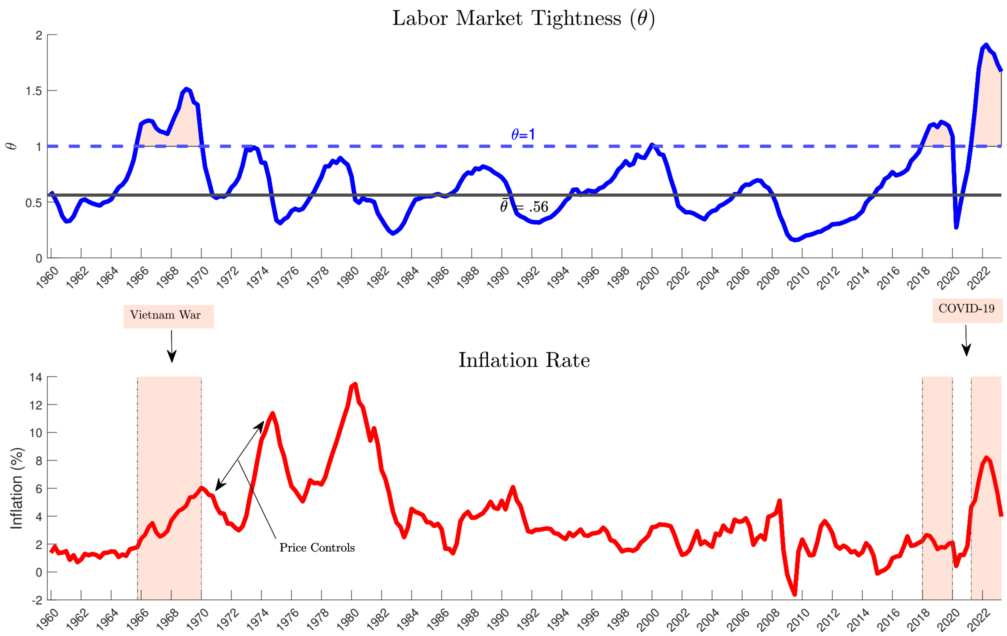

Figure 3: Top panel: (θ) vacancy-to-unemployed ratio. Bottom panel: CPI inflation rate at annual rates.

workers looking for jobs. This is how Beverdige, the founding father of the labor market literature, defined a tight labor market. It is also the criterion Maichaillat and Saez, (2023) discuss to imply that the labor market is inefficiently tight.[11] In the last 63 years, situations of labor shortage have emerged on two occasions: during the Vietnam War in the late 1960s a period also marked by a substantial demand stimulus, as documented by Blinder (2022), and the period around the COVID-19 epidemic. Both periods lie outside the typical econometric estimates of the slope of the Phillips curve. But they also correspond to inflation surges, as shown by the shaded region in the lower half of Figure 3. In other words, labor shortages go hand in hand with inflationary surges. Yet, since labor shortages are rare, our findings are consistent, for example, with Hazell et al (2022) findings and a number of other studies that focus on periods in which there are no labor shortages.

Over the past century U.S. economic history documents three other major episodes of acute labor shortages (i.e. $\theta > \theta^*$): World War I, World War II, and the Korean War. All three episodes were also associated with inflationary surges as shown in Figure 18 in the Appendix. However, as documented by Rockoff (1981), they were also accompanied by comprehensive price controls, which complicates empirical inference. For this reason, we focus on the period from 1960 to 2022.[12]

Sometimes a figure says more than a thousand words. Figure 4 shows a scatter plot between the two major data-series at the heart of the empirical analysis: raw annualized inflation rates and the

[11] While $\theta^* = 1$ is a natural benchmark, the analysis suggests that this value may fluctuate across regions, countries and time, even if we leave full exploration of this to future research.

[12] Rockoff (1981) also discusses the price controls from August 1971 to April 1974 implemented by President Nixon, as shown in Figure 3. This period of price controls is less problematic for the empirical analysis since there was no labor shortage at the time, and our emphasis is on documenting nonlinearity when $\theta > 1$.

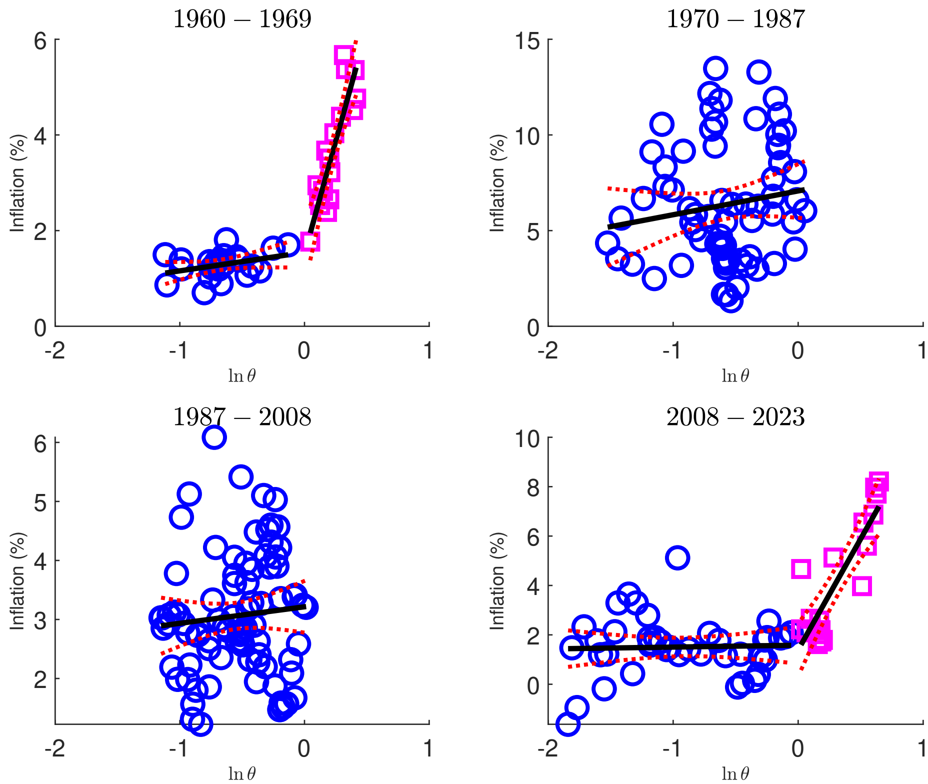

Figure 4: Inflation: CPI inflation rate at annual rates. $\ln\theta$: log of the vacancy-to-unemployed ratio.

logarithm of labor market tightness at quarterly frequency.[13] The periods of labor shortage, namely 1960-1969 and 2008-2022, strongly suggest that an inverted L shaped Phillips curve lurks behind the scene. The other two periods, typically used in studies of Phillips curves, show $\log\theta < 0$ and – at least at first glance – a relatively flat Phillips curve. Figure 19 in Appendix B shows similar patterns when price inflation is replaced with wage inflation.

The first main contribution of this paper is empirical. We add more structure to the data, in line with the earlier literature, by considering a series of regressions with both fixed and time-varying coefficients, adding controls, creating proxies for expectations, using an instrumental variable approach and suggesting two alternative ways of testing for nonlinearities.

The empirical analysis leads to two major findings. First, there is evidence in favor of significant non-linearities in the Phillips curve that are statistically significant. This is in line with the visual impression given by Figure 4: In periods of labor shortage, labor market tightness has much larger impact on inflation. Second, supply shocks have a larger impact when there is labor shortage and this increase is statistically significant.

The second main contribution of the paper is to resurrect Phillips' original idea of an inverted-L curve as in Figure 1 within the canonical New Keynesian framework. We think it is important to retain the

[13] By expressing this ratio in logs, it is irrelevant which variable enters in the denominator or the numerator. See the empirical section and the Appendix for a description of how this variable is constructed.

core elements of the New Keynesian framework because it highlights the role of the anchoring of inflation expectations. Our objective is a theory that accounts for both the Great Inflation of the 1970s (when the labor market was slack but inflation expectations unanchored) and the inflation surge of the 2020s (when the labor market was tight but inflation expectations firmly anchored). Accounting for both episodes is necessary if one is to have any hope of using the model to interpret the time series data and obtain statistically significant results.

While the elements of our model that generate inflation expectations in the Phillips curve are standard, drawn from the New Keynesian literature, the non-linearities arise due to an imperfect labor market. Households decide how many people enter the labor force but some workers cannot find jobs. This is modeled via standard search and matching function. In the spirit of Phillips, there are non-linearities in how wages are set. While firms are tempted to outbid one another when the labor market is tight, and workers are happy to accept higher wages, the margin for cutting wages is limited during times of high unemployment. This is because workers refuse to accept jobs that pay much below the prevailing wage despite the elevated levels of unemployment. Our assumption relies on the extensive empirical literature documenting that wages are "downward rigid,"providing support for Phillips' hypothesis.[14]

We derive a Phillips curve of the same form as (1) with three major differences. First, instead of the output or unemployment gap, the explanatory variable, i.e. x_t in equation (1), is labor market tightness, measured by the log deviation of θ from its steady state. Second, when there is labor shortage, i.e. $\theta > \theta^*$, then the slope κ changes and becomes steeper than normal. This captures the key non-linearity that Phillips emphasized and arises due to the same force that he proposed. Third, when the labor market is tight, supply shocks are transmitted to inflation with greater force. Meanwhile, inflation expectations play a major role in either regime, their relative strength depending on the details of the model specification.

Our work is related to several strands of the literature. We contribute to labor market models in the spirit of the search and matching framework of Mortensen and Pissarides (1994), among others. We introduce employment agencies that optimally choose how many vacancies to post, which pins down wages, resolving the classic problem of wage indeterminacy (see for one, Hall, 2005). In addition, we impose a constraint on the employment-agency optimization problem by introducing a wage norm that only adjusts gradually under normal circumstances, yet has as an ancor the wages that would prevail under flexible wage setting.

We cast our model in a NK framework with price rigidities, thus presenting a theory of the relationship between unemployment and monetary policy alternative to Blanchard and Galì (2010), Galì (2009) and Michaillat (2014) among others. We also contribute to the extensive literature following

[14]The idea of downward rigid nominal wages dates back at least to Malthus who noted that "it very rarely happens that the nominal price of labour universally falls", Malthus (1798). Bewley (1999) interviewed corporate executives documenting their reluctance to cut nominal wages. More recently, substantial nominal wage rigidity has been studied in U.S, administrative data by Fallick, Lettau and Wascher (2011), in worker surveys by Barattieri, Basu and Gottschalk (2014), and in cross-country data by Schmitt-Grohe and Uribe (2016).

Phillips (1958), which formalizes the relationship between inflation and various measures of economic activity. Our contribution is to derive the Inv-L NK Phillips curve when the measure of economic slack is given by the vacancy-to-unemployed ratio combined with assumptions about wage setting that give rise to non-linearities.

The idea that Blinder (2022) labels crude Keynesianism, depicted in Figure 1, is also closely related to Friedman (1964, 1993)'s "plucking model", according to which "output is viewed as bumping along the ceiling of maximum feasible output every now and then." This is also the implication of the crude Keynesian view, and corresponds closely to the non-linearity in the original Phillips curve modeled here. Dupraz, Nakamura and Steinsson (2019) provide evidence in favor of the plucking model and a theoretical framework to support it appealing, like us, to downward rigid wages. The idea of an inverse L shaped supply function is also found in much of the literature on secular stagnation, such as Eggertsson, Mehrotra and Robbins (2019) using similar modeling devices as well as works by Benigno and Ricci (2011) and Schmitt-Grohe and Uribe (2016) emphasizing downward rigid wages. Our theoretical contribution relative to these works is twofold. First, we incorporate an inverse-L aggregate supply mechanism into an otherwise standard New Keynesian model with forward looking price setting. The forward looking price setting is critical to account for both the inflation surge of the 2020s as well as that of the 1970s. Second, we explicitly integrate v/u into the model, which has been found to be empirically relevant in recent years, as discussed in the next subsection.

Gagliardone and Gertler (2023) also seek to account for the recent inflation surge, but where we explain it via nonlinearities in the Phillips curve, they see the cause as oil price spikes coupled with easy monetary policy. Another study highlighting nonlinearities in the Phillips curve is Harding, Linde and Trabandt (2023). A major difference from our work is that instead of generating nonlinearities through labor shortage, they trace them to quasi-kinked demand for goods, as in Kimball (1995).

Bernanke and Blanchard (2023) present an econometric model that examines the factors contributing to inflationary surges. They differentiate between the impact of supply shocks and labor-market tightness on inflation. According to their findings, supply shocks have a significant impact at the outset of the recent inflationary surge, while labor-market tightness exerts more persistent effects. These empirical findings are broadly consistent with ours, as we will see, even if there are some differences in the details. A key difference is that our empirical strategy allows for supply shocks to have increased amplification when there is labor shortage, a property that follows naturally from our empirical anlysis.[15]

There is substantial empirical literature estimating the Phillips curve using time-series data. Recent closely related works include Ball et al. (2022), Barnichon and Shapiro (2022), Blanchard et al. (2015), Domash and Summers (2022), Furman and Powell (2021), Gordon (1977, 2013) and McLeay and Tenreyro (2019). Importantly, Barnichon et al. (2021), Furman and Powell (2021), Ball et al (2022), Domash

[15]While, on the one hand, their analysis may underestimate the effect of supply shocks by not interacting with labor shortages, it could also overstate it by assuming complete price flexibility. In contrast, our model incorporates sticky prices, and inflation depends on real marginal costs, specifically on the wages of new hires rather than all employed workers. This allows us to disentangle the positive relationship between inflation and labor market share, as characterizing the benchmark New Keynesian model. One possible advantage of our framework is its ability to account for both the missing disinflation following the 2008 financial crisis and the inflationary surge in the 2020s.

and Summer (2022) argue that the measure of labor market tightness we use provides a better fit than more traditional ones, like unemployment, an insight we build on. Within this literature, some have also emphasized nonlinearities, including Ball et al. (2022) and Gagnon and Collins (2019). Other have stressed time variation in the slope, such as Benati (2010), Blanchard et al. (2015), Blanchard (2016) and Matheson and Stavrev (2013).

Part of the literature tries to address the question of whether supply shocks were the main culprit or demand shocks. One interpretation of our work, is that it reconciles the findings reported in these papers. Demand shocks are a plausible candidate for why the labor market became so tight, even if the model also predicts that a drop in labor force participating may have played a role. Yet, our estimation suggest that supply shocks also played an important role, despite that our measures are not outside historical norms, because according to the empirical analysis, as well as our theoretical model, supply shocks have a much larger impact if the labor market is tight.

Given the limited sample size of time-series data, cross sectional data investigation is becoming increasingly used. Cerrato and Gitti (2023) produce a new analysis in the spirit of Hazzell et al. (2022) with cross-sectional data of Metropolitan Statistical Areas using a shift-share instrument for identification and finding that the slope of the Phillips curve tripled in the wake of the pandemic. Recent work by Gitti (2023), using similar methods and relying on a measure of labor market tightness as we do here, finds results consistent with our own. In a similar vein, Smith et al. (2023) use both cross sectional U.S. and E.U. data and find evidence of a kink point when the labor market is running hot. Generally, we interpret all these results as broadly consistent with the evidence we present.[16]

This work is structured as it follows. Section 2 presents our empirical motivation. Section 3 sets out the theoretical model. Section 4 discusses the model's implications for the Phillips curve, Section 5 its policy implications. Section 6 concludes.

2 Empirical Results

Although Figure 4 hints at the possibility of a nonlinear Phillips curve, each point on a scatter plot represents an equilibrium outcome when viewed through the prism of a general equilibrium model. In the model introduced in the second part of the paper, for instance, each data point is determined by the intersection of the Phillips Curve (aggregate supply) and the aggregate demand. Aggregate demand is influenced by the spending decisions of households and the government, as well as by monetary policy. Consequently, from a theoretical perspective, one may not have strong reasons to believe that the scatter plot, viewed in isolation, provides compelling evidence one way or the other.

The identification problem inherent in a scatter plot of this kind has been well understood for over a century, dating back at least to Lenoir (1913).[17] If only the supply curve shifts, the data traces out

[16] Adrjan and Lydon (2023) provide evidence of a particularly hot labor market for lower paid workers.

[17] For the historical context of the identification problem in macroeconomic models, see Christ (1994).

demand. Conversely, if only demand shifts, the data traces out aggregate supply. Viewed in this light, the data for the 1960s, displayed in the upper-left panel of Figure 4, is particularly intriguing. According to the measures discussed below, the factors influencing the Phillips curve's shift (such as various proxies for inflation expectations and supply disturbances) remained relatively stable during this period. Meanwhile, shifts in aggregate demand during the 1960s are well documented (see, for example, Blinder, 2022). The figure speaks for itself.

The literature on Phillips curve estimation and the conditions under which it is identified is extensive. We do not attempt to survey it here. McLeay and Tenreyro (2019) provide a recent, lucid account that highlights various challenges to identification and possible solutions. For our purposes, an important takeaway (see Section 5 of their paper) is that if supply shocks and inflation expectations are adequately controlled for, the Phillips curve can be empirically recovered.[18]

Our preferred benchmarks consist of two empirical specifications. The first is an ordinary least-squares regression with constant coefficients, allowing for piecewise nonlinearity through a dummy variable. The second benchmark specification permits time variation in all coefficients and is estimated using a Kalman filter. In summary, our main conclusion is that we find statistically significant nonlinearity in the Phillips curve when there is a labor shortage under both specifications (i.e. $\theta > \theta^*$). Furthermore, this result is robust across various alternative specifications, which are summarized in Section 2.4 and detailed in the Appendix. Before proceeding for the first empirical specification, however, we summarize why we adopt the strategy of the recent literature cited in the introduction that emphasizes v/u as a better measure of labor market tightness than more traditional ones such as the unemployment rate alone.

2.1 Why is it useful to use vacancy-to-unemployment ratio as measure of economic slack?

The surge in the 2020s began in March 2021 as the US economy was recovering from the recession triggered by Covid-19. During this period, both the core Personal Consumption Index (PCE) and the Consumer Price Index (CPI) exceeded the Federal Reserve's 2 percent target, measured in year-on-year inflation. The core PCE reached its peak at 5.6 percent in February 2022, when the Federal Reserve started increasing Federal Funds rates. The core CPI, on the other hand, peaked at 6.6 percent in September 2022.

One reason the Federal Reserve delayed rate increases for a full year after inflation exceeded its target was that traditional measures of slack, such as unemployment, remained well above pre-pandemic levels. For instance, in March 2021, the unemployment rate stood at 6 percent, while the Federal Reserve Open Market Committee considered maximum employment to be 4 percent at the time. Another widely used metric, the prime-age-to-population ratio, was 76.9 in March 2021, considerably below the pre-pandemic ratio of 80.5.

[18] See also two recent survey articles: Mavroeidis, Plagborg-Moller, and Stock (2014) and Coibion and Gorodnichenco (2019).

It wasn't until these metrics returned to their pre-pandemic levels in March 2022 that the Federal Reserve began raising rates (for a more detailed discussion of the Federal Reserve's policy decisions during this period, see Eggertsson and Kohn (2023)).

However, despite traditional measures of slack indicating little reason for inflationary pressures from the labor market, the ratio of vacancy to unemployment was flashing red. Many firms were desperately seeking to hire more workers, with numerous establishments partially closing and displaying signs using "labor shortage" as reason for the temporary closure. The vacancy-to-unemployment ratio exceeded 1 in May 2021, reaching its peak in March 2022 when the Federal Reserve initiated rate hikes. By that time, the v/u ratio had surpassed 2, signifying the tightest labor market since World War II. It is due to the signal of tight labor markets embedded in v/u during the recent inflationary surge that a number of recent empirical studies have opted to use it instead of more traditional measures of labor tightness, such as unemployment, to account for the rise in inflation.[19] As illustrated in Figure 20 in the Appendix, in the decade leading up to the inflation surge, the v/u ratio essentially provided the same information as unemployment alone. It was only during the inflation surge in the 2020s that the vacancy-to-unemployed ratio began to convey fundamentally different information compared to unemployment rate alone.

2.2 Empirical results using a benchmark regression

In the first benchmark empirical framework we consider the following ordinary least squares regression:

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + (\beta_\theta + \beta_{\theta_d} D_t) \ln \theta_t + (\beta_v + \beta_{v_d} D_t) \nu_t + \beta_{\pi^e} \pi_t^e + \varepsilon_t, \tag{2}$$

where β_c, β_π, β_θ, β_{θ_d}, β_v, β_{v_d}, β_{π^e} are parameters, and ε_t is a zero-mean normally-distributed error. D_t is a dummy variable that takes value one if $\theta_t \geq 1$. $\pi_t \equiv \ln P_t / \ln P_{t-1}$ is inflation, π_{t-1} is its one-quarter lag, $\ln \theta_t$ is the logarithm of the vacancy-to-unemployed ratio, ν_t is a supply shock, and π_t^e is inflation expectations.

In formulating (2), we follow as closely as possible the recent literature. Our empirical contribution is simple: accounting for nonlinearity in a parsimonious way. The nonlinearity takes a special form, as can be seen from equation (2), i.e. it is piecewise linear in logs. When $\theta > 1$, the slope of the regression can differ from that under normal circumstances.

The contribution most closely related to ours is Ball, Leigh, and Mishra (2022). They instead allow for nonlinearity that applies at all times by including squared and cubed terms for θ_t. They find the non-linear terms to be statistically significant. We view their findings as complementary to ours. The motivation for our alternative approach is the suggestive evidence in Figure 4, where the data appears as if it can be closely approximated by a piecewise linear regression. In addition, the model we present in the next section is naturally expressed as piecewise linear in logs.

[19] Furman and Powell (2021) argue that the best measure of economic slack for forecasting nominal wage and price growth is the vacancy-to-unemployed ratio, θ. Recent literature has corroborated this finding; see, for example, Ball et al. (2022), Barnichon and Shapiro (2022), and Domash and Summers (2022).

Table 1 presents the estimates of an OLS regression of U.S. quarterly data from 1960 Q1 to 2023 Q2. The dependent variable is the core Consumer Price Index (CPI), which excludes food and energy prices. For ease of interpretation, all inflation variables are annualized quarterly rates and are expressed relative to a constant 2 percent annual inflation rate.[20] We have already discussed the key explanatory variable θ, which is expressed in logs. The next subsection discusses how we proxy the other explanatory variables.

The first major takeaway from Table 1 is that the nonlinearity of the Phillips curve is statistically significant (at the 1% level) and large. This is shown by the third row of Table 1, in column (3) for the full sample and column (4) for the sub-sample 2008-2023. The slope of the curve when $\theta > 1$ is the sum of the second and third rows.[21] The estimate for the slope when $\theta < 1$ is given by the second row of columns (3) and (4). While the point estimate has the expected sign, one cannot reject the hypothesis that the Phillips curve is completely flat when $\theta < 1$, as implied by the "crude" Keynesians model (see panel (a) of Figure 1 in the introduction). The slope coefficient is larger and statistically significant in columns (1) and (2) when one does not allow the slope to change when $\theta > 1$. This suggests that what is driving the statistical significance of the slope in columns (1) and (2) is largely periods in which $\theta > 1$.

[20] Estimates are invariant to the adjustment, except for the constant. However, as clarified below, considering the data as a deviation from 2 percent (the current inflation target of the Federal Reserve) is meaningful for interpreting the estimates of the constant.

[21] For example the slope is 0.5199+5.2042=5.7241 in column (4).

Table 1: Phillips Curve Estimates

	(1) 1960-2023	(2) 2008-2023	(3) 1960-2023	(4) 2008-2023
Inflation lag	0.3768*** (0.0946)	0.2765 (0.2465)	0.2601*** (0.0933)	−0.1355 (0.2052)
$\ln \theta$	0.6822*** (0.183)	0.7062* (0.3806)	0.2422 (0.1998)	0.5199 (0.3257)
$\theta \geq 1$			3.727*** (0.8346)	5.2042*** (0.9298)
μ shock	0.0372* (0.0192)	0.0101 (0.0379)	0.0444** (0.0204)	−0.0096 (0.0236)
$\theta \geq 1$			0.0831 (0.1073)	0.2771** (0.1385)
Inflation expectations	0.6524*** (0.106)	1.0613 (0.6352)	0.8033*** (0.1016)	0.5324 (0.5777)
Constant	0.5629*** (0.1585)	1.0303** (0.4621)	0.2046 (0.1679)	0.4072 (0.4037)
R^2 adjusted	0.816	0.530	0.828	0.663
Observations	254	60	254	60

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2023 Q2

· (2) and (4): sample 2008 Q3 – 2023 Q2

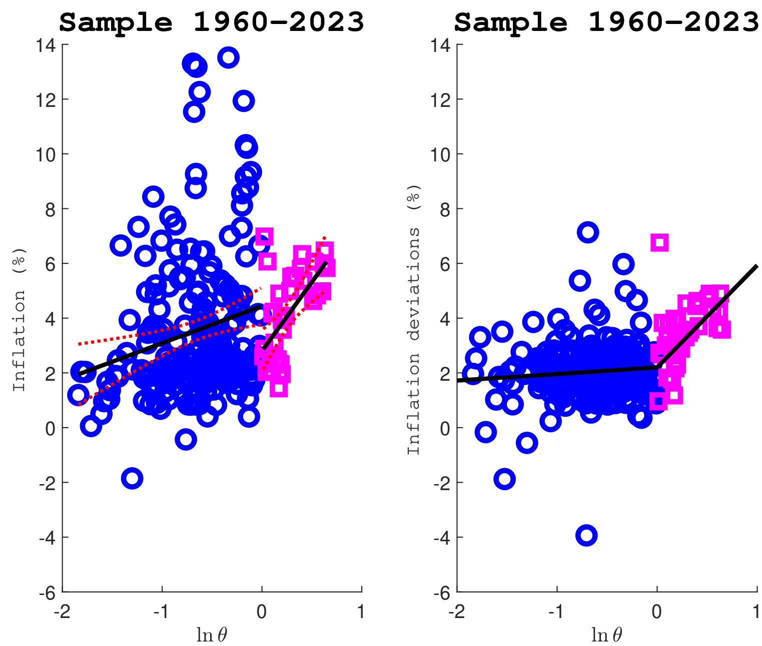

Figure 5: Left panel: scatter plot of the raw data of inflation and $\ln\theta$ used in the regression (3) of Table 1, sample 1960 Q1 - 2023 Q2. Right panel: scatter plot of 'inflation deviations' and $\ln\theta$, sample 1960 Q1 - 2023 Q2. 'Inflation deviations': inflation in deviations from its lagged value, the supply shock and inflation expectations using regression (3) of Table 1.

A second major takeaway from Table 1 is that the coefficient for lagged inflation is statistically significant for the entire sample, but not for the period 2008-2023, when $\theta > 1$ for a significant part of the period. As we will see, this empirical result is a prediction of our model.

The third major takeaway pertains to the increased significance of supply shocks when $\theta > 1$. The point estimate for the entire sample indicates that the effect of supply shocks is roughly three times larger when $\theta > 1$ than under normal circumstances. However, the standard errors associated with this effect when $\theta > 1$ are notably large, and as a result, the data lacks the statistical power to reject the hypothesis that the coefficient is zero. This is primarily due to the fact that in the full sample, there are only two periods in which $\theta > 1$: the 1960s and the most recent period. Notably, during the 1960s, there were virtually no supply shocks, as shown in Figure 17 in the Appendix. This scarcity of supply shocks in the 1960s makes it challenging to precisely identify the coefficient using the full sample. However, when we restrict the sample to the most recent time period, i.e. 2008-2023, we discover a substantial and statistically significant coefficient on the supply shock when $\theta > 1$. In contrast, when $\theta < 1$, the impact of the supply shock is not statistically distinguishable from zero, and the point estimate is negligible.

Finally, for the full sample we find a statistically significant coefficient for inflation expectation according to the baseline regression. If we restrict the sample to 2008-2023, however, this coefficient is smaller, and no longer statistically significant. The main reason is that inflation expectations have been relatively stable according to our measures in the later part of this period. There is not sufficient

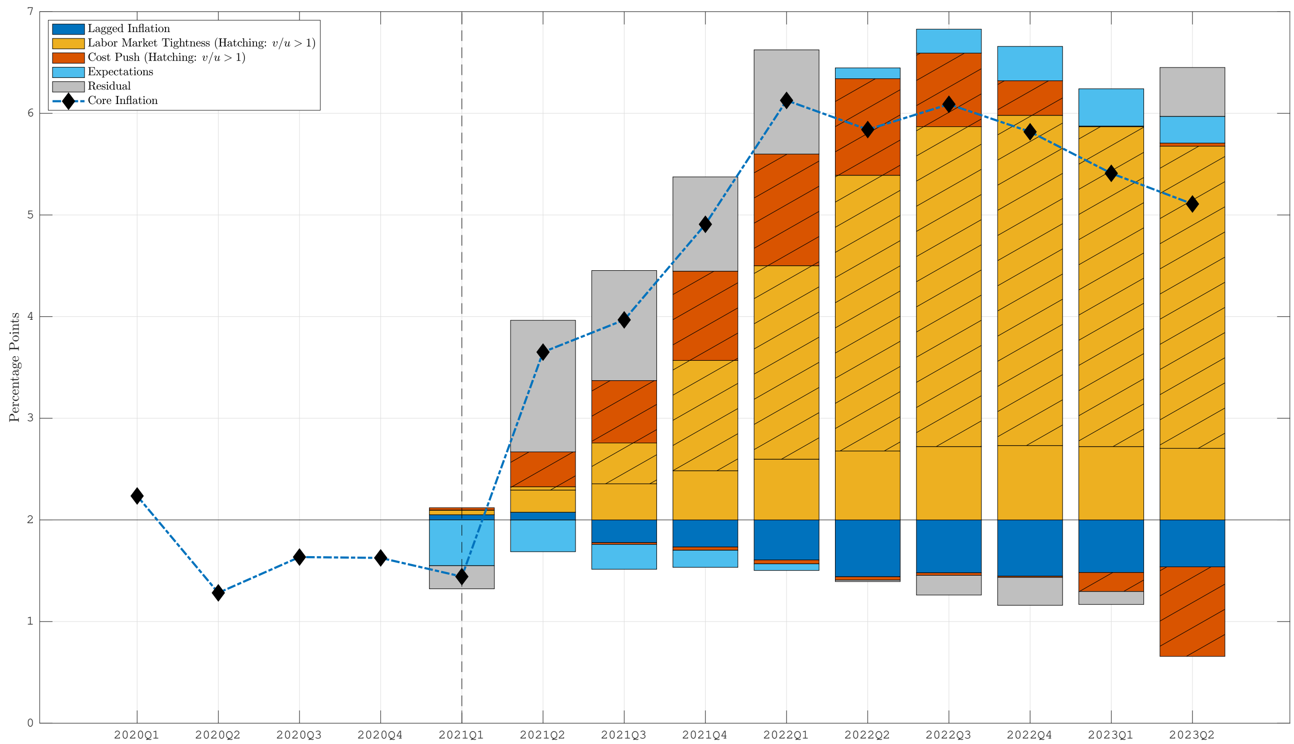

Figure 6: Decomposition of the regression (4) of Table 1, sample 2008 Q3 - 2023 Q2, among the various regressors of equation (2). For the variable $\ln \theta$ and the supply shock v, hatching corresponds to the contributions of the variable when $\theta > 1$. Core inflation and all the components are plotted at annual rates.

variation in the data to obtain a statistically significant estimate.

Figure 5 provides a visual interpretation of the regression that parallels Figure 4. The left panel combines all the data points from Figure 4 into a single scatter plot. The right panel, on the other hand, subtracts from inflation all the right-hand side explanatory variables of equation (2), with the exception of $\ln \theta$ and the constant. What is traced out by the solid line is the estimated INV-L NK Phillips Curve we would draw in $(\ln \theta_t, \pi_t)$ space in the absence of shocks.

Figure 6 is useful to interpret the quantitative contribution of the single regressors in equation (2), estimated on the sample 2008 Q3 - 2023 Q2. This figure plots up with a dashed blue line with diamonds the actual data represented as year-on-year percentage increase in core inflation. For each time period, core inflation is decomposed into the contributions given by the various regressors. Consider first the role of labor tightness in the absence of nonlinearities. As shown by the sold yellow bars, an increase in labor tightness alone would account for a modest increase in inflation, which is below 3 percent. What accounts for the lion share of the inflation surge is the labor tightness arising due to the change in the slope of the Phillips Curve once $\theta > 1$. We see similarly, that cost push shocks, once interacted with $\theta > 1$ play an important role in the surge, especially towards the beginning of the surge. Towards the end, however, they put downward pressures on inflation.

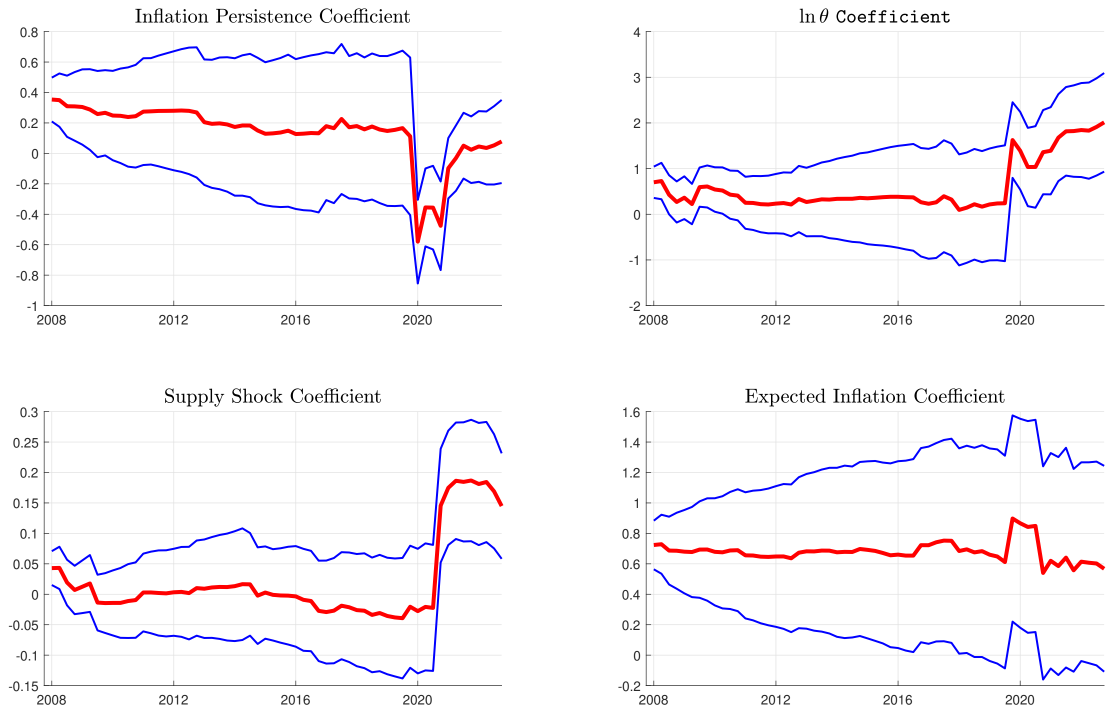

Figure 7: Estimates of the Kalman Filter with time-varying parameters on sample 2008 Q3 – 2023 Q2 with one-standard-deviation confidence bands.

The constant of the regression suggests an interesting economic interpretation. It can be related to the value for θ at which inflation is equal to 2% in the absence of shocks, which we denote by $\bar{\theta}$. We obtain $\bar{\theta} = 0.4382$ for the full sample and 0.4569 for the 2008-2023 sub-sample.[22]

2.3 Empirical results using a regression with time-varying coefficients

As an alternative benchmark to capture the nonlinearities in our benchmark regression, we consider a specification that allows for time-varying coefficients, focusing on the period from 2008 Q3 to 2023 Q2. Generally, the results support our previous findings. We follow closely the existing literature, see Blanchard, Cerutti and Summers (2015). We consider the regression reported in Table 1, but the parameters are now allowed to vary over time by a random walk. The model is estimated by a Kalman filter using as initial conditions the OLS estimates generated by a regression up to 2008 Q2. Figure 7 shows how the estimated coefficients vary over time from 2008 Q3 to 2023 Q2 with red lines. The blue lines correspond to one-standard-deviation confidence bands.

The main conclusion is that the estimated coefficients shift sharply towards the end of the sample, once $\theta > 1$. The slope of the curve steepens significantly in the post-COVID period, ending with a

[22] To see this, first observe that the mean of the supply shock is close to zero, and all inflation measures are considered as deviations from the 2% target rate. The variable $\ln\theta$, however, is not measured as a deviation from its average value. This implies that we can back it up through the equation $\ln\bar{\theta} = -\beta_c/\beta_\theta$, obtaining $\bar{\theta} = 0.4382$ and $\bar{\theta} = 0.4569$, for the third and fourth specifications of Table 1, respectively.

value close to 2. This is consistent with the results in Table 1, although smaller in magnitude. The coefficient on the supply shock also increases from zero to over 0.18. This is of the same order, even if slightly smaller, than the estimated value of the supply shock in Table 1 when $\theta > 1$.

The inflation-persistence coefficient declines over time and hovers near zero at end of the sample. This, too, is one of our model's main predictions when $\theta > 1$, and is consistent with the benchmark regression.

Figure 21, in Appendix B, highlights how poorly a forecaster would have done using either our benchmark regression or our regression with time varying co-efficients if the forecaster fails to incorporate the non-linearities. The results reported in that figure give a natural explanation for why both policy makers and professional forecasters consistently failed to forecast the scope of the surge in inflation, as well as its persistence as shown in Figures 14 and 15 already mentioned in the introduction.

2.4 Robustness to alternative measures of θ^*, supply shocks, inflation expectations and other alternative specifications

It is widely recognized that Phillips curve estimates are highly sensitive to the specific empirical specification and the choice of variables. For a recent survey, please refer to Mavroeidis, Plagborg-Moller, and Stock (2014). While conducting a comprehensive exploration of this sensitivity is beyond the scope of this paper, we present the precise specifications for each variable in Table 1 and demonstrate how alternative assumptions can influence the results. In essence, while the coefficients' magnitudes are sensitive to the exact specification, in line with existing literature, our two key empirical findings remain robust: i) there is statistically significant evidence suggesting that the Phillips curve's slope increases when $\theta > \theta^*$, and, under the same conditions, ii) supply shocks have a more substantial impact on inflation.

Our selection of $\theta^* = 1$ was primarily motivated by its compelling theoretical interpretation, as discussed in the introduction. This choice was also supported by the observation that readers can verify from Figure 4, which suggests that $\theta^* = 1$ serves as a reasonable approximation. To formally justify this choice, we take an agnostic approach about the threshold and select it by maximizing the likelihood function across piecewise linear models with varying threshold values. Using the number obtained by this procedure we replicate the results presented in Table 1. The outcomes are summarized in Table 2. The results for the latter sample are essentially the same. The results for the entire sample, however, indicate that the additional effect of the supply shock when $\theta > \theta^*$ is statistically significant.

As illustrated in Figure 22, the likelihood function across different values for the threshold exhibits a relatively flat profile between the optimal choice of θ^* and 1, especially for the sample period 2008-2023. This is one of the reasons we chose to use 1 as the benchmark, given its historical usage in the literature and its appealing theoretical interpretation. As we will see in the next section, however, our theory does not make a strong prediction about the specific value of θ^*. It may vary across regions, countries and over time. We leave further exploration of this topic for future research.

To determine the slope of the Phillips curve, it is critical to control for supply shocks. The literature has considered several proxies. One approach is to use so-called "headline shocks", i.e. the difference between headline and core CPI inflation. The Personal Consumption Expenditures (PCE) price index is an alternative to CPI to generate these shocks. Another common approach is to compute the difference between the changes in the import prices and in the GDP deflator. Figure 17, in the Appendix, presents the raw data using these three approaches. We are agnostic about the appropriate measure of supply shocks. In the benchmark specification we do a principal component analysis and use the first principal component of these three series as a proxy. As shown in Appendix A, the main result of interest (evidence of non-linearity when $\theta > 1$) is robust to considering each measure separately (see Tables 3 and 4).

Another explanatory variable is inflation expectations, which we derive through direct measures as in the recent literature.[23]

The benchmark analysis uses the two-year quarterly inflation expectations measure provided by the Federal Reserve Bank of Cleveland, i.e. the rate that inflation is expected to average over the next two years.[24] This series is available only since 1982 Q1. For the earlier period we use the Livingston inflation expectations survey; since this survey is conducted only twice yearly, we interpolate it to obtain quarterly observations.[25] Figure 23 in the Appendix shows this data. Appendix A shows that our main results are robust to various alternative measures of inflation expectations: 1-year CPI inflation expectations in the Survey of Professional Forecasters, five-year inflation expectations of the Federal Reserve Bank of Cleveland and a five-year five-year forward measure developed by Groen and Middledorp (2013); see Tables 5-7.

A common approach in estimating Phillips curves is using lagged variables as instruments. We adopt this approach in Table 8, where we instrument inflation lags and the natural logarithm of the vacancy-to-unemployed ratio with the fitted values of their first lags.

Our baseline regression employs core CPI as the dependent variable. In Table 9, we present the results using core PCE instead.

[23] In the canonical Keynesian model, the expectation on the right-hand side of the Phillips curve, as in equation (1), is expected inflation in the next quarter. Since we consider various types of expectation, a simple way of interpreting regressions using longer-term expectations is to posit the one-quarter-ahead inflation expectation as

$$E_t \pi_{t+1} = h \cdot \underbrace{\pi_t}_{\text{Current Inflation}} + (1-h) \cdot \underbrace{\pi_t^e}_{\text{Longer term inflation}}$$

where h is the weight of current inflation in predicting next-quarter inflation, while π_t^e represents a measure of some longer-term expectation, which serves as an anchor. What is the best measure then becomes an empirical question. As shown below, the key result is robust to various alternative measures of inflation expectations as proxies of π_t^e. Bernanke (2007), in discussing the inflation forecasting model of the Federal Reserve, argues that long-term expectations seem more important for the price-setting behavior.

[24] In their estimates, they use Treasury yields, inflation data, inflation swaps, and survey-based measures of inflation expectations.

[25] The 12-month CPI forecast of the Livingston Survey better represents a 14-month forecast (see Carlson, 1977).

Our choice of using the natural logarithm of $\theta = v/u$ is driven by the theoretical model we derive in the following section. Additionally, taking logarithms has the advantageous property that the order of the ratio v/u or u/v becomes irrelevant. Table 10 displays the results when using the level of θ instead of the log.

To align with the existing literature, lagged inflation enters the baseline regression. In the theoretical model, however, the lagged state variable is the real wage rate. In Table 11, we report the results when lagged inflation is replaced with lagged real wages.

3 The model

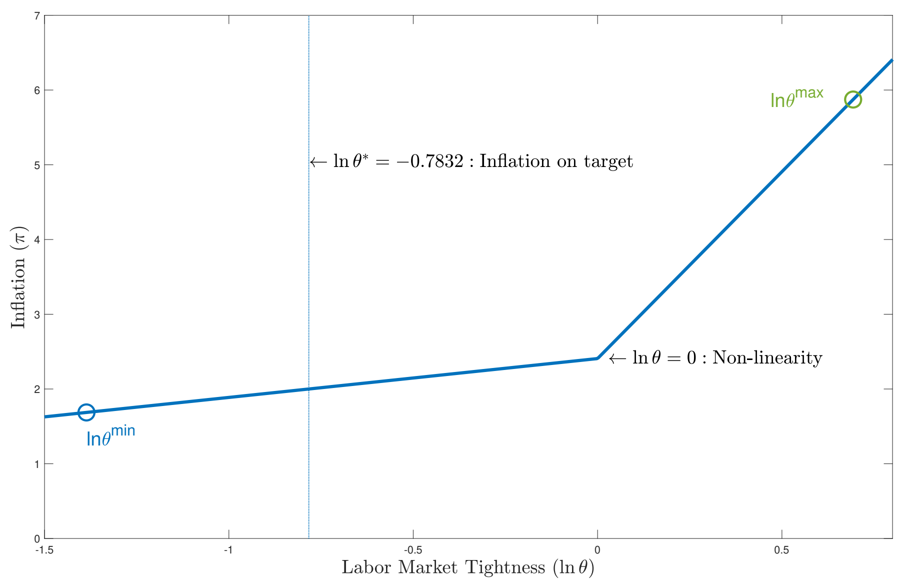

Figure 8: A Model of an Inv-L NK Phillips Curve as a function of labor market tightness.

Just like the canonical New Keynesian model, ours is designed to capture the forward-lookingness of firms, which gives rise to expectations about future inflation as in equation (1). The main difference in our model with respect to the standard New-Keynesian account involves the labor market, which is the new element that generates nonlinearities.

The labor market is modeled via search and matching. As is well known there is a variety of ways of modeling the real wage rate in this class of models, since each employment match generates a "surplus" shared between firms and workers. Here we propose a wage-setting mechanism that is motivated by the observation of Phillips (1958), that wages appear to be highly flexible when the market is tight and fall only slowly under regular circumstances.

A road map, i.e. statement of the bottom line up-front, enables the reader to anticipate what the model delivers. Using the same notation as in the introduction, the Inv-L NK Phillips curve is given by

$$
\pi_t = \begin{cases} \kappa^{tight}\hat{\theta}_t + \kappa_\varrho^{tight}\hat{\varrho}_t^{tight} + \beta E_t \pi_{t+1} & \text{labor shortage } (\hat{\theta}_t > \hat{\theta}_t^*) \\ \\ \kappa_w \hat{w}_{t-1} + \kappa \hat{\theta}_t + \kappa_\varrho \hat{\varrho}_t + \kappa_\beta E_t \pi_{t+1} & \text{normal } (\hat{\theta}_t \leq \hat{\theta}_t^*)) \end{cases} \tag{3}
$$

in which $\hat{\varrho}_t$ includes all the shocks perturbing the model which we will discuss in detail in this section.[26]

We plot this relationship in Figure 8 in the absence of shocks using the regression results (column (4) of Table 1) to capture the slope of the Phillips curve and the point at which θ hits the inflation target. The key result is that $\kappa^{tight} > \kappa$, as shown in Figure 8. At a given level of tightness, denoted by the vertical line, a tighter labor market increases inflation. However, the curve is clearly quite flat. Once θ reaches $\theta^* = 1$ (or $\ln\theta = 0$), inflation is barely above target at 2.4 %. It is only when θ crosses $\theta = 1$ and enters a regime of labor shortage that it exerts significant inflationary pressure. Increasing θ from 0.4569, which is the estimate for labor tightness being neither inflationary nor deflationary, to 1 only increases inflation from 2% to 2.4%. But, in labor shortage territory, raising θ from 1 to 2 increases inflation from 2.4% to 5.8%.

When there is labor shortage, the response of inflation to supply shocks, $\hat{\varrho}_t$, too is more pronounced, i.e. $\kappa_\varrho^{tight} > \kappa_\rho$. The effect of inflation expectations, however, is not clear-cut. It depends on how wages react to expectations of future inflation. As we will see, there are some parameter values that capture the idea of a wage-price spiral, in which case, in a slack labor market, inflation is even more responsive to expectations of future inflation than during labor shortage, i.e. $\kappa_\beta > \beta$.

Section 4 summarizes the microfoundations of equation (3). Section 5 discusses policy implications where the model is cast in a form familiar to most readers.

3.1 Households

There is a continuum of representative households of measure one. The members have different disutilities of working. No decision is made about each member's hours of work (intensive margin), but the household does decide how many members work (extensive margin). In other words, the household j chooses the labor market participation rate. The utility flow at time t of household j is given by preferences as in Greenwood, Hercowitz and Huffman (1988):[27]

$$
U(C_t^j, F_t^j, \chi_t, \Psi_t, \xi_t) = \frac{1}{1-\sigma}\left(C_t^j - \chi_t \int_0^{F_t^j} f^\omega df + \Psi_t\right)^{1-\sigma} \xi_t \tag{4}
$$

where C_t^j is consumption of household j and F_t^j is the number of members who decide to participate in the labor market. Each household member is indexed by f and has fixed disutility f^ω from taking

[26] Variables with a hat represent log-deviations from their respective steady states. In this context, w denotes the real wage, and all other parameters will be explained later in the text.

[27] The use of GHH preferences allows us to abstract from wealth effects in labor force participation, which simplifies the algebra.

part in the labor force, as in Galì (2009), with $\omega > 0$. The variable χ_t is an exogenous shock to labor force participation, and $\sigma > 0$ is a parameter. The variables Ψ_t and ξ_t are treated as exogenous by the household. Ψ_t is introduced to simplify the Euler Equation for consumption as is clarified in Section 5, and ξ_t is an intertemporal disturbance that moves the natural rate of interest.

Household members are ordered by their disutility from working. For example, it may be more costly to have an aging grandmother in the labor force than a prime age woman. Integrating the cost of labor force participation yields

$$\int_0^{F_t^j} f^\omega df = \frac{(F_t^j)^{1+\omega}}{1+\omega}. \tag{5}$$

The household decides labor force participation. Not all of the labor force is employed, however, owing to frictions in the labor market, which are modeled by a search and matching function.

The supply of labor of household j is divided between the employed and the unemployed

$$F_t^j = N_t^j + U_t^j, \tag{6}$$

where N_t^j is workers employed by firms and U_t^j is unemployed workers at the end of period t – *after job search in period t*.

At the beginning of each period a fraction $(1-s)$ of people participating in the labor market is attached to firms. Implicitly, we think of this as employment based on existing relationships. For the purpose of this paper, however, we do not model how these relationships are formed or keep track of them over time. [28]

The total number of employed households j at time t is

$$N_t^j = N_t^{ex}(j) + N_t^{new}(j) \tag{7}$$

where $N_t^{ex}(j)$ represents workers from existing work relationships given by

$$N_t^{ex}(j) = (1-s)F_t^j \tag{8}$$

while $N_t^{new}(j)$ represents workers who have formed new work relationship with firms, and we interpret as having entered them either via job-to-job transitions, from unemployment to work, or as representing new members to the labor force who obtained work immediately.

These workers who form a new employment relationship are drawn from pool of workers represented by sF_t^j. Their ability to enter employment is determined by the aggregate matching function

[28] It is as if each labor force member entered the period unattached. We discuss later, however, that variation over time of s could be relevant together with the parameter measuring the matching efficiency to understand why v/u can provide more information than u alone to understand inflationary pressures.

which determines total employment matches

$$M_t = m_t U_t^{\eta} V_t^{1-\eta}, \tag{9}$$

where $m_t > 0$ is matching efficiency, which can be time varying, and V_t is aggregate vacancies posted by employment agencies (described shortly) and U_t is aggregate unemployment. Since these variables represent aggregate variables (and thus do not have superscript j), household j takes them as given; η is a parameter with $0 < \eta < 1$. That s can range from zero to one is attractive, because it allows us to nest two special cases. The standard NK model assumes that $s = 0$ so that the labor market is perfectly flexible. On the opposite extreme $s = 1$ represents the case in which all people have to search for a job in every period. Moreover, we will argue that this exogenous parameter has interesting economic interpretation which will be important later on.

Define the aggregate tightness of the market by $\theta_t \equiv V_t/U_t$. A tight labor market means higher θ_t, as there are relatively more vacancies than job seekers. The probability of a job seeker finding a job is $f(\theta_t) = \frac{M_t}{sF_t} = u_t \frac{m_t \theta_t^{1-\eta}}{s}$ taken as given by the household as it depends on aggregate variables. The number of successful employment matches of the members of household j that are not attached to existing firms at the beginning of the period is then[29]

$$N_t^{new}(j) = M_t^j = f(\theta_t) s F_t^j = u_t m_t \theta_t^{1-\eta} F_t^j.$$

The budget constraint can thus be written as:

$$B_t^j + P_t C_t^j + T_t = (1 + i_{t-1}) B_{t-1}^j + W_t^{ex}(1-s) F_t^j + W_t^{new} u_t m_t \theta_t^{1-\eta} F_t^j + Z_t^F + Z_t^E + P_t q_t \bar{O}_t, \tag{10}$$

where B_t^j is a risk-free nominal bond denominated in units of currency at time t at the nominal interest rate i_t, P_t is the price index associated with consumption basket C_t^j, T_t are lump-sum taxes, and Z_t^F and Z_t^E are the firms' and the employment agencies' profits divided equally across households. Finally, $P_t q_t \bar{O}_t$ represents the revenues that the households receive by selling an intermediate input, denoted by $\bar{O}_t$, to firms at the exogenous real price q_t. A natural interpretation of this endowment is oil. We assume that workers with existing employment relationships receive a salary W_t^{ex} while newly hired workers are paid salary W_t^{new}.

Substituting equation (5) into the household utility, the household maximizes (4) subject to (10) by its choice of C_t^j, B_t^j, F_t^j taking as given all the variables not indexed by j in maximization problem.

The necessary and sufficient conditions for the household maximization problem are straightforward to derive and are summarized below. Since all households behave the same in equilibrium we suppress the superscript j going forward.

[29] The number of matches can be interpreted as encompassing the sum of job-to-job hires, unemployment-to-employment as well as those that enter the labor force and find a job. As documented by Sedlacek (2016) approximately three quarters of all new matches represents job-to-job transitions and new entrants to the labor force.

The household's optimal labor-force participation implies:

$$F_t = \left(\frac{(1-s)w_t^{ex} + u_t m_t \theta_t^{1-\eta} w_t^{new}}{\chi_t} \right)^{\frac{1}{\omega}} \tag{11}$$

which says that participation is increasing both in tightness and in the real wage, defined as $w_t^i \equiv W_t^i / P_t$ where $i = new, ex$, but can be negatively affected by the shock χ_t. The optimal consumption decision implies:

$$X_t^{-\sigma} = \beta(1+i_t) E_t \left\{ X_{t+1}^{-\sigma} \Pi_{t+1}^{-1} \frac{\xi_{t+1}}{\xi_t} \right\}, \tag{12}$$

in which β is the rate of time preference, $\Pi_t \equiv P_t / P_{t-1}$ and

$$X_t \equiv C_t - \chi_t \frac{F_t^{1+\omega}}{1+\omega} + \Psi_t. \tag{13}$$

Finally, a necessary condition for optimality is that the household's intertemporal budget constraint holds with equality.[30]

We assume that C_t is a consumption basket given by a Dixit-Stiglitz aggregator of the form

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\epsilon_t - 1}{\epsilon_t}} di \right]^{\frac{\epsilon_t}{\epsilon_t - 1}},$$

where i indexes consumption of a good of variety i, and $\epsilon_t > 1$ is the elasticity of substitution among the differentiated goods. The household's optimal choice of good variety i at time t implies

$$c_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\epsilon_t} C_t,$$

where $p_t(i)$ is the price of variety i and

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\epsilon_t} di \right]^{\frac{1}{1-\epsilon_t}}.$$

3.2 Firms

There is a continuum of firms of measure one, each firm i producing a good of variety i. The source of demand is household and government consumption, C_t and G_t plus the cost, $\gamma_t^c V_t$, to the employment agency of posting vacancies, V_t, measured in units of the consumption good, where $\gamma_t^c > 0$ can be stochastic. We assume that the government spending bundle and the vacancy cost, $\gamma^c V_t$, take the same form as the Dixit-Stiglitz household consumption basket.

[30] Or equivalently we can state a transversality condititionm; see e.g. Woodford (2003) for a discussion.

A representative firm i thus faces the following demand, $y_t(i)$, for its output

$$y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\epsilon_t} Y_t, \tag{14}$$

where $p_t(i)$ is the price of goods of variety i and $Y_t = C_t + G_t + \gamma_t^c V_t$. Firms use labor and intermediate input to produce goods according to the technology

$$y_t(i) = A_t N_t(i)^\alpha O_t(i)^{1-\alpha}, \tag{15}$$

with $0 < \alpha < 1$ where A_t is productivity, $N_t(i) = N_t^{ex}(i) + N_t^{new}(i)$ is the labor employed by firm i, while $O_t(i)$ is an intermediate input of production (e.g. oil).

The firms' discounted value of current and expected future profits are:

$$E_t \sum_{T=t}^{\infty} Q_{t,T} \left\{ p_T(i) y_T(i) - W_T^{ex} N_T^{ex}(i) - (1+\gamma_T^b) W_T^{new} N_T^{new}(i) - P_T q_t O_T(i) - \frac{\varsigma}{2} \left(\frac{p_T(i)}{p_{T-1}(i)} - 1 \right)^2 P_T Y_T \right\} \tag{16}$$

where $Q_{t,T} \equiv \beta^{T-t} (X_T^{-\sigma}/P_T)/(X_t^{-\sigma}/P_t)$ is the stochastic discount factor the household uses at time t to value future nominal income at time T. As in the price-adjustment model of Rotemberg (1983), ς is a parameter measuring the cost of adjusting prices.[31] The term γ_t^b represents the fee on the wage bill of new workers that the firm has to pay to the employment agency to hire them.

The maximization of the firm is constrained by the limit that the firm cannot hire more existing workers at the existing wage rate than the number of workers attached to the firm, i.e.

$$N_t^{ex}(i) \leq N_t^E \tag{17}$$

where $N_t^E = (1-s)F_t$, see equation (8). Moreover, while existing workers can be fired, new workers can only be added, i.e.

$$N_t^{new}(i) \geq 0. \tag{18}$$

The problem of the firm can be stated as choosing all variables sub-scripted with i to maximize (16) subject to (14), (15), (17), (18) and $N_t(i) = N_t^{ex}(i) + N_t^{new}(i)$. The first order conditions are shown in Appendix D. We will consider an equilibrium in which constraint (17) is binding while (18) is not, therefore

$$N_t(i) > N_t^E, \tag{19}$$

which can be ensured to be the case by choosing a sufficiently large s (if $s = 1$, then $N_t^E = 0$). As we show in the Appendix, a sufficient condition for the firm to always choose to first hire its existing workforce before hiring new workers is that

$$(1+\gamma_t^b) W_t^{new} > W_t^{ex}. \tag{20}$$

[31] Calvo's price-setting model leads to the same AS equation in a first-order approximation. However, we use Rotemberg's assumption in order to simplify the presentation.

This condition arises from the observation that, given the perfect substitutability of new and existing labor forces, the firm opts to first hire from its existing workforce whenever the cost of hiring a new worker for production exceeds that of the existing workforce. The important implication of distinguishing between new and existing workers is that it clarifies that, provided $N_t(i) > N_t^E$ the marginal costs of production for the firm depends only on the wages of the new workers as well as the hiring cost.

Since all firms face the same problem, there is a symmetric equilibrium in which $p_t(i) = P_t$ and $y_t(i) = Y_t$. The aggregate Phillips curve is derived directly by combining the first-order conditions detailed in Appendix D:

$$(\Pi_t - 1)\Pi_t = \frac{\epsilon_t - 1}{\varsigma}\left(\frac{\mu_t}{A_t}\left(\frac{(1+\gamma_t^b)w_t^{new}}{\alpha}\right)^{\alpha}\left(\frac{q_t}{1-\alpha}\right)^{1-\alpha} - 1\right) + \beta E_t\left\{\left(\frac{X_{t+1}}{X_t}\right)^{-\sigma}\frac{Y_{t+1}}{Y_t}(\Pi_{t+1}-1)\Pi_{t+1}\right\}, \tag{21}$$

where we have defined $\mu_t \equiv \epsilon_t/(\epsilon_t - 1)$ and w_t^{new} is the real wage paid to new workers. Since all firms behave in the same way then aggregate output is given by

$$Y_t = A_t N_t^{\alpha} \bar{O}_t^{1-\alpha}, \tag{22}$$

where we have used that in equilibrium the market for the intermediate input endowment (e.g., oil) is given by $O_t = \bar{O}_t$. Finally, note that

$$N_t = [(1-s) + u_t m_t \theta^{(1-\eta)}]F_t, \tag{23}$$

and

$$N_t = [1 - u_t]F_t. \tag{24}$$

3.3 Wage determination

The central idea in Phillips (1958) is that the relationship between nominal wage growth and labor market tightness is non-linear. He argues that even if demand for labor is low workers are unwilling to take jobs at wages below the "prevailing rates." This implies that despite low demand for workers, and high unemployment "wage rates fall only very slowly". Yet the converse is not true according to Phillips. Workers are perfectly happy to accept wages that are *higher* than the prevailing wage rate. For this reason when labor markets are sufficiently tight "we should expect employers to bid wage rates up quite rapidly". To capture Phillips idea we assume that the wage of a worker hired at time t is:

$$W_t^{new} = \max\{W_t^{ex}, P_t w_t^{flex}\} \tag{25}$$

where w_t^{flex} is the *flexible real wage*, i.e. the wage rate that clears the market in the absence of any constraints, while W_t^{ex} is wages of workers that are attached to existing firm relationships. To see how this captures Phillips idea, consider the possibility that the demand for labor is weak. In this case, as we will see shortly, then the prevailing wage rate, i.e. ${W_t}^{ex}$, is greater than the wage rate that would clear the market, i.e. $P_t w_t^{flex}$. The max operator says that newly hired workers are unwilling to work for a lower wage rate than what is prevailing at the firm, so that if ${W_t}^{ex} > P_t w_t^{flex}$ then $W_t^{new} = {W_t}^{ex}$. Yet, since workers are perfectly happy to work for *higher* wages than the existing workforce, then if the labor market is sufficiently tight, and ${W_t}^{ex} < P_t w_t^{flex}$, we have that $W_t^{new} = P_t w_t^{flex}$.[32] We can restate equation (25) in real terms

$$w_t^{new} = \max\{{w_t}^{ex}, w_t^{flex}\} \tag{26}$$

As is well known in search and matching literature the wage rate is not in general determined, since each employment relationship generates a surplus. How the surplus is divided between the employer and firms can be done in several different ways, the most common in the labor-search literature is Nash bargaining. Models incorporating price stickiness typically assume that the real wage is exogenous. One of the theoretical contributions of this paper is to move away from these assumptions in favor of Phillips idea of asymmetric response of wages to tightness and slack.

To do this, we propose a simple model of employment agencies, who have access to the matching technology, and screen workers to be eligible for employment. This will provide the foundation for the flexible wage rate. Once we derive the flexible wage rate, we show how existing wage rates evolve as a function of the flexible wage rate. There is a good reason to start with the flexible wage for it will serve as an anchor for existing wages. We will assume, as observed in the data, that when the wages of new employees increase, then wages of existing employees increase as well, but to a lesser extent.

3.4 Employment agencies

There is a continuum of employment agencies of measure one. The employment agencies find workers suitable for employment by firms. Since each agency is small, they take as given the wage rate and the rate of matches per vacancy posted. The number of matches per vacancies posted is

$$n(\theta_t) = \frac{M_t}{V_t} = \frac{m_t(U_t)^{\eta}(V_t)^{1-\eta}}{V_t} = m_t(\theta_t)^{-\eta}. \tag{27}$$

Consider the problem of a representative agency j. It charges a fee γ_t^b to the firm that is proportional to the salary of the worker it screens for employment, while incurring a cost given by γ_t^c. The number of matches it generates is given by $n(\theta_t)V_t(j) = m_t(\theta_t)^{-\eta}V_t(j)$. The revenues and the cost of the employment agency are measured in units of the final good. Profits are given by:

$$\underbrace{\gamma_t^b w_t^{flex} m_t \theta_t^{-\eta}}_{\text{Marginal benefit}} V_t(j) - \underbrace{\gamma_t^c}_{\text{Marginal cost}} V_t(j) \tag{28}$$

[32] Note that consistently with the derivation of the AS equation (21) $(1+\gamma_t^b)W_t^{new} > W_t^{ex}$ always.

As long as the marginal benefit is greater or equal to the marginal cost, the agency will post vacancies. This leads to a fall in wages of new hires, as well as increase in tightness θ_t which lowers the number of matches for each vacancy posted. Equilibrium is reached when marginal benefits equal marginal costs, i.e.:

$$\underbrace{\gamma_t^b w_t^{flex} m_t \theta_t^{-\eta}}_{\text{Marginal benefit}} = \underbrace{\gamma_t^c}_{\text{Marginal cost}} . \tag{29}$$

Let us suppose the wages of both existing and new workers are determined by the flexible wage:

$$w_t^{ex} = w_t^{new} = w_t^{flex}, \tag{30}$$

which is then given by

$$w_t^{flex} = \frac{\gamma_t^c}{\gamma_t^b} \frac{1}{m_t} \theta_t^{\eta}. \tag{31}$$

If we assume that there is no constraint on how much wages can decline, this equation closes the model once we add the goods market equilibrium:[33]

$$Y_t = C_t + G_t + \gamma_t^c u_t \theta_t F_t, \tag{32}$$

and a specification of the monetary policy rule

$$i_t = \psi(\Pi_t, \xi_t, A_t, G_t, ...) \tag{33}$$

for some function $\psi(\cdot)$. A full specification of the equilibrium if the wage of workers are flexible can be stated formally.[34].

In this equilibrium the employment agencies post vacancies as long as the marginal benefit is higher than the marginal cost. In general equilibrium, wages and labor market tightness adjust so that this condition is satisfied.

When all wages are fully flexible we learned that the hiring agencies post vacancies until wages fall, and tightness increases sufficiently, so that marginal cost is equal to marginal benefit. We now consider the possibility that the wages of existing workers is greater than the flexible wage rate, i.e.

[33]We have used $Y_t = C_t + G_t + \gamma^c V_t$, noticing that $V_t = F_t(V_t/U_t)(U_t/F_t) = u_t \theta_t F_t$. Moreover we abstract from the resources taken by the quadratic cost of price setting; see Eggertsson and Singh (2019) for various ways in which this has been justified.

[34]An equilibrium with flexible prices is defined by a collection of stochastic processes for $\{i_t$, X_t, Π_t, Y_t, C_t, F_t, θ_t, N_t, U_t, w_t^{ex}, w_t^{new}, $w_t^{flex}\}_{t=0}^{\infty}$ that satisfy (11), (12), (13), (21), (22), (23), (24), (30), (31), (32), (33) and $u_t = U_t/F_t$ given an exogenous process for $\{A_t, G_t, \xi_t, \chi_t, \Psi_t, m_t, q_t, \gamma_t^c, \gamma_t^b, \epsilon_t, \bar{O}_t\}_{t=0}^{\infty}$

$w_t^{ex} > w_t^{flex}$. We follow Phillips' suggestion that new workers refuse to work for a wage that is below the existing wage rate so that $w_t^{new} = w_t^{ex}$. Suppose that the vacancies required to satisfy firms labor demand at the existing wage rate is V_t^{ex}. Any vacancy beyond V_t^{ex} will never be filled, since firms have already satisfied their labor demand which is a function of w_t^{ex}. Therefore, the employment agency has no incentive to post further vacancies since the marginal benefit, which is zero, is below the marginal cost.[35] What remains to close the model, then, is to specify the evolution of existing wages.

We assume that existing wages evolve as follows:

$$W_t^{ex} = (W_{t-1}^{ex}(\Pi_{t+1}^e)^{\delta})^{\lambda}(P_t w_t^{flex})^{1-\lambda}\phi_t. \tag{34}$$

To understand this specification, consider the special case in which $\lambda = 1$ and $\delta = 0$. Then $W_t^{ex} = W_{t-1}^{ex}$ so that existing wages stay constant at their nominal value capturing Keynes idea that wages are downward rigid. Once $\lambda < 1$ wages get pulled upward towards the flexible wage rate in a tight market. Of particular interest is that in a slack labor market the new wages are equivalent to the prevailing wage rate, which is pulled down towards the flexible wage rate at a speed that depends upon how close λ is to zero. For high enough λ the wage rate falls slowly as suggested by Phillips, even if the unemployment rate is high. Conversely, in a tight labor market, both existing and new wages are pulled upwards by w_t^{flex} which is determined by market forces. Unless the market is extremely tight, however, and $w_t^{flex} > w_t^{ex}$, this adjustment will only happen gradually depending on how close λ is 0.

We introduce an additional feature by including the variable Π_{t+1}^e, relevant if $\delta > 0$. This captures the idea that inflation expectations affect wage-setting behaviour. These expectations can, for example, be anchored by the inflation target of the central bank. Alternatively, this variable allows us to model "price-wage" spiral commonly thought to have played a role in the 1970s. Finally the shock ϕ_t allows for flexibility in the specification when non-linearities are relevant.

Writing the wages of new hires in real term:

$$w_t^{new} = \begin{cases} w_t^{flex} & \text{for } \theta_t > \theta_t^* \\ \\ w_t^{ex} = \left(w_{t-1}^{ex}\frac{(\Pi_{t+1}^e)^{\delta}}{\Pi_t}\right)^{\lambda}(w_t^{flex})^{1-\lambda}\phi_t & \text{for } \theta_t \leq \theta_t^*. \end{cases} \tag{35}$$

When wages are flexible, the optimizing behavior of the employment agency plays a central role in determining them by equating the marginal cost with the marginal benefit of posting vacancies. This solves the classic wage indeterminacy of standard labor-market search-matching models. Yet, the flexible wage rate is also important in the determination of existing wages. It serves as an anchor which pulls the existing wages towards the market clearing one.

[35]Observe that in this equilibrium the hiring agency is actually making a profit on its last vacancy posting.

To summarize: If the wage of new hires are constrained by the wages of existing workers, then households refuse to accept salary below "prevailing wage rate" of the existing workers. At the same time, households make an active labor market decision by choosing their participation rate. For any given wage rate and labor market tightness θ_t, the household chooses F_t optimally. Meanwhile, the gradual movement of the wage rates of existing workers constrains the number of workers firms employ. Wages fall only gradually towards the flexible wage rate when the market is weak, so N_t is determined by firms' demand for labor at the prevailing wage rate. This constraint, in turn, becomes binding for the employment agency, which supplies vacancies elastically in response, and up, to firms' labor demand, but earns a profit on the marginal vacancy.

What remains to close the model is to determine the value of θ_t^* which is implied by setting $w_t^{flex} = w_t^{ex}$ yielding:

$$\theta_t^* = \frac{\gamma_t^b}{\gamma_t^c} m_t \left(w_{t-1}^{ex} \frac{(\Pi_{t+1}^e)^\delta}{\Pi_t} \right)^{\frac{1}{\eta}} (\phi_t)^{\frac{1}{\lambda\eta}}. \tag{36}$$

As suggested by this formula θ_t^* varies with time and can depend on institutional features of different regions and countries. Since our analysis was conducted using only time series data, we lack the statistical power to estimate how θ_t^* varies with time. Consequently, we took the pragmatic approach of approximating it by 1 in the empirical analysis (one can always pick the shock ϕ_t for this to be true). However, as suggested by equation (36), one should expect this variable to vary with shocks. Obtaining empirical evidence to address this important issue is left for future research.[36]

We can now formally define the equilibrium in the full model.[37]

So far, we have described the employment agency problem as separate from the firms problem. Yet, since we have described the marginal cost and benefits of the employment agency in term of the final good, the agency can in principle represent an independent Human Resource department within each firm.[38].

4 The Inv-L NK Phillips Curve

The model we have just sketched out can be presented closer to the spirit of Phillips' original suggestion (depicted in Figure 1) than later incarnations of his work. The central result is that this modified curve now takes an inverse-L shape, as in Figure 1, in a first order approximation around the steady

[36] Examining cross-sectional data based on MSA, Gitti (2023) permits variation in θ^* across states and yields promising results. Allowing for different thresholds across states, as opposed to assuming they are the same, provides a better fit.

[37] An equilibrium is defined by a collection of stochastic processes for $\{i_t, X_t, \Pi_t, Y_t, C_t, F_t, \theta_t, N_t, U_t, w_t^{new}, w_t^{ex}, w_t^{flex}, \theta_t^*\}_{t=0}^{\infty}$ that satisfy (11), (12), (13), (21), (22), (23), (24), (30), (31) (32), (33), (35), (36) and $u_t = U_t/F_t$ given exogenous processes $\{A_t, G_t, \xi_t, \chi_t, \Psi_t, m_t, q_t, \gamma_t^c, \gamma_t^b, \epsilon_t, \bar{O}_t, \phi_t\}_{t=0}^{\infty}$.

[38] The optimal pricing decision of the firm, and the optimal vacancy posting by the employment agency do not depend on each other directly. We keep these as different entities to make the presentation simpler. If the cost of recruitment depends on the *labor* hired by the firms then pricing decisions interact with vacancy creation. Michaillat (2014) is an example of this approach. In his model the labor the firm recruits can either be devoted to production or to hiring more people

state. Two main economic propositions underlie the inverse-L shape. The first is simply that while, given sufficient time and price incentives, most factors of production can typically be increased in one way or another, one factor will always be in limited supply over any reasonable time horizon: the number of people who can work. Second, it has long been recognized that more than other prices the price of labor (wages) falls slower even when it is in excess supply (high unemployment). Together, these observations imply that over some range, an increase in demand increases production without significant inflation pressures, as more people are drawn into the labor force and are gainfully employed. Firms increase their production while their marginal costs only modestly increase. However, given the first proposition, this process is bound to hit a wall once the labor force is fully employed. Then higher demand expresses itself mostly in higher inflation.

Our first characterization of the Phillips curve is a log-linear approximation of 21:

$$\pi_t = \frac{1-\epsilon}{\varsigma}\alpha(\underbrace{z_\gamma \hat{\gamma}_t^b + \hat{w}_t^{new}}_{\text{Marginal Cost of Labor}}) + \frac{1-\epsilon}{\varsigma}(\underbrace{\hat{\mu}_t - \hat{A}_t + (1-\alpha)\hat{q}_t}_{\text{Cost Push Shocks}}) + \beta E_t \pi_{t+1} \tag{37}$$

where a hat denotes the log-deviation of a variable with respect to the steady state, $\pi_t \equiv \ln \Pi_t$, and the parameter z_γ is defined in Appendix D. The first part of this expression, highlighted by the first curly bracket, underscores that the primary driving force behind inflation is the marginal cost of hiring a new worker. This cost is composed of two terms, the wage bill of the new workers, $\hat{w}_t^{new}$, as well as the cost of hiring new workers, i.e., $\hat{\gamma}_t^b$. The second term represents what the literature typically identifies as cost push or supply shocks.

An important implication of this characterization is that it demonstrates that, while marginal costs have an effect on inflation, as in the standard New Keynesian model, they are expressed differently. In the New Keynesian literature, marginal costs typically refer to wages of all employees, usually measured by time series such as the aggregate labor share or the Employment Cost Index (ECI), a BLS survey of employers' payrolls that measures total compensation. In contrast, our model suggests that the relevant marginal costs for pricing decisions of firms are instead approximated by the cost of adding new workers to the labor force, which may or may not correspond to the wages of existing workers, depending on the state of the labor market.[39]. This cost, in turn, is summarized by the wages of new hires as well hiring costs. [40]

The non-linearity of the Phillips curve arises due to the wage rate of new hires depending on whether the labor market is tight or not, i.e., $\hat{w}_t^{new} = \max(\hat{w}_t^{ex}, \hat{w}_t^{flex})$, which can be written using (35) as:

[39]Under normal circumstances, i.e. in the absence of labor shortage, the two will move together, consistent with the work of Gertler, Huckfeldt and Trigari (2020). It is during periods of labor shortage that the distinction between the wages of new and existing hires becomes critical in the model

[40]Bernanke and Blanchard (2023), for example, suggest that labor market tightness did not, at the very beginning of the inflation surge of 2022, play a fundamental role. Their result, however, depends upon the assumption that they measure marginal costs via ECI, in contrast to our model. Perhaps more importantly, our framework highlights the interaction between labor tightness and supply shock. One advantage of our approach is that we can explain why supply shocks had such an outsized effect during this period. Moreover, our model is consistent with the lack of disinflation post-2008.

$$\hat{w}_t^{new} = \begin{cases} \hat{w}_t^{flex} = \eta\hat{\theta}_t + \hat{\gamma}_t^c - \hat{\gamma}_t^b - \hat{m}_t & \hat{\theta}_t > \hat{\theta}_t^* \\ \hat{w}_t^{ex} = \lambda(\hat{w}_{t-1}^{ex} - \pi_t + \delta E_t\pi_{t+1}) + (1-\lambda)\hat{w}_t^{flex} & \hat{\theta}_t \leq \hat{\theta}_t^* \end{cases} \tag{38}$$

assuming $\pi_{t+1}^e = E_t\pi_{t+1}$. [41]

Since equations (29), (34) and (36) are linear in logs, these expressions are exact, i.e. there is no approximation error. Using them we can write the Phillips curve as:

$$\pi_t = \begin{cases} \kappa^{tight}\hat{\theta}_t + \kappa_v^{tight}(\hat{v}_t + \hat{\vartheta}_t^{tight}) + \beta E_t\pi_{t+1} & \hat{\theta}_t > \hat{\theta}_t^* \\ \kappa_w\hat{w}_{t-1} + \kappa \underbrace{\hat{\theta}_t}_{\text{tightness}} + \kappa_v(\underbrace{\hat{v}_t}_{\text{cost-push}} + \underbrace{\hat{\vartheta}_t}_{\text{matching}}) + \kappa_\beta E_t\pi_{t+1} & \hat{\theta}_t \leq \hat{\theta}_t^* \end{cases} \tag{39}$$

where the coefficients satisfy $\kappa^{tight} > \kappa > 0$, $\kappa_v^{tight} > \kappa_v > 0$ and $\kappa_w > 0$. It is ambigious if κ_β is greater or less than β. These results can be confirmed by the analytical expression of the coefficients.[42]

The cost push shocks, or supply shocks, are denoted by $\hat{v}_t$ and defined as

$$\underbrace{\hat{v}_t}_{\text{Cost push/supply shocks}} \equiv \underbrace{\hat{\mu}_t}_{\text{Markups}} - \underbrace{\hat{A}_t}_{\text{Productivity}} + (1-\alpha)\underbrace{\hat{q}_t}_{\text{Oil price}} \tag{40}$$

The shocks to which the hiring agency is subject, i.e., shocks to the matching technology, are summarized by $\hat{\vartheta}_t$

$$\underbrace{\hat{\vartheta}_t}_{\text{Matching shocks}} \equiv \alpha(z_\gamma - (1-\lambda)) \underbrace{\hat{\gamma}_t^b}_{\text{Firm hiring cost}} + \alpha(1-\lambda) \underbrace{\hat{\gamma}_t^c}_{\text{Vacancy cost}} - \alpha(1-\lambda) \underbrace{\hat{m}_t}_{\text{Matching efficiency}} + \alpha \underbrace{\hat{\phi}_t}_{\text{Wage shock}}, \tag{41}$$

in which $z_\gamma \equiv \gamma^b/(1+\gamma^b)$.

Unlike the cost-push shock $\hat{v}_t$, which is the same regardless the tightness of the labor market, the shocks to the matching technology take a different form when the labor market is tight in which case they are summarized by

$$\hat{\vartheta}_t^{tight} = \alpha(\hat{\gamma}_t^c - (1-z_\gamma)\hat{\gamma}_t^b - \hat{m}_t).$$

[41] The expression for $\hat{\theta}_t^*$ is

$$\hat{\theta}_t^* = \hat{\gamma}_t^b - \hat{\gamma}_t^c + \hat{m}_t + \eta^{-1}\hat{w}_{t-1} + \eta^{-1}\delta E_t\pi_{t+1} - \eta^{-1}\pi_t + \lambda^{-1}\eta^{-1}\hat{\phi}_t.$$

[42] The analytic expressions for the coefficients are given in the Appendix.

The reason is that these shocks operate through the cost of new hires. And this cost depends on whether the labor market is tight or not.

The theoretical results shown in equation (39) highlight four major theoretical predictions the empirical analysis in Section 2 confirmed.

The first main theoretical result is that $\kappa^{tight} > \kappa$. This is a key prediction of the model: It says that when the labor market is sufficiently tight, i.e. $\hat{\theta}_t > \hat{\theta}_t^*$, then inflation responds much more strongly to labor market tightness than under normal circumstances. This prediction is the central focus of our empirical analysis in Section 2. The empirical finding is shown using a basic regression, with several alternative specifications for robustness, as well as allowing for time variation in the coefficients estimated using the Kalman.

The second main theoretical result is that $\kappa_v^{tight} > \kappa_v$. This implies that cost-push and markup shocks exert a greater impact on inflation when the labor market is tight. This theoretical prediction is supported by our empirical analysis and is statistically significant. In the empirical analysis, the shocks we proxy are most closely represented by what we label as cost-push shocks, while the shock to labor matching has not received the same amount of attention in the literature. Investigating more direct measures of shocks to the labor market matching technology is an important topic for future research, allowing for a more direct empirical confrontation of the model's prediction. We briefly discuss this issue in Section 5.3.

The third main theoretical result is that while the real wages enters as a lagged variable when $\hat{\theta}_t < \hat{\theta}_t^*$, the Phillips curve is perfectly forward looking when $\hat{\theta}_t \geq \hat{\theta}_t^*$. This prediction is confirmed by the empirical analysis using either as lagged variable inflation (following existing literature) or lagged real wages as suggested by the theory (see Tables 1 and 11 respectively). The lagged variables are statistically significant using the full sample, but they are no longer statistically significant once we considering the latest part of the sample that included the period in which there was labor shortage. An alternative way we confirmed this theoretical prediction was by allowing for time varying coefficient and estimating the model using Kalman filter. As shown in Figure 7 inflation persistence dropped sharply once there is labor shortage.

Let us now turn to the policy implications.

5 The policy framework for the Inv-L NK Phillips Curve

We have shown that the Phillips curve can be written in terms of $\hat{\theta}_t$ to relate the prediction of the model to our, and other recent empirical work, that suggest this measure of tightness forecasts inflation better than other common proxies of aggregate demand. There is, however, a direct link between θ and output. Given that most policy discussion turns on output and inflation, there is some advantage in casting the model in this more familiar terms, which we do here. Our objective is to show that our general framework can be used to explain three major episodes that have been the focal point in monetary policy, the Great Inflation of the 1970's, the "missing inflation" following the financial crisis of 2008 and finally the inflation surge of 2020s. We use the results from our regression analysis to parameterize the model to address the degree to which the model, at least in broad orders of magnitudes, can rationalize the observed patters in US data.

To streamline the model and focus more closely on the key points, we make the simplifying assumption that in equilibrium $X_t = C_t = Y_t - G_t$.[43] This implies that the Euler Equation for consumption takes the traditional form from the New Keynesian literature:

$$\hat{Y}_t - \hat{G}_t = E_t\hat{Y}_{t+1} - E_t\hat{G}_{t+1} - \sigma^{-1}(\hat{\imath}_t - E_t\pi_{t+1} - \hat{r}^e_t),$$

where $\hat{r}^e_t \equiv \hat{\xi}_t - E_t\hat{\xi}_{t+1}$ is the part of the natural rate of interest that is generated by shocks to preferences. We simplify the wage of new hires to

$$w^{new}_t = \begin{cases} w^{flex}_t & \text{if } \theta_t > \theta^* \\ \bar{w}^{\lambda}(\Pi^{-1}_t)^{\lambda}(\Pi^e_{t+1})^{\delta\lambda}(w^{flex}_t)^{1-\lambda} & \text{if } \theta_t \leq \theta^* \end{cases}$$

in which $\bar{w}$ is the steady-state real wage. This implies there are no lagged variable in the Phillips curve when the labor market is slack, which, while empirically relevant for the full sample in our estimation, is not essential to the policy analysis. Making the model perfectly forward-looking allows a tighter analytic characterization which can be displayed graphically, as we soon will see.

Denote by Y^*_t the level of output when Phillips becomes steeper, i.e. when $\theta_t > \theta^*_t$.

The Phillips curve is then given by:

$$\pi_t = \begin{cases} \tilde{\kappa}^{tight}\left(\hat{Y}_t + \frac{\alpha}{\omega}\hat{\chi}_t\right) + \tilde{\kappa}^{tight}_v\hat{v}_t + \tilde{\kappa}^{tight}_\beta E_t\pi_{t+1} & \text{if } \hat{Y}_t > \hat{Y}^*_t \\ \tilde{\kappa}\left(\hat{Y}_t + \frac{\alpha}{\omega}\hat{\chi}_t\right) + \tilde{\kappa}_v\hat{v}_t + \tilde{\kappa}_\beta E_t\pi_{t+1} & \text{if } \hat{Y}_t \leq \hat{Y}^*_t, \end{cases}$$

where the coefficients, which are detailed in Appendix E, satisfy $\tilde{\kappa}^{tight} > k > 0$, $\tilde{\kappa}^{tight}_v > \tilde{\kappa}_v$ if $\lambda > 0$. The relationship between $\tilde{\kappa}^{tight}_\beta$ and $\tilde{\kappa}_\beta$ is again ambiguous. To simplify the analysis we set $\tilde{\kappa}^{tight}_\beta = \tilde{\kappa}_\beta = 1$ so that there is no long-run trade-off between inflation and output, but this is not essential to our main results.[44] To simplify the analysis we only consider, for now, a subset of the supply shocks given by $\hat{v}_t = \hat{\mu}_t + (1-\alpha)\hat{q}_t$, but abstract for now from matching shocks, $\hat{\vartheta}_t$ that we will get back to in the context of the inflation surge of 2020s. Relative to our earlier exposition we see presence of a new shock, which was previously embodied in $\hat{\theta}_t$. This is the shock to labor participation, $\hat{\chi}_t$. An

[43]To obtain this result, we assume that in equilibrium $\Psi_t = F^{1+\omega}_t/(1+\omega)$.

[44]By assuming that $\beta = \tilde{\kappa}_\beta = 1$ then in steady state, where all variables take a constant value, the Phillips curve implies that inflation cancels out and output is equal to zero.

increase in $\hat{\chi}_t$ represents higher dis-utility of working, which for a given output will lead to higher inflationary pressures.

We close the model with a simple policy rule:

$$\hat{\imath}_t = \hat{r}_t^e + \phi_\pi(\pi_t - \pi^*) + e_t$$

where $\phi_\pi > 1$ is a reaction coefficient of inflation deviating from the target, denoted by π^*. Recall that $\hat{r}_t^e$ contains movements in aggregate demand explained by the demand disturbance ξ_t. In other words, we assume that the central bank fully offsets any exogenous demand shock, but we leave open the question of whether it will do so in response to supply shocks, or demand shocks like government spending. The central bank response to these variables can be incorporated into the monetary policy shock e_t, which for now we leave unspecified.

One prominent hypothesis on the causes of inflation in the 1970s, set forth by Clarida, Gali and Gertler (2000), is that the central bank did not react strongly enough to inflation by raising the interest rate, so $\phi_\pi < 1$. This assumption leads to equilibrium indeterminacy, meaning that there are infinite possible paths of inflation that qualify as a solution. Our purpose here, however, is to highlight the effect of inflation expectations becoming unanchored. That is, we are interested in comparative statics with respect to the central bank's long-run inflation target. If the model has an infinite number of equilibria, comparative statics are meaningless. So it is useful to assume that $\phi_\pi > 1$. Moreover, since we allow for a policy shock e_t, the policy rule is in any case still rich enough to encompass the possibility that monetary policy did not respond strongly enough to the rise in inflation, thus capturing the spirit of their hypothesis.

The discussion that follows is highly stylized and can be boiled down to simple AD-AS diagrams. Yet, the results from the regression in Section 2 can be mapped directly into the parameters of the Phillips curve. This, in turn, means that we can sketch up our diagrams in a way that has quantitative interpretation, even if a full quantitative simulation of these episodes is beyond the scope of the paper. We use the empirical result for the entire period when considering the Great Inflation of the 1970s but the result from 2008 to the present day when considering the "missing deflation" during in the aftermath of the Financial Crisis and the inflation surge of 2020s.

Table A shows the values of the parameters we take as given. The model is written in quarterly frequencies to match our data observation. Most of the values are typical relative to the literature, such as σ, and ϕ_π, require little explanation, given how common they are in the literature. The matching parameter η is taken from Blanchard, Domash and Summers (2022). Steady steady state unemployment is 4 percent approximating the current estimate of the Federal Reserve of the level of unemployment consistent with inflation being on target (see e.g. Eggertsson and Kohn (2022)). We infer the fraction of the labor force attached to existing labor force based upon labor flow data. The key interpretation of the intermediate input in the production function is that it represents oil. The value we use is higher than typical estimates of the share of oil of output (e.g. Blanchard and Gali (2007) report it as

3 percent of output). We assume the higher value to allow for a richer interpretation of intermediate goods more broadly. We assume $\omega = 1$, a value common in the monetary literature even if this value is typically chosen in models where the labor choice of the households is on the intensive margin as opposed to the extensive margin. The value of λ measure how strongly existing wages are pulled to the wage rate that would prevail if wages were flexible and we set it at 0.5. The values of $\tilde{\kappa}$'s is constrained by the numerical value we estimate them to be when θ_t is the argument appearing in the Phillips Curve. For this reason we have found that the results are not particularly sensitive to λ.

Table A: Parameters based on existing literature

Calibrated parameters	**Symbol**	**Value**
Intertemporal rate of substitution	σ	0.5
Taylor rule coefficient	ϕ_π	1.5
Matching technology parameter	η	0.4
Steady State Unemployment	u	0.04
Fraction of labor force attached to existing firms	$1-s$	0.93
Share of intermediate input (oil) in the production function	$1-\alpha$	0.1
Disutility of entering the labor force	ω	2
Response of existing wages to flexible wages	λ	0.5

Given these parameters in Table A, we can use the expressions in the Appendix, together with empirical analysis, to back out the implied values for the key parameters of the Phillips curve, which are summarized in Table B. We these numerical values to give quantiative interpretation to the simple exposition in the AS-AD diagrams in the next few sections.

Table B: Coefficient in the Phillips curve based on the estimation and the calibrated parameters

Parameters derived from estimation	**(1) 1960-2023**	**(2) 2008-2023**
$\tilde{\kappa}$	0.003	0.0065
$\tilde{\kappa}_v$	0.0444	-0.0096
$\tilde{\kappa}^{tight}$	0.0491	0.0716
$\tilde{\kappa}_v^{tight}$	0.6315	1.338

5.1 Understanding the inflation surge in the 1970s the disinflation and Volcker

recession in the 1980s

We first consider how the model explains the increase in inflation in the 1970s, often referred to as the "Great Inflation". This will be a familiar story to most readers, and our version does not differ substantially from the conventional account (see e.g. Erceg and Levin (2003) and Goodfriend and King (2006)). Nevertheless, it is helpful to spell it out clearly within our model, so that we can contrast with our account of the inflationary surge of the 2020s. A convenient analytical device is to split the model into the short run, denoted by S, and the long run, denoted by L. A major simplification, which allows us to illustrate the main points analytically, is to assume that all shocks occur in the short run and in the long run revert to zero with a fixed probability $(1-\tau)$. This implies an expected duration of the "short run" being $\frac{1}{\tau}$.

The short run is described by two shocks. First, we assume a short run supply shock given by $\hat{v}_S = (1-\alpha)\hat{q}_S$, i.e. we consider the possibility that inflation was triggered by the the large oils shocks observed during the 1970's. Second, we assume that in the short run, the public has a different belief about the central bank's long-run inflation target, which we assume is π^*. This means that even if long-run inflation will eventually stabilize, i.e. $\pi_L = \pi^*$, as was eventually the case of the U.S., we assume that peoples people's expectation the 1970s did not co-incide with how things actually turned out. This means that π_L^e, is assumed to be different from what turned out to be the case ex post. The motivation for this is straightforward: several long-term expectation measures in the 1970s and early 1980s cited in the introduction suggested that five-to-ten year inflation expectations reached 10 % and only declined gradually during the 1980s (see especially footnote 6). This means that people's expectation about long run inflation expectation in the 1970's turned out to be "incorrect" ex post.[45]

In the long run, we assume the labor market is back to "normal". In the absence of shocks, the unique bounded solution is simply given by $\pi_L = \pi^*$ and $\hat{Y}_L = 0 < \hat{Y}^*$.

Let us now consider the short run. Suppose that expected long-term inflation expectations are above π^*, as observed in the data, and constant at $\pi_L^e > \pi^*$. Then inflation expectations in the short run are

$$E_S \pi_{S+1} = \tau \pi_S + (1-\tau)\pi_L^e.$$

Since our model is characterized by long-run monetary policy neutrality, then even though people expect $\pi_L^e > \pi^*$, it remains the case that a unique bounded solution is given by $\hat{Y}_L = 0$.

Setting all shocks to zero (except the oil-price/energy shock) the Euler Equation for consumption is:

$$\hat{Y}_S = -\sigma^{-1}\frac{\phi\pi - \tau}{1-\tau}(\pi_S - \pi^*) + \sigma^{-1}(\pi_L^e - \pi^*) \tag{42}$$

while the Inv-L NK Phillips Curve is:

[45] Of course there is nothing that says that these believes have to have been irrational.

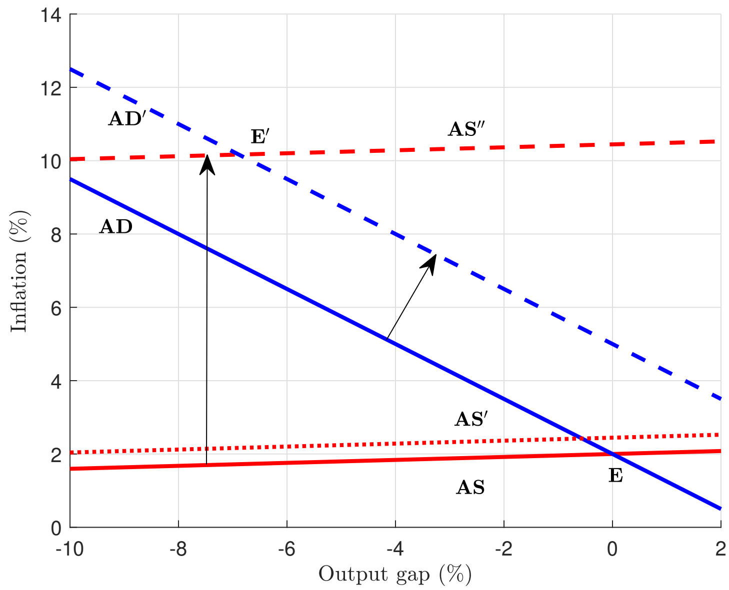

Figure 9: The 1970s Great Inflation: Inflation and output determination using the AS-AD model in response to an increase in long-run inflation expectations and an oil shock. Inflation is measured in annual rates and presented as a percentage, while the output gap is expressed in percentage points. The AS curve shifts from AS to AS′ following a 30% rise in oil prices. Subsequently, with a 10% increase in long-run inflation expectations, the AS′ curve moves to AS″ and the AD curve to AD′. The equilibrium transitions from E to E′.

$$\pi_S - \pi^* = \begin{cases} \frac{\tilde{\kappa}^{tight}}{1-\tau}\hat{Y}_S + \frac{\tilde{\kappa}_v^{tight}}{1-\tau}\hat{v}_S + \pi_L^e - \pi^* & \text{if } \hat{Y}_t > \hat{Y}^* \\ \\ \frac{\tilde{\kappa}}{1-\tau}\hat{Y}_S + \frac{\tilde{\kappa}_v}{1-\tau}\hat{v}_S + \pi_L^e - \pi^* & \text{if } \hat{Y}_t \leq \hat{Y}^*, \end{cases} \tag{43}$$

See Appendix E for further details of the derivations.

These two equations are plotted in Figure 9, the aggregate demand equation (42) in blue and the Inv-L NK Phillips Curve in red. Entering the 1970s, θ was less than one and held at that level during that period so that that $Y_t < Y_t^*$. Equilibrium is determined on the flat part of the Phillips curve which is depicted in the figure. Consider first the the initial equilibrium when inflation is at target and output at potential at point E. The numerical example captures that the Inv-L NK Phillips curve is very flat. There are two ingredients that we have not spelled out. First, the supply shock together with by how much inflation expectation becoming "unanchored" but these two shocks define the short-run. Second, the expected duration the short-term is expected to last. We use the principal component measure of the supply shock which peaked at 30 percent, while we assume that inflation expectation rose to 10 percent. Moreover, we assume that expected duration of the short run is just shy of a year, or 3 and 1/3 quarters.

Consider first the effect of a supply shock. This will directly shift the red curve upward, with higher inflation and lower output. The oil shocks of the 1970s clearly played a role in the inflation surge.

Of greater interest, however, is the effect of a rise in the central bank's inflation target perceived by the private sector, i.e. $\pi_L^e > \pi^*$. If the public believes that the central bank will set a higher inflation target in the long run, this results in a one-to-one upward shift of the red curve. Hence, it immediately generates inflation just like a supply shock, for a given level of demand. This is not the whole story, however. As shown by equation (42), the rise in long-term inflation expectations also increases demand in the short run by reducing the real interest rate and so making borrowing cheaper (real rates were negative through much of the 1970s.) This shifts the blue curve up. It is easy to show, however, that the effect on output is always negative if the model has a unique equilibrium, i.e. under the condition that $\phi_\pi > 1$.[46]

As we can see, our numerical example can capture – in rough orders of magnitudes – the rise in inflation expectations. While we see that the supply shocks played a role, the major contributor is the unanchoring of inflation expectations.

We can use the same framework to see why disinflation can be very costly if the private sector believes that the long-term inflation target is high, despite the central bank's claims to the contrary. How costly this is depends fundamentally on how the perceptions are formed about future inflatoin. Consider the possibility that people will only reconsider their perception of the long-run inflation target if they see some significant reduction in *current inflation*. In this case, because of the flatness of the Phillips curve, it will be very costly to bring inflation down. This is one popular narrative for the Volker recession in the early 1980's. While we will not spell out this process explicitly here, it should be easy to see that we have all the main ingredients to tell that story by linking inflation expectations to realized output and inflation which coudl be interpreted by the public as measures of the central banks resolve in bringing down inflation.

5.2 The missing deflation following 2008

As illustrated in figure 10 the model does not have much difficulty rationalizing the fact that inflation did not decline significantly following the financial crisis of 2008. (More to be added)

5.3 Understanding the inflationary surge and a potential soft landing in the 2020s

Let us now contrast this fairly conventional account of the Great Inflation with the surge in inflation in the 2020s, in the framework of our model. The fundamental difference is that in this case $\hat{\theta}_t > \theta^*$ so that the demand intersects the steeper segment of the Phillips curve.

[46] It would be wrong, however, to draw the conclusion that $\phi_\pi < 1$ implies that higher long-term inflation expectations increase output. If $\phi_\pi < 1$ there is an infinite number of equilibria. The model cannot predict which one will equilibrium is to be chosen in response to a change in any of the exogenous variables if $\phi_\pi < 1$. Comparative statics are thus meaningless.

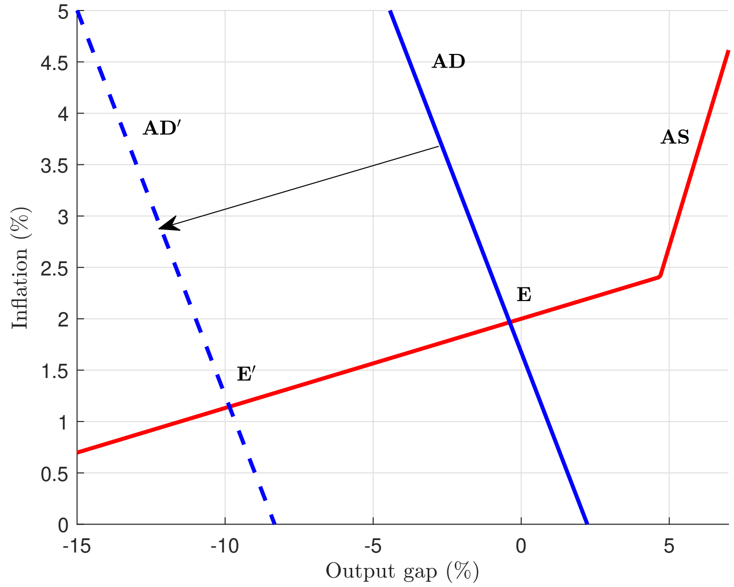

Figure 10: The 2008 Missing Disinflation: Inflation and output determination using the AS-AD model in response to a fall in aggregate demand. Inflation is measured in annual rates and presented as a percentage, while the output gap is expressed in percentage points. The AD curve shifts from AD to AD′ following a negative demand shock. The equilibrium transitions from E to E′.

The model can be simplified as before, dividing it into short and long run. Now, however, we assume that long-run inflation expectations are anchored, so that $\pi_L^e = \pi_L = \pi^*$. Moreover, we now focus on the short-run demand (fiscal) shock $\hat{G}_S$, the supply shock $\hat{q}_S$ and the monetary policy shock e_S. We will also consider a shock to labor force participation, $\hat{\chi}_S$[47]

As shown in Appendix E, in the model the short run can then be summarized by the following two equations:

$$\pi_S - \pi^* = \begin{cases} \frac{k^{tight}}{1-\tau}(\hat{Y}_S + \frac{\alpha}{\omega}\hat{\chi}_S) + \frac{k_v^{tight}}{1-\tau}(1-\alpha)\hat{q}_S & \text{if } \hat{Y}_t > \hat{Y}^* \\ \\ \frac{k}{1-\tau}(\hat{Y}_S + \frac{\alpha}{\omega}\hat{\chi}_S) + \frac{k_v}{1-\tau}(1-\alpha)\hat{q}_S & \text{if } \hat{Y}_t \leq \hat{Y}^*. \end{cases} \tag{44}$$

The equilibrium is shown in Figure 10. Consider first the equilibrium at point A. We think of this as representing the pre-pandemic situation, say end of 2019. As we have seen, inflation took off in 2021. Our main hypothesis is that this was due to the labor market tightness, which in the current framework shows up as an increase in aggregate demand through government spending, G_S, and an expansionary monetary policy, represented by negative e_S. Note also that supply shocks, such as a reduction in labor force participation, will also produce labor market tightness – a point we return to shortly. That there had been a substantial demand stimulus was relatively well known by 2021.

[47] This can in principle be permanent, which involves some minor complications.

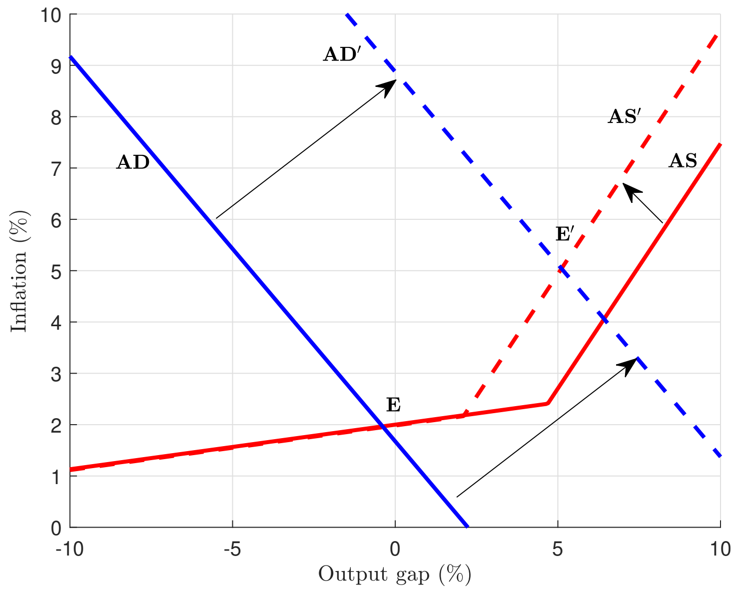

Figure 11: The 2020s Inflationary Surge: Inflation and output determination using the AS-AD model in response to an increase in demand and a supply shock. Inflation is measured in annual rates and presented as a percentage, while the output gap is expressed in percentage points. The AS curve shifts from AS to AS' following a 5% rise in supply shock. The AD moves to AD'. The equilibrium transitions from E to E' moving from the flat part of the AS equation to the steep one.

Indeed, it was the ground for widespread criticism of the administration, most notably a series of articles by Lawrence Summers (2021). Our hypothesis on why the surge in inflation nevertheless caught policymakers and private forecasters by surprise is that they assumed the Phillips curve was given by the flat part of Figure 10, so that even if the stimulus was indeed excessive, the impact on inflation would be minor, as illustrated by point B, the intersection of demand and the flat part of the Phillips curve. With labor market tightness measured by θ rising to a level not seen since measurements have been available, our key hypothesis is that the economy was instead on the upward sloping segment of the Phillips curve, a region that was a key prediction of Phillips himself in the original article in 1958, but one that may have been overlooked, since the Great Inflation of the 1970s was driven by different forces. This degree of tightness or labor shortage had not been seen since the Korean and Vietnam wars.

If one accepts our basic premise that the inflationary surge was driven by labor shortage, there is a silver lining. If the surge is driven not by expectations – which may be hard to rein in – but instead by a steep Phillips curve, it should be much less costly to bring inflation down to target. A steep Phillips curve implies "easy up" – i.e. a relatively small output gain is associated with the inflation – but also the converse, "easy down" – small output losses from bringing inflation under control. Leaving aside the trade-off implied by the supply shock, all that is needed is for the central bank to raise the interest rate to offset the increase in demand, which is summarized by $\hat{G}_S$ plus the shock to labor force participation $\hat{\chi}_S$. It is easy to confirm that if we select $e_S = \sigma(1-\tau)(\hat{G}_S + \frac{\alpha}{\omega}\hat{\chi}_S)$ then inflation is on target and the output gap is zero. Translated to interest rates, this means that if a central bank

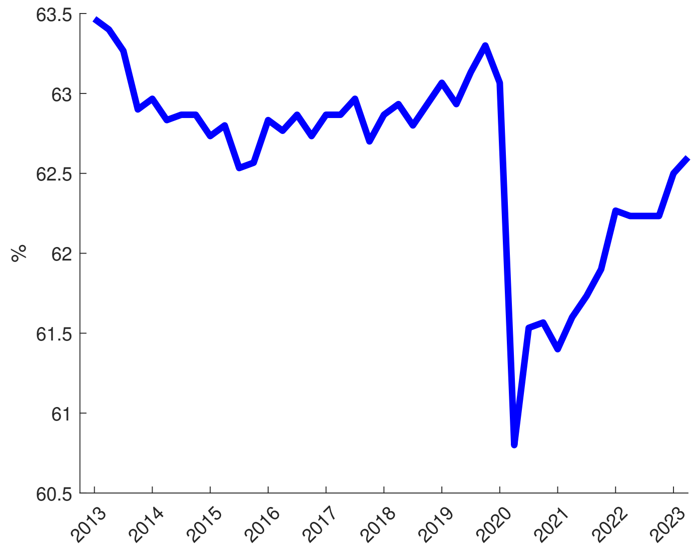

Figure 12: Labor force participation in the U.S. during the last decade.

finds itself on the upward-sloping segment of the Phillips curve it can hit its inflation target and attain potential output by raising interest rates according to the formula:

$$\hat{\imath}_t = \hat{r}_t^e + \sigma(1-\tau)\left(\hat{G}_S + \frac{\alpha}{\omega}\hat{\chi}_S\right). \tag{45}$$

The nonlinearity of the Phillips curve implies that reaching point D in Figure 10 can reduce inflation significantly with a relatively small sacrifice of output – a soft landing. It is an open question, however, how far the non-inflationary level of output is from the kink point, defining as non-inflationary output as that consistent with target inflation. What that output actually is, and the degree of labor tightness associated with it, remains an open question to which we return in the conclusion.

6 Conclusion

In this paper we have proposed a reformulation of what has become known as the canonical New Keynesian Phillips curve and replaced it with one that admits significant nonlinearities. Our hypothesis is that the nonlinearity is responsible for the increase in inflation in the 2020s. We conjecture that a key reason why policymakers and market participants alike failed to foresee the surge in inflation, or its persistence is that they implicitly or explicitly assumed a "flat" Phillips curve. Even after substantial inflation had already occurred, the reassurance that expectations were holding stable further induced the belief that the surge was merely transitory. One question is why the Federal Reserve did not raise interest rates more quickly. Possibly the new policy framework announced in 2020 put greater emphasis on the employment side of the Fed's dual objective. Yet, at the same time, it acknowledged that there was no agreement on any precise measure of how close the US economy was to full employment at any given point in time. This, of course, contrasts very sharply with the other side of the mandate, i.e. inflation, for which there is broad consensus on how the Fed can attain its objective.

Figure 12 sheds some light on why policymakers may have believed in 2021 that even though the traditional gauge of labor slack, i.e. unemployment, was very low, this did not capture the full picture. The unemployment rate only tells us how many active job seekers there are. As the figure reveals, however, participation collapsed with the COVID-19 epidemic, which might have suggested to many that there was still considerable room for employment to grow further. Moreover, given the flat Phillips curve – the professional consensus at the time – and stable inflation expectation, it might have been tempting for policymakers to explore the possibility that the US economy could attract greater labor force participation, e.g. similar to pre-pandemic level with relatively low risk of inflation. In terms of the dual mandate, conditional on a flat Phillips curve, this could easily have been seen at the time as a situation with possible high reward and relatively limited downside risk. The bottom line of this paper, however, is that the inflationary risk of allowing the labor market to tighten too much, to a degree we have defined as labor shortage, generates much greater upside risk for inflation than has been commonly thought. An important reason for this underestimation of inflation risk is no doubt the unprecedented labor shortage, historically unprecedented except in wartime, and the countless estimates of the slope of the Phillips curve that did not incorporate wartime. We have sought first to show this empirically and then to build a model to explain it. The good news, in any case, is that if our theory is correct the cost of taming inflation triggered by a labor shortage, but with stable inflation expectations, can be expected to be much lower than it was in the 1970s.

As in any account of events, there is a sharp trade-off between parsimony and detailed quantiative account. We have here choosen to

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A Appendix: Additional Tables

Table 2 presents the OLS estimates of regression (2) with the same variables as Table 1, except that the threshold for the dummy is set at $\theta^* = 0.5041$ for the sample 1960-2023, and at $\theta^* = 0.7962$ for the sample 2008-2023, which correspond to the thresholds, in the respective sample, that maximize the likelihood of the regressions across different thresholds, as shown in Figure 22.

Table 2: **Phillips Curve Estimates when θ^* is chosen to maximize the likelihood function of the model**

	(1) 1960-2023	(2) 2008-2023	(3) 1960-2023	(4) 2008-2023
Inflation lag	0.3768^{***} (0.0946)	0.2765 (0.2465)	0.3321^{***} (0.0925)	-0.1580 (0.2088)
$\ln\theta$	0.6822^{***} (0.183)	0.7062^{*} (0.3806)	0.7281^{***} (0.1931)	0.5493 (0.3030)
$\theta \geq \theta^*$			0.7010^{***} (0.2641)	5.3307^{***} (0.9281)
μ shock	0.0372^{*} (0.0192)	0.0101 (0.0379)	-0.0323 (0.0314)	-0.0074 (0.0242)
$\theta \geq \theta^*$			0.1095^{***} (0.0471)	0.2753^{**} (0.1346)
Inflation expectations	0.6524^{***} (0.106)	1.0613 (0.6352)	0.6754^{***} (0.1032)	0.4996 (0.5770)
Constant	0.5629^{***} (0.1585)	1.0303^{**} (0.4621)	0.6884^{***} (0.1681)	0.4289 (0.3622)
R^2 adjusted	0.816	0.530	0.835	0.673
Observations	254	60	254	60

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.
· Newey-West standard errors.
· (1) and (3): sample 1960 Q1 – 2023 Q2
· (2) and (4): sample 2008 Q3 – 2023 Q2

Table 3 presents the OLS estimates of regression (2) with the same variables as Table 1, except that we proxy the supply shock with the four-quarter average of the CPI headline shock.

Table 3: **Phillips Curve Estimates using only CPI headline shock as proxy for supply disturbances**

	(1) 1960-2023	(2) 2008-2023	(3) 1960-2023	(4) 2008-2023
Inflation lag	0.3976^{***}	0.258	0.3046^{***}	-0.1113
	(0.1017)	(0.2432)	(0.1132)	(0.2032)
$\ln \theta$	0.681^{***}	0.7329^{*}	0.3415	0.5565
	(0.1817)	(0.3781)	(0.2058)	(0.3385)
$\theta \geq 1$			2.7553^{***}	3.2039^{**}
			(1.0291)	(1.458)
μ *shock*	0.121	0.1726	0.098	0.0152
	(0.0834)	(0.1915)	(0.0871)	(0.1265)
$\theta \geq 1$			0.1409	0.9436
			(0.334)	(0.612)
Inflation expectations	0.6562^{***}	0.8719	0.7841^{***}	0.3631
	(0.1089)	(0.6663)	(0.1172)	(0.6667)
Constant	0.5185^{***}	0.9787^{**}	0.2273	0.3977
	(0.1552)	(0.4368)	(0.1716)	(0.3912)
R^2 adjusted	0.811	0.5412	0.819	0.6365
Observations	254	60	254	60

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2023 Q2

· (2) and (4): sample 2008 Q3 – 2023 Q2

Table 4 presents the OLS estimates of regression (2) with the same variables as Table 1, except that we proxy the supply shock with the four-quarter average import-price shock.

Table 4: **Phillips Curve Estimates using import-price relative to GDP deflator as measure of supply shock**

	(1) 1960-2023	(2) 2008-2023	(3) 1960-2023	(4) 2008-2023
Inflation lag	0.3761^{***}	0.2766	0.2584^{***}	-0.1394
	(0.0945)	(0.2467)	(0.0928)	(0.203)
$\ln\theta$	0.6841^{***}	0.7052^{*}	0.2458	0.5528
	(0.1833)	(0.3805)	(0.2032)	(0.3368)
$\theta \geq 1$			3.8259^{***}	5.4658^{***}
			(0.8491)	(0.9406)
μ *shock*	0.0379^{*}	0.0095	0.0457^{**}	-0.0091
	(0.0195)	(0.0387)	(0.0206)	(0.0246)
$\theta \geq 1$			0.0783	0.2835^{**}
			(0.1065)	(0.1366)
Inflation expectations	0.6528^{***}	1.0673^{*}	0.8037^{***}	0.5321
	(0.1059)	(0.6339)	(0.1011)	(0.5701)
Constant	0.5807^{***}	1.0348^{**}	0.2291	0.4433
	(0.16)	(0.469)	(0.1726)	(0.4131)
R^2 adjusted	0.816	0.530	0.828	0.663
Observations	254	60	254	60

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2023 Q2

· (2) and (4): sample 2008 Q3 – 2023 Q2

Table 5 presents the OLS estimates of regression (2) with the same variables as Table 1, except that 2-year Cleveland-Fed inflation expectation is replaced by the 1-year CPI inflation expectations of the U.S. Survey of Professional Forecasters until 1981 Q3, which is patched backward using the interpolated 12-month Livingston survey until 1960 Q1.

Table 5: **Phillips Curve Estimates using 1-year CPI expectation of the U.S. Survey of Professional Forecasters**

	(1) 1960-2023	(2) 2008-2023	(3) 1960-2023	(4) 2008-2023
Inflation lag	0.3108*** (0.0972)	0.1302 (0.2761)	0.2224** (0.0994)	−0.1832 (0.206)
$\ln \theta$	0.7024*** (0.1778)	0.4882 (0.3612)	0.3508* (0.1849)	0.4451 (0.3273)
$\theta \geq 1$			2.9809*** (0.8703)	5.1187*** (1.2407)
μ shock	0.0351* (0.0195)	0.0121 (0.0377)	0.0403* (0.0205)	−0.0065 (0.0238)
$\theta \geq 1$			0.0813 (0.0951)	0.2728** (0.1356)
Inflation expectations	0.7613*** (0.1093)	1.6869** (0.6753)	0.8763*** (0.1109)	0.7202 (0.4777)
Constant	0.482*** (0.1452)	0.3493 (0.4036)	0.1865 (0.1478)	0.1074 (0.3056)
R^2 adjusted	0.821	0.541	0.829	0.662
Observations	254	60	254	60

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2023 Q2

· (2) and (4): sample 2008 Q3 – 2023 Q2

Table 6 presents the OLS estimates of regression (2) with the same variables as Table 1, except that inflation expectations are proxied by the 5-year inflation expectations of the Cleveland Fed until 1982 Q2, which are patched with PFS 1-year inflation expectations for the GDP deflator until 1970 Q2 and the interpolated 12-month Livingston inflation expectations until 1960 Q1.

Table 6: **Phillips Curve Estimates Using Cleveland Fed's 5-year inflation expectation measure**

	(1) 1960-2023	(2) 2008-2023	(3) 1960-2023	(4) 2008-2023
Inflation lag	0.4032***	0.3441	0.2861***	−0.1415
	(0.1004)	(0.2434)	(0.1033)	(0.2007)
$\ln\theta$	0.7453***	0.8417**	0.3375	0.5544*
	(0.1889)	(0.4125)	(0.2053)	(0.3289)
$\theta \geq 1$			3.6526***	5.9245***
			(0.8838)	(0.9909)
μ shock	0.0477**	0.0272	0.0569***	0.0021
	(0.019)	(0.0375)	(0.0203)	(0.0243)
$\theta \geq 1$			0.0869	0.2734**
			(0.1064)	(0.1322)
Inflation expectations	0.6349***	0.5614	0.7926***	−0.0033
	(0.1203)	(0.7261)	(0.1219)	(0.6704)
Constant	0.5933***	0.9477**	0.2515	0.2083
	(0.1607)	(0.4687)	(0.1729)	(0.4109)
R^2 adjusted	0.808	0.495	0.820	0.6542
Observations	254	60	254	60

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2023 Q2

· (2) and (4): sample 2008 Q3 – 2023 Q2

Table 7 presents the OLS estimates of regression (2) with the same variables as Table 1, except that the 2-year Cleveland-Fed inflation expectations are replaced by the five-year five-year forward inflation expectations back-casted by Groen and Middledorp (2013) until 1971 Q4. The expectations are patched with the interpolated 12-month Livingston inflation expectations until 1960 Q1.

Table 7: **Phillips Curve Estimates using five-year five-year forward inflation expectations**

	(1) 1960-2023	(2) 2008-2023	(3) 1960-2023	(4) 2008-2023
Inflation lag	0.5878^{***} (0.0839)	0.136 (0.2388)	0.5749^{***} (0.0893)	-0.1691 (0.2404)
$\ln\theta$	0.8574^{***} (0.2017)	1.8952^{***} (0.5538)	0.7551^{***} (0.2154)	1.3114^{**} (0.6099)
$\theta \geq 1$			0.9787 (0.6767)	4.3577^{***} (1.2093)
μ *shock*	0.0672^{***} (0.0213)	-0.0011 (0.0255)	0.0694^{***} (0.0223)	-0.0175 (0.023)
$\theta \geq 1$			0.0497 (0.078)	0.2578^{**} (0.1156)
Inflation expectations	0.3291^{***} (0.0825)	1.7337^{***} (0.4676)	0.3428^{***} (0.0859)	1.108^{**} (0.5432)
Constant	0.5389^{***} (0.1274)	0.9216^{**} (0.4191)	0.4508^{***} (0.144)	0.459 (0.3739)
R^2 adjusted	0.785	0.594	0.784	0.6902
Observations	254	60	254	60

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2023 Q2

· (2) and (4): sample 2008 Q3 – 2023 Q2

Table 8 presents the OLS estimates of regression (2) with the same variables as Table 1, except that the inflation lag and the log of the vacancy rate to unemployment rate θ are instrumented with the fitted values of their first lags.

Table 8: **Phillips Curve Estimates using an instrumental variable approach.**

	(1) 1960-2023	(2) 2008-2023	(3) 1960-2023	(4) 2008-2023
Inflation lag (Fitted)	0.2918^{***} (0.1052)	0.4386^{**} (0.1880)	0.1878^{*} (0.0979)	0.2403 (0.1469)
$\ln\theta$ *(Fitted)*	0.8151^{***} (0.2188)	0.3795 (0.3923)	0.4461^{*} (0.2684)	0.3922 (0.3910)
$\theta \geq 1$ *(Fitted)*			3.8985^{***} (1.0038)	2.3705 (1.5323)
μ shock	0.0506^{**} (0.0235)	0.0123 (0.0409)	0.0525^{**} (0.0244)	-0.0098 (0.0210)
$\theta \geq 1$			0.1580 (0.1463)	0.3091^{**} (0.1539)
Inflation expectations	0.7766^{***} (0.1067)	1.2247^{*} (0.7137)	0.8920^{***} (0.1007)	0.9882 (0.7307)
Constant	0.6214^{***} (0.1933)	0.7884^{*} (0.4065)	0.3559 (0.2296)	0.6484 (0.4611)
R^2 adjusted	0.797	0.493	0.809	0.567
Observations	254	60	254	60

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2022 Q3

· (2) and (4): sample 2008 Q3 – 2023 Q2

Table 9 presents the OLS estimates of regression (2) with the same variables as Table 1, except that PCE core inflation rate replaces CPI core inflation as the dependent variable.

Table 9: **Phillips Curve Estimates using PCE core.**

	(1) 1960-2023	(2) 2008-2023	(3) 1960-2023	(4) 2008-2023
Inflation lag	0.5489^{***} (0.0712)	0.3835^{*} (0.2188)	0.4329^{***} (0.0796)	0.0905 (0.2401)
$\ln\theta$	0.2938^{**} (0.1141)	0.3995 (0.2809)	-0.0289 (0.151)	0.2984 (0.2712)
$\theta \geq 1$			2.5772^{***} (0.6559)	2.9382^{***} (0.988)
μ *shock*	0.0352^{***} (0.0133)	0.0201 (0.032)	0.0428^{***} (0.0143)	0.021 (0.0207)
$\theta \geq 1$			0.0395 (0.0798)	0.1873 (0.1163)
Inflation expectations	0.3861^{***} (0.0685)	0.8572 (0.5471)	0.5068^{***} (0.078)	0.5736 (0.4497)
Constant	0.2042^{**} (0.1003)	0.5561^{**} (0.2764)	-0.0713 (0.1357)	-0.4036 (0.3757)
R^2 adjusted	0.864	0.560	0.872	0.627
Observations	254	60	254	60

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2023 Q2

· (2) and (4): sample 2008 Q3 – 2023 Q2

Table 10 presents the OLS estimates of regression (2) with the same variables as Table 1, except that the level of θ is used rather than its log.

Table 10: **Phillips Curve Estimates using the level of θ**

	(1) 1960-2023	(2) 2008-2023	(3) 1960-2023	(4) 2008-2023
Inflation lag	0.3352^{***} (0.0928)	0.1524 (0.2534)	0.2741^{***} (0.0922)	0.0321 (0.2733)
θ	1.202^{***} (0.2523)	1.5676^{**} (0.7303)	0.2252 (0.3604)	0.9664 (0.6056)
$\theta \geq 1$			0.9512^{***} (0.3231)	0.7498 (0.6072)
μ *shock*	0.0391^{**} (0.0195)	0.0169 (0.0384)	0.0446^{**} (0.0204)	-0.0166 (0.0209)
$\theta \geq 1$			0.0875 (0.1086)	0.2575^{*} (0.1509)
Inflation expectations	0.7086^{***} (0.1025)	0.7036 (0.6959)	0.8027^{***} (0.1011)	0.8092 (0.5879)
Constant	-0.6009^{***} (0.1514)	-0.5831^{**} (0.4537)	-0.1355 (0.2083)	-0.4036 (0.3757)
R^2 adjusted	0.823	0.5628	0.830	0.635
Observations	254	60	254	60

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2023 Q2

· (2) and (4): sample 2008 Q3 – 2023 Q2

Table 11 presents the OLS estimates of regression (2) with the same variables as Table 1, except that the lag of the inflation rate is replaced by the lag value of the detrended real wage.

Table 11: **Phillips Curve Estimates using lagged real wages instead of lagged inflation.**

	(1) 1960-2023	(2) 2008-2023	(3) 1960-2023	(4) 2008-2023
Real wage lag	12.4967**	−4.7157	15.0302***	−1.7364
	(4.9608)	(8.3158)	(3.9894)	(8.9739)
$\ln\theta$	0.7666***	1.0602**	−0.0686	0.5415
	(0.2818)	(0.4189)	(0.2572)	(0.4337)
$\theta \geq 1$			5.7805***	4.2291***
			(0.750)	(1.3590)
μ shock	0.051**	0.0060	0.0618**	−0.0099
	(0.0258)	(0.0440)	(0.0241)	(0.0202)
$\theta \geq 1$			0.0683	0.2637*
			(0.1160)	(0.1343)
Inflation expectations	1.0212***	1.2877**	1.0517***	0.5192
	(0.0504)	(0.5916)	(0.0487)	(0.6315)
Constant	0.7803***	1.3802***	0.1021	0.4577
	(0.2282)	(0.3744)	(0.2044)	(0.4690)
R^2 adjusted	0.788	0.490	0.828	0.658
Observations	254	60	254	60

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.
· Newey-West standard errors.
· (1) and (3): sample 1960 Q1 – 2023 Q2
· (2) and (4): sample 2008 Q3 – 2023 Q2

B Appendix: Additional Figures

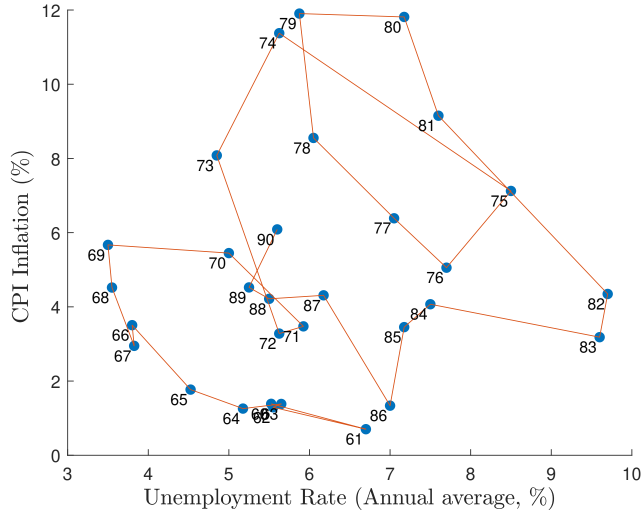

Figure 13: Empirical breakdown of the Phillips Curve in the 1970s as discussed in the Introduction, sample 1960-1990.

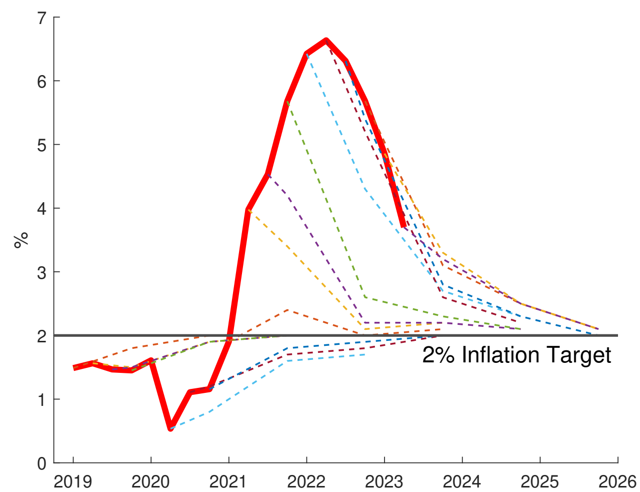

Figure 14: PCE-index inflation at annual rate (red line) and the inflation forecast of the Summary of Economic Projections (SEP) (dashed lines) of the Federal Reserve up to and during the inflation surge.

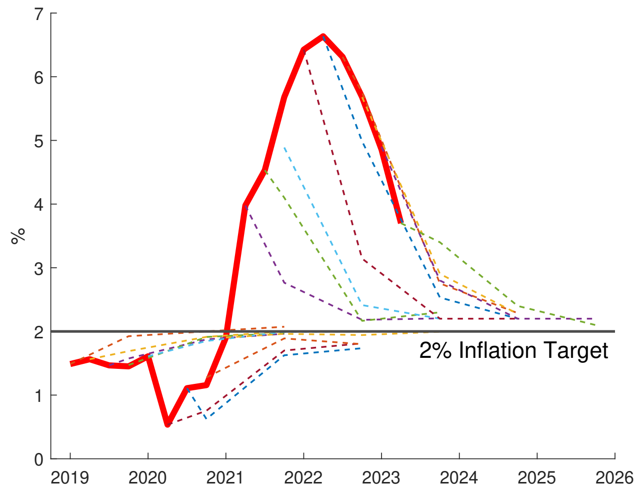

Figure 15: PCE-index inflation at annual rate (red line) and the inflation forecast of the Survey of Professional Forecasters (SPF) (dashed lines) up to, and during, the inflation surge.

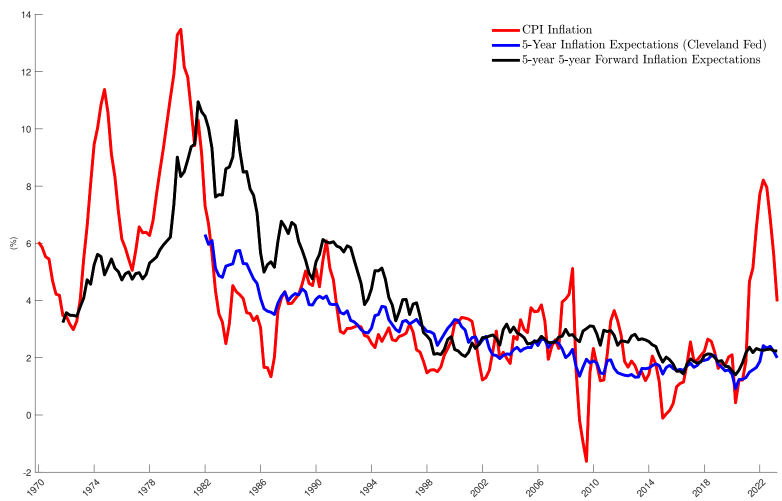

Figure 16: This figure contrasts CPI inflation at annual rates with the five-year expected inflation rate compiled by the Cleveland Fed and five-year five-year forward inflation expectations, which are market-based from 1997 and back-casted by Groen and Middledorp (2013) to 1970.

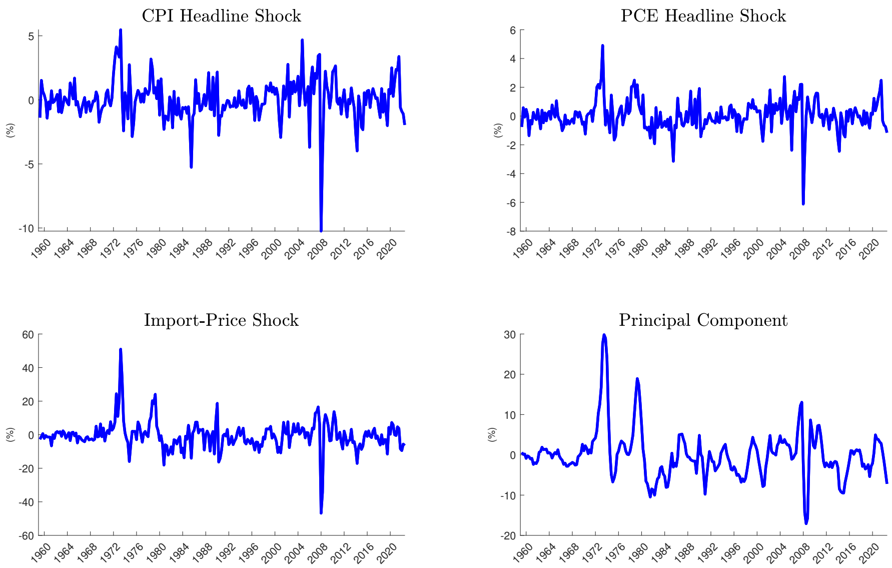

Figure 17: Measures of supply shock and their principal component (four-quarter average)

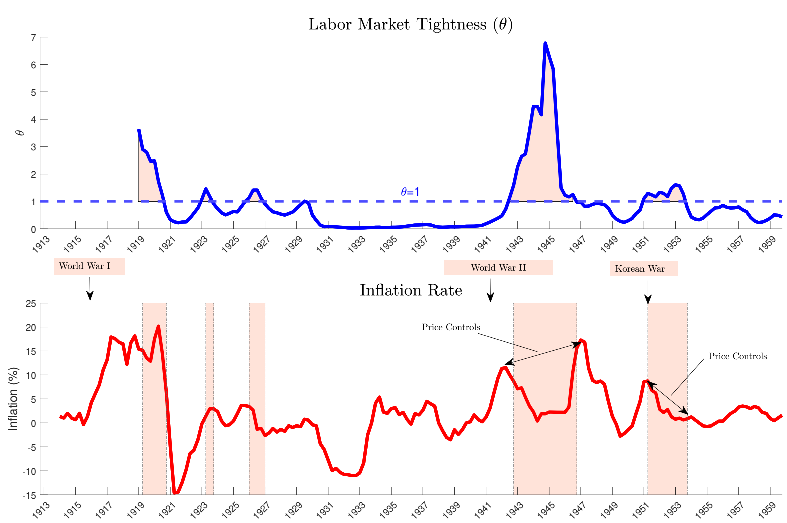

Figure 18: Top panel: (θ) vacancy-to-unemployed ratio. Bottom panel: CPI inflation rate at annual rates. Sample 1913-1959.

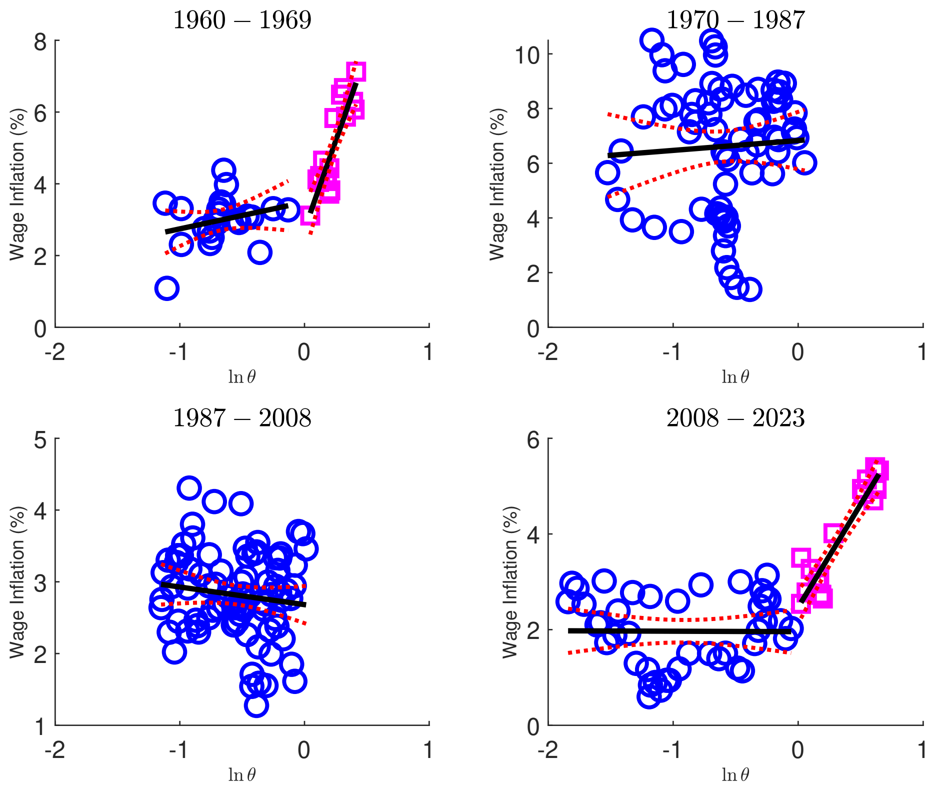

Figure 19: Wage inflation: growth rate of average hourly earnings of production and nonsupervisory employees. $\ln\theta$: log of the vacancy-to-unemployed ratio.

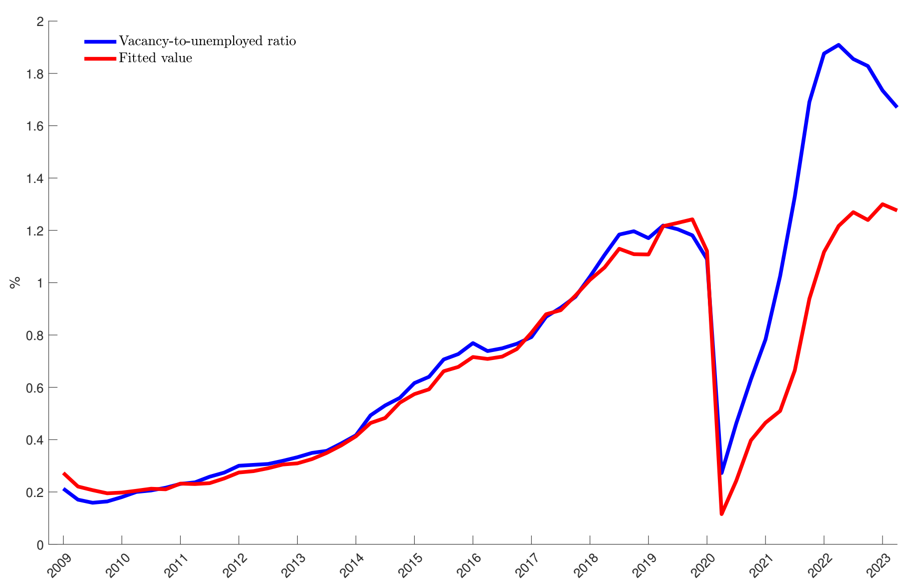

Figure 20: Vacancy-to-unemployed ratio and its fitted value using the regression $\ln\theta_t = a + b\ln(u_t/(1-u_t)) + \varepsilon_t$ on the sample 2001 Q1 – 2023 Q2. The Figure shows the time-series for the sample 2009 Q1 – 2023 Q2.

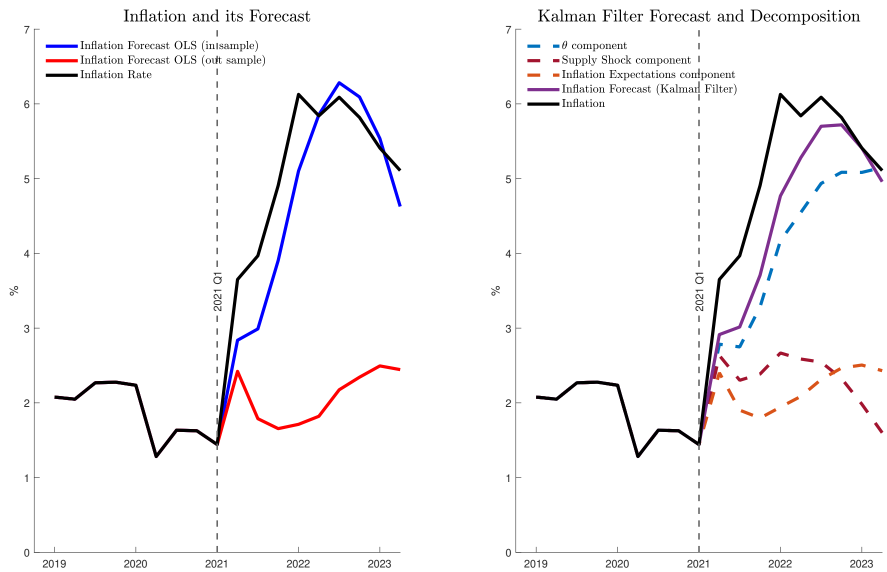

Figure 21: Left panel: CPI inflation rate at annual rates (black line); out-of-sample inflation forecast (red line) using OLS regression (2) without the dummy variable on the sample 2008 Q3 – 2021 Q1; in-sample inflation forecast (blue line) using OLS regression (2) on the sample 2008 Q3 – 2023 Q2. Right panel: CPI inflation rate at annual rates (black line); in-sample inflation forecast (purple line) using Kalman-Filter estimation with time-varying coefficients on the sample 2008 Q3 – 2023 Q2. The three dashed lines represent the inflation forecasts using the Kalman-Filter estimates by restricting only to the variable θ, or the supply shock or the inflation expectations, respectively.

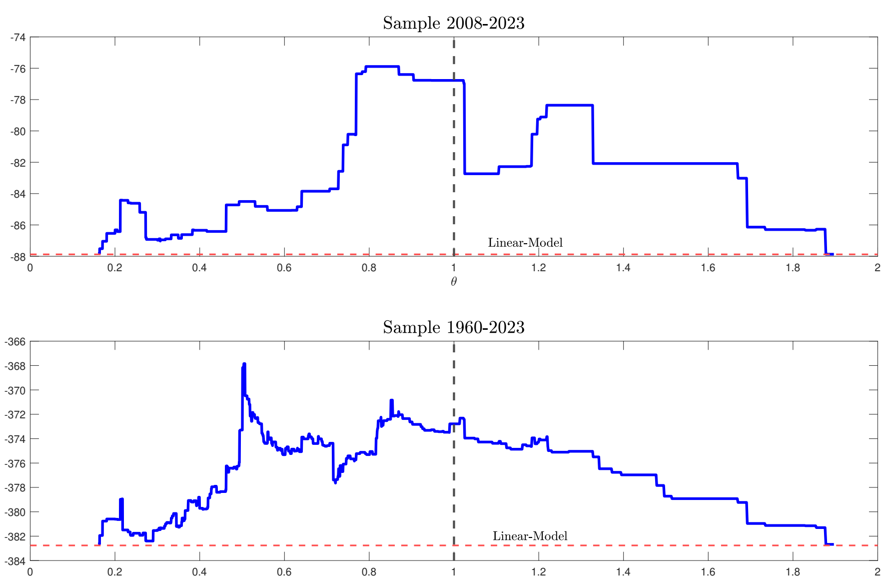

Figure 22: Maximum likelihood of OLS regression of equation (2) by varying the threshold for the dummy at different values of θ. Sample 1960-2023 and Sample 2008-2023. The red dashed line reports the maximum likelihood of the OLS regression of equation (2) without the dummy.

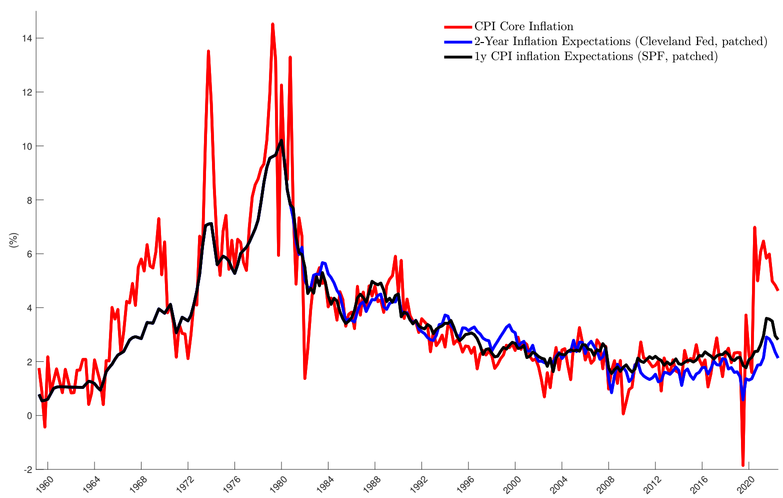

Figure 23: CPI core inflation (annualized quarterly rates). 2-year inflation expectations of the Cleveland Fed patched, before 1982 Q1, with 12-month Livingston inflation expectations. 1-year CPI inflation expectations of SPF patched, before 1981 Q3, with 12-month Livingston inflation expectations.

C Appendix: Data Description

Table 1

Table 1 presents the estimates of equation

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + (\beta_\theta + \beta_{\theta_d} D_t) \ln \theta_t + (\beta_v + \beta_{v_d} D_t) \cdot v_t + \beta_{\pi^e} \cdot \pi_t^e + \varepsilon_t, \tag{C.1}$$

in which π_t is the annualized quarterly inflation rate, computed as log changes, in deviation from a 2% inflation target. The rate is computed using the CPI core component (net of energy and food); π_{t-1} is its lagged value. Data on CPI are from FRED, collected quarterly, using the average of monthly observations for each quarter.

$\ln \theta_t$ is the log of the ratio of vacancies to unemployed workers provided by Barnichon (2011) and updated by the author. Data are monthly. Accordingly, the quarterly series used in the regression is log of the average of the relevant monthly observations. D_t is a dummy variable taking the value one when $\theta_t \geq 1$.

v_t is the four-quarter average of the principal component of the following three series: headline shocks, both to CPI and PCE, and import shock. The CPI or PCE headline shock is the difference between the annualized quarterly inflation rate computed using the CPI or PCE price index and that computed using the CPI or PCE price index excluding energy and food. The import shock is the difference between the annualized quarterly inflation rate computed using the import-price deflator and that computed using the GDP deflator. Data are from FRED and collected quarterly, using the average of the relevant monthly observations. We proxy the supply shock with the four-quarter average of the principal component of the three series. Let z_t be the principal component of the three series described above; then v_t is given by:

$$v_t = (z_t + z_{t-1} + z_{t-2} + z_{t-3})/4.$$

We proxy inflation expectations (π^e) with the 2-year inflation expectations of the Federal Reserve Bank of Cleveland, which are collected from FRED quarterly and available since 1982 Q2. The series is patched backward to 1960 Q1 with the 12-month inflation expectations from the Livingston survey. Since the latter is twice yearly, missing observations are interpolated though a spline curve-preserving function. In all regressions, π_t and π_t^e are deviations with respect to a 2% annual inflation target.

Table 2

Table 2 repeats the regressions in Table 1, except that the threshold for the dummy is set at $\theta^* = 0.5041$ for the sample 1960 Q1-2023 Q2, and at $\theta^* = 0.7962$ for the sample 2008 Q3-2023 Q2, which correspond to the thresholds, in the respective sample, that maximize the likelihood of the regressions across different thresholds, as shown in Figure 22.

Table 3

Table 3 uses as a measure of supply shock the four-quarter average of the CPI headline shock, described under Table 1.

Table 4

Table 4 uses as a measure of supply shock the four-quarter average of the import-price shock, described under Table 1.

Table 5

Table 5 uses as a proxy of inflation expectations the 1-year CPI inflation expectations of the U.S. Professional Forecasters Surveys, retrieved from Thompson Reuters Datastream, which starts in 1981 Q3. We patch this series backward to 1960 Q1 again using interpolated 12-month Livingston inflation expectations.

Table 6

Table 6 uses as a proxy of inflation expectations the 5-year inflation expectations of the Federal Reserve of Cleveland, which starts in 1982 Q1, collected from the FRED database. The series is patched with the 1-year GDP-deflator inflation expectations of the U.S. Professional Forecasters Surveys, retrieved from Thompson Reuters Datastream, which starts in 1970 Q2, and finally patched backward to 1960 Q1 again using interpolated 12-month Livingston inflation expectations.

Table 7

Table 7 uses as a proxy of inflation expectations the 5-year 5-year forward inflation expectations backcasted by Groen and Middleddorp (2013) until 1971 Q4. The series is patched backward to 1960 Q1, again using interpolated 12-month Livingston inflation expectations.

Table 8

Table 8 presents the OLS estimates of regression (2) with the same variables as Table 1, except that the inflation lag and the log of the vacancy rate to unemployment rate θ are instrumented with the fitted values of OLS regression on their first lags. Namely, the regressors π_{t-1} and $\ln\theta_t$ are replaced with the fitted values of the respective OLS estimates:

$$\pi_t = \beta_0 + \beta_1\pi_{t-1} + \varepsilon_t,$$
$$\ln\theta_t = \gamma_0 + \gamma_1 \ln\theta_{t-1} + \varepsilon_t.$$

Table 9

Table 9 uses inflation measures from the core PCE at annualized quarterly rate. The core PCE price index is collected from the FRED database quarterly, as the average of the relevant monthly observations.

Table 10

Table 10 uses the level of θ rather than the $\ln\theta$.

Table 11

Table 11 repeats the estimation of Table 1, but in which the lag of inflation is replaced with the detrended real wage. The real wage is built deflating the series "Nonfarm Business Sector: Unit Labor Costs for All Workers (ULCNFB)" retrieved from the BLS with the GDP deflator from FRED database. The series is not stationary and it is deetrended using the procedure of Hamilton (2018) to obtain the cyclical component.

Figure 2

Figure 2 plots the annual inflation rate computed using the quarterly CPI. CPI quarterly observations are the average of the relevant monthly observations. Inflation expectations are from the Livingston Survey of the Federal Reserve Bank of Philadelphia for the 12-month horizon on CPI. The frequency of the graph is twice yearly, like the Livingston Survey data.

Figure 3

Figure 3 plots the annual inflation rate computed using the quarterly CPI (top panel). CPI quarterly observations are the average of the relevant monthly observations. θ is the ratio of vacancies to unemployed workers (bottom panel) provided by Barnichon (2011) and updated by the author. Data are monthly. Accordingly, the quarterly series is the average of the relevant monthly observations.

Figure 4

Figure 4 uses the same data as Figure 3, except that it plots $\ln\theta$ rather than θ.

Figure 5

The left panel of Figure 5 uses the same data for inflation and θ as in Table 1, described above. The variable 'inflation deviations', π_t^d, on the right panel is built as

$$\pi_t^d = \pi_t - \beta_\pi \pi_{t-1} - (\beta_v + \beta_{v_d} D_t) \cdot v_t - \beta_{\pi^e} \pi_t^e$$

using the estimates of Table 1, column (3).

Figure 6

Figure 6 builds a decomposition among the different regressors of Table 1 column 4. Consider a situation in which inflation is on target, expectation on target, and $\ln\theta_t = \ln\bar{\theta}$ where $\ln\bar{\theta}$ correspond to $\ln\theta_t$ being neither inflationary or deflationary. In this case

$$0 = \beta_c + \beta_\theta \ln\bar{\theta}$$

Hence:

$$\ln\bar{\theta} = -\frac{\beta_c}{\beta_\theta}.$$

This implies that the fitted value for inflation can be written as follows:

$$\begin{aligned}\pi_t = \quad & \underbrace{\beta_\pi \pi_{t-1}}_{\text{Lagged Inflation}} + \underbrace{\beta_\theta(\ln\theta_t - \ln\bar{\theta})}_{\text{Labor Market Tightness}} + \underbrace{D_t\left[\beta_{\theta d}(\ln\theta_t - \ln\bar{\theta}) - \left(\frac{\beta_{\theta d}\beta_c}{\beta_\theta}\right)\right]}_{\text{Labor Market Tightness when } \theta \geq 1} \\ & + \underbrace{\beta_v v_t}_{\text{Cost Push Shock}} + \underbrace{\beta_{\theta_v} D_t v_t}_{\text{Cost Push Shock when } \theta \geq 1} + \underbrace{\beta_\pi \pi_t^e}_{\text{Inflation Expectations}}\end{aligned}$$

Figure 6 shows this decomposition together with core inflation, both at annual rates.

Figure 7

Figure 7 presents the estimates through Kalman Filter of the measurement equation

$$\pi_t = \beta_{c,t} + \beta_{\pi,t}\pi_{t-1} + \beta_{\theta,t}\ln\theta_t + \beta_{v,t}v_t + \beta_{\pi^e,t}\pi^e_t + \varepsilon_t,$$

in which ε_t is distributed as $N(0, \sigma^2_\varepsilon)$ with the state equations given by

$$\begin{aligned}
\beta_{c,t} &= \beta_{c,t-1} + \epsilon_{c,t} \\
\beta_{\pi,t} &= \beta_{\pi,t-1} + \epsilon_{\pi,t} \\
\beta_{\theta,t} &= \beta_{\theta,t-1} + \epsilon_{\theta,t} \\
\beta_{v,t} &= \beta_{v,t-1} + \epsilon_{v,t} \\
\beta_{\pi^e,t} &= \beta_{\pi^e,t-1} + \epsilon_{\pi^e,t}
\end{aligned}$$

in which $\epsilon_{c,t} \sim N(0, \sigma^2_\epsilon)$, $\epsilon_{\pi,t} \sim N(0, \sigma^2_{\epsilon_\pi})$, $\epsilon_{\theta,t} \sim N(0, \sigma^2_{\epsilon_\theta})$, $\epsilon_{v,t} \sim N(0, \sigma^2_{\epsilon_v})$, $\epsilon_{\pi^e,t} \sim N(0, \sigma^2_{\epsilon_{\pi^e}})$. The Kalman Filter is initialized by running an OLS regression of the measurement equation with constant coefficients on the sample period 1960 Q1 – 2008 Q2. Then, the Kalman Filter estimation runs from 2008 Q3 to 2023 Q2. σ^2_ε is initialized as the variance of the residuals of the OLS regression on the pre-sample; β_c, β_π, β_θ, β_v and β_{π^e} are initialized with OLS estimates of the respective coefficients on the pre-sample; σ^2_ϵ, $\sigma^2_{\epsilon_\pi}$, $\sigma^2_{\epsilon_\theta}$, $\sigma^2_{\epsilon_v}$, $\sigma^2_{\epsilon_{\pi^e}}$ are initialized with the variance of the respective coefficients of the OLS regression on the pre-sample. Figure 7 plots the estimated time-varying coefficients $\beta_{\pi,t}$, $\beta_{\theta,t}$, $\beta_{v,t}$ and $\beta_{\pi^e,t}$ using the Kalman Filter and their one-standard-deviation confidence bands.

Figure 9

The Figure uses the estimates of Table 1, column 3, for $\kappa = 0.2422$, $\kappa_v = 0.0444$, $\kappa^{tight} = 0.2422 + 3.727$, $\kappa_v^{tight} = 0.0444 + 0.0831$. The estimates κ and κ^{tight} are divided by 400 since in the OLS regression inflation is measured in percent and at annual rates while the model is interpreted at a quarterly frequency. The following parametrization is used: $\omega = 3$, $\alpha = 0.9$, $\lambda = 0.05$, $s = 0.0724$, $\bar{u} = 0.04$, $\eta = 0.4$, $\tau = 0.7$, $\phi_\pi = 1.5$, $\sigma = 0.5$, $\pi^* = 0.05$. The following parameters, derived in Appendix D, are obtained:

$$\begin{aligned}
z_\theta &\equiv (1-\lambda)\eta + \frac{s-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{s}(1+\omega)(1-\eta), \\
\tilde{z}_\theta &\equiv \left(\frac{1-s}{1-\bar{u}}(1-\lambda) + \frac{s-\bar{u}}{1-\bar{u}}\right)\eta + \frac{s-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{s}(1+\omega)(1-\eta),
\end{aligned}$$

$$\tilde{\kappa} \equiv \frac{\frac{\omega\kappa}{\alpha z_\theta}}{1-\frac{\kappa\lambda}{z_\theta}}, \tag{C.2}$$

$$\tilde{\kappa}_v \equiv \frac{\kappa_v}{1-\frac{\kappa\lambda}{z_\theta}},$$

$$\tilde{\kappa}^{tight} \equiv \frac{\frac{\omega}{\alpha}\frac{\kappa^{tight}}{\tilde{z}_\theta}}{1 - \frac{\kappa^{tight}\lambda}{\tilde{z}_\theta}\frac{1-s}{1-\bar{u}}},$$

$$\tilde{\kappa}_v^{tight} \equiv \frac{\kappa_v^{tight}}{1 - \frac{\kappa^{tight}\lambda}{\tilde{z}_\theta}\frac{1-s}{1-\bar{u}}}.$$

The Figure plots the AD equation given by:

$$\hat{Y}_S = -\sigma^{-1}\frac{\phi\pi - \tau}{1-\tau}(\pi_S - \pi^*) + \sigma^{-1}(\pi_L^e - \pi^*).$$

and the inv-L NK Phillips curve:

$$\pi_S - \pi^* = \begin{cases} c + \frac{\tilde{\kappa}^{tight}}{1-\tau}\hat{Y}_S + \frac{\tilde{\kappa}_v^{tight}}{1-\tau}(1-\alpha)\hat{q}_S + (\pi_L^e - \pi^*) & \hat{Y}_t \geq \hat{Y}^* \\ \\ \frac{\tilde{\kappa}}{1-\tau}\hat{Y}_S + \frac{\tilde{\kappa}_v}{1-\tau}(1-\alpha)\hat{q}_S + (\pi_L^e - \pi^*) & \hat{Y}_t < \hat{Y}^* \end{cases}.$$

where $\hat{Y}^*$ is set such that when $\hat{Y}_S = \hat{Y}^*$, $\pi_S = (2.2046/400)$ consistently with the estimates of Table 1, column 3. The supply shock is set at $\hat{q}_S = 0.0725$, consistently with the maximum 30% increase, during the 1970s inflationary surge, in the proxy of the supply shock used in the regression as shown in Figure 17; the inflation expectations shock is set at $\pi_L^e = 0.025$, consistently with the 10% increase in inflation expectations observed during the 1970s, according to Figure 2. The parameter c is such that

$$c = \left(\frac{\tilde{\kappa}}{1-\tau} - \frac{\tilde{\kappa}^{tight}}{1-\tau}\right)\hat{Y}^*.$$

In Figure **9**, inflation, π_S, is at annual rates and in percent, output gap $\hat{Y}_S$ is in percentage points.

Figure 10

The Figure uses the estimates of Table 1, column 4, for $\kappa = 0.5199$, $\kappa_v = -0.0096$, $\kappa^{tight} = 0.5199 + 5.2042$, $\kappa_v^{tight} = -0.0096 + 0.2771$. Figure 10 uses calibration of the other parameters as in Figure 9.

The Figure plots the AD equation given by:

$$\hat{Y}_S = \hat{G}_S - \sigma^{-1}\frac{\phi\pi - \tau}{1-\tau}(\pi_S - \pi^*)$$

and the Inv-L NK Phillips curve:

$$\pi_S = \begin{cases} c + \frac{\tilde{\kappa}^{tight}}{1-\tau}\hat{Y}_S + \frac{\tilde{\kappa}_v^{tight}}{1-\tau}(1-\alpha)\hat{q}_S + \pi^* & \hat{Y}_t \geq \hat{Y}^* \\ \\ \frac{\tilde{\kappa}}{1-\tau}\hat{Y}_S + \frac{\tilde{\kappa}_v}{1-\tau}(1-\alpha)\hat{q}_S + \pi^* & \hat{Y}_t < \hat{Y}^* \end{cases},$$

where $\hat{Y}^*$ is set such that when $\hat{Y}_S = \hat{Y}^*$, $\pi_S = (2.4072/400)$ consistent with the estimates of Table 1, column 4. The demand shock is set at $\hat{G}_S = -0.0198$ and the supply shock $\hat{q}_S$ to zero. In Figure **10**, inflation, π_S, is at annual rates and in percent, output gap $\hat{Y}_S$ is in percentage points.

Figure 11

The Figure uses the calibration and AS-AD schedules of Figure **10**. The demand shock is set at $\hat{G}_S = 0.018$, the supply shock is set at $\hat{q}_S = 0.0125$, consistently with the maximum 5% increase, observed during the 2020s inflationary surge, in the proxy of the supply shock used in the regression, as shown in Figure 17. In Figure **11**, inflation, π_S, is at annual rates and in percent, output gap $\hat{Y}_S$ is in percentage points.

Figure 12

Data are taken from the FRED Database.

Figure 13

Data are taken from the FRED Database. Inflation is computed using the CPI annual inflation rate (Q4 on Q4) for the reference year. The unemployment rate is the annual average.

Figure 14

Data for the PCE-index inflation and its forecasts of the Summary of Economic Projections are from the FRED database.

Figure 15

Data on PCE-index inflation and its forecasts of the Survey of Professional Forecasters are from the FRED database.

Figure 16

Figure 16 plots the annual inflation rate computed using the quarterly CPI. CPI quarterly observations are the average of the relevant monthly observations. Inflation expectations are the 5-year inflation expectations of the Federal Reserve of Cleveland. Data are from the FRED database. The Figure also plots the 5-year 5-year forward inflation expectations of Groen and Middledorp (2013).

Figure 17

Figure 17 presents the three different measures of the supply shock that we use to build the proxy for v_t, namely the four-quarter averages of the principal component of the two headline shocks (using CPI and PCE price index) and the import-price shock, as described under Table 1.

Figure 18

Figure 18 plots the annual inflation rate computed using the quarterly CPI (top panel). CPI quarterly observations are the average of the relevant monthly observations. θ is the ratio of vacancies to unemployed workers (bottom panel) derived by Petrosky-Nadeau and Zhang (2021) back to 1919. Data are monthly. Accordingly, the quarterly series is the average of the relevant monthly observations.

Figure 19

Figure 19 shows scatter plots of wage inflation and $\ln \theta$ at quarterly frequency and for different samples. The wage inflation is the annual growth rate using Average Hourly Earnings of Production and Nonsupervisory Employees, Manufacturing, Dollars per Hour, retrieved from the FRED database. $\ln \theta$ is the logarithm of the ratio of vacancies to unemployed workers as in Figure 4.

Figure 20

Figure 20 plots the vacancy-to-unemployment ratio (θ) and its fitted valued using the regression

$$\ln \theta_t = a + b \ln(u_t/(1-u_t)) + \varepsilon_t$$

estimated with OLS on the sample 2001 Q1 – 2023 Q2. Data are from the Job Openings and Labor Turnover Survey of the BLS. Data are monthly. Accordingly, the quarterly series is the average of the relevant monthly observations.

Figure 21

The black line of the left panel of Figure 21 is the annual CPI inflation rate excluding food and energy sectors. The red line represents the out-of-sample prediction of equation

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + \beta_\theta \ln \theta_t + \beta_v v_t + \beta_{\pi^e} \pi_t^e + \varepsilon_t,$$

estimated for the sample 2008 Q3 – 2021 Q1 for the period 2021 Q2 – 2023 Q2. The model produces forecasts for the quarterly inflation rate in deviations of a 2% target. Accordingly, we build the corresponding predicted inflation at annual rates.

The blue line represents the in-sample prediction of equation

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + (\beta_\theta + \beta_{\theta_d} D_t) \cdot \ln \theta_t + (\beta_v + \beta_{v_d} D_t) v_t + \beta_{\pi^e} \pi_t^e + \varepsilon_t,$$

estimated for the sample 2008 Q3 – 2023 Q2 for the period 2021 Q2 – 2023 Q2. The model produces predictions for the quarterly inflation rate in deviaiton of a 2% target. Accordingly, we build the corresponding predicted inflation at annual rates.

The black line in the right panel of Figure 21 is again the annual CPI inflation rate excluding food and energy. The purple line is the in-sample prediction using the non-linear Kalman Filter estimates, while 'θ component', 'Supply Shock component', 'Inflation Expectations component' correspond, respectively, to the in-sample prediction derived from the following three equations using the Kalman Filter estimates.

$$\begin{aligned} \pi_t^\theta &= \beta_{\pi,t} \pi_{t-1}^\theta + \beta_{\theta,t} (\ln \theta_t - \ln \bar{\theta}_t), \\ \pi_t^v &= \beta_{\pi,t} \pi_{t-1}^v + \beta_{v,t} v_t, \\ \pi_t^* &= \beta_{\pi,t} \pi_{t-1}^* + \beta_{\pi^e,t} \pi_t^e. \end{aligned}$$

in which

$$\ln \bar{\theta}_t = -\beta_{c,t}^{-1} \beta_{\theta,t},$$

and initial conditions are given by the inflation rate in 2021 Q1. The model produces inflation predictions at quarterly frequency in deviations of a 2% target, so we build the corresponding annual inflation predictions plotted in the Figure.

Figure 22

Figure 22 reports the likelihood value of the regressions

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + (\beta_\theta + \beta_{\theta_d} D_t) \ln \theta_t + (\beta_v + \beta_{v_d} D_t) \cdot v_t + \beta_{\pi^e} \cdot \pi_t^e + \varepsilon_t, \tag{C.3}$$

in which $D_t = 1$ whenever $\theta_t \geq \theta^*$ otherwise $D_t = 0$. The threshold θ^* can take values between $\theta_{\min} = \min \theta_t$ and $\theta_{\max} = \max \theta_t$. The Figure plots the likelihood value as a function of the threshold θ^* estimating the regressions under the samples 1960 Q1- 2023 Q2 (bottom panel) and 2008 Q3- 2023 Q2 (top panel).

Figure 23

Figure 23 plots the inflation rate and inflation expectations used in Table 1 and Table 5. Inflation rate is the annualized quarterly inflation rate computed using core CPI. Inflation expectations, used in Table 1, are the 2-year inflation expectations of the Federal Reserve Bank of Cleveland, collected from FRED quarterly and available since 1982 Q2. The series is patched backward to 1960 Q1 with the 12-month inflation expectations from the Livingston survey. Since the latter is twice yearly, missing observations are interpolated though a spline curve-preserving function. Inflation expectations in Table 5 are the 1-year CPI inflation expectations of the Survey of Professional Forecasters, retrieved from Thompson Reuters Datastream, which starts in 1981 Q3. We patch this series backward to 1960 Q1 again using interpolated 12-month Livingston inflation expectations.

D Appendix: The Model

D.1 Steady state

Let us first consider a steady state in which $\xi_t = \xi$, $A_t = A$, $\epsilon_t = \epsilon$, $q_t = q$, $\bar{O}_t = \bar{O}$, $G_t = G$, $m_t = m$, $\chi_t = \chi$, $\Psi_t = \Psi$, $\gamma_t^c = \gamma^c$, $\gamma_t^b = \gamma^b$, $\phi_t = \phi$. The steady-state versions of equations (11), (21), (23), (24) and (29) imply:

$$\chi F^{\omega} = (1-s)w^{ex} + um\theta^{1-\eta}w^{new},$$

$$w^{new} = \frac{\alpha}{1+\gamma^b}\left(\frac{\epsilon-1}{\epsilon}A\right)^{\frac{1}{\alpha}}\left(\frac{q_t}{1-\alpha}\right)^{-\frac{1-\alpha}{\alpha}},$$

$$1 = \frac{N}{F} + u$$

$$N = F(1-s+um\theta^{1-\eta}),$$

$$w^{flex} = \frac{1}{m}\frac{\gamma^c}{\gamma^b}\theta^{\eta}.$$

Note that existing wages are set in the steady state at:

$$w^{ex} = (w^{ex}\Pi^{-1}(\Pi^e)^{\delta})^{\lambda}(w^{flex})^{1-\lambda}\phi.$$

We assume that ϕ is such that $\phi = \Pi^{\lambda}(\Pi^e)^{-\lambda\delta}$ and therefore $w^{ex} = w^{flex}$ in the steady state. It follows that $w^{new} = w^{ex} = w^{flex} = \bar{w}$. We can then solve the above equation for the steady state $(\bar{N}, \bar{F}, \bar{u}, \bar{\theta}, \bar{w})$. We make assumptions such that $\bar{\theta} < 1$ in the steady state.

D.2 Derivation of the AS equation (21)

The firms discounted value of current and expected future profits are:

$$E_t \sum_{T=t}^{\infty} Q_{t,T}\left\{p_T(i)y_T(i) - W_T^{ex}N_T^{ex}(i) - (1+\gamma_T^b)W_T^{new}N_T^{new}(i) - P_Tq_TO_T(i) - \frac{\varsigma}{2}\left(\frac{p_T(i)}{p_{T-1}(i)} - 1\right)^2 P_TY_T\right\}$$

where $Q_{t,T} \equiv \beta^{T-t}(X_T^{-\sigma}/P_T)/(X_t^{-\sigma}/P_t)$ is the stochastic discount factor the household uses at time t. Note that the maximization problem is subject to the following constraints:

$$y_t(i) = A_t(N_t^{ex}(i) + N_t^{new}(i))^{\alpha}O_t(i)^{1-\alpha}, \tag{D.4}$$

$$y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\epsilon_t} Y_t, \tag{D.5}$$

$$0 \leq N_t^{ex}(i) \leq (1-s)F_t(i), \tag{D.6}$$

$$N_t^{new}(i) \geq 0. \tag{D.7}$$

We can use (D.4) to solve for $N_t^{new}(i)$ to obtain

$$N_t^{new}(i) = \left(\frac{y_t(i)}{A_t}\right)^{\frac{1}{\alpha}} O_t(i)^{\frac{\alpha-1}{\alpha}} - N_t^{ex}(i),$$

which substituted into the objective function yields to:

$$E_t \sum_{T=t}^{\infty} Q_{t,T} \left\{ p_T(i) y_T(i) - (W_T^{ex} - (1+\gamma_T^b) W_T^{new}) N_T^{ex}(i) - (1+\gamma_T^b) W_T^{new} \left(\frac{y_T(i)}{A_T}\right)^{\frac{1}{\alpha}} O_T(i)^{\frac{\alpha-1}{\alpha}} + \right.$$
$$\left. - P_T q_T O_T(i) - \frac{\varsigma}{2}\left(\frac{p_T(i)}{p_{T-1}(i)} - 1\right)^2 P_T Y_T \right\}.$$

Note that whenever $W_t^{ex} < (1+\gamma_t^b) W_t^{new}$, it follows, using (D.6), that the optimal choice for $N_t^{ex}(i)$ is $N_t^{ex}(i) = (1-s)F_t(i)$. Given this result, first-order conditions with respect to $p_t(i)$ and $O_t(i)$ imply

$$\begin{aligned} 0 &= (1-\epsilon_t) y_t(i) + \epsilon_t (1+\gamma_t^b) W_t^{new} \frac{1}{\alpha} \left(\frac{y_t(i)}{A_t}\right)^{\frac{1}{\alpha}-1} \frac{y_t(i)}{A_t p_t(i)} O_t(i)^{\frac{\alpha-1}{\alpha}} + \\ & -\varsigma \left(\frac{p_t(i)}{p_{t-1}(i)} - 1\right) \frac{1}{p_{t-1}(i)} P_t Y_t + \varsigma E_t \left\{ \beta \left(\frac{X_{t+1}}{X_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}} \left(\frac{p_{t+1}(i)}{p_t(i)} - 1\right) \frac{p_{t+1}(i)}{(p_t(i))^2} P_{t+1} Y_{t+1} \right\} \end{aligned}$$

and

$$\frac{1-\alpha}{\alpha}(1+\gamma_t^b) W_t^{new} \left(\frac{y_t(i)}{A_t}\right)^{\frac{1}{\alpha}} O_t(i)^{-\frac{1}{\alpha}} = P_t q_t.$$

We can combine the second first-order condition into the first to substitute for $O_t(i)$ and obtain

$$\begin{aligned} 0 &= (1-\epsilon_t) y_t(i) + \epsilon_t \left(\frac{(1+\gamma_t^b) W_t^{new}}{\alpha}\right)^{\alpha} \left(\frac{P_t q_t}{1-\alpha}\right)^{1-\alpha} \frac{y_t(i)}{A_t p_t(i)} + \\ & -\varsigma \left(\frac{p_t(i)}{p_{t-1}(i)} - 1\right) \frac{1}{p_{t-1}(i)} P_t Y_t + \varsigma E_t \left\{ \beta \left(\frac{X_{t+1}}{X_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}} \left(\frac{p_{t+1}(i)}{p_t(i)} - 1\right) \frac{p_{t+1}(i)}{(p_t(i))^2} P_{t+1} Y_{t+1} \right\}. \end{aligned}$$

All firms are going to set the same price therefore $p_t(i) = P_t$ and $y_t(i) = Y$. We can then obtain

$$\begin{aligned} 0 &= (1-\epsilon_t) + \frac{\epsilon_t}{A_t} \left(\frac{1+\gamma_t^b}{\alpha} \frac{W_t^{new}}{P_t}\right)^{\alpha} \left(\frac{q_t}{1-\alpha}\right)^{1-\alpha} - \varsigma \left(\Pi_t - 1\right) \Pi_t + \\ & + \varsigma E_t \left\{ \beta \left(\frac{X_{t+1}}{X_t}\right)^{-\sigma} \frac{Y_{t+1}}{Y_t} \left(\Pi_{t+1} - 1\right) \Pi_{t+1} \right\}, \end{aligned}$$

from which we can obtain equation (21), here restated as

$$\begin{aligned}(\Pi_t - 1)\,\Pi_t &= \frac{\epsilon_t - 1}{\varsigma}\left(\frac{\mu_t}{A_t}\left(\frac{1+\gamma_t^b}{\alpha}\frac{W_t^{new}}{P_t}\right)^{\alpha}\left(\frac{q_t}{1-\alpha}\right)^{1-\alpha} - 1\right) + \\ &\quad + \beta E_t\left\{\left(\frac{X_{t+1}}{X_t}\right)^{-\sigma}\frac{Y_{t+1}}{Y_t}\,(\Pi_{t+1} - 1)\,\Pi_{t+1}\right\}, \end{aligned} \tag{D.8}$$

in which we have defined $\mu_t \equiv \epsilon_t/(\epsilon_t - 1)$.

Note that whenever $W_t^{ex} = (1+\gamma_t^b)W_t^{new}$, the above derivation applies too implying the same AS equation.

D.3 Inv-L Phillips curve characterization

In a log-linear approximation of equation (21), the AS equation is:

$$\pi_t = \frac{(\epsilon - 1)}{\varsigma}(\hat{\mu}_t + \alpha(\hat{w}_t^{new} + z_\gamma\hat{\gamma}_t^b) - \hat{A}_t + (1-\alpha)\hat{q}_t) + \beta E_t\pi_{t+1}, \tag{D.9}$$

in which $z_\gamma \equiv \gamma^b/(1+\gamma^b)$.

Consider first the case in which $\theta_t \geq \theta_t^*$, then $\hat{w}_t^{new} = \hat{w}_t^{flex}$ with the flexible wages, (29), given, in a log-linear approximation, by

$$\hat{w}_t^{flex} = \eta\hat{\theta}_t + \hat{\gamma}_t^c - \hat{\gamma}_t^b - \hat{m}_t. \tag{D.10}$$

Using (D.10), into the AS equation we obtain:

$$\pi_t = \kappa^{tight}\hat{\theta}_t + \kappa_v^{tight}(\hat{v}_t + \hat{\vartheta}_t^{tight}) + \beta E_t\pi_{t+1}, \tag{D.11}$$

given the following parameters

$$\kappa^{tight} = \frac{(\epsilon-1)\alpha\eta}{\varsigma},$$

$$\kappa_v^{tight} = \frac{(\epsilon-1)}{\varsigma},$$

having defined

$$\hat{v}_t \equiv \hat{\mu}_t - \hat{A}_t + (1-\alpha)\hat{q}_t,$$

$$\hat{\vartheta} = \alpha(\hat{\gamma}_t^c - (1-z_\gamma)\hat{\gamma}_t^b - \hat{m}_t).$$

Now consider the state in which $\theta_t < \theta_t^*$, then $\hat{w}_t^{new} = \hat{w}_t^{ex}$, and therefore:

$$\begin{aligned}\hat{w}_t^{new} &= \lambda\hat{w}_{t-1} - \lambda\pi_t + \lambda\delta E_t\pi_{t+1} + (1-\lambda)\hat{w}_t^{flex} + \hat{\phi}_t \\ &= \lambda\hat{w}_{t-1} - \lambda\pi_t + \lambda\delta E_t\pi_{t+1} + (1-\lambda)(\eta\hat{\theta}_t + \hat{\gamma}_t^c - \hat{\gamma}_t^b - \hat{m}_t) + \hat{\phi}_t,\end{aligned}$$

in which we have used $\hat{w}_{t-1}$ in place of $\hat{w}^{ex}_{t-1}$.

We can then substitute the wage norm into (D.9) to write it as

$$\begin{aligned}\pi_t &= \frac{(\epsilon-1)}{\zeta}\Big\{\hat{\mu}_t + \alpha[\hat{w}_{t-1} - \lambda\pi_t + \lambda\delta E_t\pi_{t+1} + z_\gamma\hat{\gamma}^b_t + (1-\lambda)(\eta\hat{\theta}_t + \hat{\gamma}^c_t - \hat{\gamma}^b_t - \hat{m}_t) + \hat{\phi}_t] \\ &\quad -\hat{A}_t + (1-\alpha)\hat{q}_t\Big\} + \beta E_t\pi_{t+1},\end{aligned}$$

which can be written more compactly as

$$\pi_t = \kappa_w\hat{w}_{t-1} + \kappa\hat{\theta}_t + \kappa_v(\hat{v}_t + \hat{\vartheta}_t) + \kappa_\beta E_t\pi_{t+1}, \tag{D.12}$$

given the following parameters

$$\kappa_w = 1-\psi,$$

$$\kappa = (1-\lambda)\psi\kappa^{tight},$$

$$\kappa_v = \psi\kappa_v^{tight},$$

$$\kappa_\beta = (1-\psi)\delta + \psi\beta,$$

with ψ being a positive parameter with $0 < \psi \leq 1$ defined as

$$\psi \equiv \frac{1}{1 + \frac{(\epsilon-1)}{\zeta}\alpha\lambda}.$$

in which

$$\hat{\vartheta}^{tight}_t = \alpha(1-\lambda)(\hat{\gamma}^c_t - \hat{m}_t) + \alpha\hat{\phi}_t + \alpha(z_\gamma - (1-\lambda))\hat{\gamma}^b_t$$

Note that $\kappa < \kappa^{tight}$ and $\kappa_v < \kappa_v^{tight}$, since $0 < \psi \leq 1$ and $0 < \lambda \leq 1$.

Note that

$$\pi_t = \kappa^{tight}\hat{\theta}_t + \kappa_v^{tight}(\hat{v}_t + \hat{\vartheta}^{tight}_t) + \beta E_t\pi_{t+1},$$

applies when $\hat{\theta}_t \geq \hat{\theta}^*_t$ while

$$\pi_t = \kappa_w\hat{w}_{t-1} + \kappa\hat{\theta}_t + \kappa_v(\hat{v}_t + \hat{\vartheta}_t) + \kappa_\beta E_t\pi_{t+1},$$

whenever $\hat{\theta}_t < \hat{\theta}^*_t$. To make them continuous at $\hat{\theta}_t = \hat{\theta}^*_t$ and $\hat{v}_t = \hat{\vartheta}^{tight}_t = \hat{\vartheta}_t = 0$, we should correct by c_{θ,t^*} so that

$$\pi_t = c_{\theta,t^*} + \kappa^{tight}\hat{\theta}_t + \kappa_v^{tight}(\hat{v}_t + \hat{\vartheta}^{tight}_t) + \beta E_t\pi_{t+1},$$

given

$$c_{\theta,t^*} = -(\kappa^{tight} - \kappa)\hat{\theta}^*_{t^*} - (\beta - \kappa_\beta)E_{t^*}\pi_{t^*+1} + \kappa_w\hat{w}_{t^*-1}$$

and

$$\hat{\theta}^*_{t^*} = \hat{\gamma}^b_{t^*} - \hat{\gamma}^c_{t^*} + \hat{m}_{t^*} + \eta^{-1}\hat{w}_{t^*-1} + \eta^{-1}\delta E_{t^*}\pi_{t^*+1} - \eta^{-1}\pi_{t^*} + \lambda^{-1}\eta^{-1}\hat{\phi}_{t^*}$$

in which t^* is the time in which the economy switches from the flat part of the AS equation to the steep part.

E Derivations of Section 5

In Section 5, we make the assumption that the wage norm is

$$w_t^{new} = \begin{cases} w_t^{flex} & \text{for } \theta_t > \theta_t^* \\ \bar{w}^{\lambda}(\Pi_t^{-1})^{\lambda}(\Pi_{t+1}^{e})^{\delta\lambda}(w_t^{flex})^{1-\lambda} & \text{for } \theta_t \leq \theta_t^*, \end{cases}$$

We take a first-order approximation of equations (11), (23), (24) to obtain

$$\omega \hat{F}_t + \hat{\chi}_t = \frac{1-s}{1-\bar{u}}\hat{w}_t^{ex} + \frac{s-\bar{u}}{1-\bar{u}}\hat{w}_t^{new} - \frac{\bar{u}}{1-\bar{u}}\hat{u}_t, \tag{E.13}$$

$$\hat{N}_t = \hat{F}_t - \frac{\bar{u}}{1-\bar{u}}\hat{u}_t \tag{E.14}$$

$$\hat{u}_t = -\frac{s-\bar{u}}{s}(\hat{m}_t + (1-\eta)\hat{\theta}_t). \tag{E.15}$$

Therefore we can combine (E.14) and (E.13) to obtain

$$\hat{N}_t = \frac{1}{\omega}\left(\frac{1-s}{1-\bar{u}}\hat{w}_t^{ex} + \frac{s-\bar{u}}{1-\bar{u}}\hat{w}_t^{new}\right) - \frac{1}{\omega}\hat{\chi}_t - \frac{1+\omega}{\omega}\frac{\bar{u}}{1-\bar{u}}\hat{u}_t, \tag{E.16}$$

which holds independently of the wage mechanism.

Consider first the case when $\theta_t \leq \theta_t^*$ and therefore

$$\hat{w}_t^{new} = \hat{w}_t^{ex} = (1-\lambda)(\eta\hat{\theta}_t - \hat{m}_t) - \lambda(\pi_t - \delta E_t \pi_{t+1}), \tag{E.17}$$

having set $\hat{\gamma}_t^c = \hat{\gamma}_t^b = \hat{\phi}_t = 0$.

Note that

$$\hat{N}_t = \frac{1}{\alpha}(\hat{Y}_t - \hat{A}_t), \tag{E.18}$$

having set $\hat{O}_t = 0$. We can then plug (E.15), (E.17) and (E.18) into (E.16) to obtain

$$\frac{1}{\alpha}(\hat{Y}_t - \hat{A}_t) = \frac{1}{\omega}\left((1-\lambda)(\eta\hat{\theta}_t - \hat{m}_t) - \lambda(\pi_t - \delta E_t\pi_{t+1})\right) - \frac{1}{\omega}\hat{\chi}_t + \frac{1+\omega}{\omega}\frac{s-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{s}(\hat{m}_t + (1-\eta)\hat{\theta}_t)$$

which can be simplified to obtain

$$\frac{1}{\alpha}(\hat{Y}_t - \hat{A}_t) = \frac{z_\theta}{\omega}\hat{\theta}_t - \frac{\lambda}{\omega}(\pi_t - \delta E_t\pi_{t+1}) - \frac{1}{\omega}\hat{\chi}_t + \frac{z_m}{\omega}\hat{m}_t$$

in which we have defined

$$z_\theta \equiv (1-\lambda)\eta + \frac{s-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{s}(1+\omega)(1-\eta),$$

$$z_m \equiv -(1-\lambda) + \frac{s-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{s}(1+\omega).$$

Therefore, we can write

$$\hat{\theta}_t = \frac{\lambda}{z_\theta}(\pi_t - \delta E_t \pi_{t+1}) + \frac{1}{z_\theta}\hat{\chi}_t - \frac{z_m}{z_\theta}\hat{m}_t + \frac{\omega}{\alpha z_\theta}(\hat{Y}_t - \hat{A}_t).$$

Using it into (D.12) we can obtain

$$\pi_t = \tilde{\kappa}\left(\hat{Y}_t + \frac{\alpha}{\omega}\hat{\chi}_t - \hat{A}_t - \frac{\alpha z_m}{\omega}\hat{m}_t\right) + \tilde{\kappa}_v\left(\hat{\mu}_t - \hat{A}_t + (1-\alpha)\hat{q}_t - \alpha(1-\lambda)\hat{m}_t\right) + \tilde{\kappa}_\beta E_t \pi_{t+1},$$

having defined:

$$\tilde{\kappa} \equiv \frac{\frac{\omega\kappa}{\alpha z_\theta}}{1-\frac{\kappa\lambda}{z_\theta}},$$

$$\tilde{\kappa}_v \equiv \frac{\kappa_v}{1-\frac{\kappa\lambda}{z_\theta}},$$

$$\tilde{\kappa}_\beta \equiv \frac{\kappa_\beta - \frac{\delta\lambda\kappa}{z_\theta}}{1-\frac{\kappa\lambda}{z_\theta}}.$$

We now characterize the case in which $\theta_t > \theta_t^*$ where:

$$\hat{w}_t^{new} = \eta\hat{\theta}_t - \hat{m}_t \tag{E.19}$$

$$\hat{w}_t^{ex} = (1-\lambda)(\eta\hat{\theta}_t - \hat{m}_t) - \lambda(\pi_t - \delta E_t\pi_{t+1}) \tag{E.20}$$

We can then plug (E.15), (E.18), (E.19), (E.20) into (E.16) to obtain

$$\begin{aligned}\frac{1}{\alpha}(\hat{Y}_t - \hat{A}_t) \quad &= \quad \frac{1}{\omega}\frac{1-s}{1-\bar{u}}\left((1-\lambda)(\eta\hat{\theta}_t - \hat{m}_t) - \lambda(\pi_t - \delta E_t\pi_{t+1})\right) + \frac{1}{\omega}\frac{s-\bar{u}}{1-\bar{u}}(\eta\hat{\theta}_t - \hat{m}_t) \\ &\quad -\frac{1}{\omega}\hat{\chi}_t + \frac{1+\omega}{\omega}\frac{s-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{s}(\hat{m}_t + (1-\eta)\hat{\theta}_t)\end{aligned}$$

which can be simplified to obtain

$$\frac{1}{\alpha}(\hat{Y}_t - \hat{A}_t) = \frac{\tilde{z}_\theta}{\omega}\hat{\theta}_t - \frac{\lambda}{\omega}\frac{1-s}{1-\bar{u}}(\pi_t - \delta E_t\pi_{t+1}) - \frac{1}{\omega}\hat{\chi}_t + \frac{\tilde{z}_m}{\omega}\hat{m}_t$$

in which we have defined

$$\tilde{z}_\theta \equiv \left(\frac{1-s}{1-\bar{u}}(1-\lambda) + \frac{s-\bar{u}}{1-\bar{u}}\right)\eta + \frac{s-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{s}(1+\omega)(1-\eta),$$

$$\tilde{z}_m \equiv -(1-\lambda)\frac{1-s}{1-\bar{u}} - \frac{s-\bar{u}}{1-\bar{u}} + \frac{s-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{s}(1+\omega).$$

Therefore, we can write

$$\hat{\theta}_t = \frac{\lambda}{\tilde{z}_\theta}\frac{1-s}{1-\bar{u}}(\pi_t - \delta E_t \pi_{t+1}) + \frac{1}{\tilde{z}_\theta}\hat{\chi}_t - \frac{\tilde{z}_m}{\tilde{z}_\theta}\hat{m}_t + \frac{\omega}{\alpha \tilde{z}_\theta}(\hat{Y}_t - \hat{A}_t).$$

Using it into (D.11) we can obtain

$$\pi_t = \tilde{\kappa}^{tight}\left(\hat{Y}_t + \frac{\alpha}{\omega}\hat{\chi}_t - \hat{A}_t - \frac{\alpha \tilde{z}_m}{\omega}\hat{m}_t\right) + \tilde{\kappa}_v^{tight}\left(\hat{\mu}_t - \hat{A}_t + (1-\alpha)\hat{q}_t - \alpha \hat{m}_t\right) + \tilde{\kappa}_\beta^{tight} E_t \pi_{t+1},$$

having defined:

$$\tilde{\kappa}^{tight} \equiv \frac{\frac{\omega}{\alpha}\frac{\kappa^{tight}}{\tilde{z}_\theta}}{1 - \frac{\kappa^{tight}\lambda}{\tilde{z}_\theta}\frac{1-s}{1-\bar{u}}},$$

$$\tilde{\kappa}_v^{tight} \equiv \frac{\kappa_v^{tight}}{1 - \frac{\kappa^{tight}\lambda}{\tilde{z}_\theta}\frac{1-s}{1-\bar{u}}},$$

$$\tilde{\kappa}_\beta^{tight} \equiv \frac{\beta - \frac{\lambda}{\tilde{z}_\theta}\frac{1-s}{1-\bar{u}}\delta \kappa^{tight}}{1 - \frac{\kappa^{tight}\lambda}{\tilde{z}_\theta}\frac{1-s}{1-\bar{u}}}.$$

Note that the steeper curve applies whenever $\theta_t > \theta_t^*$ ($\hat{\theta}_t > \hat{\theta}_t^*$) and, therefore, when $Y_t > Y_t^*$ ($\hat{Y}_t > \hat{Y}_t^*$) for an appropriately defined Y_t^* and $\hat{Y}_t^*$. The AS schedule should be appropriately adjusted for a constant c to make it continuous at the kink point.

E.1 The 1970s

To characterize the 1970s, we consider a short run in which the only shock affects positively the price of oil, $\hat{q}_S > 0$, and we allow for a shock to the policy rate e_S. Shocks revert to normal in the long run, which is an absorbing state that occurs with probability $1-\tau$. In the long run $\hat{Y}_L = 0$ $\pi_L = \pi^*$ in which π^* is the central bank inflation target. In the short run, however, we also assume that private agents fear that the central bank may have changed its long-run inflation target, so their belief is $\pi_L^e > \pi^*$.

In this case, the short-run Euler equation, substituting for the policy rule, is given by

$$\hat{Y}_S = \tau \hat{Y}_S - \sigma^{-1}(\pi^* + \phi_\pi(\pi_S - \pi^*) + e_S - \tau \pi_S - (1-\tau)\pi_L^e)$$

which implies that

$$\hat{Y}_S = -\sigma^{-1}\frac{\phi_\pi - \tau}{1-\tau}(\pi_S - \pi^*) - \frac{\sigma^{-1}}{1-\tau}e_S + \sigma^{-1}(\pi_L^e - \pi^*).$$

while the Inv-L NK Phillips curve is:

$$\pi_S = \begin{cases} c(1-\tau) + \tilde{\kappa}^{tight}\hat{Y}_S + \tilde{\kappa}_v^{tight}(1-\alpha)\hat{q}_S + \tau\pi_S + (1-\tau)\pi_L^e & \hat{Y}_t \geq \hat{Y}_t^* \\ \tilde{\kappa}\hat{Y}_S + \tilde{\kappa}_v(1-\alpha)\hat{q}_S + \tau\pi_S + (1-\tau)\pi_L^e & \hat{Y}_t < \hat{Y}_t^* \end{cases}$$

implying

$$\pi_S - \pi^* = \begin{cases} c + \frac{\tilde{\kappa}^{tight}}{1-\tau}\hat{Y}_S + \frac{\tilde{\kappa}_v^{tight}}{1-\tau}(1-\alpha)\hat{q}_S + (\pi_L^e - \pi^*) & \hat{Y}_t \geq \hat{Y}_t^* \\ \frac{\tilde{\kappa}}{1-\tau}\hat{Y}_S + \frac{\tilde{\kappa}_v}{1-\tau}(1-\alpha)\hat{q}_S + (\pi_L^e - \pi^*) & \hat{Y}_t < \hat{Y}_t^* \end{cases},$$

in which

$$c = \left(\frac{\tilde{\kappa}}{1-\tau} - \frac{\tilde{\kappa}^{tight}}{1-\tau}\right)\hat{Y}_t^*.$$

During the 1970s, θ was below the unitary value, so that $\theta_t < \theta^*$ and $\hat{Y}_t < \hat{Y}_t^*$. Therefore, the inflation rate, looking at the flat segment of the Inv-L NK curve, is given by

$$\pi_S - \pi^* = \frac{\frac{\tilde{\kappa}_v}{k}(1-\alpha)\hat{q}_S - \frac{\sigma^{-1}}{1-\tau}e_S + \sigma^{-1}(\pi_L^e - \pi^*) + \frac{1-\tau}{\tilde{\kappa}}(\pi_L^e - \pi^*)}{\frac{1-\tau}{\tilde{\kappa}} + \sigma^{-1}\frac{\phi_\pi - \tau}{1-\tau}}.$$

Short-run inflation is pushed above the target by the supply shock, the disanchoring of inflation expectations, and an accommodative monetary policy.

E.2 The 2020s

To characterize the 2020s, we consider a short run in which $\hat{G}_S > 0$, $\hat{\chi}_S > 0$, $\hat{q}_S > 0$ and allow for variations in the policy shock e_S. Shocks revert to zero in the long run. This is an absorbing state that occurs with probability $1-\tau$. In the long run $\hat{Y}_L = 0$ and $\pi_L = \pi^*$.

The short-run Euler equation, substituting for the policy rule, can accordingly be written as

$$\hat{Y}_S = \hat{G}_S + \tau(\hat{Y}_S - \hat{G}_S) - \sigma^{-1}(\pi^* + \phi_\pi(\pi_S - \pi^*) + e_S - \tau\pi_S - (1-\tau)\pi^*)$$

which implies:

$$\hat{Y}_S = \hat{G}_S - \sigma^{-1}\frac{\phi_\pi - \tau}{1-\tau}(\pi_S - \pi^*) - \frac{\sigma^{-1}}{1-\tau}e_S$$

while the Inv-L NK Phillips curve is:

$$\pi_S = \begin{cases} c(1-\tau) + \tilde{\kappa}^{tight}\left(\hat{Y}_S + \frac{\alpha_n}{\omega}\hat{\chi}_S\right) + \tilde{\kappa}_v^{tight}(1-\alpha)\hat{q}_S + \tau\pi_S + (1-\tau)\pi^* & \hat{Y}_t \geq \hat{Y}_t^* \\ \tilde{\kappa}\left(\hat{Y}_S + \frac{\alpha_n}{\omega}\hat{\chi}_S\right) + \tilde{\kappa}_v(1-\alpha)\hat{q}_S + \tau\pi_S + (1-\tau)\pi^* & \hat{Y}_t < \hat{Y}_t^* \end{cases}$$

implying

$$\pi_S = \begin{cases} c + \frac{\tilde{\kappa}^{tight}}{1-\tau}\left(\hat{Y}_S + \frac{\alpha}{\omega}\hat{\chi}_S\right) + \frac{\tilde{\kappa}_v^{tight}}{1-\tau}(1-\alpha)\hat{q}_S + \pi^* & \hat{Y}_t \geq \hat{Y}_t^* \\ \\ \frac{\tilde{\kappa}}{1-\tau}\left(\hat{Y}_S + \frac{\alpha}{\omega}\hat{\chi}_S\right) + \frac{\tilde{\kappa}_v}{1-\tau}(1-\alpha)\hat{q}_S + \pi^* & \hat{Y}_t < \hat{Y}_t^* \end{cases}.$$

We can now combine the two equations to obtain

$$\pi_S - \pi^* = \begin{cases} \left(\frac{\hat{G}_S - \frac{\sigma^{-1}}{1-\tau}e_S + \frac{\alpha}{\omega}\hat{\chi}_S + \frac{\tilde{\kappa}_v^{tight}}{\tilde{\kappa}^{tight}}(1-\alpha)\hat{q}_S + \frac{1-\tau}{\tilde{\kappa}^{tight}}c}{\frac{1-\tau}{\tilde{\kappa}^{tight}} + \sigma^{-1}\frac{\phi\pi-\tau}{1-\tau}}\right) & \hat{Y}_t \geq \hat{Y}_t^* \\ \\ \left(\frac{\hat{G}_S - \frac{\sigma^{-1}}{1-\tau}e_S + \frac{\alpha}{\omega}\hat{\chi}_S + \frac{\tilde{\kappa}_v}{\tilde{\kappa}}(1-\alpha)\hat{q}_S}{\frac{1-\tau}{\tilde{\kappa}} + \sigma^{-1}\frac{\phi\pi-\tau}{1-\tau}}\right) & \hat{Y}_t < \hat{Y}_t^* \end{cases}$$

showing that inflation will be higher when the curve is steeper and that a monetary policy in which

$$e_S = \sigma(1-\tau)\left(\hat{G}_S + \frac{\alpha}{\omega}\hat{\chi}_S\right)$$

will be able to stabilize inflation and output, conditional on $\hat{q}_S = 0$.

E.3 The 2008 missing disinflation

To characterize the 2008 missing disinflation, we consider a short run in which $\hat{G}_S < 0$. Shocks revert to zero in the long run. This is an absorbing state that occurs with probability $1-\tau$. In the long run $\hat{Y}_L = 0$ and $\pi_L = \pi^*$. The analysis is similar to that of the previous section, with the appropriate modifications.