Negative nominal interest rates and the bank lending channel*

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Abstract

We investigate the bank lending channel of negative nominal policy rates from an empirical and theoretical perspective. For the empirical results we rely on Swedish data, including daily bank-level lending rates. We find that retail household deposit rates are subject to a lower bound (DLB). Empirically, once the DLB is met, the pass-through to lending rates and credit volumes is substantially lower and bank equity values decline in response to further policy rate cuts. We construct a banking sector model and use our estimate of the pass-through of negative policy rates to lending rates as an identified moment to parameterize the model and assess the impact of negative policy rates in general equilibrium. Using the theoretical framework, we derive a sufficient statistic for when negative policy rates are expansionary and when they are not.

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1 Introduction

Between 2012 and 2016, a handful of central banks reduced their policy rates below zero for the first time in history. While *real* interest rates have been negative on several occasions, *nominal* rates have not. The recent experience suggests that negative policy rates have become part of the central banker's toolbox, and calls into question the relevance of the zero lower bound (ZLB) as a constraint on monetary policy. However, the question of whether and when negative policy rates are an *effective* tool for macroeconomic stabilization remains largely unresolved.

Understanding the impact of negative policy rates on the macroeconomy is of urgent current interest. Policy rates have been declining steadily since the early 1980s, resulting in worries about secular stagnation (see e.g. Summers 2014, Eggertsson and Mehrotra 2014 and Caballero and Farhi 2017). Kiley and Roberts (2017) estimate that the ZLB will bind 30-40 percent of the time going forward, and the recent outbreak of COVID-19 has again brought interest rates close to zero in several economies.[1]

In this paper we investigate the impact of negative policy rates on the macroeconomy through the bank lending channel, both from an empirical and a theoretical perspective. We use a combination of aggregate and bank level data to empirically examine the pass-through of negative policy rates to deposit and lending rates, credit volumes and bank equity, using Sweden as a case study.[2] We then construct a theoretical model with a banking sector that can rationalize our empirical findings and parameterize it using, among other things, our empirical pass-through estimate as an identified moment. The model allows us to derive a sufficient statistic - discussed below - for when negative policy rates are expansionary and when they are not. We conclude by providing estimates for the quantitative impact of the bank lending channel of negative policy rates on key macroeconomic aggregates in the context of Sweden.

In the empirical section we investigate the pass-through of negative policy rates to banks assets and liabilities. We start by documenting virtually full pass-through of negative policy rates to money market rates, that is, interbank lending rates. This is not surprising, as the interbank rate is effectively equivalent to the policy rate (the "repo-rate") itself. The transmission of negative policy rates to other highly liquid assets such as government bonds is also strong. A high degree of pass-through to interbank rates can be expansionary by lowering bank funding and opportunity costs. Yet, since interbank lending measures lending from one bank to another, rather than lending to the bank sector as a whole, its impact on aggregate funding costs is limited. Moreover, and as highlighted in our model, since banks invest in liquid assets (e.g. government bonds) to insurance against liquidity risk, lower returns on these assets directly *reduce* bank profitability.

[1] An alternative to negative interest rates is unconventional monetary policy measures. There are several reasons, however, why it is important to consider policy measures beyond these tools. Some of the credit policies used by the the Federal Reserve, the FDIC and the Treasury were severely constrained by Congress following the financial crisis, as stressed by Bernanke et al. (2018). Moreover, there remains little, if any, consensus among economists on how effective quantitative easing and forward guidance is. Plausible estimates range from considerable effects to none (see e.g. Greenlaw et al. (2018) for a somewhat skeptical review, Swanson (2017) for a more upbeat assessment, and Greenwood et al. (2014) for a discussion of debt management at the zero lower bound).

[2] The Swedish case is useful to study for a number of reasons, most importantly because we have access to high-frequent microdata on bank-level mortgage rates. In addition, the Swedish central bank - the Riksbank - cut interest rates four times into negative territory, providing relatively rich variation in the data which can be used to estimate the pass-through of negative rates and how it depends on the lower bound on the deposit rate. Sweden also has relatively low cash-dependence. Hence, to the extent that cash provides a friction on the effectiveness of negative policy rates, Sweden should be one of the countries where such policies are relatively powerful.

We document a collapse in the pass-through of the policy rate to retail household deposit rates once the policy rate turns sufficiently negative. This is in sharp contrast to regular circumstances, in which the two move closely together. Because deposits are the main source of bank financing (approximately 50 % in our data), the transmission of policy rates to banks marginal funding costs is impaired once this point is reached. An important finding is that this collapse in pass-through does not need to happen at exactly zero. In our empirical setting, the deposit rate remained responsive to policy rate cuts, albeit less so, until the policy rate reached -0.25 percent.[3]

We proceed by considering the pass-through of negative policy rates to lending rates. To do so, we conduct an event study around days of policy rate cuts using daily bank level mortgage rates. Under regular circumstances, we show that the pass-through of policy rate cuts is around 80 percent within 30 days. Once policy rates reach a level at which deposit rates are no longer responsive however, the pass-through collapses. In fact, taking a weighted average over the different mortgage contracts offered across all banks in our sample, the average pass-through to aggregate mortgage rates becomes slightly *negative*, i.e. the empirical evidence suggests a modest increase in lending rates once the policy rate falls sufficiently below zero.

In addition to a collapse in the pass-through to lending rates, we also document an increase in the dispersion of lending rates as the policy rate turns negative. The rise in dispersion is linked to banks financing structures. Banks which rely more heavily on deposit financing are less likely to reduce their lending rates once the deposit rate has stopped responding. Moving to bank-level lending *volumes*, we show that Swedish banks that rely more heavily on deposit financing also have lower credit growth once the deposit rate has reached its lower bound.

A key question in the debate surrounding negative interest rates is whether they have detrimental effects on banks net worth. We conclude the empirical section by conducting an event study using equity values for publicly listed Swedish banks. In contrast to policy rate cuts in normal times, policy rate cuts in negative territory are found to negatively impact bank equity values.

Motivated by the empirical results, we construct a bank model which rationalizes the empirical findings, and provides additional theoretical predictions for when negative policy rates are effective. The model is also used to produce quantitative predictions on the impact of negative rates and to discuss policy interventions which can make negative rates more effective once retail deposit rates reach a lower bound.

Negative nominal interest rates can arise in our model because banks and households are potentially willing to pay for storage and liquidity services provided by reserves and deposits. Taking this into account, our model derives in place of the conventional zero lower bound two new lower bounds: a lower bound on the policy rate (the "PLB") and a lower bound on the deposit rate banks can offer (the "DLB"). Both arise due the the presence of paper currency, although evidence on banks cash holdings suggests that the PLB has remained non-binding. Unlike the PLB, the available empirical evidence indicates that the DLB is close to zero for small depositors and only modestly negative for larger depositors.

In our baseline model, policy rate cuts are perfectly transmitted to deposit rates and lending rates when the lower bounds are non-binding, thus stimulating lending and aggregate demand. Once the DLB is reached,

[3]In countries where the spread between deposit and policy rates are higher, for instance in some Eurozone countries, deposit rates could fall by more, and hence the policy rate can go lower, before deposit rates reach a DLB.

however, the transmission of policy rates to the main financing source of banks breaks down – as in the data. In this case, we show that the impact of negative policy rates on bank profits is a sufficient statistic for the qualitative impact of negative policy rates on bank lending.

Whether negative policy rates reduce or increase bank profits depends on the balance sheet composition of banks. In addition to lending (B) and reserves (R) on the asset side of the balance sheet, we assume that banks hold liquid assets (A), such as government bonds, to insure against liquidity shocks. On the liability side, in addition to deposits (D), banks finance themselves via "external financing" (F), which refers to all funding which is not subject to the DLB (e.g. bank bond issuance and larger wholesale depositors). The sufficient statistic implies that negative policy rates expand bank lending if

$$\rho^a A + R < \rho^f F \tag{1}$$

where ρ^a and ρ^f measure the effective pass-through of the policy rate to liquid assets and external financing respectively. Intuitively, bank profits increase (and hence lending expands) if the effective pass-through of negative policy rates is larger to bank liabilities ($\rho^f F$) compared to bank assets ($\rho^a A + R$). Looking at Swedish data, we find that this condition is *not* satisfied, suggesting that further policy rate cuts at the DLB was ultimately counterproductive through the bank lending channel. This is consistent with the negative impact on bank equity values found in the data, and the collapse of pass-through to lending rates. We discuss how this sufficient statistic can explain why different empirical studies provide seemingly conflicting evidence on the bank lending channel of negative policy rates across different banking systems.

After discussing our partial equilibrium results, we embed the banking model into a dynamics stochastic general equilibrium model, which nests the standard New Keynesian model as a special case. We use our empirical estimates of the pass-through of negative policy rates to lending rates, combined with existing empirical evidence on the link between bank net worth and bank lending[4], as identified moments to pin down the key parameters of the model. The model can then be used to quantitatively evaluate the effect of negative rates on output and inflation depending on whether the DLB is binding or not, and on the initial balance sheet composition of the bank sector. Overall, we find that the effect of policy rate cuts on output at the DLB can be both positive and negative. In a benchmark parameterization where the balance sheet composition is set to match the Swedish banking sector, the impact is negative and implies that a 100 basis points reduction in the policy rate once the DLB is binding results in an approximately 22 basis points contraction in output. Our quantitative exercise suggests that for negative policy rates to have a non-trivial positive impact on the economy, other channels – such as an asset price channel, an exchange rate channel, or an increase in the government's fiscal space due to negative rates – would have to be important.

A key simplification in our benchmark specification is that there is perfect competition in deposit markets, which allows us to illustrate the key mechanism in a parsimonious way. It implies, however, that policy rates always move deposit rates one-to-one away from the DLB, which is counterfactual. We consider two natural extensions of the model. First, we extend the model by assuming adjustment costs to changing the quantity of deposits. Second, we consider an extension with monopolistic competition in the deposit market. Such extensions are needed to capture the relationship between the policy rate and the deposit rate prior to the DLB. Using the same procedure to calibrate the model as in our benchmark, we find that the negative output effect

[4]See the macroeconomic assessment group MAG (2010).

becomes stronger. Hence, our benchmark model provides more conservative estimates of the contractionary effects.

Literature review Our paper relates to a growing empirical literature on the effects of negative interest rates on bank outcomes. The empirical literature has focused primarily on three topics, namely the impact of negative policy rates on deposit rates, lending rates and bank equity values.

We document that household deposit rates are clearly subject to a DLB in Sweden. This is consistent with Heider et al. (2016), which documents that median overnight deposit rates in the Eurozone did not fall below zero following the introduction of negative policy rates. Basten and Mariathasan (2018) and Hong and Kandrac (2018) show similar findings for Switzerland and Japan, respectively. The zero lower bound on deposit rates in the Eurozone appears to be most prevalent for household and other retail deposits. Boucinha and Burlon (2020), Eisenshmidt and Smets (2019) and Altavilla et al. (2019) document that an increasing fraction of *corporate* depositors are charged a negative interest rate, although the pass-through is substantially weakened relative to when rates are positive. Altavilla et al. (2019) document that firms facing negative deposit rates withdraw their funds and increase fixed assets instead.

In terms of bank lending, Bech and Malkhozov (2016) document how lending rates in Switzerland increased after the introduction of negative policy rates. Basten and Mariathasan (2018) further show that the lending margin for Swiss banks increased following negative rates, and that banks with larger reserve holdings increased interest rates by more. Amzallag et al. (2019) find that Italian banks with higher deposit ratios charged higher rates on fixed-rate mortgages in response to policy rates turning negative. Hong and Kandrac (2018) document lending rate increases in Japan. Eisenshmidt and Smets (2019) on the other hand, find no evidence that high-deposit banks in Germany increased loan rates relative to low-deposit banks. Bottero et al. (2019) show that Italian banks with more liquid balance sheets expanded lending in response to negative policy rates. Adolfsen and Spange (2020) document a reduction in pass-through to lending rates in Denmark and show that it is relatively uniform across bank funding structures.[5] We show evidence of a reduction in pass-through to lending rates in Sweden, and find that this reduction in pass-through crucially depends on bank funding structures. To complement the existing literature - which is largely cross-sectional in nature and hence silent about aggregate affects - we also provide novel evidence on the transmission of negative policy rates to *aggregate* lending rates, using daily data and an event-study design for identification.

We document that policy rate cuts below zero reduces the market value of bank equity. This is consistent with Ampudia and Van den Heuvel (2018), which show that while policy rate cuts normally expand bank equity values in the Eurozone, policy rate cuts in negative territory *lowers* bank equity values. Moreover, we show that the effect is larger for banks with higher deposit shares.[6] Hong and Kandrac (2018) show related evidence from Japan, documenting that Japanese banks' equity values declined by five percent within an hour of the announcement of negative rates by the Bank of Japan.

To summarize: Our findings on the impact of negative policy rates on deposit rates and bank equity

[5]In addition, Wang (2018) documents that low (but positive) interest rates are associated with lower lending rate pass-through in the US and then builds a model which can rationalize these findings. He does not explicitly focus on the pass-through of negative policy rates, however, and how a DLB can affect the transmission of further policy rate cuts.

[6]Heider et al. (2016) report similar findings.

values are largely consistent with the existing literature for other countries. Our main contribution to the *empirical* literature is to use daily interest rate data from Sweden in an event study to estimate the pass-through of negative policy rates to lending rates. In contrast to much of the existing literature, our empirical estimates are informative about the *absolute* effect of negative policy rates on lending rates, rather than a *relative* difference between treated and control banks. For instance, Basten and Mariathasan (2018) study the impact on lending margins in Switzerland using a difference in difference analysis and find that banks with higher reserve holdings have lower pass-through *relative* to other banks. In contrast, we use an event study setup to estimate the *absolute* pass-through of negative policy rates to lending rates. This estimate is not only of particular policy interest, it also serves as an identified moment to match in our banking model.

The *theoretical* literature on negative interest rates is, perhaps surprisingly, somewhat smaller, given the high stakes in the policy debate.[7] Three studies are especially relevant for the theoretical analysis in this paper.

Rognlie (2015) provided a first analysis of the normative aspects of negative policy rates. A key distinction between his paper and ours is that in his model households face only one interest rate, and the central bank can control this interest rate directly. Since we are interested in theoretically evaluating the pass-through of negative policy rates to other interest rates, our model differs from his by having multiple interest rates and explicitly modeling the transmission mechanism of monetary policy through the bank sector.

Brunnermeier and Koby (2017) propose a theoretical model in which there is a reversal rate at which point further interest rate cuts become contractionary. There are three main differences between their model and ours. First, the main mechanism in their paper is not motivated by the DLB, which in our model is derived theoretically from the households portfolio allocation problem due to the existence of money as a nominal asset. Thus our focus is primarily on the problem of negative policy rates in the presence of paper currency, while the reversal rate arises independently of paper currency, and can occur at either positive or negative policy rates. Second, the reversal rate arises mainly due to maturity mismatch on the banks balance sheet, which gives rise to capital gains/losses due to unanticipated interest rate cuts. We abstract from banks balance sheet mismatch, as we aim to evaluate whether negative policy rates can substitute for regular policy rate cuts. This consideration arises independently of whether interest rate cuts are anticipated or not. Third, banks in our model have access to not only deposit financing and net worth, but also rely on market funding – which is potentially subject to a different pass-through. This gives rise to the second key mechanism for understanding the effectiveness of negative policy rates in our model, namely the banks exposure to negative policy rates on the asset side *relative* to the liability side (i.e. our sufficient statistic), which is the key condition for determining if negative policy rates are effective or not in our model.

Finally, Ulate (2019) embeds a monopolistically competitive banking sector into an otherwise standard New Keynesian model and investigates the extent to which negative policy rates are expansionary. Our theoretical analysis differs from Ulate (2019) in that we model a richer bank balance sheet, allowing banks to finance themselves with non-deposit funding, as well as specifying an explicit role for liquid assets. The presence of external financing allows for substantial pass-through to banks overall funding costs, even when the DLB is binding. Our model highlights the importance of considering the net exposure of banks to negative

[7]There is however a large literature on the effects of the zero lower bound. See for example Krugman (1998) and Eggertsson and Woodford (2006) for two early contributions.

policy rates, based on *both* assets and liabilities, rather than focusing on deposit shares alone. In Ulate (2019), negative policy rates always reduce bank profits in partial equilibrium. In our setting however, negative policy rates can both increase and decrease bank profits depending on banks' net exposure to negative rates. Our model can therefore reconcile the sometimes contradicting findings of different empirical studies.

Although our results indicate that the transmission of negative interest rates through the bank sector can have diminishing returns depending on i) the distance to the DLB and ii) banks net exposure to negative rates, there are ways in which negative policy rates can have further expansionary effects which we do not capture in our model. Most importantly, we do not study other transmission mechanisms such as exchange rate effects, the effect on asset prices or the role negative rates may play in providing additional fiscal space for the government. Second, if the DLB is overcome, our model predicts that negative policy rates should be an effective way to stimulate the economy. This could happen if banks over time become more willing to experiment with negative deposit rates, and depositors do not substitute to cash, or if there are institutional changes which affect the DLB. In Section 4 we consider under which conditions this could happen. Examples of such policies include a direct tax on paper currency, as proposed first by Gesell (1916) and discussed in detail by Goodfriend (2000) and Buiter and Panigirtzoglou (2003) or actions that increase the storage cost of money, such as eliminating high denomination bills. Another possibility is abolishing paper currency altogether. These policies are discussed in, among others, Agarwal and Kimball (2015), Rogoff (2017a) and Rogoff (2017c), who also suggest more elaborate policy regimes to circumvent the zero lower bound. The results presented here do not contradict these ideas. Rather, they suggest that given the current institutional framework, and conditional on bank balance sheets, negative interest rates may not be an effective way to stimulate aggregate demand via the bank lending channel.

2 Empirical analysis

Both the empirical and the theoretical sections are organized around the simplified bank balance sheet illustrated in Figure 1.

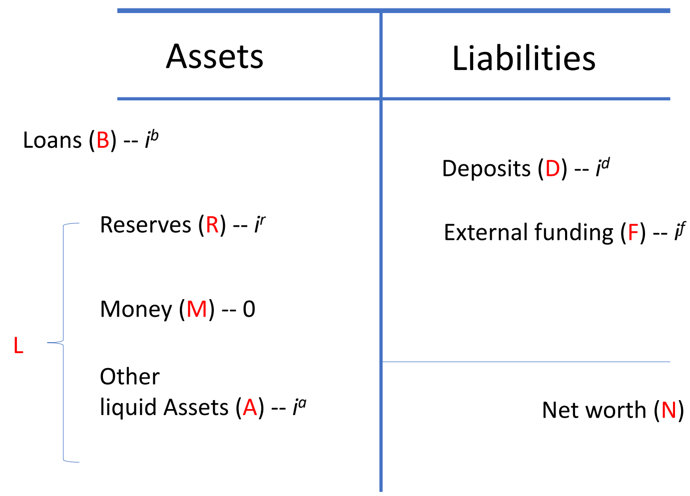

Figure 1: Bank balance sheet.

The policy instrument is the interest rate the central bank pays on reserve accounts that commercial banks hold to execute interbank transactions, i_t^r.[8] We denote the pass-through of the policy rate to an interest rate i by ρ^i, where i refers to either liquid assets (A), deposits (D), external financing (F) or bank lending (B). The main objective of this section is to empirically account for the pass-through of the policy rate to these interest rates.

We first illustrate how the policy rate is transmitted to the interest rate on liquid assets before moving on to the liability side of the balance sheet. This part of the analysis is relatively straight forward and largely descriptive. The main focus, however, is on the impact of negative policy rates on lending rates and volumes. We conclude the empirical section by estimating the impact of negative policy rates on the price of equity (N). The objective of Section 3 is to provide a theoretical framework which outlines how each component of the balance sheet is determined under negative policy rates, using the empirical evidence to discipline the model.

2.1 Liquid assets ($L = M + R + A$)

We start by investigating the pass-through of negative policy rates to banks' liquid assets. Figure 2 depicts the evolution of the repo-rate over the past 10 years, along with money market rates (STIBOR) and the interest rate on government bonds.

As the figure reveals, there has been approximately full pass-through from the policy rate to money market rates. The money market rate measures the rate at which commercial banks lend their excess reserves

[8]The exact implementation of negative policy rates differ across the countries that have implemented them, see Bech and Malkhozov (2016) for a detailed overview across countries. In the case of Sweden, which we focus on, the Riksbank operates a corridor system and the policy rate is referred to as the repo-rate. The repo-rate is essentially the interest rate banks receive for holding transaction balances at the Riksbank. Banks can borrow from the Riksbank at 75 basis points above the policy rate and central bank reserves earn an interest rate 75 basis points below the policy rate. Consider for example a policy rate of - 0.5 percent. In order to implement this rate, the Riksbank sells certificates in repo transactions that pay - 0.5. As the banks are obtaining -1.25 on their reserves, they will use the reserves to purchase these certificates. In this sense the repo-rate is essentially equivalent to the Riksbank directly paying - 0.5 on bank reserves.

to other banks. It is not surprising that the money market rate follows the policy rate into negative territory, as long as the supply of reserves is sufficiently large.

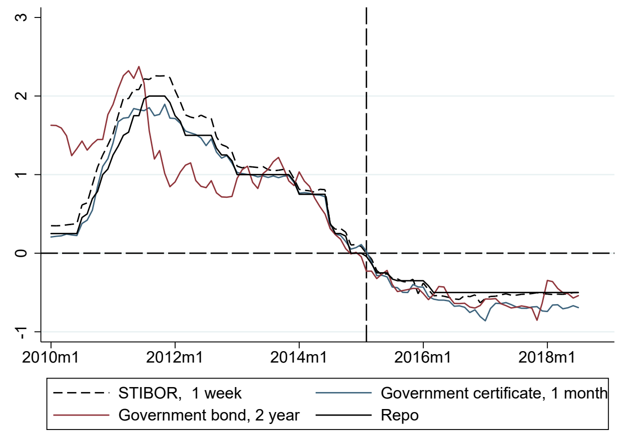

Figure 2: Interest on liquid assets.

Notes: This figure shows the repo-rate together with the interest rates on 1-week interbank rate (STIBOR, 1 week), 1 month government certificates (Government certificate, 1 month) and 2 year government bonds (Government bond, 2 year). Source: Riksbank.

What can banks do with their excess reserves? Apart from lending them to other commercial banks, they can also buy liquid assets, such as government bonds. Viewed in this light, it is easy to see why there is also relatively strong pass-through to liquid asset rates. According to the figure, the short term risk-free interest rate on government bonds falls virtually one-to-one with the repo-rate and the money market rate. The pass-through to longer term government bond rates also seems to have stayed strong, see Figure A.5 in the appendix. In fact, the average spreads between government bond rates of 2,5 and 10 year maturities relative to the policy rate all declined in the post-zero period.

2.2 Bank financing

2.2.1 Deposits (D)

Deposit financing accounts for about half of bank liabilities for large Swedish banks, as illustrated in Figure A.1 in Appendix A. For smaller banks the deposit share is larger. The deposit rate is therefore especially important for evaluating banks' marginal costs of funds and overall profitability.

The left panel of Figure 3 depicts aggregate deposit rates in Sweden.[9] Prior to the policy rate turning negative, the aggregate deposit rate is below the policy rate and moves closely with it. As the policy rate turns negative, this relationship breaks down. Instead of following the policy rate into negative territory, the deposit rate appears bounded at a level close to zero. This bound persists throughout the whole period for which the policy rate is negative, i.e. roughly five years.

[9] The aggregate deposit rate is a weighted average of the interest rate on different deposit accounts. It thus includes both highly liquid checking accounts, as well as less liquid fixed deposit accounts with minimum deposit amounts.

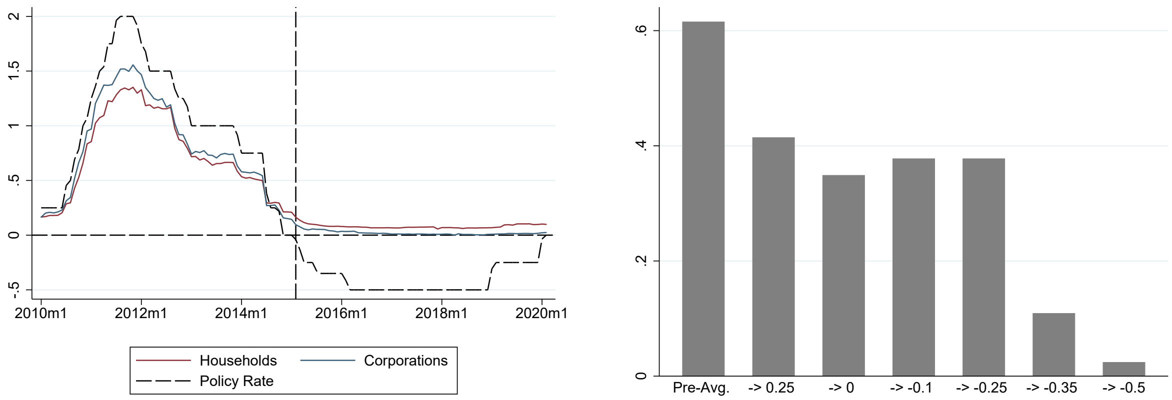

Figure 3: Deposit rates (left panel) and the relative change in deposit rates (right panel).

Notes: This figure shows the policy rate, household deposit rate and corporate deposit rate (left panel) and the pass-through of policy rate cuts to deposit rates (right panel). The policy rate is defined as the repo-rate. In the right panel, we show the change in the deposit rate relative to the change in the policy rate (i.e. $\Delta i_d/\Delta i_r$) for the last six policy rate reductions. The numbers on the x-axis denote the policy rate level to which the repo-rate was reduced. Source: The Riksbank, Statistics Sweden.

It is useful to take a closer look at the last six policy rate reductions. The change in the deposit rate relative to the change in the repo-rate is illustrated in the right panel of Figure 3. The first bar captures the average relative change in deposit rates prior to 2014. In this case, the pass-through to the aggregate deposit rate was approximately 60 percent. For the post-2014 data, the pass-through is lower. For policy rate cuts in positive territory, the pass-through is approximately 40 percent. For the first two cuts in negative territory, i.e. to -0.1 percent and to -0.25 percent, the pass-through remains relatively unchanged. For the final two interest rate cuts, however, the pass-through collapses to roughly zero. As the deposit rate reaches its lower bound (DLB), further reductions in the policy rate do not lead to continued reductions in the deposit rate. We refer to the period after the deposit rate has stopped responding, i.e. the last two policy rate reductions, as the period in which the DLB is binding.

Even with the DLB binding, an increase in fees could decrease the effective deposit rate. In Appendix A, we document that the observed evolution of commission income is not consistent with fees increasing sufficiently to compensate for the DLB on the nominal deposit rate.

2.2.2 Other financing sources (F)

For larger Swedish banks, almost half of liabilities are non-deposits. The largest remaining component of non-equity bank funding is covered bonds.[10] The left panel of Figure A.5 in Appendix A depicts the interest rates on covered bonds alongside the policy rate, and shows sign of somewhat reduced pass-through after policy rates turn negative.

Even if the pass-through to covered bond rates is weaker, the interest rate on covered bonds with shorter

[10]Comparing the volume of outstanding covered bonds to the volume of total mortgages, covered bonds are 70 % of mortgages. However, over the relevant time period, covered bonds have increased by only 38 % as much as mortgages, suggesting that covered bonds is not the main funding source for evaluating marginal funding costs - which is the costs that matter in out model in Section 3. See https://www.riksbank.se/globalassets/media/rapporter/ekonomiska-kommentarer/engelska/2020/the-funding-of-the-major-swedish-banks-and-its-effect-on-household-mortgage-rates.pdf.

maturities eventually becomes negative (Figure A.5), suggesting a stronger pass-through than for deposit rates. If banks respond to negative policy rates by shifting away from deposit financing, they might therefore reduce their marginal financing costs. However, as we show in Figure A.6 in Appendix A, this did not happen. There is no increase in the issuance of covered bonds for Swedish banks, and the deposit share in fact *increases*.

There are at least four possible explanations for why banks did not shift away from deposit financing despite an apparent cost advantage associated with non-deposit funding: i) maintaining a base of depositors creates some synergies which other financing sources do not, ii) other funding sources are associated with higher liquidity risk, iii) the room for new issuances of covered bonds may be limited by the availability of bank assets to use for collateral, and iv) Basel III regulation makes deposit financing more attractive in terms of satisfying new requirements. In Section 3, these consideration are taken into account by assuming that banks insurance against refinancing risk by investing in liquid assets, and that the refinancing risk associated with external financing is larger than insured deposit-financing. Since the rate on liquid assets follows the policy rate into negative territory, an implication of this is that external financing does not necessarily become cheaper, which can explain the patterns observed in Figure A.6 as we shall see.

2.3 Bank loans (B)

2.3.1 Bank lending rates

Official bank lending rates are only available at a monthly frequency, and not at the bank level. Here, we therefore use daily bank level data based on *listed* mortgage rates.[11]

Figure 4 plots the raw data for daily mortgage rates since 2014, with each line corresponding to one of the eleven banks in our sample. The upper left panel depicts lending rates for mortgages with three month fixed interest rate periods, referred to as floating rates. This is the most common mortgage contract in the Swedish market, accounting for roughly 70 percent of the market.[12] Consider first the four interest rate cuts which occur prior to the deposit rate reaching its lower bound, i.e. the policy rate reductions all the way down to -0.25 percent. These are the pre-DLB policy rate cuts in our sample. For these policy changes, banks respond to policy rate cuts by consistently reducing their lending rates. This stands in stark contrast to the last two policy rate cuts, i.e. to -0.35 and to -0.5 percent, in which there is virtually no reduction in bank lending rates. While one bank cuts its lending rate in response to the policy rate being reduced to -0.35, the lending rate is increased again shortly thereafter. Overall, the pass-through to lending rates appears severely weakened.

While floating rate mortgages are the most common mortgage contracts, Figure 4 also reports mortgage contracts with fixed rate periods of one year, three years and five years. Again, there is a collapse in

[11]The mortgage rates are the interest rates listed by banks on mortgages with floating rates or fixed rate periods from one to five years. Some banks also list lending rates on mortgages with fixed rate periods of ten years. However, we focus on the mortgage contracts used by most of the banks in our sample in order to increase power. Our sample consists of the eleven largest Swedish banks or financial institutions in terms of mortgage lending. According to The Swedish Bankers Association, the six largest banks account for 91 percent of the mortgage market – all of which are in our sample. The largest banks as of 2016 in terms of mortgage lending are Swedbank (24 %), Handelsbanken (23 %), Nordea (15 %), SEB, (15 %), SBAB (8 %), Lansforsakringar (6 %). The remaining 9 percent are attributed to among others Danske Bank and Skandiabanken. See the 2016 report: https://www.swedishbankers.se/media/1310/1611bolaanemarknaden_eng.pdf. All of the banks mentioned are in the sample.

[12]See the 2018 report on the mortgage market in Sweden: https://www.swedishbankers.se/ media/3906/1809_bolaanemarknad-2018_en.pdf

pass-through for the two last policy rate reductions. In addition, the dispersion in bank lending rates increases. Below, we investigate the underlying sources for this increase in dispersion.

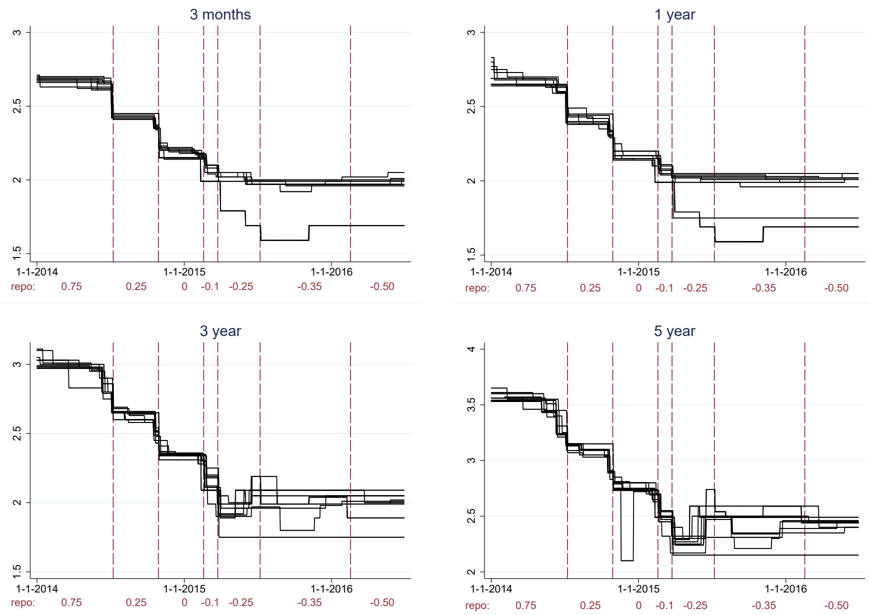

Figure 4: Bank-level mortgage rates for different maturities

Notes: This figure shows bank level mortgage rates for different maturities – 3 months, 1 year, 3 years and 5 years. 01.01.2014 - 31.12.2018. Dashed vertical lines indicate days of repo rate reductions. Source: compricer.se.

Event study

To formalize the pass-through of policy rate cuts to lending rates and investigate whether it changes at the DLB, we estimate the regression

$$\bar{i}^l_t = \alpha + \sum_{k=-30}^{30} \beta_k I_k + I^{DLB}_t \sum_{k=-30}^{30} \beta^{DLB}_k I_k + \epsilon_t \tag{2}$$

where $\bar{i}^l_t$ is the average, daily aggregate bank lending rate. We construct this variable by aggregating the lending rates of each bank, using their total assets as weight. Furthermore, each bank's lending rate has been constructed by weighting each of their lending contracts by their respective market share[13].

The fourteen interest rate cuts since 2009 were of different sizes. To account for this, we scale the lending rate by the change in the repo-rate. Thus a value of -100 implies full pass-through of the policy rate cut.

The dummy variables I_k indicate the number of days k since the repo-rate cut occurred.[14] We consider 30 days prior to and following each policy rate cut. Because the lending variable is scaled, the dummy coefficients, (the β_k's), reflect the degree of pass-through. For instance, a value of $\beta_{10} = -75$ means there has

[13]The floating mortgage rate receives a weight of 0.7 to reflect its large market share, whereas the mortgage contracts with fixed interest rate periods of one year, three years and five years receives a weight of $0.3/3$ each.

[14]Note that we could also have used the bank level lending rates, and instead run the regression $i^l_{i,t} = \alpha + \sum_{k=-30}^{30} \beta_k I_k + \epsilon_{i,t}$ (with standard errors clustered at the bank level and using weights to capture bank i's market share). This leads to the same coefficient estimates, but results in smaller confidence intervals. We therefore use the more conservative approach of not using the panel dimension of the data in the regression.

been 75 percent pass-through of the policy rate to the lending rate 10 days after the cut in the policy rate.

The third term in the regression addresses the question of whether the DLB changes the pass-through of the policy rate to the lending rate. This term interacts the daily indicator variable I_k with a dummy variable capturing whether the DLB is binding or not, I_k^{DLB}. The coefficients β_k^{DLB}'s thus measure if there has been a change in pass-through at the DLB. The sum $\beta_k^{DLB} + \beta_k$ measures the total pass-through of policy rate cuts once the DLB becomes binding.

The regression results are reported in Table 1. To conserve space we only report the coefficient estimates for every five days. Consider first the second column, which reports estimated values of β_k. In normal times, i.e. from 2009 and until the deposit rate reaches its lower bound in late 2015, the lending rate starts declining slightly prior to the policy rate reduction. At the day of the policy rate change, the average lending rate has fallen by 40 percent as much as the policy rate. Thirty days after the policy change, the pass-through has reached 78 percent, on par with international evidence (Gregora et al., 2019).

The third and fourth columns contain the main empirical result of the paper. The third column reports the effect of the DLB on the pass-through of policy rate cuts. The estimated coefficient is positive and statistically significant from the day of the policy rate cut and throughout the event window. This suggest that the DLB has a statistically significant negative effect on the pass-through of policy rates to lending rates. Moreover, the fourth column indicates that the point estimate of the effect of the DLB is strongly economically significant. Policy rate cuts, once the DLB is binding, have no significant pass-through to lending rates and the point estimates suggest that, if anything, further policy rate cuts increase lending rates.

Event time	β_k	β_k^{DLB}	Pass-through post-bound: $\beta_k + \beta_k^{DLB}$
- 30	0.00	0.00	0.00
- 25	- 2.94	3.32	0.38
- 20	- 0.45	8.93	8.48
- 15	0.06	8.43	8.48
- 10	- 4.89	13.6	8.72
- 5	- 23.2*	31.5	8.33
0	- 40.3***	48.7*	8.33
5	- 62.4***	68.3***	5.92
10	- 72.4***	78.4***	5.92
15	- 73.6***	79.5***	5.92
20	- 67.4***	73.3***	5.92
25	- 71.4***	77.3***	5.92
30	- 77.6***	83.4***	5.82

Table 1: Regression results from estimating equation (2).

Notes: This table reports the regression results from estimating equation (2). * denotes p< 0.1 and *** denotes p<0.01.

Figure 5 shows the observed interest rates for the policy rate cuts at the DLB compared to the average pass-through (and its associated confidence interval) when the policy rate is in positive territory. Each black dot captures the average pass-through k days after the repo-rate cut, and corresponds to the numbers in the second column of Table 1. The gray area captures a 95 percent confidence interval around the estimates. Note that bank lending rates start to fall some days prior to the policy rate cut, reflecting that the Riksbank announces the policy rate cut some days before the new repo rate is implemented. The coefficient estimates do not show sign of any anticipation effects prior to 5-7 days before the policy rate cut. The red and blue lines depict average bank lending rates for the two final policy rate cuts, i.e. the policy changes which occur in the post-bound period. In both cases, there is a slight increase in bank lending rates, in stark contrast to the normal pass-though.[15]

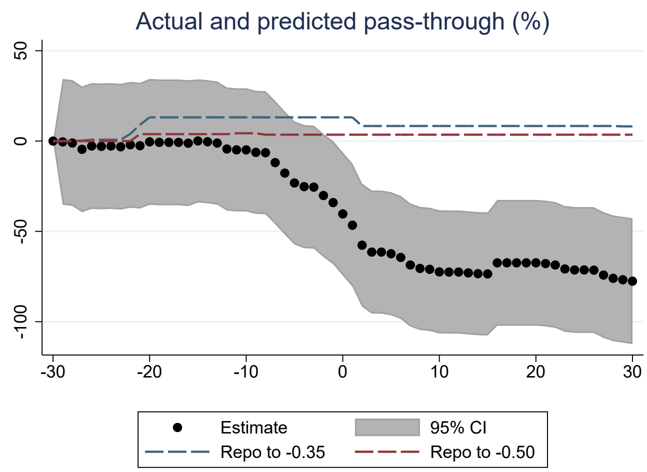

Figure 5: Event study on bank level mortgage rates.

Notes: This figure compares the actual and predicted pass-through for the two last policy rate cuts in our sample (repo rare reduced to -0.35 and repo reduced to -0.5). The interest rates are weighted average of 3 months, 1 year, 3 year and 5 year fixed interest rate period contracts. All twelve interest rate cuts in the period 2009 - 2015m4 are used to estimate the average pass-through and the corresponding confidence interval.

An important question is whether the evolution of the listed rates used here provides an accurate picture of the evolution of the *actual* transaction rates borrowers face. Erikson and Vestin (2019) show that observed lending rates - in contrast to listed rates as indicated in Figure B.1 in Appendix B - fell in 2017 and 2018 and attribute this to policy rate cuts in 2015 and 2016. This would entail a *substantial* delay in pass-through relative to pre-DLB levels. Still, this could be the case if, for instance, banks initially colluded on not lowering rates at the DLB, and this collusive equilibrium eventually broke down. However, a break-down of a collusive equilibrium should also result in lower listed rates - which are the rates that banks post on their websites and use to attract new customers and gain market shares. Listed rates did not fall in 2017 and 2018.

What then, can explain the divergence between listed rates and transaction rates two years after the last policy rate cut? In Appendix B, we aggregate the listed rates and show that there are minimal differences between the implied aggregate listed rates and the transaction rates in the time period around the repo rate cuts. We argue that the subsequent decline in transaction rates observed in 2017 and 2018, and the divergence relative to the listed rates, can be explained by macroprudential regulation which lowered household leverage. The intuition is that lower leverage enabled more households to negotiate a lower interest rate than the listed rate. In order to evaluate whether this explanation holds quantitatively, we use an LTV-regulation introduced in 2010 to predict the fall in observed lending rates following the LTV-regulation introduced in 2016. We show in the appendix that the divergence between listed rates and transaction rates can be fully explained by macroprudential regulation.

[15]In Appendix Figure A.7 we include plots similar to Figure 5 for mortgage contracts with fixed interest rate periods of 3 months, 1 year, 3 years and 5 years separately. They all show a collapse in pass-through once the deposit rate has reached its lower bound.

Cross-sectional correlation with deposit shares

In addition to a decline in the average pass-through, there is an increase in dispersion once the deposit rate has reached its lower bound. Moreover, this dispersion appears connected with the differential reliance of banks on deposit financing. To illustrate this, we estimate the following regression

$$\text{Pass through}_{ir} = \alpha + \text{Deposit share}_i + \epsilon_{ir} \tag{3}$$

where pass-through is defined as before, i denotes the bank and r indexes one of the last two repo-rate cuts. Data on deposit shares are from Statistics Sweden. We use deposit shares as of 2014, in order to not capture any changes in deposit shares in response to the policy rate cuts.[16] Observations are weighted by bank size. In the baseline regression, the deposit share is a continuous variable. As an alternative approach, we also include a specification where the treatment variable is an indicator variable for whether bank i has a deposit share above the sample median.

The regression results are reported in Table 2. They indicate a statistically significant negative correlation between deposit shares and pass-through, despite the low number of observations. From the first column, we see that a ten percentage point increase in the deposit share is associated with a decrease in pass-through of ten percentage points.[17] The negative correlation remains statistically significant, at the ten percent level or higher, when any one bank is dropped from the sample – see Figure A.9 in the Appendix.

	(1) Pass-through	(2) Pass-through
Deposit share	-0.957**	-12.49
	(-2.15)	(-1.68)
Deposit share variable:	continuous	high or low
N	18	18

Table 2: Pass-through and deposit shares.

Notes: This table shows regression results from estimating equation (3). t statistics in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Figure 6 illustrates the negative correlation between pass-through and deposit share, which suggests that banks which are more reliant on deposit financing are less willing to reduce their lending rates once the DLB is reached. Because the pass-through to deposit rates has been smaller than the pass-through to other financing sources, banks which rely heavily on deposit financing are likely to experience smaller reductions in their total funding costs once rates turn negative, a key feature of the model we present in Section 3.

[16]This data is available for nine of the banks in the sample.

[17]When using only an indicator variable for having low or high deposits in the second column, the regression coefficient becomes borderline insignificant. However, the coefficient estimate suggests that on average, banks with high deposit reliance have twelve percentage points lower pass-through than banks with low deposit reliance.

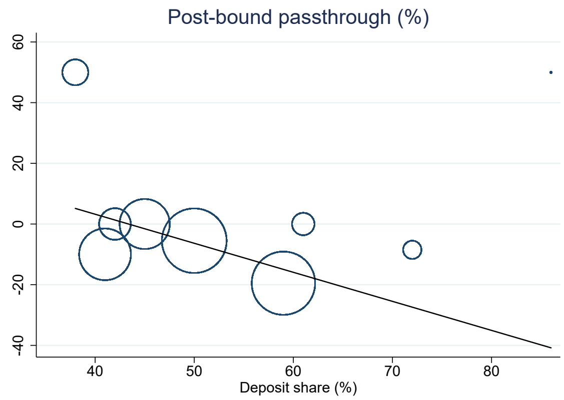

Figure 6: Pass-through and deposit shares.

Notes: This figure shows the average pass-through in the post-bound period as a function of 2014 deposit shares. Size of circles represent bank size. The line is a linear regression of bank-level pass-through on deposit share, where each bank is weighted by size.

Identification We use daily data to identify the impact of policy rate cuts on lending rates. However, one might worry about potential confounding factors. As we use daily data it seems unlikely that macroeconomic developments unrelated to monetary policy influence our pass-through estimates. However, other monetary policy announcements, such as quantitative easing (QE) or forward guidance, could potentially affect our findings.

The Riksbank adopted QE and negative rates simultaneously in February 2015. Quantitative easing was subsequently increased in March 2015 and October 2015, when the policy rate was reduced further to -0.25 and -0.35 percent respectively. There was no further QE announcements related to the last policy rate cut to -0.50 percent in February 2016. Could quantitative easing explain the breakdown in pass-through? First, we note that the intention behind QE would be to lower lending rates. Hence, this policy would tend to work against our results. Second, the timing of QE in Sweden does not coincide with the DLB, and hence with the collapse in pass-through to lending rates. While quantitative easing was in place for three of the four policy rate cuts in negative territory, the DLB was binding for the two last policy rate cuts only. The collapse in pass-through to lending rates therefore, does not coincide with any changes to QE policies.

Another potential confounding factor is forward guidance and/or information about underlying economic conditions. In Table A.2 in Appendix A, we summarize the forward guidance released at all policy rate changes from July 2014 and onwards. In their monetary policy reports, the Riksbank consistently wrote that they expected to begin increasing the repo rate after 1-1.5 years. This communication did not change as the repo rate fell below zero for the first time, nor when the DLB became binding. It therefore seems unlikely that forward guidance substantially affects our results.

A related but distinct issue is whether the negative interest rate policy itself conveyed information about economic fundamentals which was new to the bank sector. This might very well be the case. However, in order to explain the breakdown in pass-through to lending rates, this effect must have materialized not when negative rates were initially implemented, but rather only for the two last policy rate cuts only. That is, reducing the interest rate to -0.35 and -0.50 percent must have entailed a substantially different signal about economic conditions than reducing the policy rate to -0.10 and -0.25 percent. This seems unlikely. Moreover, it is unclear why this would generate the observed correlation between pass-through and deposit shares.

To summarize, our daily data and the lack of confounding factors which occurred simultaneously with the DLB becoming binding, leads us to conclude that the the event study is a relatively robust way to identify the collapse in pass-through at the DLB.

2.3.2 Bank lending volumes

We documented a negative relationship between lending rate pass-through and deposit shares. Accordingly, we should expect banks with higher deposit shares to also have lower lending growth once the DLB is binding. In this section we confirm that this pattern holds empirically and is statistically significant. Monthly lending volumes for household lending are available at the bank-level from Statistics Sweden, and we focus on the period from January 2014 to December 2017. Because the loan volume data is monthly, we cannot rely on an event study setup to identify the causal impact of negative interest rates on lending volumes - as we did with lending rates. Hence, we rely on cross-sectional variation, allowing us to estimate the relative impact of negative interest rates on lending volumes. A key disadvantage fo this approach, however, is that it does not allow us to estimate the aggregate impact of policy rate changes. Hence we view these result as complementary to the empirical results on lending rates using daily data, which we consider as the main empirical finding.

We restrict our sample to only include the banks used in the previous analysis, and run the following regression

$$\Delta \log(\text{lending}_{it}) = \alpha_i + \delta_t + \beta_t \text{ Deposit share}_i \times I_t^{DLB} + \epsilon_{it} \tag{4}$$

where $\Delta \log(\text{lending}_{it})$ is the 3-month percentage growth in lending rates for bank i at the monthly date t, α_i captures bank fixed effects to control for bank specific factors and δ_t captures time fixed effects to control for factors common to all banks. Standard errors are clustered at the bank level and observations are weighted by bank size.[18]

The regression results, reported in Table 3, suggest that banks with higher deposit shares had lower lending growth once the DLB became binding. The first column says that a ten percentage points increase in the deposit share, say from 50 to 60 percent, reduces lending growth by on average 2.4 percentage points. This compares to a mean lending growth of 2.2 percent in 2014. The second column compares banks with above or below median deposit shares, and says that banks with above median deposit shares on average have 3.5 percentage points lower lending growth than banks with below median deposit shares in the post-bound period. Hence, we conclude that not only are banks which rely more heavily on deposit financing likely to increase mortgage rates in response to policy rate cuts once the DLB is reached, they also experience lower growth in household lending volumes.

[18] Weighting the observations by bank size gets us closer to the aggregate effect on lending growth, by placing less weight on the very small banks. If we do not weight observations by bank size, the negative effect becomes larger but ceases to be statistically significant. Alternatively, we can drop the two smallest banks - with a combined market share of roughly three percent - from our sample, which gives us a larger (negative) coefficient which is statistically significant also without weighing. See Table A.3 in the appendix.

	(1) Δ log(loans)	(2) Δ log(loans)
Post $\times$ deposit share	-0.240*	
	(0.118)	
Post $\times$ deposit share		-3.468**
		(1.096)
N	360	361
No. of clusters	10	10
Mean of dependent variable	2.231	2.231
SD of dependent variable	7.052	7.052
Bank FE	Yes	Yes
Time FE	Yes	Yes
measure	Cont.	High or low

Table 3: Regression results from estimating equation (4).

Notes: * p<0.1, ** p<0.05, ***p<0.01. Post = 1 for the period when the repo-rate is below -0.25, and zero otherwise. Standard errors clustered at the bank level. Summary statistics taken over whole sample period (2014 - 2017).

A potential concern is that the estimated coefficients in Table 3 reflect structural differences in lending growth across banks with different deposit shares, and as such are not related to policy rate cuts at the DLB. To ensure that this is not the case, we conduct two falsification tests where we investigate the lending response of banks for earlier periods with policy rate reductions, including the initial cut into negative territory. The results are reported in Table A.1 in Appendix A. Reassuringly, the deposit share is only informative about the response of bank lending to policy rate cuts once the DLB is binding.

2.4 Bank profits

Finally, we investigate how negative policy rates affect bank net worth. We perform a simple event study using data on stock prices to investigate whether market participants consider negative policy rates as good or bad news for bank profitability. Specifically, we use daily stock market data on four publicly listed Swedish banks.[19] We compare the excess return on these stocks relative to the return on the main index (OMX 30) in a window around policy rate announcement days.

To formalize this we run the regression specified in equation (5), in which we define the excess return on bank stock i as $\text{Excess Return}_{it} = [\log(\text{Stock price})_{i,t} - \log(\text{Stock price})_{i,t-1}] \times 100 - [\log(\text{Stock price})^{\text{index}}_{t} - \log(\text{Stock price})^{\text{index}}_{t-1}] \times 100$.

$$\text{Excess Return}_{it} = \alpha + \gamma_k\,\text{Event Day}_k + \beta_k\,\text{Event Day}_k \times I^{\text{post-zero}}_t + \epsilon_{it} \tag{5}$$

The $\hat{\beta}_k$ coefficients capture the differential effect on bank stock excess returns in the post-zero period and are plotted in Figure 7. On the announcement day, bank stocks on average have an excess return of almost -1.5 percentage points in the post-zero period relative to normal times.[20] This negative effect is statistically

[19] The four banks are Nordea, SEB, Handelsbanken and Swedbank, and the stock market data can be found at the Nordic Nasdaq. There are three other publicly listed banks or credit companies on the Swedish stock exchange (Arion Banki, Avanza and TF Bank), but the combined mortgage market share of these three banks is virtually zero, and so we drop them from our sample.

[20] The significant negative effect on the announcement day is robust to replacing $I^{post-zero}$ with I^{DLB}, although the results become

significant at the five percent level, despite our small sample size. We thus conclude that policy rate cuts in negative territory have a detrimental impact on the excess return on Swedish bank stocks, suggesting that market participants view them as bad news for the profitability of Swedish banks.

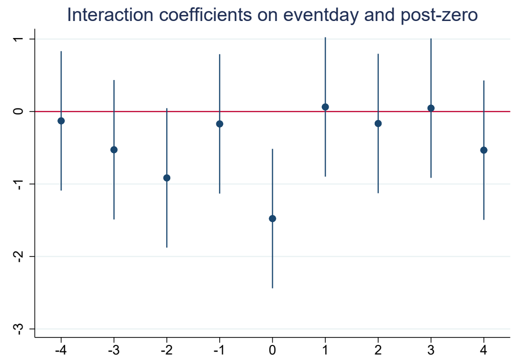

Figure 7: Excess stock return and negative policy rates.

Notes: This figure shows coefficient estimates $\hat{\beta}_k$ from estimating equation (5). The coefficient estimates can be interpreted as the relative excess return of bank stocks in the post-zero period in a window around policy rate cut announcement days ($k = 0$). Vertical bars correspond to 95 % confidence intervals.

2.5 External validity

An important question is how generalizable our findings are. We focus on three factors which could reduce the external validity, namely i) the representativeness of the Swedish banking system , ii) the relatively low cash-dependency of the Swedish economy, and iii) the Riksbank's motivation for introducing negative policy rates.

First, external validity could be low if the Swedish banking system is not comparable to the banking system of other countries. This could, for instance, be due to Swedish banks being overly reliant on deposit-financing or having different market structures, in which the latter could materialize in the form of different monetary policy pass-through rates. Reassuringly, the funding structures of Swedish banks are relatively similar to those in other countries. In terms of deposits, large Swedish banks have a deposit share of 47 %, similar to that of the Euro Area - which is 46 % according to Heider et al. (2016). The deposit reliance in Sweden is however lower than in the US - which is 79 % according to Drechsler et al. (2017). This means that, all else equal, we would expect negative interest rates to work *better* in Sweden than in the US, as will become evident in the theoretical analysis.

The monetary policy pass-through to deposit markets in Sweden is comparable to the pass-through in other countries. Our estimate of 40 % close to the DLB is in line with the pass-through found for US banks in Drechsler et al. (2017). In terms of pass-through to lending rates, our pre-DLB pass-through of approximately 78 % is very similar to international evidence. For instance, Gregora et al. (2019) show that the average pass-through to lending rates is approximately 80 % across 52 different studies. Moreover, Gregora et al.

somewhat noisier as there are only two policy rate cuts in the post-bound period. While the relative return on bank stocks seems to become negative as the policy rate turns negative, the absolute return on bank stocks remains positive until the DLB is met. See Table A.4 in Appendix A.

(2019) explicitly test if small, open economies (such as Sweden) have a significantly different pass-through than other countries, and conclude that this is not the case.

Second, Sweden has a relatively digitized payment system, and cash-dependency is low. Again, this should, if anything, imply that negative policy rates might work *better* in Sweden than in other countries. We note, however, that cash does play a non-negligible role also in the Swedish payment system. For instance, according to a Riksbank note[21], in 2014 (i.e. right before the onset of negative policy rates) 23 % of Swedish households had used cash as a means of payment for their latest purchase and 83 % had used cash in the past 30 days. This suggests that cash represents a viable payment option also in Sweden, meaning that households should be able to switch into cash if deposit rates become sufficiently negative (conditional on storage costs). As such, the existence of a deposit lower bound in Sweden is not surprising, and is likely to be driven by similar factors as in other countries.

Third and finally, a concern is that Sweden - as a small open economy - has limited independence in its monetary policy decisions. For instance, some European countries such as Denmark and Switzerland, introduced negative policy rates in response to ECB policy, motivated by exchange rate considerations. The Riksbank however, introduced negative policy rates as a response to low inflation rates and low inflation expectations. As such, their decision to cut policy rates below zero was in line with that one typically has in mind for larger central banks which have contemplated this policy option.

In sum, we believe our results should be of general interest, and that the findings are not driven by institutional features specific to Sweden.

3 Theoretical analysis

We now move on to construct a model of bank lending with negative policy rates based on the preceding empirical analysis. We first consider a partial equilibrium model where all interest rates are exogenous. This setting allows us to clarify a number of issues, including the existence of a *policy rate bound* (PLB) and how a binding deposit rate bound (DLB) affects the transmission of monetary policy also when the PLB is not binding. We then embed the partial equilibrium model into a general equilibrium framework in order to close the model and endogenise all interest rates. To conserve space, much of the derivation of the model is relegated to Appendix D.

3.1 Bank lending and negative policy rates in partial equilibrium

Consider a perfectly competitive risk-neutral bank that maximizes the discounted present value of dividends

$$\max E_t \sum_{t=0}^{\infty} \delta^t DIV_t \tag{6}$$

[21] https://www.riksbank.se/en-gb/statistics/statistics-on-payments-banknotes-and-coins/payment-patterns/

where DIV_t is dividends and $0 < \delta < 1$ is a discount factor. The bank maximizes the path of dividends subject to the following flow budget constraint:

$$
\begin{aligned}
& DIV_t + B_t + R_t + A_t + M_t - D_t - F_t \\
= & (1+i^b_{t-1})B_{t-1} + (1+i^r_{t-1})R_{t-1} + (1+i^a_{t-1})A_{t-1} \\
& + M_{t-1} - (1+i^d_{t-1})D_{t-1} - (1+i^f_{t-1})F_{t-1} \\
& - C(F_t, L_t, N_t) - \Psi(R_t, M_t) - \Gamma(B_t, N_t) - S(M_t)
\end{aligned} \tag{7}
$$

following the notation in Figure 1, i.e. B_t is loans the bank extends at interest rate i^b_t, R_t is reserves with interest rate i^r_t, M_t is paper currency which pays a zero return and A_t is "other" liquid assets with interest rate i^a_t. To finance its asset holdings, the bank raises funds via deposit financing D_t at an interest rate cost of i^d_t, and via other funds F_t at an interest rate cost of i^f_t . Throughout the analysis, D_t captures funding that is subject to a lower bound, whereas F_t captures all other funding which in principle is not subject to a (strict) lower bound. We first assume that D_t is a frictionless source of funding and then consider two alternative models, namely the case where there are adjustment costs associated with adjusting D_t and the case where banks engage in monopolistic competition for deposits and set the deposit rate i^d_t. Both models - once calibrated to yield a pass-through of negative policy rates to borrowing rates in line with the empirical evidence in Table 1 - yields similar results as the baseline model considered here, which has the added virtue of simplicity.

The net worth of the bank, N_t, is defined as:

$$
N_t \equiv (1+i^b_t)B_t + (1+i^r_t)R_t + (1+i^a_t)A_t + M_t - (1+i^d_t)D_t - (1+i^f_t)F_t \tag{8}
$$

while liquid assets are defined as

$$
L_t \equiv R_t + M_t + A_t \tag{9}
$$

The bank can hold non-loan assets only in positive quantities, i.e. we impose the non-negativity constraints

$$
A_t \geq 0, R_t \geq 0, M_t \geq 0. \tag{10}
$$

Bank intermediation costs are captured by the four functions $C(.)$, $\Psi(.)$, $\Gamma(.)$ and $S(.)$ corresponding to different aspects of the bank's intermediation process. Since the model is solved using a log-linear approximation, only assumptions about the elasticity of each function with respect to their arguments are needed to characterize dynamics. We give examples of functional forms for $C(.)$, $\Psi(.)$, $\Gamma(.)$ and $S(.)$ which the reader may find useful to interpret the restrictions we impose on these functions below.

In line with the banking literature, e.g. Freixas and Rochet (2008), banks hold liquid assets to insure against liquidity risk. Liquidity risk arises because loans B_t are illiquid while external financing F_t is subject to refinancing risk. The cost of liquidity risk is captured by the function $C(L_t, F_t, N_t)$.[22] To capture the costs associated with liquidity risk and banks' insurance motive in a reduced-form way, we assume that $\frac{\partial C}{\partial F} > 0$, $\frac{\partial C}{\partial N} < 0$ and $\frac{\partial C}{\partial L} \leq 0$. Liquid assets decreases the costs associated with liquidity risk up until a satiation point $L^*(N_t, F_t)$ and has zero impact for $L > L^*$. Away from this satiation point, the elasticity of C with respect

[22]See Freixas and Rochet (2008) for micro foundations of a cost function that captures liquidity risk. We note, however, that given its reduced-form nature we can interpret C more broadly. For instance, we can think of it as more generally capturing the costs of external financing.

to F is $\gamma_f \geq 0$ and the negative of the elasticities of C with respect to L and N are denoted by $\gamma_l \geq 0$ and $\gamma_n \geq 0$.[23] We assume that the cost associated with liquidity risk is weakly decreasing in net worth.[24] As we will see in section 2.2.2 assuming constant elasticities away from the satiation point of these functions can generate an evolution of L and F in line with the data.

While all liquid assets reduce liquidity risk, reserves and paper currency also contribute to reducing banks operational costs due their special "money role", e.g. in settling inter-bank transactions and servicing the public's demand for cash. We capture this by the function $\Psi(.)$. For simplicity, money and reserves are perfect substitutes for the purpose of liquidity provision.[25] Define the monetary base $MB_t \equiv R_t + M_t$. We assume that $\Psi'(MB_t) \leq 0$ up to a satiation point - denoted MB^* - and $\Psi'(MB_t) = 0$ for $MB_t > MB^*$. The negative of the elasticity of the banks intermediation function with respect to MB_t, when $MB_t < MB^*$, is denoted $\gamma_{mb} \geq 0$.[26]

Holding money involves a cost captured by $S(M_t)$, with $S'(M_t) > 0$. A simple interpretation of this cost is that it captures the storage costs of money. More generally it should be thought of as the additional cost of using paper currency relative to reserves in financial intermediation. The elasticity of the storage costs with respect to M is denoted γ_{ms}.[27]. As holding currency is costly, while holding reserves is not, banks choose not to hold currency whenever the interest on reserves is positive. It may, however, choose to increase its currency holdings once the reserve rate becomes negative. In this case the bank needs to consider storage costs of holding cash.[28]

The function $\Gamma(B_t, N_t)$ captures the cost of lending. We assume that $\frac{\partial\Gamma}{B} > 0$, $\frac{\partial\Gamma}{\partial N} < 0$ and $\frac{\partial^2\Gamma}{\partial B\partial N} < 0$. There are a variety of ways to micro-found these assumptions. The simplest is through regulatory capital requirements, but one could also consider for instance market based capital requirements.[29] Note that in our specification, Γ depends on current net worth. In Appendix D.7 we show that our results are robust to replacing current net worth with lagged net worth. An interpretation for why intermediation costs are strictly increasing in lending is that they arise due to borrower default, e.g. due to capacity constraints in loan monitoring (Curdia and Woodford (2011)). The elasticity of this cost function with respect to lending is

---

[23]Thus $\gamma_f \equiv \frac{\partial C}{\partial F}\frac{F}{C}$, $\gamma_N \equiv -\frac{\partial C}{\partial N}\frac{N}{C}$,$\gamma_L \equiv -\frac{\partial C}{\partial L}\frac{L}{C}$.

[24]This assumption is only important for Proposition 5 in the appendix.

[25]This simplification is not as restrictive as it may seem, as any relative disadvantage of cash to reserves can be captured by a storage cost function S().

[26]Thus $\gamma_{mb} \equiv \frac{\partial\Psi}{\partial MB}\frac{MB}{\Psi}$.

[27]Thus $\gamma_{ms} = \frac{\partial S}{\partial M}\frac{\bar{M}}{\bar{S}}$.

[28]Assuming that banks do not hold cash is a harmless abstraction as vault cash is a trivial component of banks balances. As currency is an asset with a zero return, however, explicitly accounting for it is critical when thinking about the central banks ability to charge negative interest rates on reserve balances since banks can always substitute reserves for cash – giving rise to the PLB

[29]The simplest interpretation of the role of net worth is that it comes about due to a capital requirement, often modeled as

$$\frac{N_t}{B_t} \geq \kappa$$

where κ is a parameter. A constraint of this form would be a limiting case of the smooth function assumed here, one in which the cost of intermediation goes to infinity if the constraint is breached. The dependence of lending costs on net worth is micro-founded in Holmstrom and Tirole (1997) and Gertler and Kiyotaki (2010), as well as documented empirically, for example, in Jiménez et al. (2012).

denoted by ν, while the negative of the elasticity of lending with respect to net worth is ι.[30]

We assume that the bank pays out a fixed fraction ω of its $t-1$ net worth in dividends, i.e. dividends are sticky (Juelsrud and Nenov (2020)).[31] The flow constraint can then be simplified to[32]

$$\frac{1}{1+i_t^d} N_t = (1-\omega) N_{t-1} + Z_t \tag{11}$$

where Z_t is the profit of the bank, i.e.

$$Z_t \equiv \frac{i_t^b - i_t^d}{1+i_t^d} B_t + \frac{i_t^r - i_t^d}{1+i_t^d} R_t + \frac{i_t^a - i_t^d}{1+i_t^d} A_t - \frac{i_t^d}{1+i_t^d} M_t - \frac{i_t^f - i_t^d}{1+i_t^d} F_t - C(F_t, L_t, N_t) - \Gamma(B_t, N_t) - \Psi(R_t, M_t) - S(M_t) \tag{12}$$

Equation (11) says that net worth today depends on the retained earnings from last period plus the current profits of the bank.

Denoting the value of the bank at time t by $V(N_{t-1})$, the bank's problem is

$$V(N_{t-1}) = \max_{B_t, R_t, A_t, M_t, F_t, N_t} (\omega N_{t-1} + \delta V(N_t))$$

s.t. (10) and (11). The full details of the solution to the bank's problem are included in Appendix D. In what follows, we characterize the key components.

The first order condition that determines optimal bank lending is:

$$\underbrace{\frac{i_t^b - i_t^d}{1+i_t^d}}_{\text{Marginal benefit of lending}} = \underbrace{\Gamma_B(B_t, N_t)}_{\text{Marginal cost of lending}} \tag{13}$$

This condition is the key equation of the model because it determines the lending activities of the banking sector. The left hand side denotes the marginal benefit of lending, given by the spread between the rate the bank obtains from extending loans (which it takes as given in partial equilibrium), and the interest on deposits, i_t^d.

Importantly, despite the presence of another source of external financing, the deposit rate captures the marginal cost of funding for banks. The reason is that external financing carries refinancing risk. In equilibrium, at any interior solution, the bank is indifferent between D_t and F_t. Put differently, the bank equates the marginal cost of deposits i_t^d with the marginal cost of external financing, i.e. $i_t^f + \frac{\partial C}{\partial F}$. Hence, a reduction in i_t^f relative to i_t^d leads to a shift in financing from deposits D_t to external financing F_t, while keeping the marginal cost of funding fixed at i_t^d.[33] The right hand side reflects the marginal cost of lending

[30] Thus, $\nu \equiv \frac{\partial \Gamma}{\partial B}\frac{B}{\Gamma}$ and $\iota \equiv -\frac{\partial \Gamma}{\partial N}\frac{N}{\Gamma}$. Moreover, we will assume a functional form such that the elasticity of Γ_B with respect to B is $\nu - 1$.

[31] A common specification in the literature, see e.g. Curdia and Woodford (2011), assumes that dividends are fully paid out within each period. In that case, net worth is always zero. The generalization considered here is important for our application since negative policy rates can have a negative effect on net worth, and via this channel, lending.

[32] To obtain this characterization use the expression for net worth (8), solve for D_t and substitute for D_t in the flow constraint.

[33] Note that, even though marginal costs of the two funding sources are equalized, this does not mean that the presence of external financing does not affect bank outcomes. If banks shift from D to F, for instance when the policy rate is negative and the rate on D is at the DLB, the overall profitability of the bank is affected.

which is increasing in B.[34]

Before moving on to analyzing the effect of changes to the policy rate, we first discuss whether there is a lower bound on how low the policy rate can go.

3.1.1 The policy rate bound

The policy rate bound ("PLB") arises in the model because the banks have the option of exchanging reserves for paper currency. The PLB thus depends heavily on the cost for the banks of holding paper currency relative to reserves, captured by the function $S(.)$. To understand how it emerges, it is helpful to consider the following specification

$$S(M_t) = \alpha^m M_t. \tag{14}$$

Suppose that the DLB is at exactly zero. In this case Proposition 1 defines the policy rate bound.

Proposition 1. *(The Policy Rate Bound) If the cost of using paper currency is given by equation* (14)*, the DLB is zero, and the lower bound on the policy rate is*

$$i_t^r \geq i^{PLB} \equiv -\alpha^m \tag{15}$$

The proof of this proposition follows directly from the first order conditions of the banking problem, and is shown in Appendix D.2. Since banks value the services central bank reserves provide, the central bank can charge a negative interest rate on reserves. However, money also provides these services, but at a higher relative cost captured by $S(M_t) = \alpha^m M_t$. If the central bank charges a too high price for the service provided by reserves, the banks withdraw their reserves in favor of paper currency, using it to insure against liquidity risk and settle interbank transactions outside of the central bank. The cost function in equation (14) is proportional, so that if the interest rate charged on reserves (e.g. -2 percent) is lower than the negative of the marginal cost of storing cash captured by $-\alpha^m$ (e.g. -1.5 percent), the bank will withdraw all reserves. If the marginal storage cost is increasing, then as the central bank lowers the reserve rate, banks convert reserves into cash. If the cost of holding cash increases without a bound, there is in principle no PLB.

Empirically, there has been somewhat limited substitution from reserves to cash, see Figure A.8 in the appendix. It therefore seems plausible to assume that the PLB is below the negative interest rates which have so far been implemented. We therefore proceed under the assumption that the PLB is non-binding, but note that the question of exactly what the PLB is remains unanswered.

[34] An alternative to the set-up considered here would be to assume that banks engage in monopolistic competition in deposit markets, as in for instance Ulate (2019). In such a setting, the policy rate would enter directly into (13). If the feedback from net worth to lending is small, this would imply that lending rates could fall when the policy rate is reduced, even though the deposit rate is at the DLB. Note, however, that such a scenario is not consistent with the empirical evidence in Section 2. The empirical evidence in Section 2 is, however, consistent with monopolistic competition and a *strong* feedback from net worth to lending rates. For lending and ultimately aggregate demand, such a case is isomorphic to what we consider here. We discuss and illustrate this in detail in Section 3.4.3 and in Appendix D.9.2, where we consider an alternative model where banks engage in monopolistic competition in deposit markets.

3.1.2 The effect of policy rate cuts

We now consider the effect of policy rate cuts, under the assumption that the PLB is not binding, and keeping the lending rate, i_t^b fixed. Let us suppose that i_t^d, i_t^a and i_t^f , are functions of the policy rate i_t^r, with reduced form pass-through coefficients ρ^d, ρ^a and ρ^f respectively. If $\rho^i = 1$ there is full pass-through to the interest rate i, while $\rho^i = 0$ indicates zero pass-through. For instance, if the deposit rate is not responding to further policy rate cuts due to the DLB, then $\rho^d = 0$. In general equilibrium, the pass-through of the policy rate to all rates is determined endogenously. Yet, most of the insights can be illustrated in partial equilibrium.

We use a log-linear approximation of the model around a steady state in which there is full pass-through so that $\bar{\imath}^r = \bar{\imath}^d = \bar{\imath}^f = \bar{\imath}^a$, using bars to denote steady state values. The full set of log-linearized equations are listed in Appendix D. The following proposition highlights a useful observation.

Proposition 2. *(Full pass-through and liquidity satiation) If there is full pass-through in steady state, i.e. $\bar{\imath}^r = \bar{\imath}^d = \bar{\imath}^f = \bar{\imath}^a$, then the bank is satiated in liquid assets ($L = L^*$) and holds no paper currency ($M = 0$).*

The proof of this proposition follows directly from the first order condition of the banks problem, i.e. equations (D.1.5)-(D.1.7) in Appendix D. Together, these equations imply that if $i^f = i^a = i^d = i^r$, then $C_L = C_F = \Psi_R = 0$, i.e. the bank is satiated in liquid assets. The steady state values for external financing $\overline{F}$, other liquid assets $\overline{A}$ and central bank reserves $\overline{R}$ are then implicitly defined by equations (D.1.5)-(D.1.7) in Appendix D. Because paper currency and reserves serve the same role, but money generates storage costs, banks choose to hold no paper currency.

We now analyze the impact of negative policy rates on bank lending in partial equilibrium, and how it depends on the transmission of the policy rate i_t^r to other interest rates, i.e. the pass-through coefficients ρ^i for $i \in \{d, a, f\}$.

A log-linear approximation of (11) and (13) around the steady state yields

$$\hat{B}_t = \frac{1}{\nu - 1}\left(\frac{1+\bar{\imath}^b}{\bar{\imath}^b - \bar{\imath}^d}\right)(\hat{\imath}_t^b - \hat{\imath}_t^d) + \frac{\iota}{\nu - 1}\hat{N}_t \tag{16}$$

$$\hat{N}_t = \frac{1+\bar{\imath}^d}{\Theta}\left\{(1-\omega)\hat{N}_{t-1} + \frac{\bar{B}}{\bar{N}}\hat{\imath}_t^b \frac{1+\overline{i^b}}{1+\overline{i^d}} + \Omega_\rho \hat{\imath}_t^r\right\} \tag{17}$$

where $\hat{N}_t \equiv \log N_t / \log \bar{N}$, $\hat{B}_t \equiv \log B_t / \log \bar{B}$, $\hat{\imath}_t^d \equiv \log(1+i_t^d)/(1+\bar{\imath}^d)$, $\hat{\imath}_t^b \equiv \log(1+i_t^b)/(1+\bar{\imath}^b)$, and $\Theta \equiv 1 - \frac{\iota}{\nu}(\bar{\imath}^b - \bar{\imath}^d)\frac{\bar{B}}{\bar{N}} > 0$. The key coefficient is Ω_ρ, defined and discussed in detail below.

In the remaining analysis, we make the following assumption

$$\frac{\iota}{\nu} < \frac{(1-\omega)(1+\bar{\imath}^d)}{\bar{\imath}^b - \bar{\imath}^d}\frac{\bar{N}}{\bar{B}} \tag{18}$$

which says that the feedback effect from changes to net worth to lending is not too strong, which is a necessary condition for the approximate solution to have a unique bounded solution. Observe that it implies that $\Theta > 0$.

An approximated partial equilibrium is defined as a collection of processes for $\{\hat{N}_t, \hat{B}_t\}$ which solve equations (16) and (17) given exogenous sequences for $\{\hat{\imath}_t^r, \hat{\imath}_t^d, \hat{\imath}_t^f, \hat{\imath}_t^a, \hat{\imath}_t^b\}$. An important observation is that there is no first-order impact of changes in non-loan asset holdings on the bank's lending decision determined by equation (16). Yet, as will be clear below, the steady state values of these variables are important via their impact on net worth through Ω_ρ.

The first term in equation (16) captures that bank lending is an increasing function of the spread between borrowing and deposit rates. Since $i_t^d = \rho^d i_t^r$, a lower policy rate translates into lower funding costs for the banks in proportion to the coefficient ρ^d. If $\rho^d = 0$, lower policy rates are not translated into lower deposit rates and thus the major financing cost channel of bank lending is unaffected by the decline in the policy rate. The second term shows that lending is increasing in net worth. This follows directly from the assumption that intermediation costs depend negatively on net worth. The strength of this force depends on both elasticities ι and ν, i.e. by how much intermediation costs change with higher net worth and increased lending. To understand the total effect of a policy rate cut on bank lending, we thus need to understand the effect on net worth.

Equation (17) summarizes the evolution of net worth as a function of past net worth, the lending rate and the policy rate. Net worth depends positively on its own lagged value as well as the lending rate. What about the effect of the policy rate on net worth? The key coefficient is Ω_ρ, defined as

$$\Omega_\rho \equiv \frac{\bar{A}}{\bar{N}}(\rho^a - \rho^d) + \frac{\bar{R}}{\bar{N}}(1-\rho^d) - (\rho^f - \rho^d)\frac{\bar{F}}{\bar{N}} - \rho^d\left(\frac{1+\bar{\imath}^b}{1+\bar{\imath}^d}\frac{\bar{B}}{\bar{N}} - \frac{1}{1+\bar{\imath}^d}\right) \tag{19}$$

This coefficient summarizes how changes in policy rates are translated into net worth through each component of the bank's balance sheet. Consider the term $\frac{\bar{A}}{\bar{N}}(\rho^a - \rho^d)$, for instance. If the policy rate cut changes the interest on deposits by more than the interest on liquid assets, i.e. $\rho^a < \rho^d$, then a policy rate cut increases the net interest margin on liquid asset holdings, which in turn contributes to increasing the net worth of the bank. The second term reflects this balance sheet effect due to reserves, the third term due to external financing and the final term due to bank lending.

Using equations (16) and (17), it is straightforward to show that the effect of a policy rate cut on lending at time t is

$$\frac{\partial \hat{B}_t}{\partial \hat{\imath}_t^r} = \underbrace{-\rho^d \frac{1}{\nu-1}\left(\frac{1+\bar{\imath}^b}{\bar{\imath}^b - \bar{\imath}^d}\right)}_{\text{Marginal funding cost channel}} + \underbrace{\frac{1+\bar{\imath}^d}{\Theta}\frac{\iota}{\nu-1}\Omega_\rho}_{\text{Bank capital channel}} \tag{20}$$

which then continues to affect future borrowing at time $t + j$ via the effect on net worth in future periods. In our model, policy rate cuts can therefore stimulate lending via two channels. The first channel arises because lower policy rates lower the marginal funding cost of banks. This impact relies on the pass-through to deposit rates ρ^d. Once there is no pass-through to deposit rates, this mechanism shuts down, as long as the bank remains indifferent between deposits and other sources of financing. However, policy rate cuts can still increase lending via a bank capital channel captured by Ω_ρ. If lower policy rates expand bank profitability, higher net worth will induce banks to lend more even if the deposit rate is unchanged.

The next proposition considers the case when there is full pass-through to all interest rates.

Proposition 3. *(Policy rate changes with full pass-through). Suppose that there is full pass-through to all interest rates so that $\rho^a = \rho^d = \rho^f = 1$. Then a reduction in $\hat{\imath}_t^r$ increases bank lending, i.e.* $\frac{\partial \hat{B}_t}{\partial \hat{\imath}_t^r} < 0$

To prove this proposition, note that for this special case, $\Omega_\rho = \frac{1}{1+\bar{\imath}^d} - \frac{1+\bar{\imath}^b}{1+\bar{\imath}^d}\frac{\bar{B}}{\bar{N}} < 0$. Hence, when there is full pass through, the effect of policy rate cuts on bank lending is unambiguous.

The conclusion of the last proposition changes when there is no pass-through to the deposit rate, i.e. $\rho^d = 0$. This corresponds to the case in which the DLB is binding, i.e. once policy rates move far enough into negative territory. In this case, the marginal financing cost of the bank is no longer affected. Whether or not a policy rate cut stimulates lending then depends on how net worth is affected.

Proposition 4. *(Policy rate cut when DLB is binding). Suppose there is no pass-through to deposit rates so that $\rho^d = 0$. Then a reduction in $\hat{\imath}_t^r$ increases lending if*

$$\rho^a \bar{A} + \bar{R} < \rho^f \bar{F} \tag{21}$$

and decreases lending otherwise.

To prove this note that in this case (19) becomes

$$\Omega_\rho = \frac{\bar{A}}{\bar{N}}\rho^a + \frac{\bar{R}}{\bar{N}} - \rho^f \frac{\bar{F}}{\bar{N}}$$

The condition for when policy rate cuts at the DLB have a positive effect on net worth is then given by equation (21). An important feature of Proposition 4, is that equation (21) is in fact a sufficient statistic for the sign of the bank lending channel when deposit rates are stuck the DLB. In other words, whether policy rate cuts are expansionary or not does not depend on the numerical values for all of the parameters of the banking model, but only on the sign of Ω_ρ. Notice that the sufficient statistic highlights that policy rate cuts at the DLB do not necessarily contract bank profits in partial equilibrium - as is commonly assumed in the literature. Rather, the impact on bank profits depends on banks balance sheets. The sufficient statistic is also relevant for assessing the quantitative impact in partial equilibrium. For instance, in a setting where banks hold large amounts of liquid assets, for instance due to quantitative easing (QE), the adverse profit effects are expected to be larger. In general equilibrium, however, the *quantitative* effect, will in general depend upon all of the structural parameters. Finally, Proposition 4 shows that the impact of policy rate cuts on net worth at the DLB is the *only* relevant information needed to assess the partial equilibrium impact of policy rate cuts on bank lending. We discuss in depth how this sufficient statistic can be implemented in Appendix C.

3.1.3 Implications for external financing and liquid assets

The model captures in broad terms the financial flows observed in Sweden, and the respective interest rate pass-throughs.

An important implication of the banks maximization problem is that there *has to be* full pass-through of the policy rate to liquid assets, also below the DLB. The reason is that banks can use their reserves to buy liquid assets from other banks. Accordingly, in equilibrium, they must be indifferent between reserves and liquid assets. This explains the pattern observed in Figure 2, which shows that the rates on highly liquid assets, such as government bonds, follow the policy rate essentially one-to-one as the policy rate turns negative. Because the banks are indifferent between government bonds and reserves, the actual quantity of reserves in equilibrium is indeterminate.

The model also clarifies why banks may choose not to increase their reliance on external financing at the DLB, even if the interest on external financing decreases relative to that of deposit financing. As we documented in Section 2.2.2, even when the rate on covered bank bonds eventually turned negative, there was no increase in banks issuance of covered bonds. Instead, the average deposit share increased. The model captures this by assuming, in line with the banking literature, that there is higher liquidity risk associated with external financing compared to regular deposits. The cost of this risk is captured by the function C. If a bank issues more bonds, it will in equilibrium invest in more liquid assets, which carry negative interest, so as to insure against higher liquidity risk. This implies that the return on the covered bond does not capture the true marginal cost of funding the bank faces.

We relegate to Appendix D.8 a more detailed discussion of how the C-function can be calibrated to match banks financial flows observed in Sweden, using both estimates of the cost of external funding from Wang et al. (2020) and choosing the remaining parameters to target certain moments in the data. To do this, however, we need to take account of general equilibrium effects, using the full general equilibrium model in the upcoming section. To anticipate the results, Table 4 shows that the general equilibrium model does a reasonable job in matching observed financial flows, using the procedure described in Appendix D.8. While the calibration procedure ensures that the evolution of external financing and liquid assets is in line with the evolution in the data, our model also matches the (untargeted) increase in deposits at the DLB.

Moment	Data	Model
$\Delta \frac{F_t}{\Lambda_t}$	- 0.2 %	- 0.3 %
$\Delta \frac{L_t}{\Lambda_t}$	- 7.5 %	- 6.8 %
$\Delta \frac{D_t}{\Lambda_t}$	1.6 %	1.9 %

Table 4: Change in external financing, liquid assets and deposits per 1 percentage point reduction in i_t^r when $i_t^r < 0$: data vs. model.

3.2 General equilibrium model of banking with negative policy rates

In this section, we outline the remaining building blocks needed to close the model to determine a general equilibrium. We then provide an analytical characterization of the model.

3.2.1 Model set-up

We assume that there are "patient" households which save and "impatient" households which borrow, intermediated by banks. Nominal frictions enter via staggered price setting by firms as in Calvo (1983). The way households and firms are modeled follows closely Benigno et al. (2014). Accordingly, the exposition of this aspect of the model is brief. Most derivations, such as first order conditions, are relegated to the appendix.

Households

Borrowers (b) make up a fraction χ of households, while savers (s) make up the remaining share $1-\chi$.[35] Households have at time 0 a utility function of the form

$$\mathcal{U}_t^j = \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^j)^t \left[U(C_t^j) + m\left(\frac{M_t^j}{P_t}\right) - \frac{(N_t^j)^{1+\eta}}{1+\eta} \right] \zeta_t \text{ with } j = s \text{ or } b \tag{22}$$

where E_t is the expectation operator, β^j is the discount factor with $0 \leq \beta^b \leq \beta^s < 1$, ζ_t is a preference shock and C_t^j is aggregate consumption

$$C_t^j \equiv \left[\int_0^1 C_t^j(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

where $C_t^j(i)$ is the consumption of good of variety i, $\theta > 1$ is the intratemporal elasticity of substitution between goods, N_t^j is hours worked and $\eta \geq 0$ is a parameter. To facilitate aggregation, the utility function for consumption is assumed to be $U(C_t^j) = 1 - \exp(-qC_t^j)$ where q is a parameter. The function $m\left(\frac{M_t^j}{P_t}\right)$ represents the utility of holding real money balances, which increases in real money balances up to a satiation point $\frac{M_t}{P_t} = m^*$, after which $m' = 0$.[36]

The households' budget constraint is:

$$\begin{aligned} M_t^j - B_t^j &= W_t^j N_t^j - B_{t-1}^j \left(1 + i_{t-1}^j\right) + M_{t-1}^j \\ &\quad - P_t S_h\left(\frac{M_t^j}{P_t}\right) - P_t C_t^j + \Psi_t^j + \psi_t^j - T_t^j - F_t^j \end{aligned} \tag{23}$$

where B_t^j denotes a one period banking contract j. $B_t^b > 0$ is debt with interest rate i_t^b, while $B_t^s < 0$ is bank deposits with interest rate i_t^d. $S^h(\frac{M_{t-1}}{P_t})$ captures the storage costs of holding money, measured in units of the consumption good, Ψ_t^j is firm profits, and ψ_t^j is bank dividends. Firm profits are distributed to both household types based on their population shares while only savers receive bank dividends. F_t^j represents exogenous pension payments of each agent. This pension is the source of external funds on the bank's balance sheet.

The households maximize (22) subject to (23). This leads to the standard consumption Euler equations for each household type, an optimal labor supply condition and transversality conditions, reported in the appendix. The households optimal holdings of money gives rise to the lower bound on the deposit rate – discussed next.

The deposit lower bound Consider the optimal money holdings of the saver. If $m_t \leq \bar{m}$ then money demand is given by:

$$\frac{m'\left(\frac{M_t^s}{P_t}\right)}{U'(C_t^s)} = \frac{i_t^d}{1+i_t^d} - S'\left(\frac{M_t^s}{P_t}\right) \tag{24}$$

[35] We normalize the number of agents to 1.

[36] Note that the satiation point can be set at any arbitrary level. Money in the utility function is therefore consistent with less cash-reliant economies such as the Swedish economy considered in the previous section. Moreover, the existence of a DLB does not rely on actual cash holdings, but households' ability to hold cash.

while if $m_t \geq \bar{m}$ the household is satiated in money and

$$S'(\frac{M_t^s}{P_t}) = \frac{i_t^d}{1+i_t^d} \tag{25}$$

Equation (25) generates the bound on deposit rates.

The simplest formulation of a storage costs is that it is proportional to money holdings, i.e. $S(\frac{M_t^s}{P_t}) = \alpha^H \frac{M_t}{P_t}$ for some $\alpha^H > 0$. In this case condition (25) implies

$$i_t^d = \frac{\alpha^H}{1-\alpha^H} \equiv i^{DLB}$$

which represents the effective lower bound on deposit rates for small depositors.

A straightforward interpretation of why banks have not been willing to impose negative rates on deposits is that the marginal storage cost α^H is modest for small depositors, i.e. $\alpha^H \approx 0$, and that for regular depositors cash provides a close substitute to deposits. We maintain this assumption for the remainder of the paper, and therefore set $\alpha^H = 0$ which implies a DLB for the deposit rate at zero, i.e. $i_t^d \geq i^{DLB} = 0$. Towards the end of the paper we discuss policies that can relax this bound.

Firms

Each good i is produced by a firm i. Production is linear in labor, i.e. $Y_t(i) = N_t(i)$, where $N_t(i)$ is a Cobb-Douglas composite $N_t(i) = \left(N_t^b(i)\right)^{\chi}\left(N_t^s(i)\right)^{1-\chi}$ as in Benigno et al. (2014). This implies that the two household types receive a fixed share of income. The preference specification implies that firms face a downward-sloping demand function $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t$. In each period, a fraction α of firms are not able to reset their prices as in Calvo (1983). Thus, the likelihood that a price set in period t applies in period $T > t$ is α^{T-t} and in the absence of price adjustments, prices are assumed to be indexed to the inflation target Π. A firm that resets its price chooses the price that maximizes the present value of discounted profits in the event that the price remains fixed. That is, each firm i choose $P_t(i)$ to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\alpha\beta)^t \lambda_t \left[\Pi^t \frac{P_0(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} Y_t(i)\right] \tag{26}$$

where $\lambda_T \equiv q\left(\chi \exp\left\{-qC_t^b\right\} + (1-\chi)\exp\left\{-qC_t^s\right\}\right)$, which is the weighted marginal utility of consumption and $\beta \equiv \chi\beta^b + (1-\chi)\beta^s$.[37] This maximization problem leads to the standard New Keynesian Phillips Curve.

Banks and pension funds

The bank problem has already been outlined in the partial equilibrium section and the first order conditions (D.1.4) - (D.1.9) from the appendix have to hold in equilibrium. In general equilibrium, however, the interest rates and the price level are endogenously determined.

In general equilibrium, the source of external funding for the banks are the pension funds that invest F_t and receive an interest rate i_t^f as well as government bonds A_t that pay an interest i_t^a. These pension funds are financed by a lump sum pension fee, F_t^j, in the households budget constraint. F_t should be interpreted as a stand-in for large investors, like pension funds, or large retail depositors which can possibly be charged a

[37]Recall that the firm is owned by both types of households according to their respective population shares.

negative interest rate, since storing assets in terms of paper currency might be prohibitively costly for this group. Liquid assets in the form of government bonds, A_t, are held by the banks as well as the households through the pension funds.[38]

The government

The government sets monetary policy according to the Taylor type policy rule

$$i_t = r_t^n (\frac{\Pi_t}{\bar{\Pi}})^{\phi_\pi} (\frac{y_t}{\bar{y}})^{\phi_\pi} \tag{27}$$

where r_t^n is the natural rate of interest which corresponds to the interest rate in the case of flexible prices, and ϕ_π and ϕ_y are coefficients. $\bar{\Pi}$ is the inflation target of the central bank and $\bar{y}$ is the natural level of output which is constant in the model and equal to steady state output.

We assume that, away from the lower bound, reserves are supplied so that banks are fully satiated, which implies $i_t^r = i_t^d$. However, once the interest on reserves is below the DLB, then the deposit rate is stuck at its lower bound, implying $\mathrm{i}_t^d = i^{DLB}$. Thus we impose the following constraint

$$i_t^d = \max\{i^{DLB}, i_t^r\} \tag{28}$$

which implies full pass-through to deposit rates, unless the effective lower bound is binding.

The government budget constraint is

$$A_t^g + M_t + R_t = (1 + i_t^g) A_{t-1}^g + M_{t-1} + (1 + i_{t-1}^r) R_{t-1} + G - T_t$$

Total government liabilities LB_t are assumed to be fixed and follow an exogenous process, i.e,

$$LB_t = A_t^g + M_t + R_t \tag{29}$$

Taxes adjust so that the government budget constraint is satisfied, and are equally distributed across the two agents in steady state. For simplicity, in response to shocks, only the tax on the saver is varied, so that

$$T_t = \bar{T}^b + T_t^s \tag{30}$$

Equilibrium and solution method

An equilibrium is defined as a collection of stochastic processes for the endogenous variables that solve the household problem, the firm problem, the bank problem, while monetary and fiscal policy follow the policy regimes specified in the previous section. The equilibrium is defined in the appendix. The model is solved via a log-linear approximation around the steady state, while explicitly respecting the DLB.

From here on, we rewrite the model in real terms. Lower case letters refer to the real value of their nominal (big case letters) counterparts. Hat represents percentage deviation from steady state.

3.2.2 Analytic characterization

Before moving on to the numerical simulations, we briefly characterize the log-linear approximated model. We show that our model can be written in the form of the standard three-equation New Keynesian

[38] Observe that if the interest on government bonds goes negative, then individual household will substitute out of government bonds in favor of deposits that carry zero interest.

model, with two important differences: First, it is the deposit rate which enters the dynamic IS equation rather than the policy rate. Second, the natural rate of interest is now endogenous and depends on credit market outcomes.

The household and firm problems, together with the monetary policy rule, can be summarized as follows:

$$\hat{y}_t = E_t\hat{y}_{t+1} - \sigma\{\hat{\imath}_t^d - E_t\hat{\pi}_{t+1} - \hat{r}_t^n\} \tag{31}$$

$$\hat{\pi}_t = \kappa\hat{y}_t + \beta E_t\hat{\pi}_{t+1} \tag{32}$$

$$\hat{\imath}_t^r = \hat{r}_t^n + \phi_\pi\hat{\pi}_t + \phi_y\hat{y}_t \tag{33}$$

$$\hat{\imath}_t^d = \max(\bar{\imath}^{DLB}, \hat{\imath}_t^r) \tag{34}$$

where $\sigma \equiv \frac{1}{qy}$, $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(\eta+\sigma^{-1})}$, $\hat{y}_t \equiv \log\frac{y_t}{\bar{y}}$, $\hat{\imath}_t^d \equiv \log\frac{1+i_t^d}{1+\bar{\imath}^d}$, $\hat{\pi}_t \equiv \log\frac{\Pi_t}{\bar{\Pi}}$, $\bar{\imath}^{DLB} \equiv -\log(1+\bar{\imath}^d)$ and $\hat{r}_t^n$ is the natural rate of interest, defined below.

Equation (31) is identical to the standard IS equation typically derived in the representative household framework. Here it is obtained by combining the consumption Euler equation of the two agents with the aggregate resource constraint. Equation (32) is the standard New Keynesian Phillips curve which is identical to the standard model, while equation (33) is the central bank's policy rule. Equation (34) specifies the deposit rate, at the lower bound and otherwise.

A major insight from the New Keynesian literature is that if the central bank tracks the natural rate of interest it stabilizes inflation and (under certain conditions) output. This remains the case in this model. A key difference, however, is that the natural rate of interest is not exogenous as in the standard model, but instead given by

$$\hat{r}_t^n \equiv -\chi\{\hat{\imath}_t^b - \hat{\imath}_t^d\} + \hat{\zeta}_t - E_t\hat{\zeta}_{t+1} \tag{35}$$

where $\hat{\zeta}_t \equiv \log\frac{\zeta_t}{\bar{\zeta}}$, and $\hat{\imath}_t^b \equiv \log\frac{1+i_t^b}{1+\bar{\imath}^b}$. While the natural rate of interest depends on the exogenous process for the household's preferences, it also depends on the endogenous credit market spread. Thus in response to a shock that increases the spread, the central bank needs to cut interest rates to stabilize inflation.

The spread between borrowing and lending rates is derived through the optimal lending decision of the bank yielding:

$$\frac{1+i^b}{1+i^s}\left(\hat{\imath}_t^b - \hat{\imath}_t^s\right) = \nu\frac{\Gamma}{b}\left[(\nu-1)\,\hat{b}_t - \iota\hat{n}_t\right] \tag{36}$$

A consequence, which is novel in our set-up relative to the existing literature, is that bank's net worth, via the impact on credit spreads, is an important variable for determining the natural rate of interest.

The dynamics of net worth is

$$\beta^s\hat{n}_t = \beta^s\hat{\imath}_t^d + \bar{\Pi}\hat{z}_t + (1-\omega)\left(\hat{n}_{t-1} - \hat{\pi}_t\right) \tag{37}$$

where $\hat{z}_t$ is bank's profits given by

$$\frac{\overline{n}}{\bar{\Lambda}}\hat{z}_t = -\{\frac{\beta^b}{\beta^s}\frac{\bar{b}}{\bar{\Lambda}} + \frac{\bar{r}}{\bar{\Lambda}} + \frac{\bar{a}}{\bar{\Lambda}} - \frac{\bar{f}}{\bar{\Lambda}}\}\hat{\imath}_t^d + \frac{\beta^b}{\beta^s}\frac{\bar{b}}{\bar{\Lambda}}\hat{\imath}_t^b + \frac{\bar{r}}{\bar{\Lambda}}\hat{\imath}_t^r + \frac{\bar{a}}{\bar{\Lambda}}\hat{\imath}_t^a - \frac{\bar{f}}{\bar{\Lambda}}\hat{\imath}_t^f + \iota\frac{\bar{\Gamma}}{\bar{\Lambda}}\hat{n}_t \tag{38}$$

$\hat{\imath}_t^a \equiv \log\frac{1+i_t^a}{1+\bar{\imath}^a}$, $\hat{\imath}_t^f \equiv \log\frac{1+i_t^f}{1+\bar{\imath}^f}$, $\hat{z}_t \equiv \frac{z_t-\bar{z}}{\bar{n}}$ and $\hat{n}_t \equiv \log\frac{n_t}{\bar{n}}$. The profit equation is expressed in terms of $\bar{\Lambda} \equiv \bar{a} + \bar{r} + \bar{b}$ which is the total assets of the bank in steady state.

Because the lending of the banks is the debt of the borrower, the model is closed by the debt dynamics from the borrowers budget constraint and the borrowers consumption Euler equation below

$$\hat{b}_t = \frac{1}{\beta^b}\hat{b}_{t-1} - \frac{1}{\beta^b}\hat{\pi}_t + \frac{1}{\beta^b}\hat{\imath}_{t-1}^b - \chi\frac{\bar{y}}{\bar{b}}\hat{y}_t + \chi\frac{\bar{y}}{\bar{b}}\hat{c}_t^b \tag{39}$$

$$\hat{c}_t^b = E_t\hat{c}_{t+1}^b - \sigma(\hat{\imath}_t^b - E_t\pi_{t+1} - \hat{\zeta}_t + E_t\hat{\zeta}_{t+1}) \tag{40}$$

We assume full pass-through from the policy rate to liquid assets and external financing away from the DLB, but allow for partial pass-through at the DLB, i.e.:

$$\hat{\imath}_t^a = \rho^a\hat{\imath}_t^r \text{ if } \hat{\imath}_t^r < i^{DLB} \text{ and } \hat{\imath}_t^a = \hat{\imath}_t^r \text{ if } \hat{\imath}_t^r > i^{DLB} \tag{41}$$

$$\hat{\imath}_t^f = \rho^f\hat{\imath}_t^r \text{ if } \hat{\imath}_t^r < i^{DLB} \text{ and } \hat{\imath}_t^f = \hat{\imath}_t^r \text{ if } \hat{\imath}_t^r > i^{DLB} \tag{42}$$

An approximate equilibrium is a collection of stochastic processes for prices $\{\hat{\pi}_t, \hat{\imath}_t^r, \hat{\imath}_t^d, \hat{\imath}_t^f, \hat{\imath}_t^a, \hat{\imath}_t^b, \hat{r}_t^n\}$ and quantities $\{\hat{y}_t, \hat{c}_t^b, \hat{b}_t, \hat{n}_t, \hat{z}_t\}$ that solve equations (31) - (40) given an exogenous process $\{\hat{\zeta}_t\}$. Note that, since we log-linearize around a steady-state where the bank is satiated in liquid assets and where $C_f = 0$, the evolution of $\hat{l}_t$ and $\hat{f}_t$ can be omitted from the characterization of the general equilibrium. In Appendix D.8, we parametrize the C-function and show the evolution of $\hat{l}_t$ and $\hat{f}_t$.

Monetary policy affects aggregate demand in equation (31) through two channels. If the DLB is not binding, the most direct one is that a reduction in the policy rate reduces the deposit rate, stimulating spending. The second channel is that policy rate cuts endogenously reduce the natural rate of interest by increasing the credit spread in equation (35).

To illustrate the key mechanism analytically, we make three simplifying assumptions to clarify a point that holds more generally, namely that condition (21) for the net balance sheet exposure to negative policy rates is once again the key determinant of whether negative policy rates are expansionary or not once the DLB is binding.

Consider the case in which $\omega = 1$, and $\hat{b}_t = 0$ for $t > 0$. [39] Finally, suppose that prices are fixed so that $\kappa = 0$. Consider now the effect of a one-time reduction in the policy rate at time 0, while the deposit rate is unchanged (with the pass-through to liquid assets and external financing given by ρ_a and ρ_f).

Solving (31) through (40) yields:

[39] The simplest way of thinking about the second assumption, is that in response to a shock in period 0 there are government lump sum redistributions to bring debt back to steady state in period 1.

$$\hat{b}_0 = \frac{(\frac{\bar{r}}{\bar{n}} + \rho^a \frac{\bar{a}}{\bar{n}} - \rho^f \frac{\bar{f}}{\bar{n}})}{\frac{\beta^b}{\beta^s}\frac{\bar{b}}{\bar{n}} + \Theta\{\frac{\nu-1}{\iota}\Upsilon + \frac{1}{\iota}\frac{\beta^s}{\beta^s-\beta^b}\}}\Upsilon \hat{\imath}^r_0 \tag{43}$$

$$\hat{\imath}^b_0 = -\frac{1}{\Upsilon}\hat{b}_0 \tag{44}$$

$$\hat{y}_0 = -\sigma\chi\hat{\imath}^b_0 \tag{45}$$

where $\Upsilon \equiv \sigma\chi(1-\chi)\frac{\bar{y}}{\bar{b}} > 0$ and the parameter $\Theta > 0$ is defined as in the partial equilibrium model. Thus, just as in the partial equilibrium model, policy rate cuts once the DLB is binding reduce lending according to equation (44) if the condition in equation (21) is violated. In general equilibrium, however, as shown in equation (44) this increases borrowing rates, in line with our empirical evidence from Sweden. Moreover, as shown in equation (45), the result is a contraction in aggregate demand. While these results can be shown more generally, the more fundamental question is what is the quantitative implication of negative policy rates.

An inspection of the equations above reveals that the model is fully parameterized via the assignment of the banks balance sheet parameters $(\frac{\bar{n}}{\bar{\Lambda}}, \frac{\bar{b}}{\bar{\Lambda}}, \frac{\bar{f}}{\bar{\Lambda}}, \frac{\bar{r}}{\bar{\Lambda}}, \frac{\bar{a}}{\bar{\Lambda}}, \frac{\bar{y}}{\bar{b}}, \frac{\bar{\Gamma}}{\bar{b}})$, the interest rate pass-through parameters ρ^a, ρ^f, the structural parameters, $(\sigma, \kappa, \beta^s, \beta^b, \chi, \nu, \iota, \omega)$, the parameters of the policy function $(\phi_\pi, \phi_y, \bar{\Pi})$, along with the stochastic process for ξ_t. We now turn to the parameterization and quantitative predictions of the model.

3.3 Quantitative evaluation

In this section, we take the model outlined above to the data. A natural starting point is to evaluate the empirical balance sheet exposure of banks to negative policy rates.

3.3.1 Balance sheet exposure to negative rates

The analysis so far suggests that a reduction in the policy rate, once the DLB is reached, contracts lending if the transmission of negative policy rates to the asset side is larger than to the liability side, or

$$\underbrace{\frac{\bar{r}}{\bar{\Lambda}}}_{0.05} + \underbrace{\rho^a\frac{\bar{a}}{\bar{\Lambda}}}_{0.42} > \underbrace{\rho^f\frac{\bar{f}}{\bar{\Lambda}}}_{0.21} \tag{46}$$

where the relevant components of the balance sheet are expressed as a ratio of total assets.

The values reported under the curly brackets in equation (46) come from Swedish data and suggest that – in the Swedish case – banks exposure to negative policy rates is such that policy rate cuts at the DLB are expected to reduce bank profits. The condition is calibrated using balance sheet data from 2014 to capture the state of the banking system just prior to negative rates being implemented. The empirical counterpart of $\bar{a}$ is total assets less reserves and bank lending, $\bar{f}$ is total liabilities less deposits and equity, $\bar{r}$ is reserve balances with the central bank and cash.

Banking Sector Moments	Value	Empirical Counterpart
Liquid asset ratio	$\frac{\overline{a}}{\overline{\Delta}} = 0.42$	Liquid assets to total assets of 42%
Reserve ratio	$\frac{\overline{r}}{\overline{\Delta}} = 0.05$	Reserves to total assets of 5 %
External financing ratio	$\frac{\overline{f}}{\overline{\Delta}} = 0.53$	External financing to total assets of 53 %
Loans to total assets	$\frac{\overline{b}}{\overline{\Delta}} = 0.53$	Loans to total assets of 53 %
Net worth to total assets	$\frac{\overline{n}}{\overline{\Delta}} = 0.05$	Net worth to total assets of 5 %
Profit to net worth	$\frac{\overline{z}}{\overline{n}} = 0.173$	Profit to net worth of 17.3 %
Loans to GDP	$\frac{\overline{b}}{\overline{y}} = 1.4$	Loans to GDP of 140 %
Pass-through coefficient to liquid assets	$\rho^a = 1$	Pass-through of negative rates of 100 %
Pass-through coefficient to external financing	$\rho^f = 0.4$	Pass-through of external financing of 40 %

Table 5: Banking sector parameters.

Notes: This table shows key banking sector moments and pass-through coefficients necessary to calibrate the log-linear approximated model, outlined in Appendix D. Banking sector moments are based on the 2014 balance sheet of the Swedish banking system. Pass-through coefficients are computed based on Figure 3 and Figure A.5.

As shown in the previous section, the approximated model also requires assigning values to $\frac{\bar{n}}{\bar{\Lambda}}$ and $\frac{\bar{b}}{\bar{\Lambda}}$. The values assigned to these ratios come from the same data source and are reported, along with the other relevant balance sheet ratios in Table 5. In addition to these ratios, the ratio $\frac{\bar{\Gamma}}{\bar{\Lambda}}$ appears in the approximated model. From the optimal lending conditions, the steady state of the model implies that $\frac{\bar{\Gamma}}{\bar{\Lambda}} = \frac{\bar{b}}{\bar{\Lambda}} \frac{\frac{\beta^s}{\beta^b}-1}{\nu}$.

The pass-through to other liquid assets at the DLB (ρ^a) is assumed to be one, reflecting the strong pass-through of negative rates to reserve-like assets such as money market instruments and government bonds documented in Figure A.5.[40] The pass-through to external financing at the DLB (ρ^f) is set to be 0.4. This number is obtained by first computing i_t^f as the weighted average of the money market rate, weighting the money market rates and covered bond rates with weights computed from the banks balance sheets.[41]

[40] There is no indication of reduced pass-through to government bonds with 2 and 5 year maturities, see Figure A.5. The same holds for long term government bonds, with 10 year maturities. This is in contrast to the reduction in pass-through for covered bond rates (which have maturities of 2 or 5 years).

[41] To calculate this number, we first compute i^f as

$$i^f = \frac{1}{3} \times \text{Money Market Rates} + \frac{1}{3} \times \text{Covered bond, 2Y} + \frac{1}{3} \times \text{Covered bond, 5Y} \tag{47}$$

where the different interest rates used as inputs are taken from Figure A.5. We then set

$$\rho^f = \frac{\Delta i_t^f}{\Delta i_t^r} \tag{48}$$

for the period after $i_t^d \approx 0$, i.e. the post-bound period.

Parameter	Value	Source/Target
Intertemporal elasticity of substitution	$\sigma = 0.66$	Smets and Wouters (2003)
Share of borrowers	$\chi = 0.61$	Justiniano et al. (2015)
Steady-state gross inflation rate	$\bar{\Pi} = 1.005$	Match annual inflation target of 2 %
Discount factor, saver	$\beta^s = 0.9963$	Annual real savings rate of 1.5%
Discount factor, borrower	$\beta^b = 0.991$	Annual real borrowing rate of 3.5% (Mehra and Prescott, 2008)
Slope of AS equation	$\kappa = 0.02$	Eggertsson and Woodford (2003)
Taylor coefficient on inflation gap	$\phi_\Pi = 1.5$	Gali (2008)
Taylor coefficient on output gap	$\phi_Y = 0.5/4$	Gali (2008)
Marginal cost of lending	$\nu = 4.5$	Match pass-through of negative rates from Table 1.
Elasticity of lending costs wrt. net worth	$\iota = 2.28$	$\iota = (\nu - 1) \times 1.89/2.89$, based on MAG (2010) and model equations
Payout ratio	$\omega = 0.17$	Generate profit to net worth ratio of 17.3 %
Shock	**Value**	**Source/Target**
Preference shock	22.7 % temporary decrease in ζ_t	Generate a 4.5 drop in output on impact
Persistence of preference shock	$\rho = 0.92$	Duration of lower bound of 12 quarters

Table 6: Calibration

3.3.2 Assigning values to structural parameters

Numerical values are assigned to the remaining structural parameters in three steps. First, values are chosen from the existing literature in cases where they are relatively standard. Second, values for the novel parameters of our model are chosen by matching certain features of the data. Finally, given the other parameters, some structural values can be assigned exploiting steady state relationships.

We start by setting the relatively standard parameters equal to values used in the existing literature, following especially Benigno et al. (2014) closely. The values chosen for $\sigma, \kappa, \chi, \beta^s, \beta^b, \phi_\pi, \phi_y, \bar{\Pi}$ are reported in Table 6. The values for the policy rule are standard, see e.g. Gali (2008). Steady state saving and borrowing interest rates are pinned down by β^s, β^b and $\bar{\Pi}$. The risk-free rate is assumed to be 1.5 percent in steady state based on US data. The borrowing rate is pinned down based on Curdia and Woodford (2011), who report a spread of 2 percent in US data. The choice of equilibrium credit spread has limited impact on the quantitative conclusions in Section 3.4, see Figure A.10 in Appendix A. The values for κ, σ and χ reported in Table 6 are relatively conventional and values in this range can be found in a large number of sources, see for instance Benigno et al. (2014) or Justiniano et al. (2015).

The two key parameters that are specific to the model are ι and ν. We assign values to these parameters based on our own estimate of the pass-through of negative policy rates to borrowing rates at the DLB from Table 1 and empirical estimates from MAG (2010). MAG (2010) is a meta study released by the Bank of International Settlements that considers the effect of a reduction in bank equity on lending, using a large number of models. A summary finding is that a one percentage point increase in bank target capital leads to a 1.89 percentage point reduction in lending.

Using these two pieces of evidence, ι and ν are pinned down in two steps. First, defining the bank capital ratio as $\Upsilon_t = \frac{n_t}{b_t}$, and taking lending rates as given, condition (13) can be used to relate lending and bank capital to yield $\hat{b}_t = \frac{\iota}{\nu - \iota - 1}\hat{\Upsilon}_t$. MAG (2010) provides an empirical estimate for $\frac{d\hat{b}_t}{d\hat{\Upsilon}_t} = 1.89$ from which it follows that $\iota = \frac{1.89}{2.89} \times (\nu - 1)$. In the second step, we use the pass-through of policy rate cuts to borrowing rates at the DLB estimated in Table 1 as an identified moment (Nakamura and Steinsson, 2018) to set ν. Specifically, the full model is solved and ν is chosen so that in general equilibrium - if the DLB is binding - a

100 basis points reduction in the policy rate increases borrowing rates by 5.9 basis points on impact.

A remaining parameter is ω. To choose this parameter we exploit that in steady state $\bar{z}/\bar{n} = \beta^s + \omega - 1$. Using data on $\bar{z}/\bar{n}$ from Table 6 yields $\omega = 0.173$.

Finally, to conduct numerical experiments in the model, we further need to specify a shock process for ξ_t. In the numerical experiment considered next, we assume that ξ_t follows a first order auto-regressive process with persistence ρ. At time 0 there is an initial shock. The size of the shock, and its persistence, is picked to generate a 4.5 percent drop in output and a duration of the lower bound of twelve quarters assuming that the DLB is binding at zero. The output drop is calibrated to match the average decline in real output in Switzerland, the Euro Area, Sweden and Denmark in the aftermath of the financial crisis.

3.4 Simulation results

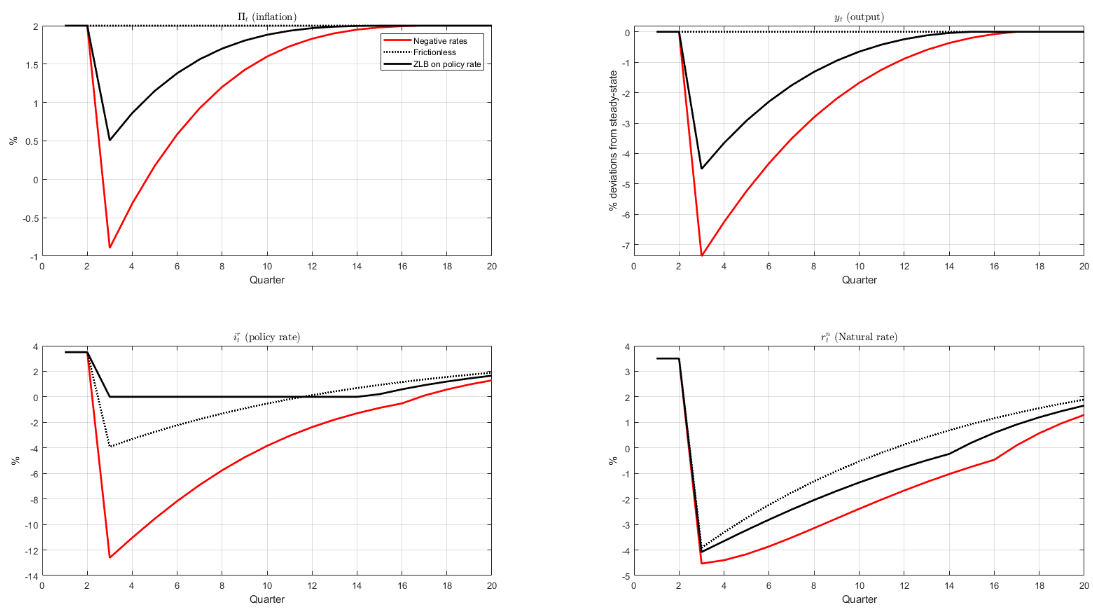

Figure 8: Response of model under the baseline calibration to a preference shock.

Notes: This figure shows the impulse response functions of inflation, output, the policy rate and the natural rate of interest in response to a preference shock. The red solid line corresponds to a model where the central bank follows the Taylor rule into negative territory, but in which the deposit rate is subject to a lower bound. The black solid line corresponds to a model where the central bank follows the Taylor rule in positive territory, but where the policy rate is subject to a zero lower bound. The dashed black line corresponds to a model in which there are no bounds on any interest rate.

Figure 8 plots the impulse responses of inflation, output, the natural rate and the policy rate to an innovation in $\hat{\xi}_0$. The solid black line shows the evolution of these variables in response to the shock, assuming the policy rate and the deposit rate are constrained at zero. The shock directly reduces the natural rate to approximately minus four percent. Since the policy rate does not follow the natural rate into negative territory, the result is a contraction in output of 4.5 percent and a drop in inflation from the target rate of 2 percent to roughly 0.5 %.

The dashed black line shows the response of the economy if there are no lower bounds. In this case, the central bank fully accommodates the shock by cutting interest rates to -4 percent. This policy rate reduction is sufficient to offset the effect of the initial shock. As a result, there is no drop in inflation or output.

The red line illustrates the case when we impose a DLB on the deposit rate but not on the policy rate, capturing the recent experience with negative policy rates. In response to the shock, the central bank cuts the policy rate to roughly -12 %, which is implied by the Taylor rule. In this case, given the balance sheet structure of the banking sector, output contracts by roughly three additional percentage points.

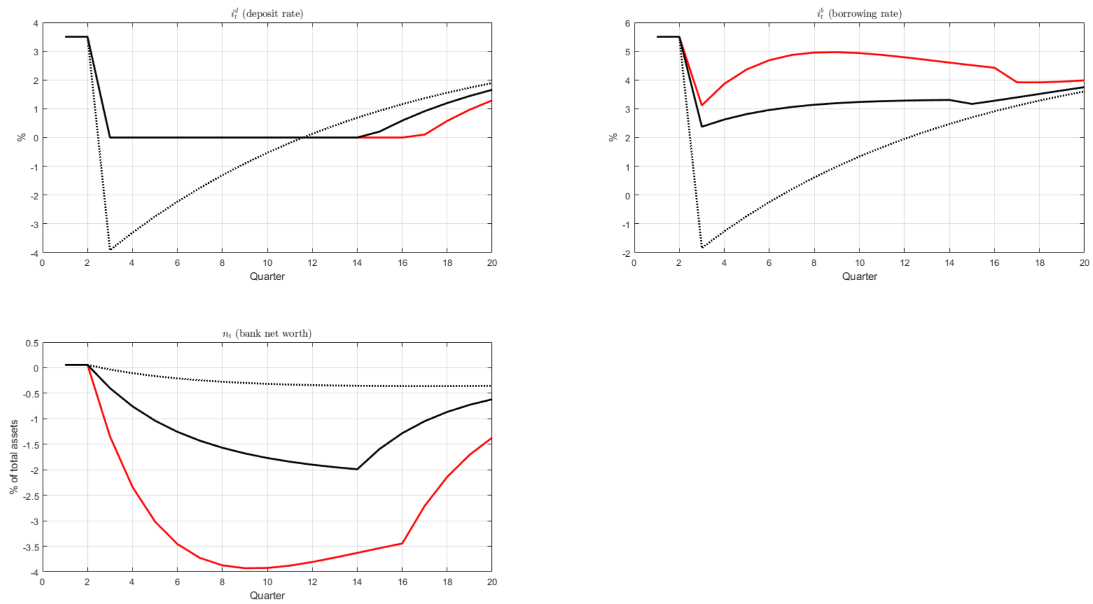

Figure 9: Response of model under the baseline calibration to a preference shock.

Notes: This figure shows the impulse response functions of the deposit rate, borrowing rate and bank net worth, in response to a preference shock. The red solid line corresponds to a model where the central bank follows the Taylor rule into negative territory, but in which the deposit rate is subject to a lower bound. The black solid line corresponds to a model where the central bank follows the Taylor rule in positive territory, but where the policy rate is subject to a zero lower bound. The dashed black line corresponds to a model in which there are no bounds on any interest rate.

Figure 9 sheds light on the underlying mechanism. Due to the DLB, the negative policy rate is not transmitted to deposit rates. As a result, there is limited pass-through to banks financing costs and so no major reduction in lending rates. Further, because banks in our economy are net holders of assets with a negative rate, the introduction of negative rates yields a reduction in bank profits. This translates into lower bank net worth and higher intermediation costs, thus increasing the interest rate spread – indeed the parameter ν was chosen so as match the estimated increase in spreads in the data. Note that, while the initial drop in the deposit rate down to the DLB reduces the borrowing rate somewhat, the reduction in borrowing rates with negative policy rates is smaller compared to the case when the policy rate does not go below zero, due to the contractionary effect on net worth. As a result, output falls by an additional amount when the central bank goes negative. Going into negative territory, once the DLB is binding, is thus contractionary.

We note that the quantitative effect of negative rates in the simulation presented here is only illustrative, and does not capture observed central bank behavior. Specifically, in our simulations, the central bank follows a Taylor rule, causing it to implement a very negative policy rate – at odds with the very modestly negative policy rates observed in practice. Subsection 3.4.2 gives a better sense of the likely quantitative effects of mildly negative rates, and highlights how this effect depends on the parameter values assumed.

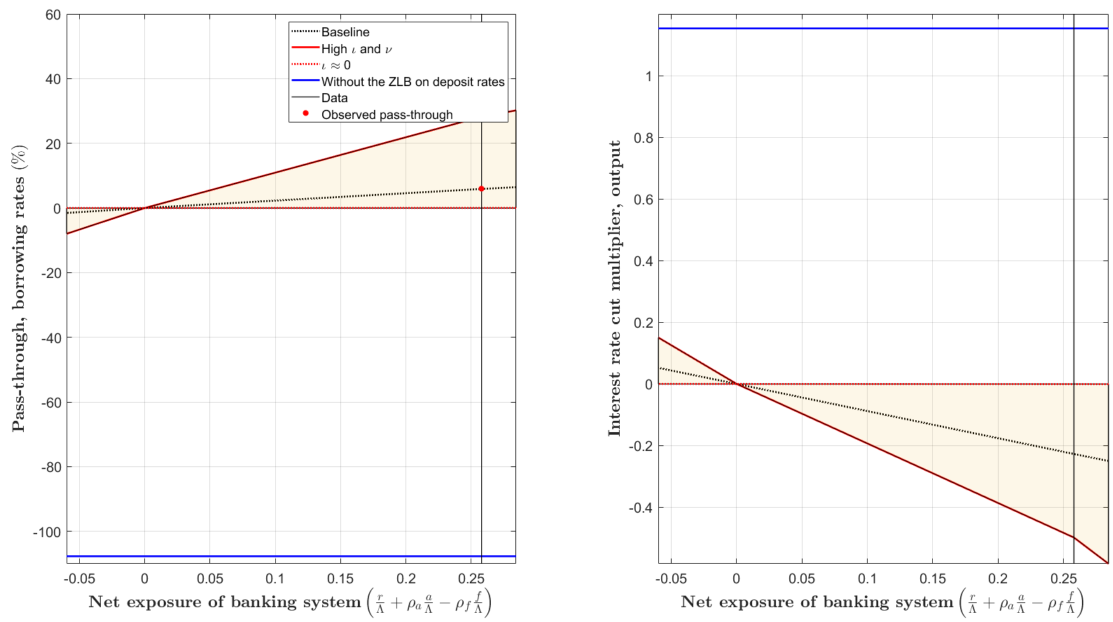

Figure 10: Effect of negative policy rates and net exposure of the banking system

Notes: This figure shows the on-impact difference in borrowing rates (left) and output (right) between two models - one model where the central bank does not impose a negative policy rate and one where the central bank follows a standard Taylor rule also below zero. The output and borrowing rate difference is scaled by absolute value of the average policy rate below zero. On the x-axis, we redo the exercise for different values of equation (46). We fix R, A and F and solve the model for different values of $\rho^f \in (0.3, 1)$. The black dashed line is the results using our baseline calibration. The red solid line is a calibration where $\nu = 51.5$, which generates a pass-through at the DLB equal to the coefficient estimate in Table 1 plus 1 standard deviation. The blue horizontal lines show the effects in the case when the DLB is non-binding.

3.4.1 Transmission of negative policy rates to lending rates

In Table 1 we reported the pass-through of policy rate cuts to lending rates at the DLB. Pass-through can also be computed in the model. The horizontal blue line in the left panel of Figure 10 shows the pass-through of interest rate cuts when the DLB is not binding. As the figure reveals, it is approximately -100, that is, borrowing rates fall approximately as much as the repo-rate in response to policy rate cuts in the quarter in which they occur.

The x-axis captures different calibrations of the exposure of the banking sector to negative rates, according to Proposition 4. In the figure, we vary ρ_f, but an alternative would be to keep ρ_f fixed and vary the balance sheet ratios, for instance $\frac{f}{\Lambda}$. In the absence of the DLB, the exposure of the banking system to negative rates is irrelevant and so the effects of policy rate cuts on lending and output is invariant to a large range of calibrations. The dashed black line shows the pass-through to borrowing rates once the DLB is binding. The red dot denotes a pass-through of 5.9, in line with our empirical estimates. The vertical line highlights the empirical exposure to negative rates using Swedish bank balance sheets. As ρ_f varies, the pass-through to borrowing rates varies.

The shaded yellow area captures variations in the pass-through resulting from different values of $\{\nu, \iota\}$.

3.4.2 Output effect of negative policy rates

To quantitatively summarize how the DLB reduces the efficiency of policy rate cuts, and how sensitive the conclusions are to parameter values, it is useful to compute an interest rate cut multiplier, defined as

$$\text{mult} = \frac{\hat{y}_1^* - \hat{y}_1^{\text{PLB}=0}}{|\hat{\iota}_1^{r^*} - \hat{\iota}_1^{r\text{PLB}=0}|} = \frac{\hat{y}_1^* - \hat{y}_1^{\text{PLB}=0}}{|\hat{\iota}_1^{r^*}|} \tag{49}$$

where $\hat{y}_1^{\text{PLB}=0}$ denotes the output on impact if the policy rate is at a hypothetical lower bound of 0 and $\hat{\iota}_1^{r\text{zero}} = 0$ is the corresponding policy rate at the bound. These numbers correspond to the initial point of the black line in Figure 10. $\iota_1^{r^*}$ is a policy intervention and the corresponding output response is $\hat{y}_1^*$ for a particular scenario. Consider, for example, the interest rate multiplier if there is no bound on any interest rate – the blue line in Figure 10. In that case, the multiplier is 0.9 and has the interpretation that a 100 basis point cut in the policy rate leads to a 0.9 percent increase in output. Moreover, it is independent of the balance sheet composition of the banking sector.

Once the DLB is reached, however, the balance sheet composition of banks is fundamental as illustrated by the condition in Proposition 4. The dashed line in Figure 10 shows the interest rate multiplier for different net exposure measures. The vertical line illustrates the benchmark parameterization, in which case the multiplier is - 0.227. Put differently, a 100 basis point interest rate cut reduces output by 22.7 basis points. The figure also illustrates two extreme cases for the choice of ι and ν. The vertical horizontal line shows the case of $\iota = 0$, in which case there is no output effect of negative rates, while the dark red line shows what we consider to be relatively high values of ν. The figure illustrates that the sign of the interest rate cut multiplier can change, depending on the banking sectors net exposure to negative rates.

3.4.3 Frictional deposits

A key abstraction in our model is that we assume perfect competition in the deposit market. The main motivation for this abstraction is that it greatly simplifies the analysis. Furthermore, it illustrates cleanly the key mechanism through which policy rate cuts at the DLB adversely affect the net worth of banks, and as a result, put upward pressure on borrowing rates and downward pressure on bank lending in general equilibrium.

A counterfactual implication of this assumption however, is that the deposit rate and the policy rate away from the DLB are always one-and-the-same. This is inconsistent with the data, since on average, the deposit rate is below the policy rate and the pass-through of policy rate cuts to the deposit rate is less than one-to-one. From the perspective of this paper, the important question is whether this simplifying assumption has meaningful effects on the key result. In this section, we briefly outline two extensions of the model which can capture the empirical relationship outlined above away from the DLB. First, we consider an extension in which there are adjustment costs to changing deposits, and second, we consider an extension in which banks engage in monopolistic competition in deposit markets.

We summarize the results of the two model extensions in Table 7. The table shows that either extension makes negative policy rates *more* contractionary than in the benchmark model – assuming the model is calibrated following the same procedure as before. The most important element of the calibration, which drives this quantitative result, is that the model is parameterized to match the increase in borrowing rates at the DLB in Sweden, which was our main empirical finding. Once this statistic is matched, our main results are robust to these extensions. To conserve on space, we leave the derivation of these extensions, as well as a more detailed discussion of the results, to Appendix D.9. Below we summarize the key intuition.

Model of deposit market	Interest cut multiplier, output
Perfect competition (baseline)	- 0.227
Adjustment costs (Appendix D.9.1)	- 0.242
Monopolistic competition (Appendix D.9.2)	- 0.492

Table 7: Interest cut multiplier (output), with different assumptions about deposit frictions

Considers first the possibility that there are adjustment costs associated with changing the stock of deposits, while preserving the assumption of perfectly competitive deposit markets. The reason this makes negative rates even more contractionary, provided the sufficient statistic (46) holds, is that deposit adjustment costs represent an additional cost of extending loans. Recall that the model is calibrated to match the evolution of bank balance sheets in Sweden, in which external financing barely changed while deposits increased. Since net worth declines as a response to policy rate cuts at the DLB and external financing is almost unchanged, the deposit reliance of banks increases. The additional cost of adjusting deposits then decreases lending by more than in the absence of these frictions, which leads to a stronger output contraction in general equilibrium.[42]

Consider next an extension with monopolistic competition, as for example in Drechsler et al. (2017). The reason this extension implies that negative rates have a more negative impact on output is as follows: In order for the model to replicate the rise in borrowing rates observed in Sweden, as a result of negative policy rates, the calibration implies that the banks are more sensitive to the evolution of net worth, i.e. the calibrated value of ι is higher. Conditional on the sufficient statistic (46), this implies that the effect of negative net worth, generated by negative policy rates, contracts bank lending by more, thus leading to a sharper output contraction in general equilibrium.

To summarize, our findings are robust to these two alternative assumptions about the structure of deposit markets. Moreover, our baseline model with perfect competition provides a more conservative estimate of the contractionary effects of policy rate cuts at the DLB. Given that the analytics are simplified considerably by the assumption of perfect competition, we opt for erring on the side of the more conservative specification. It is worth stressing, however, that extending the model along these lines sketched out above is necessary to capture the observed relationship between policy rates and deposit rates prior to reaching the DLB. We leave this important extension to future research.

3.5 Rationalizing different findings in the literature

We end this section by briefly discussing the existing empirical literature on the bank lending channel of negative policy rates in the context of the model. The literature on the bank lending channel of negative rates suggests, at first glance, that there are contrasting views as to whether negative policy rates expand bank credit. While our empirical results on credit growth volumes, consistent with Heider et al. (2016), indicate

[42]Recall that the initial increase in borrowing rates is calibrated to be in line with the empirical evidence in Table 1.

that the bank lending channel of monetary policy is weaker under negative policy rates, other studies arrive at opposite conclusions.[43] For instance, Bottero et al. (2019) and Basten and Mariathasan (2018) show evidence indicating that credit supplied increases in response to negative policy rates.

Our model can rationalize these different findings, as they are based on economies and banking systems operating under different circumstances. Bottero et al. (2019) analyze the lending response of primarily Italian banks, and show that the deposit share is not an important predictor for how banks respond to negative policy rates. This is consistent with average deposit rates in Italy at the time of the introduction of negative policy rates being above zero. Hence, as the ECB lowered the policy rate into negative territory, there was substantial scope for deposit rates to fall further, lowering bank financing costs and thereby boosting lending. This is also the prediction of the model in the case the DLB is not yet binding.

The presence or absence of a zero lower bound on deposit rates is *not* sufficient to predict whether the bank lending channel will be fully operational however. This is illustrated by comparing the findings in our paper to the findings in Basten and Mariathasan (2018). They investigate the impact on bank lending of negative policy rates in Switzerland. When the Swiss National Bank went below zero, deposit rates were already close to the zero lower bound. However, Swiss banks, according to their summary statistics, had a relatively low share of liquid assets relative to total assets, and a higher share of debt and interbank funding, suggesting a negative net exposure to negative rates.[44] In our data, the average net exposure to negative interest rates is positive, potentially explaining why the overall impact on bank net worth in Sweden and Switzerland differs. The empirical findings in both this paper and in Basten and Mariathasan (2018) are therefore consistent with our model.

4 How can negative interest rates be more effective?

Our empirical analysis focuses on how the bank lending channel responds to negative interest rates given the current institutional setting. In this section, we discuss the broader question of whether negative interest rates can stimulate aggregate demand either through other transmission channels or with institutional changes.

4.1 Other transmission channels

An important omitted channel in our analysis is exchange rate movements. Both the Swiss and Danish central banks motivated their decision to implement negative central bank rates by a need to stabilize the exchange rate. The impact on the exchange rate would depend on which interest rates are most important for explaining movements in the exchange rate, which in turn can depend on several institutional details. The empirical literature on the impact of negative rates on exchange rates is somewhat scant, but some early evidence can not reject that negative interest rates have little or no impact on exchange rates, especially over longer time-horizons (Hameed and Rose, 2016).

[43] We note, however, that most existing studies focuses on the relative differences in bank lending under negative policy rates according to different bank characteristics, while our results are also informative about the *level* of the aggregate lending response.

[44] In fact, the profit margin of Swiss banks went up under negative interest rates, according to their paper (column 12 in Table A3).

Another mechanism through which negative interest rates could affect aggregate demand is through signaling about future interest rates. De Groot and Haas (2020) build a model where negative interest rates can signal lower future deposit rates, which in equilibrium boosts aggregate demand and output.

Finally, it is also worth noting that government borrowing rates have fallen into negative territory. To the extent that this can stimulate fiscal expansions, that would be an additional way through which negative rates could have a positive effect.

Summing up, negative interest rates can affect the macroeconomy through other channels, even when the bank lending channel is relatively less efficient. Overall, however, there is limited conclusive evidence as of now with regards to the effectiveness of these other channels.

4.2 Eliminating or lowering the lower bound

The critical friction in our model is the lower bound on deposit rates. Without this friction, negative interest rates would be similar to conventional monetary policy. Consistent with our findings, Altavilla et al. (2019) find that banks which were able to impose negative deposit rates also expanded lending in response to interest rate cuts into negative territory. Unfortunately, however, so far only a minority of banks have been able to do so – at least for a meaningful fraction of their deposit customers.

The critical question is, therefore, how to remove the lower bound on deposit rates more broadly. One way of doing so in the context of our model is if the government takes action to severely increase the cost of holding paper money. There are several ways to do this. The oldest example is a tax on currency, as outlined by Gesell (1916). Gesell's idea would show up as a direct reduction in the bound on the deposit rate in our model, thus giving the central bank more room to lower the interest rate on reserves - and the funding costs of banks.

Another possibility is to ban higher denomination bills, a proposal discussed in among others Rogoff (2017a,c). To the extent that it is more costly to store small denomination bills, this too should reduce the bound on the banks' deposit rate.

An even more radical idea, which would require some extensions to our model, is to let the reserve currency and the paper currency trade at different values, rather than on par as we have assumed. This proposal would imply an exchange rate between electronic money and paper money. Agarwal and Kimball (2015), Rogoff (2017c) and Rogoff (2017b) discuss a concrete proposal, where a key pillar – but perhaps also a challenge to implementability – is that the reserve currency is the economy-wide unit of account by which taxes are paid, and accordingly what matters for firms price setting. If such an institutional arrangement is achieved, then there is nothing that prevents a negative interest rate on the reserve currency while cash in circulation is traded at a different price, given by an arbitrage condition. We do not attempt to incorporate this extension to our model but note that it seems relatively straightforward theoretically, and has the potential of solving the DLB problem. It is an open questions, however, if firms would choose to denominate the price of their goods in terms of the reserve currency which has no lower bound, rather than still denominate their pricing in the paper currency still in circulation which still has a lower bound.

Indeed, the take-away from the paper should not be that negative nominal rates are always non-expansionary, simply that they are predicted to be less stimulative than normal interest rate cuts under the current institutional arrangement (conditional on bank balance sheets, i.e. our sufficient statistic). The likely prevalence of low interest rates going forward gives strong grounds for considering departures from the current institutional framework, such as those mentioned briefly here and discussed in more detail by some of the authors cited above.

5 Conclusion

Since 2014, several countries have experimented with negative policy rates. In this paper, we have documented that negative central bank rates have not been transmitted to aggregate deposit rates, which remain stuck at levels close to zero. Using bank level data from Sweden, we documented a disconnect between the policy rate and lending rates, once the policy rate became sufficiently negative. We further showed that this disconnect was partially explained by reliance on deposit financing. Banks which rely more heavily on deposit financing were less likely to reduce their lending rates in response to policy rate cuts once the deposit rate had reached its lower bound. Consistent with this, we found that Swedish banks with high deposit shares experienced lower credit growth after the deposit rate had become unresponsive. Furthermore, we documented negative excess returns on Swedish bank stocks surrounding the announcement of policy rate cuts in negative territory, which significantly differed from the response in positive territory.

Motivated by our empirical findings, we developed a New Keynesian model with savers, borrowers, and a bank sector. By including money storage costs and central bank reserves, we captured the disconnect between the policy rate and the deposit rate at the lower bound. In this framework, we highlighted how the strength of the bank lending channel of negative policy rates depends crucially on whether (part) of banks financing costs are subject to a binding lower bound and if so, the mix of assets and liabilities with different degrees of pass-through on the banks balance sheet.

Given the long-term decline in interest rates, the need for unconventional monetary policy is likely to remain high in the future. Our findings highlight conditions for when negative policy rates can work and when they are likely to be ineffective. The question remains, however, are there other unconventional monetary pools that can be more effective? While the existing literature has made some progress in evaluating these measures, the question of how monetary policy should optimally be implemented in a low interest rate environment remains a question which should be high on the research agenda.

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A Additional figures and tables

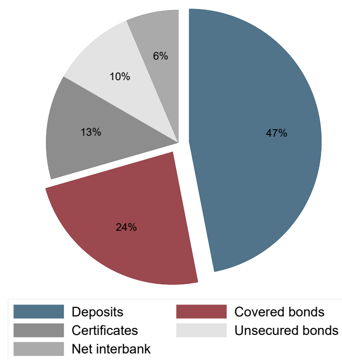


Figure A.1: Decomposition of liabilities (as of September 2015) for large Swedish banks. Source: The Riksbank

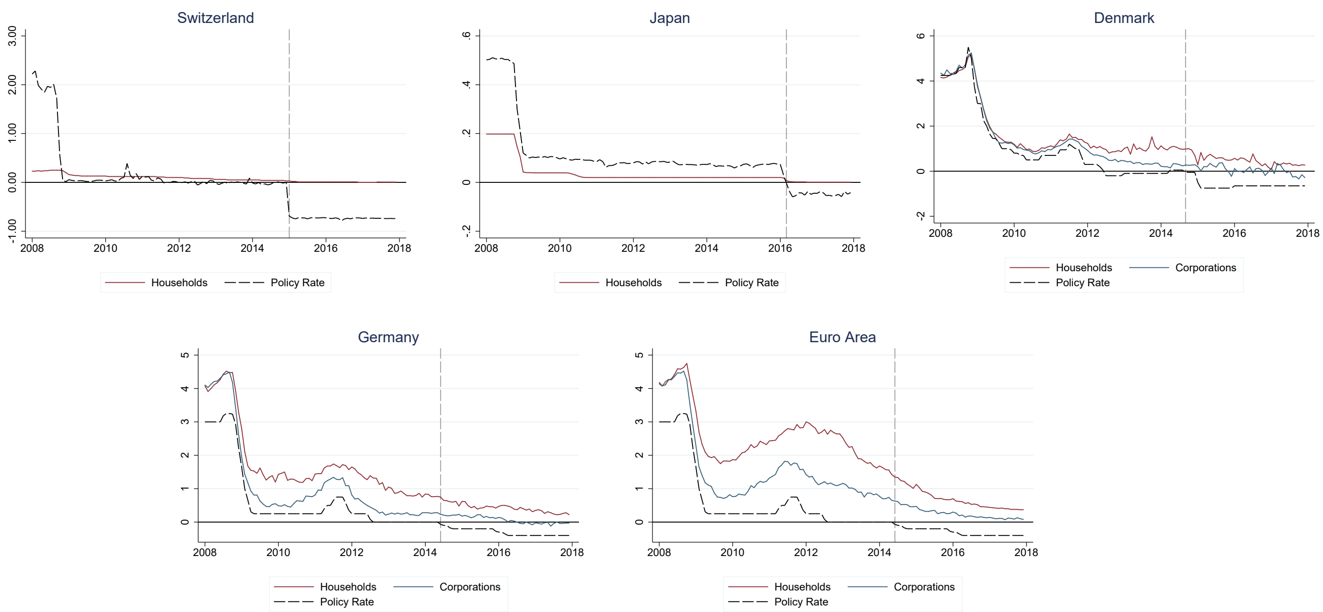


Figure A.2: Aggregate deposit rates for Switzerland, Japan, Denmark, the Euro Area and Germany. The policy rates are defined as SARON (Switzerland), the Uncollaterized Overnight Call Rate (Japan), the Certificates of Deposit Rate (Denmark) and the Deposit Rate (Euro Area and Germany). The red vertical lines mark the month in which policy rates became negative. Source: the Swiss National Bank (SNB), Bank of Japan, the Danish National Bank (DNB), and the European Central Bank (ECB).

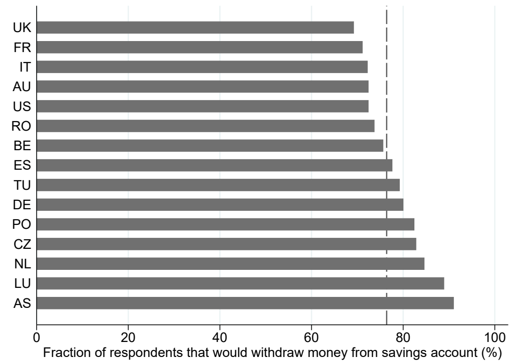


Figure A.3: Fraction of households who would withdraw money from their savings account if they were levied a negative interest rate. Solid line represent unweighted average of 76.4 . Source: ING (2015)

Figure A.4: Actual and Counterfactual Commission Income as a Share of Assets (). The counterfactual commission income is calculated as the amount of commission income that would be necessary to make up for the bound on the nominal deposit rate, all else equal. The counterfactual commission income is given by actual commission income plus $\frac{\text{Deposits}_t}{\text{Assets}_t}\left(i_t - i_t^{cf}\right)$, where i_t is the average aggregate deposit rate and i_t^{cf} is a counterfactual deposit rate calculated under the assumption that the markdown to the repo-rate is constant and equal to the pre-zero markdown. Source: Statistics Sweden and own calculations.

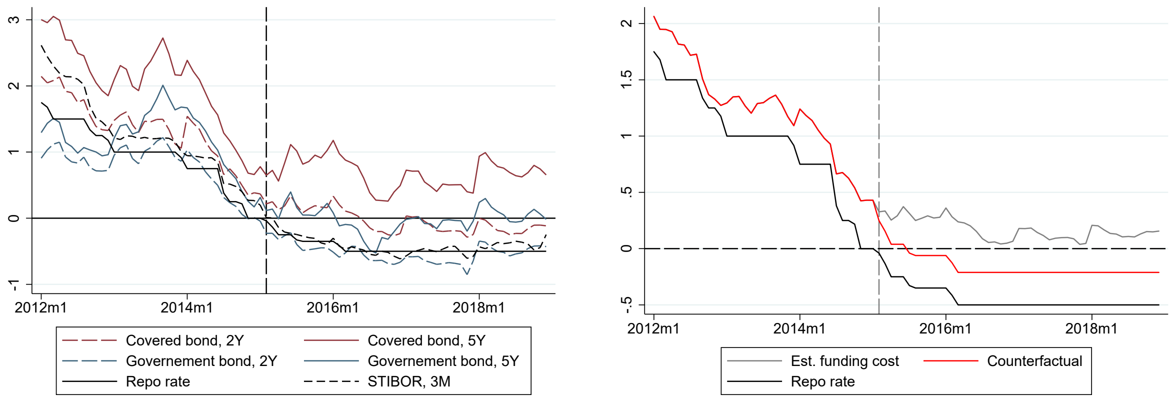


Figure A.5: Other interest rates (left panel) and an estimate of average bank funding costs (right panel).

Notes: This figure shows other interest rates (left panel) and an estimate of the average bank funding cost (right panel). The estimated average funding cost is computed by taking the weighted average of the assumed interest rates of the different funding sources of the bank. Certificates are assumed to have the same interest rate as 2Y covered bonds, while unsecured debt are assumed to have the same interest rate as 2Y covered bonds plus a 2 percent constant risk-premium. The counterfactual series corresponds to the case when the spread between the repo-rate and the estimated funding cost remains fixed at pre-negative levels. Weights are based on the liability structure of large Swedish banks, see Figure A.1. Source: The Riksbank

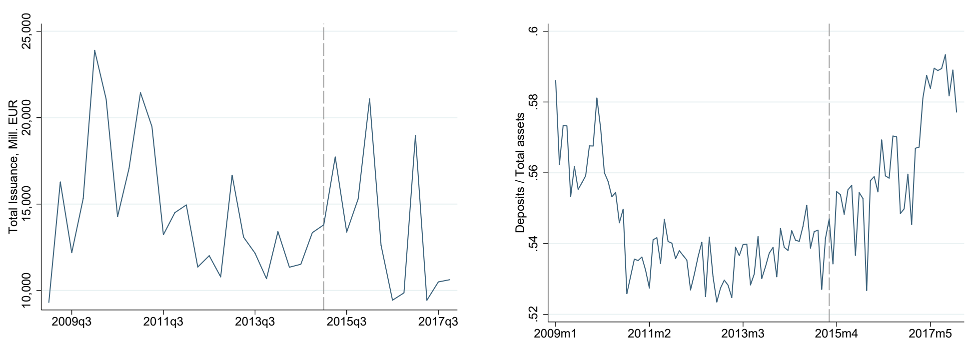


Figure A.6: Left panel: Issuance of covered bonds, Swedish banks. Right panel: Deposit share, Swedish banks. Vertical lines correspond to the date negative interest rates were implemented. Source: Association of Swedish Covered Bond Issuers, The Riksbank and Statistics Sweden

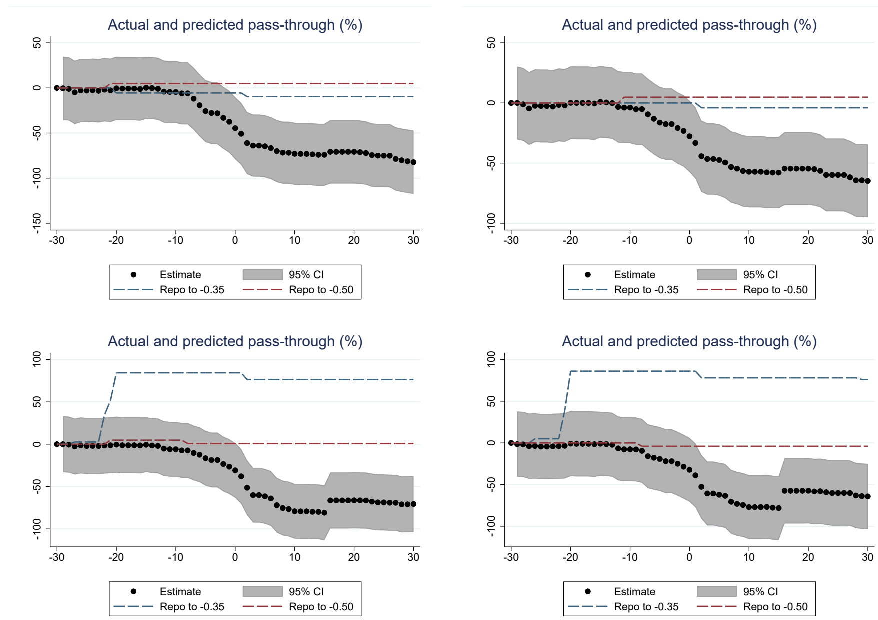


Figure A.7: Event study different mortgage rates. Upper left: 3 months. Upper right: 1 year. Lower left: 3 years. Lower right: 5 years.

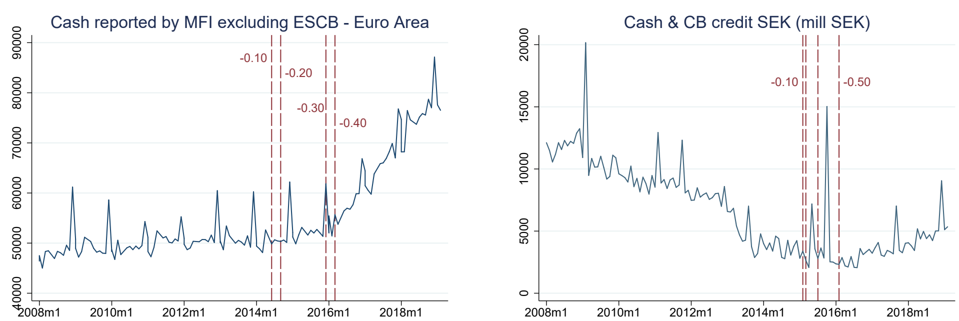


Figure A.8: Left panel: Cash held by Euro Area Banks. Variable name: Cash reported by MFIs excluding ESCB - Euro Area. Millions of Euro. Source: ECB. Right panel: Cash held by Swedish banks. Variable name: Cash and credit balances at central banks in SEK. Million SEK. Source: Statistics Sweden. The dashed, red lines refer to the interest rate cuts in negative territory.

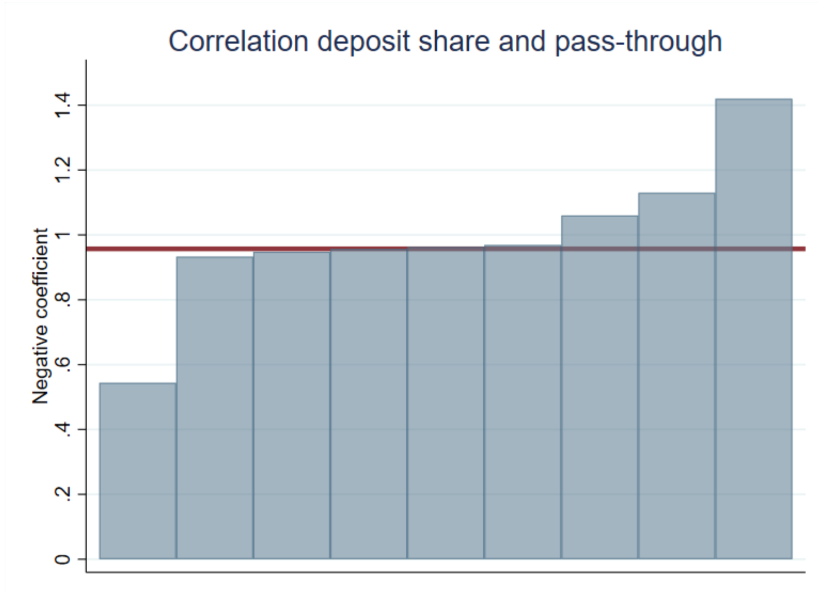


Figure A.9: We drop banks from our sample one by one, and re-estimate the regression reported in column 1 of Table 2 and visualized in Figure 6. Each bar shows the coefficient estimate from one of these regressions. The coefficient estimate in our baseline (full sample) is captured by the horizontal red line. All coefficient estimates are statistically significant at the ten percent level or higher.

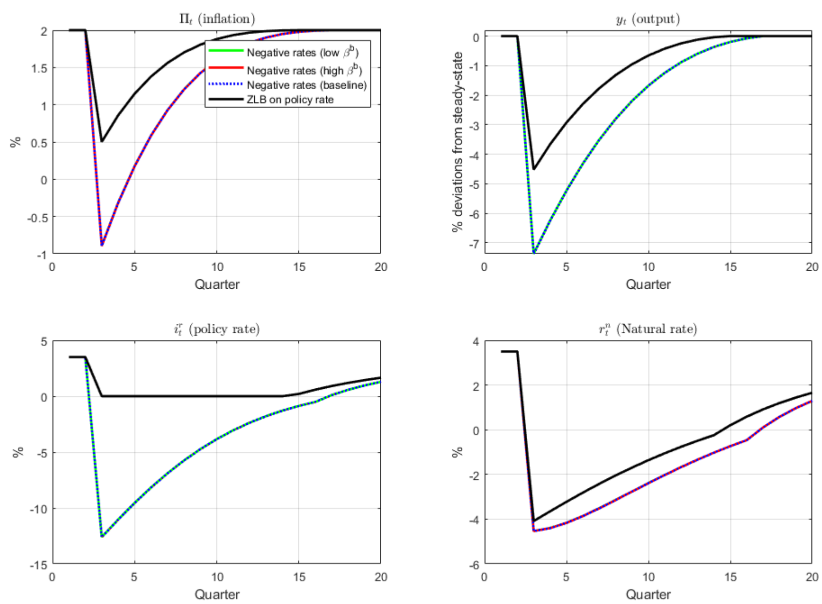


Figure A.10: This figure shows the impulse response of output, inflation, the policy rate and the natural rate in response to a preference shock but for two different calibrations of β^b. "low β^b" refers to a calibration where β^b is set to generate a steady-state borrowing rate of 8% (annualized), implying a credit spread of 4.5 pp, whereas "high β^b" refers to a calibration where β^b is set to generate a steady-state borrowing rate of 4% (annualized), implying a steady-state credit spread of 0.5%.

	(1) Δ log(loans)	(2) Δ log(loans)	(3) Δ log(loans)	(4) Δ log(loans)
Post × deposit share	0.0189			
	(0.0894)			
Post × deposit share		-0.0646		
		(0.0959)		
Post × deposit share			0.731	
			(1.476)	
Post × deposit share				-0.763
				(1.040)
N	1227	1227	1227	1227
No. of clusters	10	10	10	10
Mean of dependent variable	2.587	2.587	2.587	2.587
SD of dependent variable	7.713	7.713	7.713	7.713
Bank FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
measure	Cont.	Cont.	High or low	High or low
placeboperiod	repo-rate ∈ [1,0)	repo-rate ∈ (0,-0.25]	repo-rate ∈ [1,0)	repo-rate ∈ (0,-0.25]

Table A.1: Falsification test of lending volume regression. We consider two different "post-periods", according to the level of the repo-rate. The first period is defined as the period from 18th of December 2012 to the 17th of February 2015, when the repo-rate is in the interval between 1 and 0 percent. The second period is the period from the 17th of February 2015 to the 7th of July 2015 when the repo-rate is in negative territory but the pass-through to deposit rates are above its ZLB.

Date	**New Policy Rate**	**Forward Guidance**
July 2014	0.25	Increases in the repo rate are not expected to begin until the end of 2015
October 2014	0.00	It is assessed as appropriate to slowly begin raising the repo rate in the middle of 2016.
February 2015	-0.10	It will not be appropriate to increase the repo rate until the second half of 2016.
March 2015	-0.25	The repo rate is expected to remain at -0.25 per cent at least until the second half of 2016
July 2015	-0.35	The repo rate is expected to be around -0.35 per cent for just over a year.
February 2016	-0.50	The repo rate is expected to remain around -0.50 per cent for about a year.

Table A.2: Forward Guidance by the Riksbank

Baseline	No weighting	No weighting & smallest banks dropped	No weighting and no clustering	No clustering
-3.468**	-4.940	-8.514*	-4.940**	-3.468**
(1.096)	(4.426)	(4.381)	(2.179)	(1.562)

Table A.3: Robustness, estimation of equation (4)

	Return bank stock	Return stock market	Excess return
Pre-zero	+	+	+
Post-zero, pre-bound	+	+	-
Post-zero, post-bound	-	-	-

Table A.4: (Excess) Returns around policy rate announcements. + indicates positive effect, - indicates negative effect.

B Listed vs. transaction-based rates

An important question is whether the listed rates used in the main body of the text is a valid proxy for actual transaction rates. One approach to gauge whether this is the case, is to compare the implied aggregate listed rates with observed aggregate transaction-based rates.

Figure B.1 depicts a weighted average of the bank level floating mortgage rates aggregated to the monthly level, along with the official aggregate rates that are based upon transaction data. First note that the listed rates are somewhat higher than the transaction rates. This is not surprising, as bank customers can negotiate a lower rate based on loan characteristics. [45] The dashed vertical lines represent repo-rate reductions. Importantly, around these policy changes, the difference between listed rates and aggregate rates is stable, suggesting that the breakdown in pass-through documented in the event study is not an artifact of the listed rates.

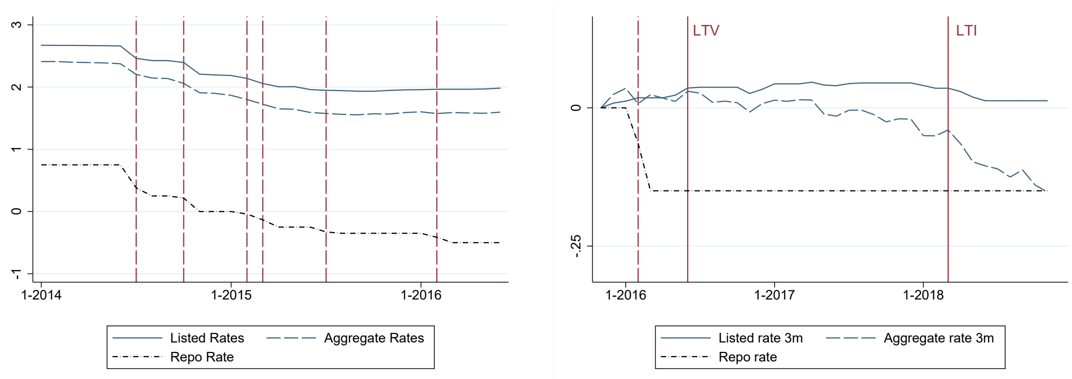


Figure B.1: Listed rates and aggregate rates (left panel) and cumulative changes (right panel).

Notes: This figure shows listed and aggregate rates (left panel) and the cumulative changes in listed and aggregate rates (right panel). Cumulative change calculated since November 2015. Dashed vertical lines correspond to repo-rate reduction dates. Solid vertical lines in the right panel correspond to June 2016 and March 2018, when mortgage market regulation was introduced.

Erikson and Vestin (2019) point out that there was some reduction in transaction rates in 2017 and 2018, and suggest that this might reflect a delayed pass-through from the repo-rate reductions in 2015-2016. While we cannot rule out that there was some long-term pass-through, we find it likely that the reduction in aggregate rates was connected to the introduction of new mortgage market regulation.

We follow Erikson and Vestin (2019) and plot the cumulative change in interest rates since November 2015 in the right panel of Figure B.1. The last repo-rate reduction is captured by the dashed vertical line. Note that there is no pass-through to either listed rates or official transaction rates in the months following the repo-rate cut. Over time however, there is a divergence between listed rates and transaction rates. As a result, by the end of 2018 – almost three years after the last repo-rate reduction – there appears to have been full pass-through aggregate rates, while listed rates remain unchanged. What explains this discrepancy?

As documented in the event study, normally full pass-through – or close to full pass-through – is achieved within a period of thirty days. Full pass-through within a period of two/three years would therefore entail a severe delay of the effectiveness of monetary policy. This could perhaps come about due to a break-down of a

[45]For example, the Swedish bank Nordea writes alongside their listed rate that “The mortgage rate we offer is an individual offer. Your mortgage rate can therefore turn out lower than the rate we list here.”.

collusive equilibrium, in which banks initially "agreed" to keep lending rates constant and then eventually deviated from this equilibrium. Note however, that the reduction in lending rates is *not* visible in listed rates, which is the interest rates that banks post on their websites and use to attract customers. It thus seems unlikely that banks would deviate from a collusive equilibrium in order to gain market shares, *without* this being observable in the listed rates.

What, then, can explain the fall in transaction rates? We believe mortgage market regulation which changed the average leverage of borrowers is a very likely explanation, supported by data. During this period, the Swedish Financial Authority implemented a new amortization requirement that encouraged lower loan-to-value (LTV) and loan-to-income (LTI) ratios. The reduction in average leverage likely enabled more households to negotiate a lower interest rates than the listed rates. These regulations were implemented in June 2016 and March 2018 - see the two vertical lines in Figure B.1. As shown in Figure B.1, the new policies introduced coincide almost perfectly with the divergence between listed rates and aggregate rates. Moreover, we have studied the effect of a similar policy change in 2010 to evaluate whether the same pattern of divergence was observed then. As shown in Figure B.2, there was a similar divergence following the new regulation in 2010. Quantitatively, the 2010-regulation reduced average LTV-ratios on new mortgages by 2 percentage points, which led to a fall in transaction rates relative to listed rates of 0.17 percentage points. The 2016-regulation led to a reduction in average LTV-ratios of 1 percentage points, i.e. half of the effect seen from the 2010-regulation. Given that the divergence between listed rates and transaction rates is due to macroprudential regulation, we would therefore expect a relative decline in transaction rates of 0.17/2 = 0.085 percentage points following the 2016 regulation - which is almost exactly what we observe in the data (0.09).[46] Hence, our assessment is that this empirical evidence strongly indicates that the reduction in aggregate lending rates two-three years after the last repo-rate cut was driven by regulatory measures, not by a delayed response to the repo-rate cuts themselves.

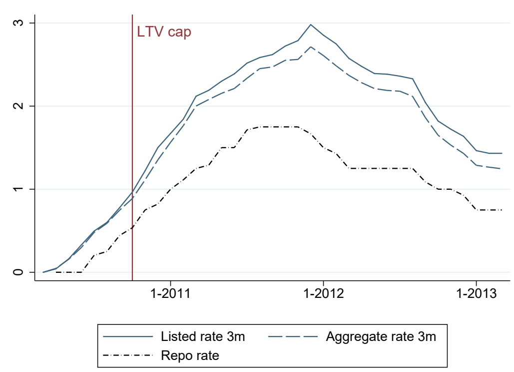


Figure B.2: Listed rates and aggregate rates – 3 months. Cumulated change since March 2010.

[46]We consider a time period of 21 months after the regulation is introduced both for the 2010 and the 2016 regulation. This maximizes the time period considered, while excluding the following loan-to-income regulation introduced in 2018 (see Figure B.1)

C Implementing the sufficient statistic

The practical implementation of the sufficient statistic developed in equation (46) depends on whether or not negative interest rates have been implemented. Here we first discuss how the sufficient statistic can be used in countries in which negative policy rates have already been used, with Sweden as our example. Secondly, we also discuss how the sufficient statistic can be useful ex ante, i.e. in countries in which negative interest rate policies have not (yet) been implemented.

Several bank balance sheet items are needed in order to evaluate equation (46). Specifically, one needs to know total assets, reserves (and cash), lending, total liabilities, deposits and equity. These items should be easily accessible in most countries. Moreover, the balance sheet ratios are relatively slow-moving variables, meaning that the sensitivity as to when they are measured should generally be relatively low. They could be measured during the period of negative interest rates, or if the country in question has not implemented negative interest rates, at the time of the analysis.

The pass-through coefficients at negative policy rates, ρ^a and ρ^f are more complicated to measure. The former captures the average pass-through to $\bar{a} \equiv$ assets $-$ reserves $-$ lending once the DLB is met, while the latter captures the average pass-through to $\bar{f} \equiv$ total assets $-$ deposits $-$ equity. Moreover, they are potentially much more fast-moving than the balance sheet ratios, and could very well be time-varying also during the period of negative policy rates. In Sweden, the policy rate never dipped below -0.5 %, implying that we can only hope to measure ρ^a and ρ^f at modestly negative interest rates. Had policy rates continued to plunge into negative territory, the pass-through coefficients may very well have changed. This would be important to keep in mind if contemplating policy rates below what we have so far seen in use.

In Sweden, the largest asset groups after lending to the general public (and central bank reserves) is lending to other credit institutions and bonds. The former is assumed to follow the money market rate, while the latter is assumed to follow government bond rates. The remaining assets consist of stocks, intangibles and "other assets". Because we do not know which interest rates to use for these items, and because they account for only a very small share of total assets, we disregard them in calculating the pass-through coefficient. As shown in Figure A.5, the pass-through to money market rates and government bond rates (of all maturities) remains high in the post-DLB period. We therefore assume for the Swedish case that $\rho^a \approx 1$. For implementation in other countries, one would want to know whether $\hat{a}$ is relatively well captured by money market instruments and government bonds, and whether the pass-through to these assets has remained high.

On the liability side, the largest items after deposits (and equity) are liabilities to credit institutions and issued securities. The former is assumed to follow money market rates, while the latter is assumed to follow covered bond rates. Because we do not know the break-down of the maturity structure of covered bonds, and because covered bonds have maturities of either 2 or 5 years, we simply use an unweighted average of the 2 and 5 year covered bond rates. In total, issued securities are about twice as large as liabilities to credit institutions. Based on this, we define a synthetic interest rate $i^f = \frac{1}{3} \times$ money market rate + $\frac{1}{3} \times$ 2y covered bond rate + $\frac{1}{3} \times$ 5y covered bond rate. We then compute the change in i^f relative to the change in the policy rate over the period from negative interest rates were implemented until the end of 2018, which gives us $\rho^f = 0.42$. Because i^f is somewhat volatile this calculation is sensitive to the end-point. However, if we instead use the average value of i^f during the period in which the policy rate was at it's minimum, i.e. -0.5 %, as the endpoint, we get a relatively similar pass-through coefficient of $\rho^f = 0.46$. Calculating ρ^f in other countries would require knowledge of the liability structure of the bank sector, as well as the relevant interest rates.

How should the sufficient statistic be used in countries where negative interest rate have not (yet) been implemented? Admittedly, the sufficient statistic is backward-looking in nature, in that it requires one to have observed the pass-through to different interest rates *after* the implementation of negative policy rates. However, with some additional assumptions, the sufficient statistic could potentially be useful also ex ante. For instance, if one is willing to assume full pass-through to $\bar{a}$ (as we do in the Swedish case, and supported by the full pass-through to government bond stipulated in among others (Brandao-Marques et al., 2021)), the condition in equation (46) can be rewritten so that negative interest rates will stimulate lending if $\rho^f > \frac{\bar{r}+\bar{a}}{\bar{f}}$. The sufficient statistic can then tell us, given the balance sheet structure of the bank sector in question, how high does the pass-through to non-deposit liabilities have to be in order for negative policy rates to stimulate lending. If $\frac{\bar{r}+\bar{a}}{\bar{f}}$ is close to one (due to for instance a high deposit share), negative interest rates will require a much higher pass-through to $\bar{f}$ than if $\frac{\bar{r}+\bar{a}}{\bar{f}}$ is close to zero.

D Additional details on the theoretical analysis

D.1 Partial equilibrium model

The bank holds liquid assets (L_t) due to liquidity risk. The banking literature typically emphasizes that small depositors (D_t) are a stable form of financing and thus not subject to large liquidity risk. The key source of liquidity risk, instead, is external financing and large retail depositors (F_t). To cost of liquidity risk is captured in reduced form by the function $C(.)$ (see Freixas and Rochet (2008) for micro foundations of a cost function that captures liquidity risk). A bank has a larger motive to hold liquid assets the more it relies on F_t for financing, due to its higher liquidity risk. Moreover, liquidity risk is weakly decreasing in net worth. For given values of N_t and F_t, the bank is satiated in liquidity for some finite value of L_t, denoted L^*. The following functional form captures these assumptions:

$$C(F_t, L_t, N_t) = \begin{cases} \lambda_c F_t^{\gamma_f} L_t^{-\gamma_l} N_t^{-\gamma_n} \text{ if } L_t < L^*(N_t, F_t) \\ C^* \text{ if } L_t \geq L^*(N_t, F_t) \end{cases}$$

where $\gamma_f, \gamma_l, \gamma_n \geq 0$ are the elasticities of the banks liquidity costs with respect to external financing, liquid assets and net worth respectively. The following restriction on the parameters is imposed

$$\gamma_f > 1 + \gamma_l \tag{D.1.1}$$

which implies a lower bound on the elasticity of liquidity costs with respect to external financing. This condition ensures that the maximization problem of the bank is well behaved.

While all liquid assets reduce liquidity risk, reserves and paper currency contribute to reducing banks operational costs due their special "money role," e.g. in settling inter-bank transaction. Bank inter-mediation costs are decreasing in R and M up to a satiation point. The following functional form captures this assumption

$$\Psi(M_t, R_t) = \begin{cases} \lambda_R (M_t + R_t)^{-\gamma_R} \text{if} M_t + R_t < R^* \\ \bar{\Psi} \text{ if } M_t + R_t \geq R^* \end{cases}$$

where $\gamma_R > 0$ measures the elasticity of bank transaction costs with respect to R+M. For simplicity, cash and reserves play the same role in facilitating transactions. The difference between the two assets is that bank reserves pay an interest of i_t^r, while paper currency pays zero interest. This implies that the bank does not choose to hold currency while the interest on reserves is positive.

Holding money entails a storage cost, which is convex in money if $M_t > 0$:

$$S(M_t) = \lambda_{MS}(M_0 + M_t)^{\gamma_{MS}} \tag{D.1.2}$$

where $\gamma_{ms} > 1$. The parameter M_0 represents a fixed cost. Once rates turn negative, equation ($D.1.2$) determines how much cash the banks will hold.

The function $\Gamma(B_t, N_t)$ captures the cost of lending. It can be rationalized as being due to default, which becomes more pronounced the higher the lending due to finite monitoring resources, see e.g. Curdia and Woodford (2011) for a discussion of this interpretation. Both the total cost of lending and the marginal cost of lending are assumed to decrease in net worth.

The intermediation function takes the form

$$\Gamma(B, N) = \lambda_B B^{\nu} N^{-\iota}$$

where $1 \leq \nu < \bar{\nu}$ measures the elasticity of intermediation costs to lending, and $0 < \iota < \bar{\iota}$ measures the elasticity with respect to net worth, and

$$\bar{\nu} \equiv (\frac{i^b - i^d}{1 + i^d})\frac{\bar{N}}{\bar{\Gamma}}, \ \bar{\iota} \equiv \frac{1}{1 + i^d}\frac{\bar{N}}{\bar{\Gamma}} \tag{D.1.3}$$

The upper bounds on $\bar{\nu}$ and $\bar{\iota}$ ensure the existence of a bounded solution to the banks problem.[47]

Provided that $V'(N_t) > 0$ the first order conditions are:

$$\begin{aligned}
B_t : \frac{i_t^b - i_t^d}{1 + i_t^d} &= \Gamma_B(B_t, N_t) && \text{(D.1.4)} \\
R_t : \frac{i_t^r - i_t^d}{1 + i_t^d} &= C_L(F_t, L_t, N_t) + \Psi_R(R_t, M_t) && \text{(D.1.5)} \\
F_t : \frac{i_t^f - i_t^d}{1 + i_t^d} &= -C_F(F_t, L_t, N_t) && \text{(D.1.6)} \\
A_t : \frac{i_t^a - i_t^d}{1 + i_t^d} &= C_L(F_t, L_t, N_t) && \text{(D.1.7)} \\
N_t : \delta\omega - (1 - \omega)\delta E_t \phi_{t+1} - \phi_t \left[\frac{1}{1 + i_t^d} + C_N + \Gamma_N\right] &= 0 && \text{(D.1.8)} \\
M_t : \frac{-i_t^d}{1 + i_t^d} &= S_M + C_L + \Psi_M - \psi_t / \phi_t && \text{(D.1.9)} \\
M_t \geq 0, \psi_t \geq 0, \psi_t M_t &= 0 && \text{(D.1.10)}
\end{aligned}$$

[47]The ratio $\frac{N}{\Gamma}$ is net worth as a fraction of the bank lending intermediation costs. Empirically this is a large number and this restriction does not impose a tight bound on the functional form assumed.

where ϕ_t is the Lagrange multiplier of equation (11) and denotes the marginal value of net worth at time t to the bank, and the Envelope has been used to substitute out for the value function in the first order condition for N_t. Condition (D.1.9) is the money demand equation, and ψ_t is the Lagrange multiplier on $M_t \geq 0$. Condition (D.1.10) is a Kuhn-Tucker complementary slackness condition. A partial equilibrium is defined by an exogenous set of interest rates $\{i_t^d, i_t^b, i_t^r, i_t^f, i_t^a\}$ taken as given by the banks, and values for $\{B_t, M_t, N_t, F_t, R_t, A_t, \phi_t, \psi_t\}$ that solve equations (D.1.4)-(D.1.10), together with the flow budget constraint in equation (11).

D.2 Proof of the PLB

Proposition 2 (The Policy Rate Bound) If the cost of using paper currency is given by equation (14), the lower bound on the policy rate is

$$i_t^r \geq i^{PLB} \equiv -\alpha^m \tag{D.B.1}$$

Proof: Combine (D.1.5) and (D.1.9), noting that $\Psi_R = \Psi_M$, and assuming the DLB binding so that $i_d = 0$ to yield

$$\psi_t = S_M + i_t^r = \alpha^M + i_t^r \geq 0 \tag{D.B.2}$$

where the last inequality follows from the complementary slackness condition (D.1.10).

D.3 Effect on Asset Holding and External Financing

The model has implications for how banks adjust their liquid asset holding and external financing in response to negative policy rates. A somewhat surprising empirical finding in Section 2 was that banks did not increase their reliance on non-deposit financing despite substantially larger pass-through to these interest rates. Below we explain this in the context of the model. Let us first summarize a useful observation.

Proposition 5. *If $\rho^d = 0$ and $i_t^r < 0$, the bank is not satiated in liquidity and $L_t < L^*$.*

The proposition follows directly from equations (D.1.5)-(D.1.7). An immediate observation from inspecting (D.1.6) and (D.1.7) is that if there is greater pass-through of policy rates to the interest rate on assets and external financing, then there is a positive spread between $i_t^a - i_t^d$ as well as $i_t^f - i_t^d$. This, in turn, implies that the bank will no longer be satiated in the liquid asset, $C_L > 0$ so that $L_t < L^*$.

The same is not true for reserves as seen from equation (D.1.5). It is possible that the bank is satiated in reserves to settle interbank transactions, i.e. $R_t \geq R^*$ so that $\chi_R = 0$, but that there is still a spread between i_t^r and i_t^d because reserves also serve a role to reduce liquidity risk via the function $C(.)$. Let us now discuss the implications of negative policy rates on external financing, reserves and other assets in the approximated equilibrium.

External financing Using the result from Proposition 5 that $L_t < L^*$, a log-linear approximation of the banks asset demand in equation (D.1.7), can be combined with a log-linear approximation of its demand for external financing in equation (D.1.6) to yield:

$$\hat{F}_t = \frac{(1+\gamma_l)(\rho^d - \rho^f)\frac{\bar{F}}{\bar{C}} + \gamma_f \frac{\bar{L}}{\bar{C}}(\rho^a - \rho^d)}{\gamma_f^2 - \gamma_f(1+\gamma_l)} \hat{\imath}_t^r + \frac{\gamma_f \gamma_n}{\gamma_f^2 - \gamma_f(1+\gamma_l)} \hat{N}_t \tag{D.3.1}$$

where the denominator is positive and the solution for $\hat{N}_t$ is given by equation (17).

Equation (*D*.3.1) has a useful interpretation. If there is full pass-through, then external financing only increases and decreases with the bank's net worth. To the extent that negative rates have negative effects on banks equity, then, this reduces the banks reliance on external financing.

Consider the implication of a collapse in pass-through to the deposit rate, i.e. $\rho^d = 0$. Then the first term says that banks will increase their reliance on external financing if

$$\frac{\gamma_f}{1+\gamma_l} \frac{\bar{L}}{\bar{F}} < \frac{\rho^f}{\rho^a} \tag{D.3.2}$$

Hence, even if the policy rate is fully passed through to the external financing rate, the bank might still choose not to rely more heavily on this funding source. To interpret this conditions, observe that the interest rate the bank pays for external financing is only one part of the cost to the bank. The other is that it needs to hold more liquid assets due to the higher liquidity risk which this source of funding generates. If there is incomplete pass-through to the external financing, while there is full pass-through to the liquid assets, then the bank may reduce its reliance on external financing once rates turn negative.

Since the pass-through in Sweden to liquid asset was stronger than to the bonds the banks could issue, this might rationalize why they did not rely more strongly on external financing in response to negative rates.

The determination of reserves Suppose that $R_t < R^*$. Combining the log-linear approximation of the the banks demand for assets and its demand for reserves yields

$$\hat{R}_t = \frac{1-\rho^a}{\gamma_R(\gamma_R+1)\frac{\bar{\Psi}}{\bar{R}}} \hat{\imath}_t^r \tag{D.3.3}$$

This suggests that if there is incomplete pass-through to liquid assets, then banks reduces their reserves when the policy rate is lowered. The reason is that reserves pay a lower interest rate than the alternative liquid assets.

In partial equilibrium, $\hat{R}_t$ measures the reserve holdings of a single bank. Once the model is integrated into general equilibrium however, the government sets both the interest on reserves, $\hat{\imath}_t^r$, *and the quantity* $\hat{R}_t$. This deserves further discussion.

Reserves measure the money balances a given bank holds on an account at the central bank. What can it do with reserves? One option is to exchange them for paper currency from the central bank. We have so far made the assumption that the storage cost of money is large enough so that the banks choose not to hold any cash, but we will revisit this assumption shortly. Another option is to use reserves to buy a liquid assets. An important observation, however, is that while this reduces the reserves of the bank, *it leaves aggregate reserves unchanged.*

Reserves and liquid assets given a negative reserve rate A natural question is what happens if the central bank chooses a negative interest on reserves when $\rho^a < 1$. As equation ($D.3.3$) suggests, this means that any single bank (in partial equilibrium) tries to reduce its reserve holdings. Yet, in general equilibrium, total reserves are pinned down by the government. How is an equilibrium ensured?

Equation ($D.3.3$) suggests that the only way $R_t \geq R^*$ can be consistent with general equilibrium is that there is full pass-through to the liquid assets, i.e. $\rho^a = 1$. In this case, all banks remain fully satiated in reserves for the purpose of interbank transactions once the rate turns negative. Yet, as suggested by Proposition 5, once rates turn negative, banks are not satiated in liquid assets. Since reserve and other liquid asset are perfect substitutes, the partial equilibrium model we have specified will only pin down the quantity of total liquid asset, not its composition. Solving the log-linear approximation of equations (D.1.5) and (D.1.7) together then yields

$$\hat{L}_t = \frac{\bar{C}}{\bar{L}} \frac{1}{(1+\gamma_l)\gamma_l} \hat{\imath}^r_t + \frac{\gamma_f}{1+\gamma_l} \hat{F}_t - \frac{\gamma_n}{1+\gamma_l} \hat{N}_t \tag{D.3.4}$$

where the solution for $\hat{F}_t$ is given by equation ($D.3.1$) and $\hat{N}_t$ by equation (17). This relationship suggests that negative rates do not have clear predictions for what happens to aggregate liquid assets, it depends on the effect it has on external financing and net worth. Meanwhile, from the perspective of the banks, the split of liquid assets between reserves and other assets is indeterminate, and ultimately depends on the choices made by the central bank which can choose both $\hat{\imath}^r_t$ and $\hat{R}_t$.

Reserves and money So far we have not considered the implication of banks' option to convert reserves into paper currency. For simplicity, we assumed that the bank held no currency because it served the same role as reserves, which in contrast to cash paid an interest, and moreover had no storage cost.

Once interest on reserves turn negative, however, converting reserves into cash can be an attractive option for the bank. Consider an equilibrium in which interest rates on reserves are sufficiently negative so that $M_t > 0$ and assume that there is no pass-through to deposit rates, so that $\rho^d = 0$. In this case $\psi_t = 0$ in equation (D.1.10) and the log-linear approximation of the banks demand for money is

$$\hat{M}_t = -\frac{1}{\gamma_{ms}(\gamma_{ms}-1)} \frac{M^*}{\bar{S}} \hat{\imath}^r_t + \gamma_R(\gamma_R+1) \frac{\bar{\chi}}{\bar{R}} \frac{M^*}{\bar{S}} \hat{R}_t$$

which suggests that as the reserve rate turns negative, banks increase cash holdings. since cash offers a gross return of zero, although entailing a storage cost. Overall, the experience so far has been that banks have not to a significant extent moved into cash, indicative of a large value of γ_{ms}.

An important assumption is the convexity of storage costs, i.e. $\gamma_{ms} > 1$. One might argue, however, that there are constant or increasing returns to building storage facilities for money, at least when cash holding are sufficiently large. Consider, for example, the assumption of constant returns, in which $\gamma_{ms} = 1$, so that cost of storage is proportional to the supply of money the bank holds for any further cash holding, i.e. $\gamma_{ms} M_t$. In this case, provided that the bank holds money, the interest rate on reserves is bounded by $i^r_t \geq -\gamma_{ms}$ as in the example we considered in the main text, since if it charges more negative interest on reserves than γ_{ms}, banks will convert reserves into paper currency and interbank transactions will be settled outside of the central bank.

The assumption that money is as effective as reserves in settling interbank transactions might seem a bit extreme. Nevertheless, it does not seem unreasonable to think that if the central bank charges very negative rates, banks might find it in their interest to exchange reserves for currency, and instead construct an alternative payment system that solves the same problem at a lower cost.

D.4 General equilibrium model

We write real variables as lower case letters of their nominal counterparts. The following defines the non-linear equilibrium of our model:

Definition D.1 (Non-linear equilibrium)**.** A non-linear equilibrium of our model is a sequence of 23 endogenous variables $\left\{y_t,\ \pi_t,\ c_t^b,\ c_t^s, m_t^b,\ m_t^s,\ b_t^b,\ b_t,\ r_t,\ a_t,\ m_t\ ,\psi_t, f_t, l_t, n_t\ , z_t, F_t,\ K_t,\ \lambda_t,\ \Delta_t, \overline{W}, T_t, \overline{f}\right\}_{t=0}^{\infty}$ and 5 endogenous prices $\left\{i_t^b,\ i_t^d, i_t^f\ , i_t^g\ i_t^r\right\}_{t=0}^{\infty}$ such that equations (D.4.1) - (D.4.28) holds.

From the household problems there is a consumption Euler equation for the borrower (D.4.1) and the saver (D.4.2), respectively, the budget constraint of the borrower (D.4.3), the money demand equations for each type of agent (D.4.4) and (D.4.5). The aggregate resource constraint (D.4.19) implies all production is consumed. b_t^b is defined as aggregate borrowing per borrower, while $b_t \equiv \chi b_t^b$ is aggregate borrowing (D.4.18).

The banks problem gives rise to (D.4.6)-(D.4.13). Relative to the text, the price level is now endogenous.

$$u'(c_t^b)\zeta_t = \beta_b(1+i_t^b)E_t u'(c_{t+1}^b)\Pi_{t+1}^{-1}\zeta_{t+1} \tag{D.4.1}$$

$$u'(C_t^s)\zeta_t = \beta_s(1+i_t^d)E_t u'(c_{t+1}^s)\Pi_{t+1}^{-1}\zeta_{t+1} \tag{D.4.2}$$

$$b_t^b = (1+i_{t-1}^b)\Pi_t^{-1}b_{t-1}^b - y_t + c_t^b - (1-\gamma^b)\Pi_t^{-1}m_{t-1}^s + m_t^s \tag{D.4.3}$$

$$\frac{\Omega'\left(m_t^s\right)}{u'(c_t^s)} = \frac{i_t^d}{1+i_t^d} \tag{D.4.4}$$

$$\frac{\Omega'\left(m_t^s\right)}{u'(c_t^b)} = \frac{i_t^b}{1+i_t^b} \tag{D.4.5}$$

$$\frac{i_t^b - i_t^d}{1+i_t^d} = \Gamma_b(b_t, n_t) \tag{D.4.6}$$

$$\frac{i_t^r - i_t^d}{1+i_t^d} = C_l(f_t, l_t, n_t) + \chi_m(r_t, m_t) \tag{D.4.7}$$

$$\frac{i_t^a - i_t^d}{1+i_t^d} = C_l(f_t, l_t, n_t) \tag{D.4.8}$$

$$\frac{i_t^f - i_d^r}{1+i_t^d} = -C_f\left(f_t, l_t, n_t\right) \tag{D.4.9}$$

$$-\frac{i_t^d}{1+i_t^d} = S_m(m_t) + C_l(ft, l_t, n_t) + \chi_m(r_t, m_t) - \psi_t/\phi_t \tag{D.4.10}$$

$$\psi_t M_t = 0 \tag{D.4.11}$$

$$n_t = \left(1+i_t^d\right)\left(z_t + (1-\omega)\,\pi_t^{-1}n_{t-1}\right) \tag{D.4.12}$$

$$z_t = \frac{i_t^b - i_t^d}{1+i_t^d}b_t + \frac{i_t^r - i_t^d}{1+i_t^d}r_t + \frac{i_t^g - i_t^d}{1+i_t^d}a_t - \frac{i_t^d}{1+i_t^d}m_t - \frac{i_t^f - i_t^d}{1+i_t^d}f_t - C(f_t, l_t, n_t) - \Gamma\left(b_t, n_t\right) - \chi(r_t, m_t) \tag{D.4.13}$$

$$i_t^r = r_t^n \Pi_t^{\phi_\Pi} y_t^{\phi_Y} \tag{D.4.14}$$

$$i_t^f = \rho^f i_t^r \tag{D.4.15}$$

$$i_t^g = \rho^a i_t^r \tag{D.4.16}$$

$$i_t^d = max\left\{-\gamma^s,\ i_t^r\right\} \tag{D.4.17}$$

$$\chi b_t^b = b_t \tag{D.4.18}$$

$$y_t = \chi c_t^b + (1-\chi)c_t^s \tag{D.4.19}$$

(D.4.20)

$$a_t + m_t + r_t = (1+i_t^a)a_{t-1} + m_{t-1} + (1+i_{t-1}^r)r_{t-1} + G - T_t \tag{D.4.21}$$

$$a_t + m_t + r_t = \bar{W} \tag{D.4.22}$$

$$f_t = \bar{f} \tag{D.4.23}$$

$$\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{1}{\theta-1}} = \frac{F_t}{K_t} \tag{D.4.24}$$

$$F_t = \lambda_t y_t + \alpha\beta E_t\left[F_{t+1}\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}\right] \tag{D.4.25}$$

$$K_t = \mu\frac{\lambda_t \Delta_t^{\eta} y_t^{1+\eta}}{q\ \exp\{-q y_t\}} + \alpha\beta E_t\left[K_{t+1}\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}\right] \tag{D.4.26}$$

$$\lambda_t = q\left(\chi\exp\{-q c_t^b\} + (1-\chi)\exp\{-q c_t^s\}\right) \tag{D.4.27}$$

$$\Delta_t = \alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta}\Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta}{\theta-1}} \tag{D.4.28}$$

D.5 Steady state

The model is approximated around a steady state when inflation is at the target level. We consider the cashless limit, in which neither the household nor the banks hold money. However, we assume that they in principle *can* hold money, which gives rise to the the bound on deposits as explained in the text.

In steady-state, $\Delta = 1$ and $\bar{F} = \bar{K}$, which implies that production is pinned down by the AS curve.

$$\mu \times \frac{y^{\eta}}{q\exp\{-qY\}} = 1 \tag{D.5.1}$$

The consumption Euler equations (D.4.2) and (D.4.2) of the saver and borrowers imply that the inverse of the discount factor of each agent is equal to the real borrowing and saving rate

$$(\beta^b)^{-1} = \frac{1+\bar{\imath}^b}{\bar{\Pi}} \tag{D.5.2}$$

and

$$(\beta^s)^{-1} = \frac{1+\bar{\imath}^s}{\bar{\Pi}} \tag{D.5.3}$$

In steady-state, we assume $\rho^f = 1$, $\rho^a = 1$ which implies that $\bar{\imath}^f = \bar{\imath}^g = \bar{\imath}^r$ where $i_t^r = r^n \Pi^{\phi_\Pi} y^{\phi_Y}$

The budget constraint (D.4.3), together with that in steady state implies

$$\frac{\bar{c}^b}{\bar{y}} = 1 - \frac{1-\beta^b}{\beta^b}\frac{1}{\chi}\bar{b} \tag{D.5.4}$$

which when combined with the resource constraint (D.4.19) implies that the steady state consumption of the saver is

$$\frac{\bar{c}^s}{\bar{y}} = 1 + \frac{1}{1-\chi}\frac{1-\beta^b}{\beta^b}\bar{b} \tag{D.5.5}$$

We directly choose the steady-state output debt $\frac{\bar{b}}{\bar{y}}$ from the data, in which case the two equations above pin down $\frac{\bar{c}^b}{\bar{y}}$ and $\frac{\bar{c}^s}{\bar{y}}$ for a given χ. Neither ratio, however, enters the linear approximation of the model shown in next subsection.

The first-order condition for lending (D.1.4) implies that

$$\frac{\bar{i}^b - \bar{i}^d}{1+\bar{i}^b} = 1 - \frac{\beta^b}{\beta^s} = \nu \frac{\bar{\Gamma}}{\bar{b}} \tag{D.5.6}$$

which, given $\frac{\bar{b}}{\bar{y}}$ and ν pins down

$$\frac{\bar{\Gamma}}{\bar{y}} = \frac{1 - \frac{\beta^b}{\beta^s}}{\nu \frac{\bar{y}}{\bar{b}}} \tag{D.5.7}$$

Steady state profit is then

$$\bar{z} = (1 - \frac{\beta^b}{\beta^s})\bar{b} - \bar{C} - \bar{\Gamma} - \bar{\chi} \tag{D.5.8}$$

Steady state net worth is

$$\bar{n} = (1 + i^d)(\bar{z} + (1-\omega)n\pi^{-1}) \tag{D.5.9}$$

or

$$\frac{\bar{z}}{\bar{n}} = \beta^s \pi^{-1} - (1-\omega)\pi^{-1} = \frac{\beta^s + \omega - 1}{\pi} \tag{D.5.10}$$

Finally, we directly pick the steady state values of $\bar{r}$, $\bar{a}$ and $\bar{f}$ from the data. These do not affect the steady state values of the variables described above, but are important for dynamics.

D.6 Log-linear (approximated) equilibrium

The model is solved via log-linear approximation. Variables are defined as: $\hat{c}_t^b \equiv \frac{c_t^b - \bar{c}^b}{\bar{y}}$, $\hat{c}_t^s \equiv \frac{c_t^s - \bar{c}^s}{\bar{y}}$, $y_t \equiv \log \frac{y_t}{\bar{y}}$, $\hat{\pi}_t \equiv \log \frac{\Pi_t}{\bar{\Pi}}$, $\hat{\imath}_t^d \equiv \log \frac{1+i_t^d}{1+\bar{\imath}^d}$, $\hat{\imath}_t^r \equiv \log \frac{1+i_t^r}{1+\bar{\imath}^r}$, $\hat{\imath}_t^b \equiv \log \frac{1+i_t^b}{1+\bar{\imath}^b}$, $\hat{b}_t \equiv \log \frac{b_t}{\bar{b}}$. Approximate equilibrium is a collection of stochastic processes for $\{\hat{c}_t^b, \hat{c}_t^s, \hat{\imath}_t^b, \hat{\imath}_t^d, \hat{\imath}_t^f, \hat{\imath}_t^r, \hat{\imath}_t^a, \hat{n}_t, \hat{y}_t, \hat{\pi}_t, \hat{b}_t, \hat{z}_t\}$ that solve equations (D.6.1)-(D.6.12)

Consumption Euler equations for the borrower (D.4.1) and the saver (D.4.2) yield

$$\hat{c}_t^b = E_t \hat{c}_{t+1}^b - \sigma(\hat{\imath}_t^b - E_t \hat{\pi}_{t+1} + \hat{\zeta}_t - E_t \hat{\zeta}_{t+1}) \tag{D.6.1}$$

$$\hat{c}_t^s = E_t \hat{c}_{t+1}^s - \sigma(\hat{\imath}_t^d - E_t \hat{\pi}_{t+1} + \hat{\zeta}_t - E_t \hat{\zeta}_{t+1}) \tag{D.6.2}$$

The budget constraint of the borrower (D.4.3) yields

$$\hat{b}_t = \frac{1}{\beta^b}\hat{b}_{t-1} - \frac{1}{\beta^b}\hat{\pi}_t + \frac{1}{\beta^b}\hat{\imath}_{t-1}^b - \chi \frac{\bar{y}}{\bar{b}}\hat{y}_t + \chi \frac{\bar{y}}{\bar{b}}\hat{c}_t^b \tag{D.6.3}$$

The lending condition (D.4.6) yields

$$\frac{1+i^b}{1+i^s}\left(\hat{\imath}_t^b-\hat{\imath}_t^s\right)=\nu\frac{\Gamma}{b}\left[(\nu-1)\,\hat{b}_t-\iota\hat{n}_t\right] \tag{D.6.4}$$

The net worth condition yields

$$\beta^s\hat{n}_t=\beta^s\hat{\imath}_t^d+\bar{\Pi}\hat{z}_t+(1-\omega)\left(\hat{n}_{t-1}-\hat{\pi}_t\right) \tag{D.6.5}$$

The definition of profits yields

$$\frac{\bar{n}}{\bar{\Lambda}}\hat{z}_t=-\{\frac{\beta^b}{\beta^s}\frac{\bar{b}}{\bar{\Lambda}}+\frac{\bar{r}}{\bar{\Lambda}}+\frac{\bar{a}}{\bar{\Lambda}}-\frac{\bar{f}}{\bar{\Lambda}}\}\hat{\imath}_t^d+\frac{\beta^b}{\beta^s}\frac{\bar{b}}{\bar{\Lambda}}\hat{\imath}_t^b+\frac{\bar{r}}{\bar{\Lambda}}\hat{\imath}_t^r+\frac{\bar{a}}{\bar{\Lambda}}\hat{\imath}_t^a-\frac{\bar{f}}{\bar{\Lambda}}\hat{\imath}_t^f+\iota\frac{\bar{\Gamma}}{\bar{\Lambda}}\hat{n}_t \tag{D.6.6}$$

The resource constraint yields

$$\hat{y}_t=\chi\hat{c}_t^b+(1-\chi)\hat{c}_t^s \tag{D.6.7}$$

The supply side yields a Phillips Curve

$$\hat{\pi}_t=\kappa\hat{Y}_t+\beta E_t\hat{\pi}_{t+1} \tag{D.6.8}$$

Policy rule:

$$\hat{\imath}_t^r=\hat{r}_t^n+\phi_\pi\hat{\pi}_t+\phi_y\hat{y}_t \tag{D.6.9}$$

Deposit bound:

$$\hat{\imath}_t^d=\max\{i^{DLB},\hat{\imath}_t^r\} \tag{D.6.10}$$

Pass-through to other rates:

$$\hat{\imath}_t^a=\rho^a\hat{\imath}_t^r \text{ if } \hat{\imath}_t^r<i^{DLB} \text{ and } \hat{\imath}_t^a=\hat{\imath}_t^r \text{ if } \hat{\imath}_t^r>i^{DLB} \tag{D.6.11}$$

$$\hat{\imath}_t^f=\rho^f\hat{\imath}_t^r \text{ if } \hat{\imath}_t^r<i^{DLB} \text{ and } \hat{\imath}_t^f=\hat{\imath}_t^r \text{ if } \hat{\imath}_t^r>i^{DLB} \tag{D.6.12}$$

Coefficient are defined as: $\sigma\equiv\frac{1}{\bar{q}\bar{y}}$, $\kappa\equiv\frac{(1-\alpha)(1-\alpha\beta)}{\alpha(\eta+\sigma^{-1})}$, $\beta\equiv\chi\beta^b+(1-\chi)\beta^s$, $i^{DLB}\equiv-\log(1+\bar{\imath}^d)$. Observe that once one chooses parameters for $(\sigma,\kappa,\beta.\beta^s,\beta^b,\chi,\nu,\iota)$ and pins from the data $(\frac{\bar{y}}{\bar{b}},\frac{\bar{b}}{\bar{\Lambda}},\frac{\bar{r}}{\bar{\Lambda}},\frac{\bar{a}}{\bar{\Lambda}},\frac{\bar{f}}{\bar{\Lambda}})$, then ω and $\frac{\bar{\Gamma}}{\bar{\Lambda}}$ are not free parameters but implied by the model equations.

The analytic characterization in the text for the IS equation is obtained by (1) combining the two consumption Euler equations with the resource constraint and (2) defining the natural rate of interest, $\hat{r}_t^n$, as the interest rate consistent with zero deviation of output from steady state and inflation on target in the absence of a lower bound.

D.7 Sensitivity to using N_{t-1} rather than N_t in $\Gamma(B_t, N)$

We assume that current net worth N_t is the relevant net worth measure in the intermediation cost function Γ in the main text. This is somewhat inconsistent with regulatory constraints, where current-period profits are not included in the measure of regulatory capital. Depending on the model setup, this assumption may or may not be innocuous, see Repullo (2020). To investigate whether this assumption is material for our main results, we here consider an alternative model where we write Γ as a function over lagged net worth N_{t-1} instead. In this case, the log-linearized model becomes

$$\hat{c}_t^b = E_t \hat{c}_{t+1}^b - \sigma(\hat{\imath}_t^b - E_t \hat{\pi}_{t+1} + \hat{\zeta}_t - E_t \hat{\zeta}_{t+1}) \tag{D.7.1}$$

$$\hat{c}_t^s = E_t \hat{c}_{t+1}^s - \sigma(\hat{\imath}_t^d - E_t \hat{\pi}_{t+1} + \hat{\zeta}_t - E_t \hat{\zeta}_{t+1}) \tag{D.7.2}$$

$$\hat{b}_t = \frac{1}{\beta^b}\hat{b}_{t-1} - \frac{1}{\beta^b}\hat{\pi}_t + \frac{1}{\beta^b}\hat{\imath}_{t-1}^b - \chi\frac{\bar{y}}{\bar{b}}\hat{y}_t + \chi\frac{\bar{y}}{\bar{b}}\hat{c}_t^b \tag{D.7.3}$$

$$\frac{1+i^b}{1+i^s}\left(\hat{\imath}_t^b - \hat{\imath}_t^s\right) = \nu\frac{\Gamma}{b}\left[(\nu-1)\,\hat{b}_t - \iota\hat{n}_{t-1}\right] \tag{D.7.4}$$

$$\beta^s \hat{n}_t = \beta^s \hat{\imath}_t^d + \bar{\Pi}\hat{z}_t + (1-\omega)\,(\hat{n}_{t-1} - \pi_t) \tag{D.7.5}$$

$$\frac{\bar{n}}{\bar{\Lambda}}\hat{z}_t = -\{\frac{\beta^b}{\beta^s}\frac{\bar{b}}{\bar{\Lambda}} + \frac{\bar{r}}{\bar{\Lambda}} + \frac{\bar{a}}{\bar{\Lambda}} - \frac{\bar{f}}{\bar{\Lambda}}\}\hat{\imath}_t^d + \frac{\beta^b}{\beta^s}\frac{\bar{b}}{\bar{\Lambda}}\hat{\imath}_t^b + \frac{\bar{r}}{\bar{\Lambda}}\hat{\imath}_t^r + \frac{\bar{a}}{\bar{\Lambda}}\hat{\imath}_t^a - \frac{\bar{f}}{\bar{\Lambda}}\hat{\imath}_t^f + \iota\frac{\bar{\Gamma}}{\bar{\Lambda}}\hat{n}_{t-1} \tag{D.7.6}$$

$$\hat{y}_t = \chi\hat{c}_t^b + (1-\chi)\hat{c}_t^s \tag{D.7.7}$$

$$\hat{\pi}_t = \kappa\hat{Y}_t + \beta E_t \hat{\pi}_{t+1} \tag{D.7.8}$$

$$\hat{\imath}_t^r = \hat{r}_t^n + \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t \tag{D.7.9}$$

$$\hat{\imath}_t^d = \max\{i^{DLB}, \hat{\imath}_t^r\} \tag{D.7.10}$$

$$\hat{\imath}_t^a = \rho^a \hat{\imath}_t^r \text{ if } \hat{\imath}_t^r < i^{DLB} \text{ and } \hat{\imath}_t^a = \hat{\imath}_t^r \text{ if } \hat{\imath}_t^r > i^{DLB} \tag{D.7.11}$$

$$\hat{\imath}_t^f = \rho^f \hat{\imath}_t^r \text{ if } \hat{\imath}_t^r < i^{DLB} \text{ and } \hat{\imath}_t^f = \hat{\imath}_t^r \text{ if } \hat{\imath}_t^r > i^{DLB} \tag{D.7.12}$$

In Figure D.7.1, we compare the evolution of output and inflation for a preference shock under the two alternative models. The first model is the baseline, identical to the model in the main text. The black dashed

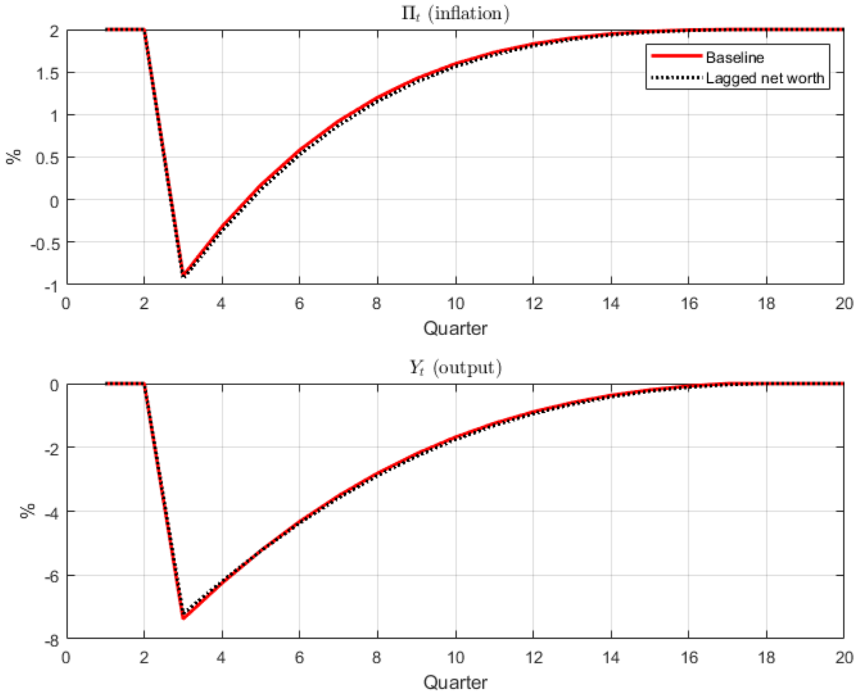


Figure D.7.1: Comparison of baseline model (red solid lines) and a model where we assume that lagged net worth is the relevant net worth measure in Γ (black, dashed lines

line reports the evolution of inflation and output in a model where we write Γ as a function of N_{t-1} rather than N_t. As highlighted by the figure, the overall dynamics are almost identical irrespective of whether we write $\Gamma(B_t, N_{t-1})$ rather than $\Gamma(B_t, N_t)$. The overall take-away from the theoretical analysis is therefore independent of the timing of N in Γ.

D.8 Dynamics of L and F

In this section, we outline the evolution of external financing F and liquid assets L for the preference shock considered in the text.

To do so, we first need to parameterize $C(F_t, L_t, N_t)$, i.e. we need to pin down γ_l, γ_f and γ_n. Moreover, from equations (D.3.4) and (D.3.1), we need to pin down $\frac{\overline{F}}{\overline{C}}$ and $\frac{\overline{L}}{\overline{C}}$.

First, note that $\frac{\overline{L}}{\overline{C}} = \frac{\overline{L}}{\overline{F}} \times \frac{\overline{F}}{\overline{C}}$, where $\frac{\overline{L}}{\overline{F}}$ is observable in the data, so this equation pins down $\frac{\overline{L}}{\overline{C}}$ given $\frac{\overline{F}}{\overline{C}}$. To pin down $\frac{\overline{F}}{\overline{C}}$, we follow Wang et al. (2020) who estimate the cost of non-deposit funding for banks. Specifically, they assume that the cost C associated with non-deposit funding F (our notation) is given by

$$\frac{C}{F} = i^r + 0.5 \times \phi \times \frac{F}{D} \tag{D.8.1}$$

where D is deposit funding. Their structural estimation implies that $\phi = 0.01$. In our setting, $\frac{F}{D} = 1.26$, with a policy rate (end of 2014) of 0. This yields a $\frac{C}{F} = 0.5 \times 0.01 \times 1.26 = 0.063$, and so $\frac{\overline{F}}{\overline{C}} = 15.87$. In our data, $\frac{\overline{L}}{\overline{F}} = 0.88$, and so $\frac{\overline{L}}{\overline{C}} = 0.88 \times 15.87 = 13.96$. These numbers imply that liquidity costs constitutes roughly 6.3 % of total non-deposit funding, and 7.1 % of total liquid assets.

Having pinned down these parameters, we are left with three unknown parameters: γ_l, γ_f and γ_n. We run a grid search over the grid $\mathcal{G} = \{0, 700\} \times \{0, 700\} \times \{0, 700\}$ with 20 equidistant nodes for each dimension to set $\left\{\gamma_l, \gamma_f, \gamma_n\right\}$ so that the evolution of external financing relative to total assets $\frac{F_t}{\Lambda_t}$ and liquid assets $\frac{L_t}{\Lambda_t}$ in our model roughly matches the evolution of these moments in the data. Specifically, we compute - in the model- the change in the ratio of external financing and liquid assets to total assets, respectively, for each percentage point the policy rate is cut below 0 and then compare that to our Swedish data.

Consistent with Wang et al. (2020), this exercise yields a $\gamma_n = 0$. In addition, it yields a $\gamma_f = 663$ and $\gamma_l = 21.9$. These coefficients generate an evolution of external financing and liquid assets that is approximately similar to the observed changes, as highlighted in Table 4. Importantly, this exercise illustrates that the calibrated model also matches the (untargeted) evolution of deposits very well. Specifically, while deposits relative to assets increased by 1.6 % in the data, it increases by 1.9 % in the model.[48] The estimated coefficients highlight that the marginal cost of adjusting external financing is quite high, which, in the context of our model, is necessary to rationalize why we did not see Swedish banks shift into external financing once the policy rate was cut at the DLB.

D.9 Frictional deposits

In the main text, we assumed that deposits are a frictionless source of financing. In this section, we consider two alternative set-ups. In Section D.9.1, we assume that there are adjustment costs associated with changing the stock of deposits. In Section D.9.2, we assume that banks engage in monopolistic competition in deposit markets, in line with for instance Drechsler et al. (2017).

D.9.1 Reduced-form adjustment cost of deposits

We start by assuming adjustment costs associated with changing the stock of deposits. Specifically, we assume that deposits now enter as an argument in a new $\tilde{C}$ function, i.e. we can write $C\left(a_t + r_t, f_t\right)$ and $\tilde{C}\left(d_t\right)$. Consider the cashless limit for the bank ($m_t = 0$). In this case, the FOC from the bank's problem becomes

[48] With a finer grid, we should in principle be able to match the data exactly. The point of this exercise is not to match the data exactly, but rather show that the model can generate changes in external financing and liquid assets approximately similar to what is observed empirically.

$$b_t: \frac{i_t^b - i_t^d}{1+i_t^d} = \Gamma_b(b_t, n_t) + \tilde{C}'(d_t) \tag{D.9.1.1}$$

$$r_t: \frac{i_t^r - i_t^d}{1+i_t^d} = C_l(a_t + r_t, f_t) + \chi'(r_t) + \tilde{C}'(d_t) \tag{D.9.1.2}$$

$$a_t: \frac{i_t^a - i_t^d}{1+i_t^d} = C_l(a_t + r_t, f_t) + \tilde{C}'(d_t) \tag{D.9.1.3}$$

$$f_t: \frac{i_t^d - i_t^f}{1+i_t^d} = C_f(a_t + r_t, f_t) - \tilde{C}'(d_t) \tag{D.9.1.4}$$

Bank profits is given by

$$z_t = \frac{i_t^b - i_t^d}{1+i_t^d} b_t + \frac{i_t^r - i_t^d}{1+i_t^d} r_t + \frac{i_t^a - i_t^d}{1+i_t^d} a_t - \frac{i_t^f - i_t^d}{1+i_t^d} f_t - C(r_t + a_t, f_t) - \tilde{C}(b_t + r_t + a_t - f_t) - \chi(r_t) - \Gamma(b_t, n_t) \tag{D.9.1.5}$$

while the law of motion for equity and the balance sheet equation is as before, i.e.

$$\frac{1}{1+i_t^d} n_t = (1-\omega) n_{t-1} \Pi_t^{-1} + z_t \tag{D.9.1.6}$$

and

$$b_t + r_t + a_t = d_t + f_t + n_t \tag{D.9.1.7}$$

Steady state

$$\frac{i^b - i^d}{1+i^d} = \Gamma_b(b, n) + \tilde{C}'(d_t) \tag{D.9.1.8}$$

$$\frac{i^r - i^d}{1+i^d} = C_l(a + r, f, n) + \chi'(r) + \tilde{C}'(d) \tag{D.9.1.9}$$

$$\frac{i^a - i^d}{1+i^d} = C_l(a + r, f, n) + \tilde{C}'(d) \tag{D.9.1.10}$$

$$\frac{i^d - i^f}{1+i^d} = C_f(a + r, f, n) - \tilde{C}'(d) \tag{D.9.1.11}$$

We assume that $i^a = i^d = i^r = i^f$ in steady-state. In that case, the steady-state satisfies the following system of equations

$$\frac{i^b - i^d}{1+i^d} = \Gamma_b(b, n) + \tilde{C}'(d) \tag{D.9.1.12}$$

$$0 = C_l(a + r, f) + \chi'(r) + \tilde{C}'(d) \tag{D.9.1.13}$$

$$0 = C_l(a + r, f) + \tilde{C}'(d) \tag{D.9.1.14}$$

$$0 = C_f(a + r, f) - \tilde{C}'(d) \tag{D.9.1.15}$$

or

$$\begin{aligned} \frac{i^b - i^d}{1+i^d} &= \Gamma_b\left(b,n\right) + \tilde{C}'\left(d\right) && \text{(D.9.1.16)} \\ 0 &= \chi'\left(r\right) && \text{(D.9.1.17)} \\ -C_l &= \tilde{C}'\left(d\right) && \text{(D.9.1.18)} \\ C_f &= \tilde{C}'\left(d\right) && \text{(D.9.1.19)} \end{aligned}$$

Note: Since $\chi'(r) = 0$, r_t vs. a_t is not pinned down. Hence, as in the main body of the text we focus on total liquidity $r_t + a_t = l_t$ from now on.

Linearized equations We linearize the model around the steady-state. The linearized banking problem is characterized by the following system of equations

$$\frac{1+i^b}{1+i^d}\left(\hat{i}_t^b - \hat{i}_t^d\right) = \Gamma_{bb} b\hat{b}_t + \Gamma_{bn} n\hat{n}_t + \tilde{C}'' d\hat{d}_t \tag{D.9.1.20}$$

$$\left(\hat{i}_t^r - \hat{i}_t^d\right) = C_{ll} l\hat{l}_t + C_{lf} f\hat{f}_t + \tilde{C}'' d\hat{d}_t \tag{D.9.1.21}$$

$$\left(\hat{i}_t^d - \hat{i}_t^f\right) = C_{fl} l\hat{l}_t + C_{ff} f\hat{f}_t - \tilde{C}'' d\hat{d}_t \tag{D.9.1.22}$$

$$\frac{b}{\Lambda}\hat{b}_t + \frac{l}{\Lambda}\hat{l}_t = \frac{f}{\Lambda}\hat{f}_t + \frac{d}{\Lambda}\hat{d}_t + \frac{n}{\Lambda}\hat{n}_t \tag{D.9.1.23}$$

$$\beta^s\hat{n}_t = \beta^s\hat{i}_t^d + \overline{\Pi}\frac{z}{n}\hat{z}_t + (1-\omega)\left(\hat{n}_{t-1} - \hat{\pi}_t\right) \tag{D.9.1.24}$$

$$\hat{z}_t\frac{n}{\Lambda} = \iota\frac{\Gamma}{\Lambda}\hat{n}_t + \frac{1+i^b}{1+i^d}\frac{b}{\Lambda}\hat{i}_t^b + \left[\left(1-\rho^d\right)\frac{r}{\Lambda} + \left(\rho^a - \rho^d\right)\frac{a}{\Lambda} - \left(\rho^f - \rho^d\right)\frac{f}{\Lambda} - \frac{1+i^b}{1+i^d}\frac{b}{\Lambda}\rho_d\right]\hat{i}_t^r \tag{D.9.1.25}$$

Simplifications We can simplify the first-order conditions with the functional form assumptions: $C = l^{-\gamma_l} f^{\gamma_f}$, $\tilde{C} = d^{\gamma_d}$ and $\Gamma = b^\nu n^{-\iota}$, which in turn implies that

$$\begin{aligned} \Gamma_{bb} b &= (\nu - 1)\Gamma_b = (\nu - 1)\nu\frac{\Gamma}{b} && \text{(D.9.1.26)} \\ \Gamma_{bn} n &= -\iota\Gamma_b = -\iota\nu\frac{\Gamma}{b} && \text{(D.9.1.27)} \\ C_{ll} l &= (-\gamma_l - 1) C_l = (\gamma_l + 1)\gamma_l\frac{C}{l} && \text{(D.9.1.28)} \\ C_{lf} f &= \gamma_f C_l = -\gamma_f\gamma_l\frac{C}{l} && \text{(D.9.1.29)} \\ C_{fl} l &= -\gamma_l C_f = -\gamma_l\gamma_f\frac{C}{f} && \text{(D.9.1.30)} \\ C_{ff} f &= \left(\gamma_f - 1\right) C_f = \left(\gamma_f - 1\right)\gamma_f\frac{C}{f} && \text{(D.9.1.31)} \\ \tilde{C}_{dd} d &= (\gamma_d - 1)\tilde{C}_d = (\gamma_d - 1)\gamma_d\frac{\tilde{C}}{d} && \text{(D.9.1.32)} \end{aligned}$$

Using these, we get:

$$\frac{1+i^b}{1+i^d}\left(\hat{i}_t^b-\hat{i}_t^d\right)=\nu\frac{\Gamma}{b}\left((\nu-1)\,\hat{b}_t-\iota\hat{n}_t\right)+(\gamma_d-1)\,\gamma_d\frac{\tilde{C}}{d}\hat{d}_t \tag{D.9.1.33}$$

$$\left(\hat{i}_t^r-\hat{i}_t^d\right)=(\gamma_l+1)\,\gamma_l\frac{C}{l}\hat{l}_t-\gamma_f\,\gamma_l\frac{C}{l}\hat{f}_t+(\gamma_d-1)\,\gamma_d\frac{\tilde{C}}{d}\hat{d}_t \tag{D.9.1.34}$$

$$\left(\hat{i}_t^d-\hat{i}_t^f\right)=-\gamma_l\gamma_f\frac{C}{f}\hat{l}_t+\left(\gamma_f-1\right)\gamma_f\frac{C}{f}\hat{f}_t-(\gamma_d-1)\,\gamma_d\frac{\tilde{C}}{d}\hat{d}_t \tag{D.9.1.35}$$

$$\frac{b}{\Lambda}\hat{b}_t+\frac{l}{\Lambda}\hat{l}_t=\frac{f}{\Lambda}\hat{f}_t+\frac{d}{\Lambda}\hat{d}_t+\frac{n}{\Lambda}\hat{n}_t \tag{D.9.1.36}$$

$$\beta^s\hat{n}_t=\beta^s\hat{i}_t^d+\overline{\Pi}\frac{z}{n}\hat{z}_t+(1-\omega)\,(\hat{n}_{t-1}-\hat{\pi}_t) \tag{D.9.1.37}$$

$$\hat{z}_t\frac{z}{\Lambda}=\iota\frac{\Gamma}{\Lambda}\hat{n}_t+\frac{1+i^b}{1+i^d}\frac{b}{\Lambda}\hat{i}_t^b+\left[\left(1-\rho^d\right)\frac{r}{\Lambda}+\left(\rho^a-\rho^d\right)\frac{a}{\Lambda}-\left(\rho^f-\rho^d\right)\frac{f}{\Lambda}-\frac{1+i^b}{1+i^d}\frac{b}{\Lambda}\rho_d\right]\hat{i}_t^r \tag{D.9.1.38}$$

We can simplify the FOCs further, by using the fact that in steady-state we have that

$$\gamma_f\frac{C}{f}\quad=\quad\gamma_d\frac{\tilde{C}}{d}=\gamma_l\frac{C}{l} \tag{D.9.1.39}$$

so the final system of equations can be written

$$\frac{1+i^b}{1+i^d}\left(\hat{i}_t^b-\hat{i}_t^d\right)=\nu\frac{\Gamma}{b}\left((\nu-1)\,\hat{b}_t-\iota\hat{n}_t\right)+(\gamma_d-1)\,\gamma_l\frac{C}{l}\hat{d}_t \tag{D.9.1.40}$$

$$\left(\hat{i}_t^r-\hat{i}_t^d\right)=\gamma_l\frac{C}{l}\left\{(\gamma_l+1)\,\hat{l}_t-\gamma_f\,\hat{f}_t+(\gamma_d-1)\,\hat{d}_t\right\} \tag{D.9.1.41}$$

$$\left(\hat{i}_t^d-\hat{i}_t^f\right)=\gamma_f\frac{C}{f}\left\{\left(\gamma_f-1\right)\hat{f}_t-\gamma_l\hat{l}_t-(\gamma_d-1)\,\hat{d}_t\right\} \tag{D.9.1.42}$$

$$\frac{b}{\Lambda}\hat{b}_t+\frac{l}{\Lambda}\hat{l}_t=\frac{f}{\Lambda}\hat{f}_t+\frac{d}{\Lambda}\hat{d}_t+\frac{n}{\Lambda}\hat{n}_t \tag{D.9.1.43}$$

$$\beta^s\hat{n}_t=\beta^s\hat{i}_t^d+\overline{\Pi}\frac{z}{n}\hat{z}_t+(1-\omega)\,(\hat{n}_{t-1}-\hat{\pi}_t) \tag{D.9.1.44}$$

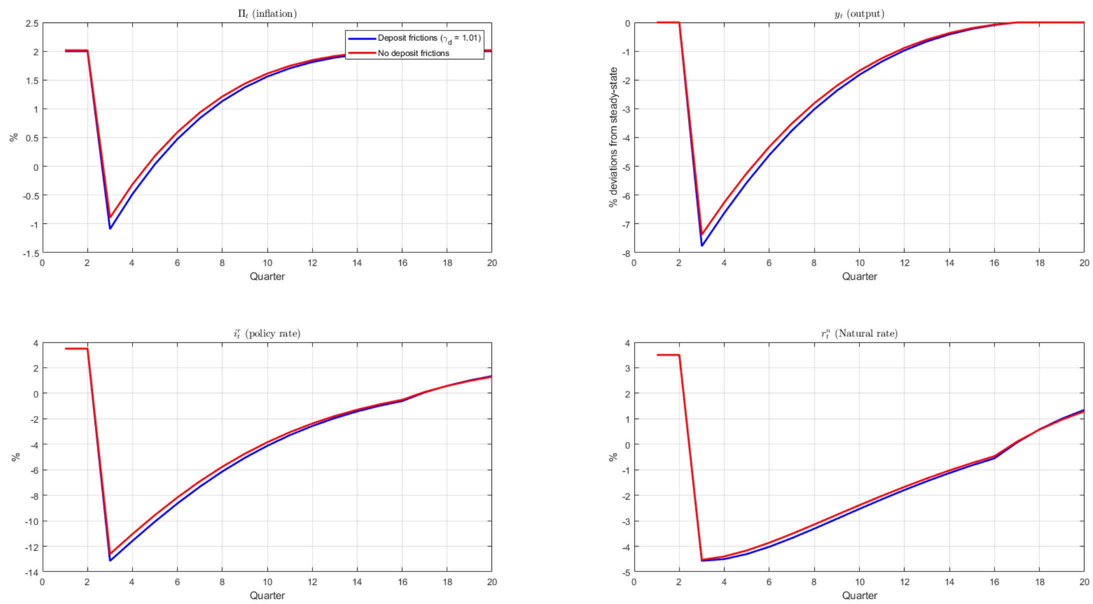
$$\hat{z}_t\frac{z}{\Lambda}=\iota\frac{\Gamma}{\Lambda}\hat{n}_t+\frac{1+i^b}{1+i^d}\frac{b}{\Lambda}\hat{i}_t^b+\left[\left(1-\rho^d\right)\frac{r}{\Lambda}+\left(\rho^a-\rho^d\right)\frac{a}{\Lambda}-\left(\rho^f-\rho^d\right)\frac{f}{\Lambda}-\frac{1+i^b}{1+i^d}\frac{b}{\Lambda}\rho_d\right]\hat{i}_t^r \tag{D.9.1.45}$$

The remainder of the model, i.e. everything outside of the banking block, remains as in the main body of the text. We parameterize C in line with the parameterization in Section D.8.

Figure D.9.1.1 and Figure D.9.1.2 shows the response of the key variables to a preference shock as in the main text, but with deposit adjustment costs. The red line corresponds to the case where adjustment costs are linear, i.e. $\gamma_d = 1$. In this case, the model (once linearized) behaves similarly to the model in the main body of the text. The blue line correspond to a case where the deposit adjustment costs are convex, i.e. $\gamma_d = 1.01$.

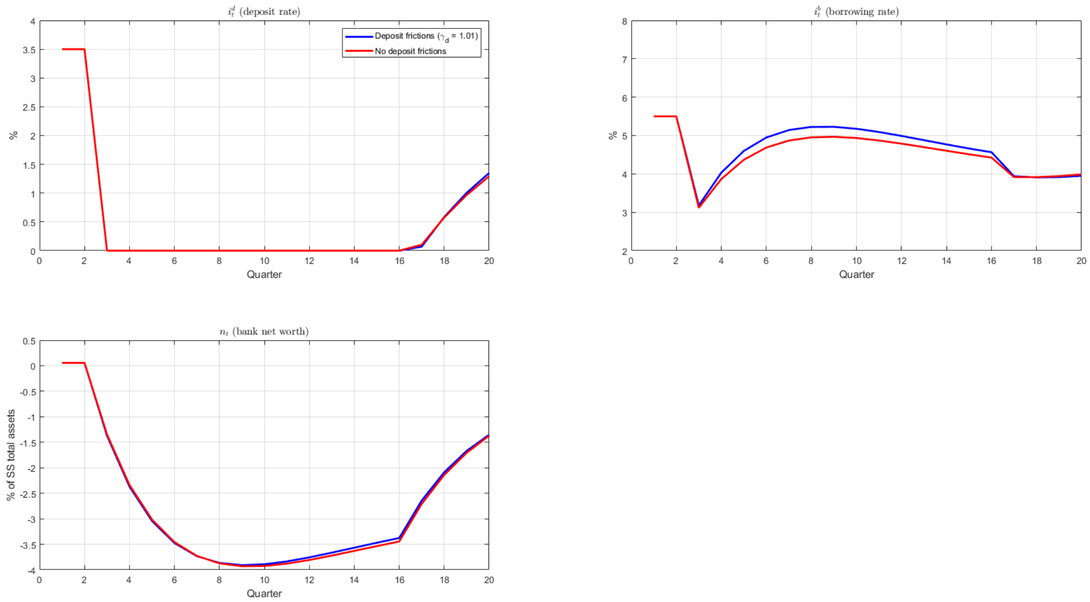
Notably, the figures suggest that the contractionary effects we documented in the main text (i.e. Figure 8 and Figure 9) is not due to deposits being a frictionless source of funding. In fact, if there are increasing adjustment costs associated with changing the stock of deposits, the contractionary effects of negative policy rates are amplified under our calibration. The intuition is the following; With increasing marginal adjustment costs, the marginal cost of issuing a loan now also reflects the marginal adjustment costs of deposits needed to fund the loan. Moreover, this adjustment cost is increasing in deposits. Once the central bank implements a negative policy rate to counter the preference shock, bank net worth is reduced. Because external financing is relatively sticky - in line with the data - the drop in net worth needs to be offset by an increase in deposits, all else equal. This is in fact consistent with the empirical evidence in Figure A.6. The increase in deposits - due to increasing adjustment costs - puts further upwards pressure on borrowing rates, ultimately amplifying the contractionary effects of lower bank net worth on borrowing rates and eventually output and inflation.

Figure D.9.1.1: Response of model under the baseline calibration to a preference shock with deposit adjustment costs



Notes: This figure shows the impulse response functions of inflation, output, the policy rate and the natural rate of interest in response to a preference shock of the model outlined in section D.9.1. The red solid line correspond to the case where $\gamma_d = 1$, while the blue line correspond to a case where $\gamma_d = 1.01$.

Figure D.9.1.2: Response of model under the baseline calibration to a preference shock with deposit adjustment costs



Notes: This figure shows the impulse response functions of several variables in response to a preference shock of the model outlined in section D.9.1. The red solid line correspond to the case where $\gamma_d = 1$, while the blue line correspond to a case where $\gamma_d = 1.01$.

D.9.2 Monopolistic competition in deposit markets

A different way of introducing frictional deposits is to assume that banks have market power in deposit markets, following for instance Drechsler et al. (2017). We implement this by assuming that we can write bank deposits as a function of the deposit rate, i.e. $D\left(i_t^d\right)$ with $D'\left(i_t^d\right) > 0$, i.e. banks have market power in deposit markets. We focus on the case where $M_t = 0$ and set $\rho_a = 1$ so that only total liquidity L_t is pinned down. In this case, we solve the flow budget constraint of the bank (7) for L_t.

$$\frac{1}{1+i_t^r} n_t = (1-\omega)\, n_{t-1} \Pi_t^{-1} + \tilde{z}_t \tag{D.9.2.1}$$

where

$$\tilde{z}_t = \frac{i_t^b - i_t^r}{1+i_t^r} b_t + \frac{i_t^r - i_t^d}{1+i_t^r} d\left(i_t^d\right) - \frac{i_t^f - i_t^r}{1+i_t^r} f_t - C\left(f_t, d_t\left(i_t^d\right) + f_t - b_t, n_t\right) - \Gamma\left(b_t, n_t\right) \tag{D.9.2.2}$$

In this case, we have the following FOCs

$$b_t: \frac{i_t^b - i_t^r}{1+i_t^r} = \Gamma_B - C_L \tag{D.9.2.3}$$

$$i_t^d: \left(\frac{i_t^r - i_t^d}{1+i_t^r} - C_L\right) d'\left(i_t^d\right) = \frac{1}{1+i_t^r} d\left(i_t^d\right) \tag{D.9.2.4}$$

$$f_t: \frac{i_t^r - i_t^f}{1+i_t^r} = C_F + C_L \tag{D.9.2.5}$$

Equation (D.9.2.4) can be rewritten as

$$i_t^d = \frac{\epsilon^d\left(i_t^d\right)}{\epsilon^d\left(i_t^d\right)+1}\left(i_t^r - \left(1+i_t^r\right)C_L\right) = \eta\left(i_t^d\right)\left(i_t^r - \left(1+i_t^r\right)C_l\right) \tag{D.9.2.6}$$

where $\epsilon^d\left(i_t^d\right) \equiv \frac{D'\left(i_t^d\right)}{D\left(i_t^d\right)} i_t^d$. and $\eta \equiv \frac{\epsilon^d\left(i_t^d\right)}{\epsilon^d\left(i_t^d\right)+1}$.

We restrict attention to a steady-state where $C_F = C_L = 0$ to facilitate comparison with the different versions of the model considered above.

Assuming that ϵ^d is constant, we can linearize the model around the steady-state to get

$$\hat{i}_t^d = \max\{i^{\text{DLB}}, \frac{1+i^r}{1+i^d}\eta\hat{i}_t^r\} \tag{D.9.2.7}$$

$$\frac{1+i^b}{1+i^r}\left(\hat{i}_t^b - \hat{i}_t^r\right) = \nu\frac{\Gamma}{b}\left[(\nu-1)\hat{b}_t - \iota\hat{n}_t\right] \tag{D.9.2.8}$$

$$\hat{n}_t = (1+i^r)\frac{1-\omega}{\overline{\Pi}}\left(\hat{i}_t^r + \hat{n}_{t-1} - \hat{\pi}_t\right) + \hat{z}_t \tag{D.9.2.9}$$

$$\hat{z}_t = \frac{1+i^b}{1+i^r}\frac{b}{n}\hat{i}_t^b - \frac{f}{n}\hat{i}_t^f + \iota\frac{\Gamma}{n}\hat{n}_t + \left(\frac{1+i^d}{1+i^r}\frac{d}{n} + \frac{f}{n} - \frac{b}{n}\right)\hat{i}_t^r \tag{D.9.2.10}$$

Notice that when we assume monopolistic competition in deposit markets, the deposit rate no longer enters the first-order condition for lending. Instead, the policy rate is the relevant rate for capturing the marginal cost of lending. This suggests that policy rate cuts can have larger stimulative effects on the supply of bank lending in this case. However, at the same time, the marginal cost of lending still depends on net worth. Thus, to the extent that negative policy rates reduce profits, it is unclear whether banks will expand or contract the supply of credit in response to a policy rate cut below zero. In practice, it depends on the strength of the effect of net worth on lending. It is also worth noticing that the model outlined above can generate a realistic deposit rate pass-through away from the DLB, in line with the empirical evidence in Figure A.2.

To see how monopolistic competition in deposit markets affect our conclusions, we simulate the model, keeping the remainder of the model unchanged. Moreover, we restrict attention to a limiting case where $\eta \longrightarrow 1$. While this is counterfactual in the sense that it yields a too large pass-through to deposit rates from policy rate cuts away from the DLB, it has several practical benefits. It allows us to retain the important feature of monopolistic competition that the policy rate and not the deposit rate enters into the first-order condition for loans (equation (D.9.2.3)), while gaining the tractability that we only have to consider one occasionally binding constraint.[49]

[49] With $\eta < 1$, the deposit rate would reach the DLB before the policy rate reached zero. In the case when we consider a ZLB on the

We simulate the model under three different parameterizations/regimes in response to a preference shock as considered in the main text. The black solid line is the "standard" case, where the policy rate is subject to a zero lower bound. The two red lines consider models where the central bank cut the rate into negative territory despite the presence of a DLB, for two alternative parameterizations. The figures illustrate that with monopolistic competition in deposit markets, a policy rate cut below the DLB can have expansionary effects through the bank lending channel. This is seen by comparing the red dashed line with the black solid line. When the policy rate goes into negative territory (the red dashed line), borrowing rates fall by more compared to the standard case (the black solid line). This increases borrowers' demand, ultimately dampening the adverse impact of the shock on output and inflation compared to not going into negative territory. Notice, however, that in this case the borrowing rate remains below the borrowing rate in the case without negative policy rates, which is inconsistent with our empirical estimates in Table 1 where we documented an *increase* in borrowing rates. This suggests that the feedback from net worth to lending rates is set too low. Therefore, the red solid line captures a case where we set $\nu = 20.12$, implying $\iota = 12.50$. These values ensure that the relative increase in borrowing rates is in line with the empirical evidence in Table 1. In other words, we follow the same calibration strategy as in our baseline. In this case, the importance of net worth for the marginal cost of lending is substantially higher compared to the red dashed line, and borrowing rates increase. This brings about a larger contraction in output and inflation.

Overall, these simulations suggest that monopolistic competition in deposit markets can make negative policy rates have both positive and negative effects on the bank lending channel of monetary policy, depending on parameterizations. A parameterization consistent with our empirical evidence in Table 1 - which documented an *increase* in borrowing rates - yields contractionary effects of negative policy rates, in line with our baseline model where deposit markets where characterized by perfect competition.

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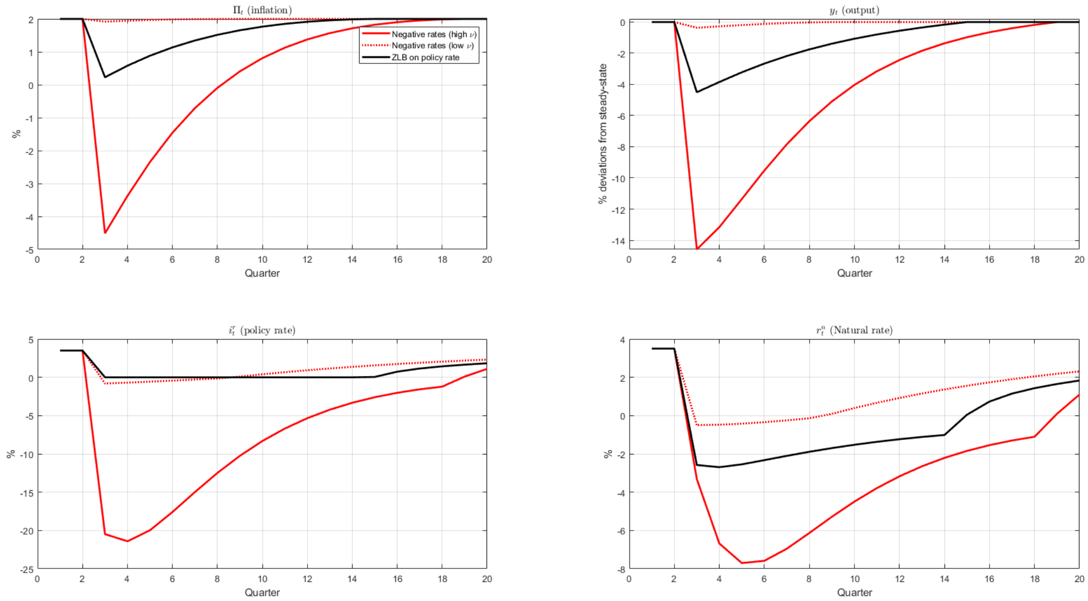
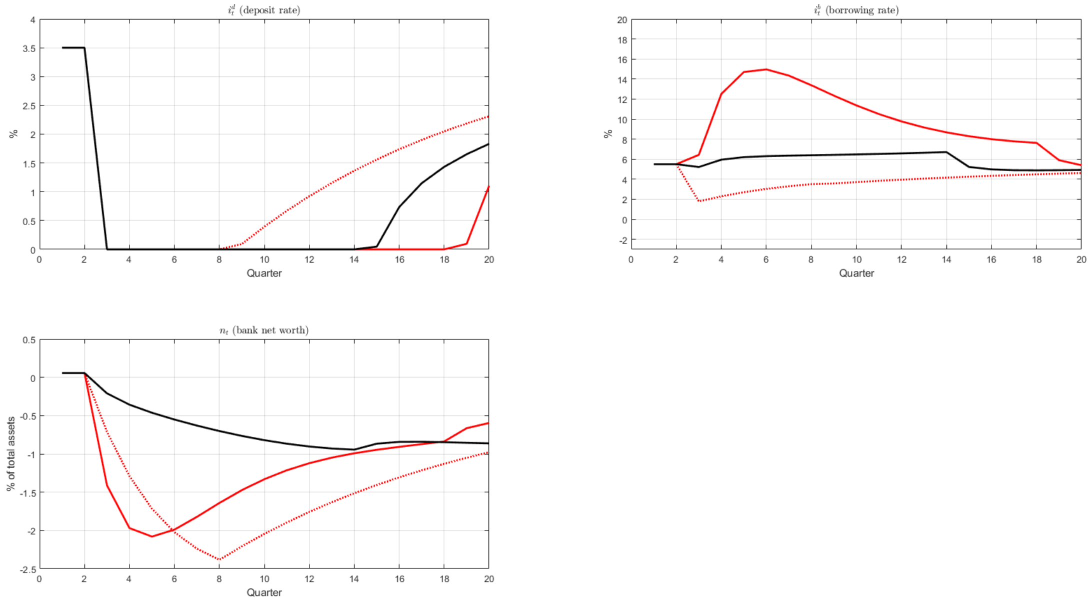
policy rate, we would then have to consider two occasionally binding constraints, i.e. one occasionally binding constraint on the deposit rate and one on the policy rate, which did not necessarily bind at the same time. Assuming that $\eta \to 1$ implies that the constraints will bind at the same time, simplifying the analysis substantially.

Figure D.9.2.1: Response of model under the baseline calibration to a preference shock with monopolistic competition in deposit markets



Notes: This figure shows the impulse response functions of inflation, output, the policy rate and the natural rate of interest in response to a preference shock of the model outlined in section D.9.2. The black solid line considers a model where there is a zero lower bound on deposit rates and the policy rate. The red solid line considers a model where there is a DLB, but not a lower bound on the policy rate. The red dashed line considers a model where there is a DLB, but not a lower bound on the policy rate and with a $\nu = 20.12$, which generates a 5.9 basis point increase in borrowing rates (relative to the model with a ZLB on the policy rate) on impact. The initial shock ζ_0 is set to -0.1275 to generate a 4.5 percent drop in output on impact in the ZLB case.

Figure D.9.2.2: Response of model under the baseline calibration to a preference shock with monopolistic competition in deposit markets



Notes: This figure shows the impulse response functions of the deposit rate, the borrowing rate and bank net worth in response to a preference shock of the model outlined in section D.9.2. The black solid line considers a model where there is a zero lower bound on deposit rates and the policy rate. The red solid line considers a model where there is a DLB, but not a lower bound on the policy rate. The red dashed line considers a model where there is a DLB, but not a lower bound on the policy rate and with a $\nu = 20.12$, which generates a 5.9 basis point increase in borrowing rates (relative to the model with a ZLB on the policy rate) on impact. The initial shock ζ_0 is set to -0.1275 to generate a 4.5 percent drop in output on impact in the ZLB case.