OPTIMAL MONETARY AND FISCAL POLICY AND THE LIQUIDITY TRAP

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ABSTRACT

This dissertation explores optimal monetary and fiscal policy at low nominal interest rates. Policy at low interest rates is challenging since the principal instrument of central banks is the short-term nominal interest rate and it cannot be set below zero. This can be particularly problematic in a deflationary environment when the government wants to stimulate the economy but is constrained by the zero bound. Recent events in Japan (where the interest rate has been close to zero for the past several years) and the lowest short-term interest rate in the US in the past 45 years make this an urgent topic of research.

The first chapter, joint with Michael Woodford, explores optimal monetary policy under the assumption that the central bank can commit to future policy. The main result is a characterization of optimal policy under commitment. Faced with temporary deflationary shocks that make the zero bound binding, we find that the central bank should commit to lower future interest rates in periods in which the deflationary pressures have subsided and the zero bounds is not binding anymore. This is useful because it creates inflation expectations, thereby lowering the real rate of return and stimulating demand. Furthermore, we show a simple price-level targeting rule that implements this equilibrium.

The second chapter explores the same problem assuming the government cannot commit to future policy. If the only instrument of policy is open-market operations in short-term bonds I show that the inability of the government to commit results in excessive deflation (when government is faced with shocks that make the zero bound binding) relative to the solution when the government can commit to future policy. This is what I call the deflation bias of discretionary policy. I propose several policies to solve this credibility problem.

This third chapter analyses fiscal policy at zero nominal interest rate in alternative institutional frameworks assuming (as in chapter 2) that the government cannot commit to future policy. Real government spending increases demand by increasing public consumption. Deficit spending increases demand by generating inflation expectations. When fiscal and monetary policy are coordinated, deficit is more effective than real government spending in a calibrated model. When the central bank is "goal independent" real government spending is still effective but deficit spending is not.

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CONTENTS

Chapter 1:

The Zero Bound on Interest Rates and Optimal Monetary Policy

(joint with Michael Woodford)

Abstract

We consider the consequences for monetary policy of the zero floor for nominal interest rates. The zero bound can be a significant constraint on the ability of a central bank to combat deflation. We show, in the context of an intertemporal equilibrium model, that open-market operations, even of "unconventional" types, are ineffective if future policy is expected to be purely forward-looking. Nonetheless, a credible commitment to the right sort of history-dependent policy can largely mitigate the distortions created by the zero bound. In our model, optimal policy involves a commitment to adjust interest rates so as to achieve a time-varying price-level target, when this is consistent with the zero bound. We also discuss ways in which other central-bank actions, while irrelevant apart from their effects on expectations, may help to make credible a central bank's commitment to its target.

The consequences for the proper conduct of monetary policy of the existence of a lower bound of zero for overnight nominal interest rates has recently become a topic of lively interest. In Japan, the call rate (the overnight cash rate that is analogous to the federal funds rate in the U.S.) has been within 50 basis points of zero since October 1995, so that little room for further reductions in short-term nominal interest rates has existed since that time, and has been essentially equal to zero for most of the past four years. (See Figure 1 below.) At the same time, growth has remained anemic in Japan over this period, and prices have continued to fall, suggesting a need for monetary stimulus. Yet the usual remedy — lower short-term nominal interest rates — is plainly unavailable. Vigorous expansion of the monetary base (which, as shown in the figure, is now more than twice as large, relative to GDP, as in the early 1990s) has also seemed to do little to stimulate demand under these circumstances.

The fact that the federal funds rate has now been reduced to only one percent in the U.S., while signs of recovery remain exceedingly fragile, has led many to wonder if the U.S. could not also soon find itself in a situation where interest-rate policy would no longer be available as a tool for macroeconomic stabilization. A number of other nations face similar questions. The result is that a problem that was long treated as a mere theoretical curiosity after having been raised by Keynes (1936) — namely, the question of what can be done to stabilize the economy when interest rates have fallen to a level below which they cannot be driven by further monetary expansion, and whether monetary policy can be effective at all under such circumstances — now appears to be one of urgent practical importance, though one with which theorists have become unfamiliar.

The question of how policy should be conducted when the zero bound is reached — or when the possibility of reaching it can no longer be ignored — raises many fundamental issues for the theory of monetary policy. Some would argue that awareness of the possibility of hitting the zero bound calls for fundamental changes in the way that policy is conducted even when the bound has not yet been reached. For example, Krugman (2003) refers to deflation as a "black hole", from which an economy cannot expect to escape once it has been entered. A conclusion that is often drawn from this pessimistic view of the efficacy of monetary policy under circumstances of a liquidity trap is that it is vital to steer far clear of circumstances under which deflationary expectations could ever begin to develop — for example, by targeting a sufficiently high positive rate of inflation even under normal circumstances.

Others are more sanguine about the continuing effectiveness of monetary policy even when the zero bound is reached, but frequently defend their optimism on grounds that again imply that conventional understanding of the conduct of monetary policy is inadequate in important respects. For example, it is often argued that deflation need not be a "black hole" because monetary policy can affect aggregate spending and hence inflation through channels other than central-bank control of short-term nominal interest rates. Thus there has been much recent discussion — both among commentators on the problems of Japan, and among those addressing the nature of deflationary risks to the U.S. — of the advantages of vigorous expansion of the monetary base even when these are not associated with any further reduction in interest rates, of the desirability of attempts to shift longer-term interest rates through purchases of longer-maturity government securities by the central bank, and even of the possible desirability of central-bank purchases of other kinds of assets. Yet if these views are correct, they challenge much of the recent conventional wisdom regarding the conduct of monetary policy, both within central banks and among monetary economists, which has stressed a conception of the problem of monetary policy in terms of the appropriate adjustment of an operating target for overnight interest rates, and formulated prescriptions for monetary policy, such as the celebrated "Taylor rule" (Taylor, 1993), that are cast in these terms. Indeed, some have argued that the inability of such a policy to prevent the economy from falling into a deflationary spiral is a critical flaw of the Taylor rule as a guide to policy (Benhabib *et al.*, 2001).

Similarly, the concern that a liquidity trap can be a real possibility is sometimes presented as a serious objection to another currently popular monetary policy prescription, namely inflation targeting. The definition of a policy prescription in terms of an inflation target presumes that there is in fact an interest-rate choice that can allow one to hit one's target (or at least to be projected to hit it, on average). But, some would argue, if the zero interest-rate bound is reached under circumstances of deflation, it will not be possible to hit any higher inflation target, as further interest-rate decreases are not possible despite the fact that one is undershooting one's target. Is there, in such circumstances, any point in having an inflation target? This has frequently been offered as a reason for resistance to inflation targeting at the Bank of Japan. For example, Kunio Okina, director of the Institute for Monetary and Economic Studies at the BOJ, was quoted by Dow Jones News (8/11/1999) as arguing that "because short-term interest rates are already at zero, setting an inflation target of, say, 2 percent wouldn't carry much credibility."

Here we seek to shed light on these issues by considering the consequences of the zero lower bound on nominal interest rates for the optimal conduct of monetary policy, in the context of an explicit intertemporal equilibrium model of the monetary transmission mechanism. While our model remains an extremely simple one, we believe that it can help to clarify some of the basic issues just raised. We are able to consider the extent to which the zero bound represents a genuine constraint on attainable equilibrium paths for inflation and real activity, and to consider the extent to which open-market purchases of various kinds of assets by the central bank can mitigate that constraint. We are also able to show how the character of optimal monetary policy changes as a result of the existence of the zero bound, relative to the policy rules that would be judged optimal in the absence of such a bound, or in the case of real disturbances small enough for the bound never to matter under an optimal policy.

To preview our results, we find that the zero bound does represent an important constraint on what monetary stabilization policy can achieve, at least when certain kinds of real disturbances are encountered in an environment of low inflation. We argue that the possibility of expansion of the monetary base through central-bank purchases of a variety of types of assets does little if anything to expand the set of feasible equilibrium paths for inflation and real activity that are consistent with equilibrium under some (fully credible) policy commitment. Hence the relevant trade-offs can correctly be studied by simply considering what can be achieved by alternative anticipated state-contingent paths of the short-term nominal interest rate, taking into account the constraint that this quantity must be non-negative at all times. When we consider such a problem, we find that the zero interest-rate bound can indeed be temporarily binding, and in such a case it inevitably results in lower welfare than could be achieved in the absence of such a constraint.[1]

Nonetheless, we argue that the extent to which this constraint restricts possible stabilization outcomes under sound policy is much more modest than the deflation pessimists presume. Even though the set of feasible equilibrium outcomes corresponds to those that can be achieved through alternative interest-rate policies, monetary policy is far from powerless to mitigate the contractionary effects of the kind of disturbances that would make the zero bound a binding constraint. The key to dealing with this sort of situation in the least damaging way is to create the right kind of expectations regarding the way in which monetary policy will be used *subsequently,* at a time when the central bank again has room to maneuver. We use our intertemporal equilibrium model to characterize the kind of expectations regarding future policy that it would be desirable to create, and discuss a form of price-level targeting rule that — if credibly committed to by the central bank — should bring about the constrained-optimal equilibrium. We also discuss, more informally, ways in which other types of policy actions could help to increase the credibility of the central bank's announced commitment to this kind of future policy.

Our analysis will be recognized as a development of several key themes of Paul Krugman's (1998) treatment of the same topic in these pages a few years ago. Like Krugman, we give particular emphasis to the role of expectations regarding future policy in determining the severity of the distortions that result from hitting the zero bound. Our primary contribution, relative to Krugman's earlier treatment, will be the presentation of a more fully dynamic analysis. For example, our assumption of staggered pricing, rather than the simple hypothesis of prices that are fixed for one period as in the analysis of Krugman, allows for richer (and at least somewhat more realistic) dynamic responses to disturbances. In our model, unlike Krugman's, a real disturbance that lowers the natural rate of interest can cause output to remain below potential for years (as shown in Figure 2 below), rather than only for a single "period", even when the average frequency of price adjustments is more than once per year. These richer dynamics are also important for a realistic discussion of the kind of policy commitment that can help to reduce economic contraction during a "liquidity trap". In our model, a commitment to create subsequent inflation involves a commitment to *keep interest rates low* for a time in the future, whereas in Krugman's model, a commitment to a higher future price level does not involve any reduction in future nominal interest rates. We are also better able to discuss questions such as how the creation of inflationary expectations during the period that the zero bound is binding can be reconciled with maintaining the credibility of the central bank's commitment to long-run price stability.

Our dynamic analysis also allows us to further clarify the several ways in which the management of private-sector expectations by the central bank can be expected to mitigate the effects of the zero bound. Krugman emphasizes the fact that increased expectations of inflation can lower the real interest rate implied by a zero nominal interest rate. This might suggest, however, that the central bank can affect the economy only insofar as it affects expectations regarding a variable that it cannot influence except quite indirectly; and it might also suggest that the only expectations that should matter are those regarding inflation over the relatively short horizon corresponding to the short-term nominal interest rate that has fallen to zero. Such interpretations easily lead to skepticism about the practical effectiveness of the expectational channel, especially if inflation is regarded as being relatively "sticky" in the short run. Our model is instead one in which expectations affect aggregate demand through several channels. First of all, it is not merely short-term real interest rates that matter for current aggregate demand; our model of intertemporal substitution in spending implies that the entire expected future *path* of short real rates should matter, or alternatively that very long real rates should matter.[2] This means that the creation of inflation expectations, even with regard to inflation that should occur only more than a year in the future, should also be highly relevant to aggregate demand, as long as it is not accompanied by correspondingly higher expected future nominal interest rates. Furthermore, the expected future path of nominal interest rates matters, and not just their current level, so that a commitment to keep nominal interest rates low for a longer period of time should stimulate aggregate demand, even when current rates cannot be further lowered, and even under the hypothesis that inflation expectations would remain unaffected. Since the central bank can clearly control the future path of short-term nominal interest rates if it has the will to do so, any failure of such a commitment to be credible will not be due to skepticism about whether the central bank is *able* to follow through on its commitment.

The richer dynamics of our model are also important for the analysis of optimal policy. Krugman mainly addresses the question whether monetary policy is completely impotent when the zero bound binds, and argues for the possibility of increasing real activity in the "liquidity trap" by creating expectations of inflation. This conclusion in itself, however (with which we agree), does not answer the question whether, or to what extent, it should actually be desirable to create such expectations, given the well-founded reasons that the central bank should have to not prefer inflation at a later time. Nor is Krugman's model well-suited to address such a question, insofar as it omits any reason for even an extremely high degree of subsequent inflation to be harmful. Our model with staggered pricing, instead, implies that inflation (whether anticipated or not) creates distortions, and justifies an objective function for stabilization policy that trades off inflation stabilization and output-gap stabilization in terms that are often assumed to represent actual central-bank concerns. We characterize optimal policy in such a setting, and show that it does indeed involve a commitment to history-dependent policy of a sort that should result in higher inflation expectations in response to a binding zero bound. We can also show to what extent it should be optimal to create such expectations, assuming that this is possible. We find, for example, that it is not optimal to commit to so much future inflation that the zero bound ceases to bind, even though this is one possible type of equilibrium; this is why the zero bound does remain a relevant constraint, even under an optimal policy commitment.

1 Is "Quantitative Easing" a Separate Policy Instrument?

A first question that we wish to consider is whether expansion of the monetary base represents a policy instrument that should be effective in preventing deflation and associated output declines, even under circumstances where overnight interest rates have fallen to zero. According to the famous analysis of Keynes (1936), monetary policy ceases to be an effective instrument to head off economic contraction in a "liquidity trap," that can arise if interest rates reach a level so low that further expansion of the money supply cannot drive them lower. Others have argued that monetary expansion should increase nominal aggregate demand even under such circumstances, and the supposition that this is correct lies behind the explicit adoption in Japan since March 2001 of a policy of "quantitative easing" in addition to the "zero interest-rate policy" that continues to be maintained.[3]

Here we consider this question in the context of an explicit intertemporal equilibrium model, in which we model both the demand for money and the role of financial assets (including the monetary base) in private-sector budget constraints. The model that we use for this purpose is more detailed in several senses than the one used in subsequent sections to characterize optimal policy, in order to make it clear that we have not excluded a role for "quantitative easing" simply by *failing to model* the role of money in the economy. The model is discussed in more detail in Woodford (2003, chapter 4), where the consequences of various interest-rate rules and money-growth rules are considered under the assumption that disturbances are not large enough for the zero bound to bind.

Our key result is an irrelevance proposition for open market operations in a variety of types of assets that might be acquired by the central bank, under the assumption that the open market operations do not change the expected future conduct of monetary or fiscal policy (in senses that we make precise below). It is perhaps worth stating from the start that our intention in stating such a result is not to vindicate the view that a central bank is powerless to halt a deflationary slump, and hence to absolve the Bank of Japan, for example, from any responsibility for the continuing stagnation in that country. While our proposition establishes that there is a sense in which a "liquidity trap" is possible, this does not mean that the central bank is powerless under the circumstances that we describe. Rather, the point of our result is to show that the key to effective central-bank action to combat a deflationary slump is *the management of expectations*. Open-market operations should be largely ineffective *to the extent that* they fail to change expectations regarding future policy; the conclusion that we draw is not that such actions are futile, but rather that the central bank's actions should be chosen with a view to signalling the nature of its policy commitments, and not in order to create some sort of "direct" effects.

1.1 A Neutrality Proposition for Open-Market Operations

Our model abstracts from endogenous variations in the capital stock, and assumes perfectly flexible wages (or some other mechanism for efficient labor contracting), but assumes monopolistic competition in goods markets, and sticky prices that are adjusted at random intervals in the way assumed by Calvo (1983), so that deflation has real effects. We assume a model in which the representative household seeks to maximize a utility function of the form

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} \left[u(C_t, M_t/P_t; \xi_t) - \int_0^1 v(H_t(j); \xi_t) dj \right],$$

where C_t is a Dixit-Stiglitz aggregate of consumption of each of a continuum of differentiated goods,

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta}{\theta-1}} di \right]^{\frac{\theta-1}{\theta}},$$

with an elasticity of substitution equal to $\theta > 1$, M_t measures end-of-period household money balances,[4] P_t is the Dixit-Stiglitz price index,

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \tag{1}$$

and $H_t(j)$ is the quantity supplied of labor of type j. (Each industry j employs an industry-specific type of labor, with its own wage $w_t(j)$.) Real balances are included in the utility function, following Sidrauski (1967) and Brock (1974, 1975), as a proxy for the services that money balances provide in facilitating transactions.[5]

For each value of the disturbances ξ_t, $u(\cdot, \cdot; \xi_t)$ is concave function, increasing in the first argument, and increasing in the second for all levels of real balances up to a satiation level $\bar{m}(C_t; \xi_t)$. The existence of a satiation level is necessary in order for it to be *possible* for the zero interest-rate bound ever to be reached; we regard Japan's experience over the past several years as having settled the theoretical debate over whether such a level of real balances exists. Unlike many papers in the literature, we do not assume additive separability of the function u between the first two arguments; this (realistic) complication allows a further channel through which money can affect aggregate demand, namely an effect of real money balances on the current marginal utility of consumption. Similarly, for each value of ξ_t, $v(\cdot; \xi_t)$ is an increasing convex function. The vector of exogenous disturbances ξ_t may contain several elements, so that no assumption is made about correlation of the exogenous shifts in the functions u and v.

For simplicity we shall assume complete financial markets and no limits on borrowing against future income. As a consequence, a household faces an intertemporal budget constraint of the form

$$E_t \sum_{T=t}^{\infty} Q_{t,T}[P_T C_T + \delta_T M_T] \leq W_t + E_t \sum_{T=t}^{\infty} Q_{t,T} \left[\int_0^1 \Pi_T(i) di + \int_0^1 w_T(j) H_T(j) dj - T_T^h \right]$$

looking forward from any period t. Here $Q_{t,T}$ is the stochastic discount factor by which the financial markets value random nominal income at date T in monetary units at date t, δ_t is the opportunity cost of holding money (equal to $i_t/(1+i_t)$, where i_t is the riskless nominal interest rate on one-period obligations purchased in period t, in the case that no interest is paid on the monetary base), W_t is the nominal value of the household's financial wealth (including money holdings) at the beginning of period t, $\Pi_t(i)$ represents the nominal profits (revenues in excess of the wage bill) in period t of the supplier of good

i, $w_t(j)$ is the nominal wage earned by labor of type j in period t, and T_t^h represents the net nominal tax liabilities of each household in period t.

Optimizing household behavior then implies the following necessary conditions for a rational-expectations equilibrium. Optimal timing of household expenditure requires that aggregate demand Y_t for the composite good[6] satisfy an Euler equation of the form

$$u_c(Y_t, M_t/P_t; \xi_t) = \beta E_t \left[u_c(Y_{t+1}, M_{t+1}/P_{t+1}; \xi_{t+1})(1+i_t)\frac{P_t}{P_{t+1}} \right], \tag{2}$$

where i_t is the riskless nominal interest rate on one-period obligations purchased in period t.

Optimal substitution between real money balances and expenditure leads to a static first-order condition of the form

$$\frac{u_m(Y_t, M_t/P_t; \xi_t)}{u_c(Y_t, M_t/P_t; \xi_t)} = \frac{i_t}{1+i_t},$$

under the assumption that zero interest is paid on the monetary base, and that preferences are such that we can exclude the possibility of a corner solution with zero money balances. If both consumption and liquidity services are normal goods, this equilibrium condition can be solved uniquely for the level of real balances $L(Y_t, i_t; \xi_t)$ that satisfy it in the case of any positive nominal interest rate.[7] The equilibrium relation can then equivalently be written as a pair of inequalities

$$\frac{M_t}{P_t} \geq L(Y_t, i_t; \xi_t), \tag{3}$$

$$i_t \geq 0, \tag{4}$$

together with the "complementary slackness" condition that at least one must hold with equality at any time. (Here we define $L(Y, 0; \xi) = \bar{m}(Y; \xi)$, the minimum level of real balances for which $u_m = 0$, so that the function L is continuous at $i = 0$.)

Household optimization similarly requires that the paths of aggregate real expenditure and the price index satisfy the bounds

$$\sum_{T=t}^{\infty} \beta^T E_t \left[u_c(Y_T, M_T/P_T; \xi_T) Y_T + u_m(Y_T, M_T/P_T; \xi_T)(M_T/P_T) \right] < \infty, \tag{5}$$

$$\lim_{T \to \infty} \beta^T E_t [u_c(Y_T, M_T/P_T; \xi_T) D_T/P_T] = 0 \tag{6}$$

looking forward from any period t, where D_t measures the total nominal value of government liabilities (monetary base plus government debt) at the end of period t. under the monetary-fiscal policy regime. (Condition (5) is required for the existence of a well-defined intertemporal budget constraint, under the assumption that there are no limitations on households' ability to borrow against future income, while the transversality condition (6) must hold if the household exhausts its intertemporal budget constraint.) Conditions (2) – (6) also suffice to imply that the representative household chooses optimal consumption and portfolio plans (including its planned holdings of money balances) given its income expectations and the prices (including financial asset prices) that it faces, while making choices that are consistent with financial market clearing.

Each differentiated good i is supplied by a single monopolistically competitive producer. There are assumed to be many goods in each of an infinite number of "industries";

the goods in each industry j are produced using a type of labor that is specific to that industry, and also change their prices at the same time. Each good is produced in accordance with a common production function

$$y_t(i) = A_t f(h_t(i)),$$

where A_t is an exogenous productivity factor common to all industries, and $h_t(i)$ is the industry-specific labor hired by firm i. The representative household supplies all types of labor as well as consuming all types of goods.[8]

The supplier of good i sets a price for that good at which it supplies demand each period, hiring the labor inputs necessary to meet any demand that may be realized. Given the allocation of demand across goods by of households in response to firm pricing decisions, on the one hand, and the terms on which optimizing households are willing to supply each type of labor on the other, we can show that the nominal profits (sales revenues in excess of labor costs) in period t of the supplier of good i are given by a function

$$\begin{aligned}\Pi(p_t(i), p_t^j, P_t; Y_t, M_t/P_t, \tilde{\xi}_t) \quad \equiv \quad & p_t(i) Y_t (p_t(i)/P_t)^{-\theta} \\ & - \frac{v_h(f^{-1}(Y_t(p_t^j/P_t)^{-\theta}/A_t); \xi_t)}{u_c(Y_t, M_t/P_t; \xi_t)} P_t f^{-1}(Y_t(p_t(i)/P_t)^{-\theta}/A_t),\end{aligned}$$

where p_t^j is the common price charged by the other firms in industry j.[9] (We introduce the notation $\tilde{\xi}_t$ for the complete vector of exogenous disturbances, including variations in technology as well as preferences.) If prices were fully flexible, $p_t(i)$ would be chosen each period to maximize this function.

Instead we suppose that prices remain fixed in monetary terms for a random period of time. Following Calvo (1983), we suppose that each industry has an equal probability of reconsidering its prices each period, and let $0 < \alpha < 1$ be the fraction of industries with prices that remain unchanged each period. In any industry that revises its prices in period t, the new price p_t^* will be the same. This price is implicitly defined by the first-order condition

$$E_t \left\{ \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi_1(p_t^*, p_t^*, P_T; Y_T, M_T/P_T, \tilde{\xi}_T) \right\} = 0. \tag{7}$$

We note furthermore that the stochastic discount factor used to price future profit streams will be given by

$$Q_{t,T} = \beta^{T-t} \frac{u_c(C_T, M_T/P_T; \xi_T)}{u_c(C_t, M_t/P_t; \xi_t)}. \tag{8}$$

Finally, the definition (1) implies a law of motion for the aggregate price index of the form

$$P_t = \left[(1-\alpha) p_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{9}$$

Equations (7) and (9) jointly determine the evolution of prices given demand conditions, and represent the aggregate-supply block of our model.

It remains to specify the monetary and fiscal policies of the government.[10] In order to address the question whether "quantitative easing" represents an additional tool of policy, we shall suppose that the central bank's operating target for the short-term nominal interest rate is determined by a feedback rule in the spirit of the Taylor rule (Taylor, 1993),

$$i_t = \phi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t), \tag{10}$$

where now $\tilde{\xi}_t$ may also include exogenous disturbances in addition to the ones listed above, to which the central bank happens to respond. We shall assume that the function ϕ is non-negative for all values of its arguments (otherwise the policy would not be feasible, given the zero lower bound), but that there are conditions under which the rule prescribes a zero interest-rate policy. Such a rule implies that the central bank supplies the quantity of base money that happens to be demanded at the interest rate given by this formula; hence (10) implies a path for the monetary base, in the case that the value of ϕ is positive. However, under those conditions in which the value of ϕ is zero, the policy commitment (10) implies only a lower bound on the monetary base that must be supplied. In these circumstances, we may ask whether it matters whether a greater or smaller quantity of base money is supplied.

We shall suppose that the central bank's policy in this regard is specified by a base-supply rule of the form

$$M_t = P_t L(Y_t, \phi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t); \xi_t)\psi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t), \tag{11}$$

where the multiplicative factor ψ satisfies

$$(i)\psi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t) \geq 1,$$

$$(ii)\psi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t) = 1 \; if \; \phi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t) > 0$$

for all values of its arguments. (Condition (ii) implies that $\psi = 1$ whenever $i_t > 0$.) Note that a base-supply rule of this form is consistent with both the interest-rate operating target specified in (10) and the equilibrium relations (3) – (4). The use of "quantitative easing" as a policy tool can then be represented by a choice of a function ψ that is greater than 1 under some circumstances.

It remains to specify which sort of assets should be acquired (or disposed of) by the central bank when it varies the size of the monetary base. We shall suppose that the asset side of the central-bank balance sheet may include any of k different types of securities, distinguished by their state-contingent returns. At the end of period t, the vector of nominal values of central-bank holdings of the various securities is given by $M_t\omega_t^m$, where ω_t^m is a vector of central-bank portfolio shares. These shares are in turn determined by a policy rule of the form

$$\omega_t^m = \omega^m(P_t/P_{t-1}, Y_t; \tilde{\xi}_t), \tag{12}$$

where the vector-valued function $\omega^m(\cdot)$ has the property that its components sum to 1 for all possible values of its arguments. The fact that $\omega^m(\cdot)$ depends on the same arguments as $\phi(\cdot)$ means that we allow for the possibility that the central bank changes its policy when the zero bound is binding (for example, buying assets that it would not hold at any other time); the fact that it depends on the same arguments as $\psi(\cdot)$ allows us to specify changes in the composition of the central-bank portfolio as a function of the particular kinds of purchases associated with "quantitative easing."

The payoffs on these securities in each state of the world are specified by exogenously given (state-contingent) vectors a_t and b_t and matrix F_t. A vector of asset holdings z_{t-1} at the end of period $t-1$ results in delivery to the owner of a quantity $a_t'z_{t-1}$ of money, a quantity $b_t' z_{t-1}$ of the consumption good, and a vector $F_t z_{t-1}$ of securities that may be traded in the period t asset markets, each of which may depend on the state of the world in period t. This flexible specification allows us to treat a wide range of types of assets that may differ as to maturity, degree of indexation, and so on.[11]

The gross nominal return $R_t(j)$ on the jth asset between periods $t-1$ and t is then given by

$$R_t(j) = \frac{a_t(j) + P_t b_t(j) + q_t' F_t(\cdot, j)}{q_{t-1}(j)}, \tag{13}$$

where q_t is the vector of nominal asset prices in (ex-dividend) period t trading. The absence of arbitrage opportunities implies as usual that equilibrium asset prices must satisfy

$$q_t' = \sum_{T \geq t+1} E_t Q_{t,T} [a_T' + P_t b_T'] \prod_{s=t+1}^{T-1} F_s, \tag{14}$$

where the stochastic discount factor is again given by (8). Under the assumption that no interest is paid on the monetary base, the nominal transfer by the central bank to the Treasury each period is equal to

$$T_t^{cb} = R_t' \omega_{t-1}^m M_{t-1} - M_{t-1}, \tag{15}$$

where R_t is the vector of returns defined by (13).

We specify fiscal policy in terms of a rule that determines the evolution of total government liabilities D_t, here defined to be inclusive of the monetary base, as well as a rule that specifies the composition of outstanding non-monetary liabilities (debt) among different types of securities that might be issued by the government. We shall suppose that the evolution of total government liabilities is in accordance with a rule of the form

$$\frac{D_t}{P_t} = d\left(\frac{D_{t-1}}{P_{t-1}}, \frac{P_t}{P_{t-1}}, Y_t; \tilde{\xi}_t\right), \tag{16}$$

which specifies the acceptable level of real government liabilities as a function of the pre-existing level of real liabilities and various aspects of current macroeconomic conditions. This notation allows for such possibilities as an exogenously specified state-contingent target for real government liabilities as a proportion of GDP, or for the government budget deficit (inclusive of interest on the public debt) as a share of GDP, among others.

The part of total liabilities that consists of base money is specified by the base rule (11). We suppose, however, that the rest may be allocated among any of a set of different types of securities that may be issued by the government; for convenience, we assume that this is a subset of the set of k securities that may be purchased by the central bank. If ω^f_{jt} indicates the share of government debt (non-monetary liabilities) at the end of period t that is of type j, then the flow government budget constraint takes the form

$$D_t = R_t' \omega^f_{t-1} B_{t-1} - T^{cb}_t - T^h_t,$$

where $B_t \equiv D_t - M_t$ is the total nominal value of end-of-period non-monetary liabilities, and T^h_t is the nominal value of the primary budget surplus (taxes net of transfers, if we abstract from government purchases). This identity can then be inverted to obtain the net tax collections T^h_t implied by a given rule (16) for aggregate public liabilities; this depends in general on the composition of the public debt as well as on total borrowing.

Finally, we suppose that debt management policy (i.e. the determination of the composition of the government's non-monetary liabilities at each point in time) is specified by a function

$$\omega_t^f = \omega^f(P_t/P_{t-1}, Y_t; \tilde{\xi}_t), \tag{17}$$

specifying the shares as a function of aggregate conditions, where the vector-valued function ω^f also has components that sum to 1 for all possible values of its arguments. Together, the two relations (16) and (17) complete our specification of fiscal policy, and close our model.[12]

We may now define a rational-expectations equilibrium as a collection of stochastic processes $\{p_t^*, P_t, Y_t, i_t, q_t, M_t, \omega_t^m, D_t, \omega_t^f\}$, with each endogenous variable specified as a function of the history of exogenous disturbances to that date, that satisfy each of conditions (2) – (6) of the aggregate-demand block of the model, conditions (7) and (9) of the aggregate-supply block, the asset-pricing relations (14), conditions (10) – (12) specifying monetary policy, and conditions (16) – (17) specifying fiscal policy each period. We then obtain the following irrelevance result for the specification of certain aspects of policy.

Proposition. The set of paths for the variables $\{p_t^*, P_t, Y_t, i_t, q_t, D_t\}$ that are consistent with the existence of a rational-expectations equilibrium are independent of the specification of the functions ψ in equation (11), ω^m in equation (12), and ω^f in equation (17).

The reason for this is fairly simple. The set of restrictions on the processes $\{p_t^*, P_t, Y_t, i_t, q_t, D_t\}$ implied by our model can be written in a form that does not involve the variables

$\{M_t, \omega_t^m, \omega_t^f\}$, and hence that does not involve the functions ψ, ω^m, or ω^f.

To show this, let us first note that for all $m \geq \bar{m}(C;\xi)$,

$$u(C, m; \xi) = u(C, \bar{m}(C;\xi); \xi),$$

as additional money balances beyond the satiation level provide no further liquidity services. By differentiating this relation, we see further that $u_c(C, m; \xi)$ does not depend on the exact value of m either, as long as m exceeds the satiation level. It follows that in our equilibrium relations, we can replace the expression $u_c(Y_t, M_t/P_t; \xi_t)$ by

$$\lambda(Y_t, P_t/P_{t-1}; \xi_t) \equiv u_c(Y_t, L(Y_t, \phi(P_t/P_{t-1}, Y_t; \xi_t); \xi_t); \xi_t),$$

using the fact that (3) holds with equality at all levels of real balances at which u_c depends on the level of real balances. Hence we can write u_c as a function of variables other than M_t/P_t, without using the relation (11), and so in a way that is independent of the function ψ.

We can similarly replace the expression $u_m(Y_t, M_t/P_t; \xi_t)(M_t/P_t)$ that appears in (5) by

$$\mu(Y_t, P_t/P_{t-1}; \xi_t) \equiv u_m(Y_t, L(Y_t, \phi(P_t/P_{t-1}, Y_t; \xi_t); \xi_t); \xi_t) L(Y_t, \phi(P_t/P_{t-1}, Y_t; \xi_t); \xi_t),$$

since M_t/P_t must equal $L(Y_t, \phi(P_t/P_{t-1}, Y_t; \xi_t); \xi_t)$ when real balances do not exceed the satiation level, while $u_m = 0$ when they do. Finally, we can express nominal profits in period t as a function

$$\tilde{\Pi}(p_t(i), p_t^j, P_t; Y_t, P_t/P_{t-1}, \tilde{\xi}_t),$$

after substituting $\lambda(Y_t, P_t/P_{t-1}; \xi_t)$ for the marginal utility of real income in the wage demand function that is used (see Woodford, 2003, chapter 3) in deriving the profit function Π. Using these substitutions, we can write each of the equilibrium relations (2), (5), (6), (7), and (14) in a way that no longer makes reference to the money supply.

It then follows that in a rational-expectations equilibrium, the variables $\{p_t^*, P_t, Y_t, i_t, q_t, D_t\}$ must each period satisfy the relations

$$\lambda(Y_t, P_t/P_{t-1}; \xi_t) = \beta E_t \left[\lambda(Y_{t+1}, P_{t+1}/P_t; \xi_{t+1})(1+i_t)\frac{P_t}{P_{t+1}} \right], \tag{18}$$

$$\sum_{T=t}^{\infty} \beta^T E_t \left[\lambda(Y_T, P_T/P_{T-1}; \xi_T) Y_T + \mu(Y_T, P_T/P_{T-1}; \xi_T) \right] < \infty, \tag{19}$$

$$\lim_{T\to\infty} \beta^T E_t [\lambda(Y_T, P_T/P_{T-1}; \xi_T) D_T/P_T] = 0, \tag{20}$$

$$q_t' = \frac{P_t}{\lambda(Y_t, P_t/P_{t-1}; \xi_t)} \sum_{T \geq t+1} \beta^{T-t} E_t \lambda(Y_T, P_T/P_{T-1}; \xi_T)[P_T^{-1} a_T' + b_T'] \prod_{s=t+1}^{T-1} F_s, \tag{21}$$

$$E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda(Y_T, P_T/P_{T-1}; \xi_T) P_T^{-1} \tilde{\Pi}_1(p_t^*, p_t^*, P_T; Y_T, P_T/P_{T-1}, \tilde{\xi}_T) \right\} = 0, \tag{22}$$

along with relations (9), (10), and (16) as before. Note that none of these equations involve the variables $\{M_t, \omega_t^m, \omega_t^f\}$, nor do they involve the functions ψ, ω^m, or ω^f.

Furthermore, this is the complete set of restrictions on these variables that are required in order for them to be consistent with a rational-expectations equilibrium. For given any processes $\{p_t^*, P_t, Y_t, i_t, q_t, D_t\}$ that satisfy the equations just listed in each period, the implied path of the money supply is given by (11), which clearly has a solution; and this path for the money supply necessarily satisfies (3) and the complementary slackness condition, as a result of our assumptions about the form of the function ψ. Similarly, the implied composition of the central-bank portfolio and of the public debt at each point in time are given by (12) and (17). We then have a set of processes that satisfy all of the requirements for a rational-expectations equilibrium, and the result is established.

1.2 Discussion

This proposition implies that neither the extent to which quantitative easing is employed when the zero bound binds, nor the nature of the assets that the central bank may purchase through open-market operations, has any effect on whether a deflationary price-level path will represent a rational-expectations equilibrium. Hence the notion that expansions of the monetary base represent an additional tool of policy, independent of the specification of the rule for adjusting short-term nominal interest rates, is not supported by our general-equilibrium analysis of inflation and output determination. If the commitments of policymakers regarding the rule by which interest rates will be set on the one hand, and the rule which total private-sector claims on the government will be allowed to grow on the other, are fully credible, then it is only the choice of those commitments that matters. Other aspects of policy should matter in practice, then, only insofar as they help to signal the nature of policy commitments of the kind just mentioned.

Of course, the validity of our result depends on the reasonableness of our assumptions, and these deserve further discussion. Like any economic model, ours abstracts from the complexity of actual economies in many respects. This raises the question whether we may have abstracted from features of actual economies that are crucial for a correct understanding of the issues under discussion.

Many readers may suspect that an important omission is the neglect of "portfolio-balance effects," which play an important role in much recent discussion of the policy options that would remain available to the Fed in the event that the zero bound is reached by the federal funds rate.[13] The idea is that a central bank should be able to lower longer-term interest rates even when overnight rates are already at zero, through purchases of longer-maturity government bonds, shifting the composition of the public debt in the hands of the public in a way that affects the term structure of interest rates. (As it is generally admitted in such discussions that base money and very short-term Treasury securities have become near-perfect substitutes once short-term interest rates have fallen to zero, the desired effect should be achieved equally well by a shift in the maturity structure of Treasury securities held by the central bank, without any change in the monetary base, as by an open-market purchase of long bonds with newly created base money.)

There are evidently no such effects in our model, resulting either from central-bank securities purchases or debt management by the Treasury. But this is not, as some might expect, because we have simply assumed that bonds of different maturities (or for that matter, other kinds of assets that the central bank might choose to purchase instead of the shortest-maturity Treasury bills) are perfect substitutes. Our framework allows for different assets that the central bank may purchase to have different risk characteristics (different state-contingent returns), and our model of asset-market equilibrium incorporates those term premia and risk premia that are consistent with the absence of arbitrage opportunities.

Our conclusion differs from the one in the literature on portfolio-balance effects for a different reason. The classic theoretical analysis of portfolio-balance effects assumes a representative investor with mean-variance preferences. This has the implication that if the supply of assets that pay off disproportionately in certain states of the world is increased (so that the extent to which the representative investor's portfolio pays off in those states must also increase), the relative marginal valuation of income in those particular states is reduced, resulting in a lower relative price for the assets that pay off in those states. But in our general-equilibrium asset-pricing model, there is no such effect. The marginal utility to the representative household of additional income in a given state of the world depends on the household's consumption in that state, not on the aggregate payoff of its asset portfolio in that state. And changes in the composition of the securities in the hands of the public *don't* change the state-contingent consumption of the representative household — this depends on equilibrium output, and while output is endogenous, we have shown that the equilibrium relations that determine it do not involve the functions ψ, ω^m, or ω^f.[14]

Our assumption of complete financial markets and no limits on borrowing against future income may also appear extreme. However, the assumption of complete financial markets is only a convenience, allowing us to write the budget constraint of the representative household in a simple way. Even in the case of incomplete markets, each of the assets that is traded will be priced according to (14), where the stochastic discount factor is given by (8), and once again there will be a set of relations to determine output, goods prices, and asset prices that do not involve ψ, ω^m, or ω^f. The absence of borrowing limits is also innocuous, at least in the case of a representative-household model, since in equilibrium the representative household must hold the entire net supply of financial claims on the government; as long as the fiscal rule (16) implies positive government liabilities at each date, then, any borrowing limits that might be assumed can never bind in equilibrium. Borrowing limits can matter more in the case of a model with heterogeneous households. But in this case, the effects of open-market operations should depend not merely on which sorts of assets are purchased and which sorts of liabilities are issued to finance the purchases, but also on the way in which the central bank's trading profits are eventually rebated to the private sector (with what delay, and how distributed across the heterogeneous households), as a result of the specification of fiscal policy. The effects will not be mechanical consequences of the change in the composition of the assets in the hands of the public, but instead will result from the fiscal transfers to which the transaction gives rise; and it is unclear how quantitatively significant such effects should be.

Indeed, leaving aside the question of whether there exists a clear theoretical foundation for the existence of portfolio-balance effects, there is not a great deal of empirical support for quantitatively significant effects. The attempt of the U.S. to separately target short-term and long-term interest rates under "Operation Twist" in the early 1960's is generally regarded as having had a modest effect at best on the term structure.[15] The empirical literature that has sought to estimate the effects of changes in the composition of the public debt on relative yields has also, on the whole, found effects that are not quantitatively large when present at all.[16] For example, Agell and Persson (1992) summarize their findings as follows: "It turned out that these effects were rather small in magnitude, and that their numerical values were highly volatile. Thus the policy conclusion to be drawn seems to be that there is not much scope for a debt management policy aimed at systematically affecting asset yields."

Moreover, even if one supposes that large enough changes in the composition of the portfolio of securities left in the hands of the private sector can substantially affect yields, it is not clear how relevant such an effect should be for real activity and the evolution of goods prices. For example, Clouse *et al.* (2003) argue that a sufficiently large reduction in the number of long-term Treasuries in the hands of the public should be able to lower the market yield on those securities relative to short rates, owing to the fact that certain institutions will find it important to hold long-term Treasury securities even when they offer an unfavorable yield.[17] But even if this is true, the fact that these institutions have idiosyncratic reasons to hold long-term Treasuries — and that, in equilibrium, no one else holds any or plays any role in pricing them — means that the lower observed yield on long-term Treasuries may not correspond to any reduction in the perceived cost of long-term borrowing for other institutions. If one is able to reduce the long bond rate only by decoupling it from the rest of the structure of interest rates, and from the cost of financing long-term investment projects, it is unclear that such a reduction should do much to stimulate economic activity or to halt deflationary pressures.

Hence we are not inclined to suppose that our irrelevance proposition represents so poor an approximation to reality as to deprive it of practical relevance. Even if the effects of open-market operations under the conditions described in the proposition are not exactly zero, it seems unlikely that they should be large. In our view, it is more important to note that our irrelevance proposition depends on an assumption that interest-rate policy is specified in a way that implies that these open-market operations have no consequences for interest-rate policy, either immediately (which is trivial, since it would not be possible for them to *lower* current interest rates, which is the only effect that would be desired), *or at any subsequent date* either. We have also specified fiscal policy in a way that implies that the contemplated open-market operations have no effect on the evolution of total government liabilities $\{D_t\}$ either — again, neither immediately nor at any later date. While we think that these definitions make sense, as a way of isolating the pure effects of open-market purchases of assets by the central bank from either interest-rate policy on the one hand and from fiscal policy on the other, it is important to note that someone who recommends monetary expansion by the central bank may intend for this to have consequences of one or both of these other sorts.

For example, when it is argued that surely nominal aggregate demand could be stimulated by a "helicopter drop of money", the thought experiment that is usually contemplated is not simply a change in the function ψ in our policy rule (11). First of all, it is typically supposed that the expansion of the money supply will be *permanent.* If this is the case, then the function ϕ that defines interest-rate policy is also being changed, in a way that will become relevant at some future date, when the money supply no longer exceeds the satiation level.[18] Second, the assumption that the money supply is increased through a "helicopter drop" rather than an open-market operation implies a change in *fiscal* policy as well. The operation increases the value of nominal government liabilities, and it is generally at least tacitly assumed that this is a permanent increase as well. Hence the experiment that is imagined is not one that our irrelevance proposition implies should have no effect on the equilibrium path of prices.

Even more importantly, we should stress that our irrelevance result applies only *given* a correct private-sector understanding of the central bank's commitments regarding future policy, which may not be present. We have just argued that the key to lowering long-term interest rates, in a way that actually provides an incentive for increased spending, is by changing expectations regarding the likely future path of short rates, rather than through intervention in the market for long-term Treasuries. As a logical matter, this need not require any open-market purchases of long-term Treasuries at all. Nonetheless, the private sector may be uncertain about the nature of the central bank's policy commitment, and so may scrutinize the bank's current actions for further clues. In practice, the management of private-sector expectations is an art of considerable subtlety, and shifts in the portfolio of the central bank could be of some value in making credible to the private sector the central bank's own commitment to a particular kind of future policy, as we discuss further in section 6. "Signalling" effects of this kind are often argued to be an important reason for the effectiveness of interventions in foreign-exchange markets, and might well provide a justification for open-market policy when the zero bound binds.[19]

We do not wish, then, to argue that asset purchases by the central bank are necessarily pointless under the circumstances of a binding zero lower bound on short-term nominal interest rates. However, we do think it important to observe that insofar as such actions can have any effect, it is not because of any necessary or mechanical consequence of the shift in the portfolio of assets in the hands of the private sector itself. Instead, any effect of such actions must be due to the way in which they change expectations regarding future interest-rate policy, or, perhaps, the future evolution of total nominal government liabilities. In sections 6 and 7 we discuss reasons why open-market purchases by the central bank might plausibly have consequences for expectations of these types. But since it is only through effects on expectations regarding future policy that these actions can matter, we shall focus our attention on the question of what kind of commitments regarding future policy are in fact to be desired. And this question can be addressed without explicit consideration of the role of open-market operations by the central bank of any kind. Hence we shall simplify our model — abstracting from monetary frictions and the structure of government liabilities altogether — and instead consider how it is desirable for interest-rate policy to be conducted, and what kind of commitments about this policy it is desirable to make in advance.

2 How Severe a Constraint is the Zero Bound?

We turn now to the question of the way in which the existence of the zero bound restricts the degree to which a central bank's stabilization objectives, with regard to both inflation and real activity, can be achieved, even under ideal policy. It follows from our discussion in the previous section that the zero bound does represent a genuine constraint. The differences among alternative policies that are relevant to the degree to which stabilization objectives are achieved having only to do with the implied evolution of short-term nominal interest rates, and the zero bound obviously constrains the ways in which this instrument can be used, though it remains to be seen how relevant this constraint may be.

Nonetheless, we shall see that it is not at all the case that there is nothing that a central bank can do to mitigate the severity of the destabilizing impact of the zero bound. The reason is that inflation and output do not depend solely upon the current level of short-term nominal interest rates, or even solely upon the history of such rates up until the current time (so that the current level of interest rates would be the only thing that could possibly changed in response to an unanticipated disturbance). The expected character of *future* interest-rate policy is also a critical determinant of the degree to which the central bank achieves its stabilization objectives, and this allows an important degree of scope for policy to be improved upon, even when there is little choice about the current level of short-term interest rates.

In fact, the management of expectations is the key to successful monetary policy at *all* times, and not just in those relatively unusual circumstances when the zero bound is reached. The effectiveness of monetary policy has little to do with the direct effect of changing the level of overnight interest rates, since the current cost of maintaining cash balances overnight is of fairly trivial significance for most business decisions. What actually matters is the private sector's anticipation of the *future* path of short rates, as this determines equilibrium long-term interest rates, as well as equilibrium exchange rates and other asset prices — all of which are quite relevant for many current spending decisions, hence for optimal pricing behavior as well. The way in which short rates are managed matters because of the signals that it gives about the way in which the private sector can expect them to be managed in the future. But there is no reason to suppose that expectations regarding future monetary policy, and hence expectations regarding the future evolution of nominal variables more generally, should change only insofar as the current level of overnight interest rates changes. A situation in which there is no decision to be made about the current level of overnight rates (as in Japan at present) is one which brings the question of what expectations regarding future policy one should wish to create more urgently to the fore, but this is in fact the correct way to think about sound monetary policy at all times.

Of course, there is no question to be faced about what future policy one should wish for people to expect if there is no possibility of committing oneself to a different sort of policy in the future than one would otherwise have pursued, as a result of the constraints that are currently faced (and that make desirable the change in expectations). This means that the private sector must be convinced that the central bank will not conduct policy in a way that is *purely forward-looking*, i.e. taking account at each point in time only of the possible paths that the economy could follow from that date onward. For example, we will show that it is undesirable for the central bank to pursue a certain inflation target, once the zero bound is expected no longer to prevent it from being achieved, even in the case that the pursuit of this target *would* be optimal if the zero bound did not exist (or would never bind under an optimal policy). The reason is that an expectation that the central bank will pursue the fixed inflation target after the zero bound ceases to bind gives people no reason to hold the kind of expectations, while the bound is binding, that would mitigate the distortions created by it. A history-dependent inflation target[20] — if the central bank's commitment to it can be made credible — can instead yield a superior outcome.

But this too is an important feature of optimal policy rules more generally (see, e.g. Woodford, 2003, chapter 7). Hence the analytical framework and institutional arrangements used to make monetary policy need not be changed in any fundamental way in order to deal with the special problems created by a "liquidity trap". As we explain in section 4, the optimal policy in the case of a binding zero bound can be implemented through a targeting procedure that represents a straightforward generalization of a policy that would be optimal even if the zero bound were expected never to bind.

2.1 Feasible Responses to Fluctuation in the Natural Rate of Interest

In order to characterize the way in which stabilization policy is constrained by the zero bound, we shall make use of a log-linear approximation to the structural equations of section 2, of a kind that is often employed in the literature on optimal monetary stabilization policy (see, e.g. Clarida *et al.*, 1999; Woodford, 2003). Specifically, we shall log-linearize the structural equations of our model (except for the zero bound (4)) around the paths of inflation, output and interest rates associated with a zero-inflation steady state, in the absence of disturbances ($\xi_t = 0$). We choose to expand around these particular paths because the zero-inflation steady state represents optimal policy in the absence of disturbances.[21] In the event of small enough disturbances, optimal policy will still involve paths in which inflation, output and interest rates are at all times close to those of the zero-inflation steady state. Hence an approximation to our equilibrium conditions that is accurate in the case of inflation, output and interest rates near those values will allow an accurate approximation to the optimal responses to disturbances in the case that the disturbances are small enough.

In the zero-inflation steady state, it is easily seen that the real rate of interest is equal to $\bar{r} \equiv \beta^{-1} - 1 > 0$, and this is also the steady-state nominal interest rate. Hence in the case of small enough disturbances, optimal policy will involve a nominal interest rate that is always positive, and the zero bound will not be a binding constraint. (Optimal policy in this case is characterized in the references cited in the previous paragraph.) However, we are interested in the case in which disturbances are at least occasionally large enough for the zero bound to bind, i.e. for it to prevent attainment of the outcome that would be optimal in the absence of such a bound. A case in which it is possible to rigorously consider this problem using only a log-linear approximation to the structural equations is that in which we suppose that the lower bound on nominal interest is not much below $\bar{r}$. We can arrange for this gap to be as small as we may wish, without changing other crucial parameters of the model such as the assumed rate of time preference, by supposing that interest is paid on the monetary base at a rate $i^m \geq 0$ that cannot (for some institutional reason) be reduced. Then the lower bound on interest rates actually becomes

$$i_t \geq i^m \tag{23}$$

We shall characterize optimal policy subject to a constraint of the form (23), in the case that *both* a bound on the amplitude of disturbances $||\xi||$ and the size of the steady-state opportunity cost of holding money $\bar{\delta} \equiv (\bar{r} - i^m)/(1 + \bar{r}) > 0$ are small enough. Specifically, both our structural equations and our characterization of the optimal responses of inflation, output and interest rates to disturbances will be required to be exact only up to a residual of order $\mathcal{O}(||\xi, \bar{\delta}||^2)$. We shall then hope (without here seeking to verify this) that our characterization of optimal policy in the case of a small opportunity cost of holding money and small disturbances is not too inaccurate in the case of an opportunity cost of several percentage points (the case in which $i^m = 0$) and disturbances large enough to cause the natural rate of interest to vary by several percentage points (as will be required in order for the zero bound to bind).

As shown in Woodford (2003), the log-linear approximate equilibrium relations may be summarized by two equations each period, a forward-looking “IS relation”

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n), \tag{24}$$

and a forward-looking “AS relation” (or “New Keynesian Phillips curve”)

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t. \tag{25}$$

Here $\pi_t \equiv \log(P_t/P_{t-1})$ is the inflation rate, x_t is a welfare-relevant output gap, and i_t is now the continuously compounded nominal interest rate (corresponding to $\log(1 + i_t)$ in the notation of section 2). The terms u_t and r_t^n are composite exogenous disturbance terms that shift the two equations; the former is commonly referred to as a “cost-push disturbance”, while the latter indicates exogenous variation in the Wicksellian “natural rate of interest”, i.e. the equilibrium real rate of interest in the case that output is at all times equal to the natural rate of output. The coefficients σ and κ are both positive, while $0 < \beta < 1$ is again the utility discount factor of the representative household.

Equation (24) is a log-linear approximation to (2), while (25) is derived by log-linearizing (7) – (9) and then eliminating $\log(p_t^*/P_t)$. We omit the log-linear version of the money-demand relation (3), since we are here interested solely in characterizing the possible equilibrium paths of inflation, output, and interest rates, and we may abstract from the question of what the required path for the monetary base may be that is associated with any such equilibrium in considering this. (It suffices that there exist a monetary base that will satisfy the money-demand relation in each case, and this will be true as long as the interest-rate bound is satisfied.) The other equilibrium requirements of section 2 can be ignored in the case that we are interested only in possible equilibria that remain forever near the zero-inflation steady state, as they are automatically satisfied in that case.

Equations (24) – (25) represent a pair of equations each period to determine inflation and the output gap, given the central bank's interest-rate policy. We shall seek to compare alternative possible paths for inflation, the output gap, and the nominal interest rate that satisfy these two log-linear equations together with the inequality (23). Note that our conclusions will be identical (up to a scale factor) in the event that we multiply the amplitude of the disturbances and the steady-state opportunity cost $\bar{\delta}$ by any common factor; alternatively, if we measure the amplitude of disturbances in units of $\bar{\delta}$, our results will be independent of the value of $\bar{\delta}$ (to the extent that our log-linear approximation remains valid). Hence we choose the normalization $\bar{\delta} = 1 - \beta$, corresponding to $i^m = 0$, to simplify the presentation of our results. In the case, the lower bound for the nominal interest rate is again given by (4).

2.2 Deflation under Forward-Looking Policy

We begin by considering the degree to which the zero bound impedes the achievement of the central bank's stabilization objectives in the case that the bank pursues a strict inflation target. We interpret this as a commitment to adjust the nominal interest rate so that

$$\pi_t = \pi^* \tag{26}$$

each period, insofar as it is possible to achieve this with some non-negative interest rate. It is easy to verify, by the IS and AS equation, that a necessary condition for this target to be satisfied is:

$$i_t = r_t^n + \pi^* \tag{27}$$

When inflation is on target, the real rate is equal to the natural real rate at all times and the output gap at its long run level. The zero bound, however, prevents (27) from holding if $r_t^n < -\pi^*$. Thus if the natural rate of interest is low, the zero bound frustrates the Central Bank's ability to implement an inflation target. Suppose the inflation target is zero so that $\pi^* = 0$. Then the zero bound is binding if the natural rate of interest is negative, and the Central Bank is unable to achieve its inflation target.

To illustrate this, let us consider the following experiment: Suppose the natural rate of interest is unexpectedly negative in period 0 and reverts back to the steady-state value $\bar{r} > 0$ with a fixed probability in every period. Figure 2 shows the state-contingent paths of the output gap and inflation in the case of three different possible inflation targets π^*. In the figure we assume in period 0 that the natural rate of interest becomes -2 percent per annum and then reverts back to the steady-state value of +4 percent per annum with a probability 0.1 each quarter. Thus the natural rate of interest is expected to be negative for 10 quarters on average at the time that the shock occurs.

The dashed lines in Figure 2 show the state-contingent evolution of the output gap and inflation if the central bank targets zero inflation.[22] The first dashed line shows the equilibrium if the natural rate of interest returns back to steady state in period 1, the next line if it returns in period 2, and so on. The inability of the central bank to set a negative nominal interest rate results in a 12 percent per output gap and 9 percent annual deflation. Since there is a 90 percent chance of the natural rate of interest to remain negative for the next quarter, this creates expectation of future deflation and negative output gap which creates even further deflation. Even if the central bank lowers the short-term nominal interest rate to zero the real rate of return is positive because the private sector expects deflation. The solid line in the figure shows the equilibrium if the central bank targets a one percent inflation target. In this case the private sector expect one percent inflation once out of the trap. This, however, is not enough to offset the minus two percent negative natural rate of interest, so that in equilibrium the private sector expect deflation instead of inflation. The result of this and a negative natural rate of interest is 3 percent annual deflation (when the natural rate of interest is negative) and an output gap of more than 5 percent.

Finally the dotted line shows the evolution of output and inflation if the central bank targets 2 percent inflation. In this case the central bank can satisfy equation (36) even when the natural rate of interest in negative. When the natural rate of interest is minus two percent, the central bank lowers the nominal interest rate to zero. Since the inflation target is two percent, the real rate is minus two percent, which is enough to close the output gap and keep inflation on target. If the inflation target is high enough, therefore, the central bank is able to accommodate a negative natural rate of interest. This is the argument given by Phelps (1972), Summers (1991), and Fischer (1996) for a positive inflation target. Krugman (1998) makes a similar argument, and suggests more concretely that Japan needs a positive inflation target of 4 percent under its current circumstances to achieve negative real rates and curb deflation.

While we see that commitment to a higher inflation target will indeed guard against the need for a negative output gap in periods when the natural rate of interest falls, the price of this solution is the distortions created by the inflation, both when the natural rate of interest is negative and under more normal circumstances as well. Hence the optimal inflation target (from among the strict inflation targeting policies just considered) will be some value that is at least slightly positive, in order to mitigate the distortions created by the zero bound when the natural rate of interest is negative, but not so high as to keep the zero bound from ever binding (see Table 1). In the case of an intermediate inflation target, however (like the one percent target considered in the figure), there is *both* a substantial recession when the natural rate of interest becomes negative, and chronic inflation at all other times. Hence no such policy allows a complete solution of the problem posed by the zero bound in the case that the natural rate of interest is sometimes negative.

Nor can one do better through commitment to any policy rule that is *purely forward-looking* in the sense discussed by Woodford (2000). A purely forward-looking policy is one under which the central bank's action at any time depends only on an evaluation of the possible paths for the central bank's target variables (here, inflation and the output gap) that are possible from the current date forward — neglecting past conditions except insofar as they constrain the economy's possible evolution from here on. In the log-linear model presented above, the possible paths for inflation and the output gap from period t onward depend only on the expected evolution of the natural rate of interest from period t onward. If we assume a Markovian process for the natural rate, as in the numerical analysis above, then any purely forward-looking policy will result in an inflation rate, output gap, and nominal interest rate in period t that depend only on the natural rate in period t — in our numerical example, on whether the natural rate is still negative or has already returned to its long-run steady-state value. It is easily shown in the case of our 2-state example that the optimal state-contingent evolution for inflation and output from among those with this property will be one in which the zero bound binds if and only if the natural rate is in the low state; hence it will correspond to a strict inflation target of the kind just considered, for some π^* between zero and two percent.

But one can actually do considerably better, through commitment to a *history-dependent* policy, in which the central bank's actions will depend on past conditions even though these are irrelevant to the degree to which its stabilization goals could in principle be achieved from then on. We characterize the optimal form of history-dependent policy, and determine the degree to which it improves upon the stabilization of both output and inflation, in the next section.

2.3 The Optimal Policy Commitment

We now characterize optimal monetary policy. We do this by optimizing over the set of all possible state-contingent paths for inflation, output and the short-term nominal interest rate consistent with the log-linearized structural relations (24) and (25), under the assumption (for now) that the expectations regarding future state-contingent policy that are required for such an equilibrium can be made credible to the private sector. In considering the central bank's optimization problem under the assumption that credible commitment is possible regarding future policy, we do not mean to minimize the subtlety of the task of actually communicating such a commitment to the public and making it credible. However, we do not believe that it makes sense to recommend a policy that would systematically seek to achieve an outcome *other* than a rational-expectations equilibrium — that is, we are interested in policies that will have the desired effect even when correctly understood by the public. Optimization under the assumption of credible commitment is simply a way of finding the best possible rational-expectations equilibrium. Once the equilibrium that one would like to bring about has been identified, along with the interest-rate policy that it requires, one can turn to the question of how best to signal these intentions to the public (an issue that we briefly address in section 5 below).

We assume that the government minimizes:

$$\min E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2) \right\} \tag{28}$$

This loss function can be derived by a second order Taylor expansion of the utility of the representative household.[23] The optimal program can be found by a Lagrangian method, extending the methods used in Clarida *et al.* (1999) and Woodford (1999; 2003, chapter 7) to the case in which the zero bound can sometimes bind, as shown by Jung *et al.* (2001). Let us combine the zero bound and the IS equation to yield the inequality:

$$x_t \leq E_t x_{t+1} + \sigma(r_t^n + E_t \pi_{t+1})$$

The Lagrangian for this problem is then:

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2}[\pi_t^2 + \lambda x_t^2] + \phi_{1t}[x_t - x_{t+1} - \sigma\pi_{t+1} - \sigma r_t^n] + \phi_{2t}[\pi_t - \kappa x_t - \beta\pi_{t+1}] \right\}$$

The first order conditions for an optimal policy commitment are shown by Jung *et al.* to be:

$$\pi_t + \phi_{2t} - \phi_{2t-1} - \beta^{-1}\sigma\phi_{1t-1} = 0 \tag{29}$$

$$\lambda x_t + \phi_{1t} - \beta^{-1}\phi_{1t-1} - k\phi_{2t} = 0 \tag{30}$$

$$\phi_{1t} \geq 0, \quad i_t \geq 0, \quad \phi_{1t} i_t = 0 \tag{31}$$

One can not apply standard solution methods for rational expectation models to solve this system due to the complications of the nonlinear constraint (31). The numerical method that we use to solve these equations is described in the appendix.[24] Here we discuss the results that we obtain for the particular numerical experiment considered in the previous section.

What is apparent from the first order conditions (29)-(30) is that optimal policy is history dependent, so that the optimal choice of inflation, the output gap and the nominal interest rates depends on the past values of the endogenous variables. This can be seen by the appearance of lagged value of the Lagrange multipliers in the first order conditions. To get a sense of how this history dependence matters, it is useful to consider the numerical exercise from the last section: Suppose the natural rate of interest becomes negative in period 0 and then reverts back to steady state with a fixed probability in each period.

Figure 3 shows the optimal output gap, inflation and the price level from period 0 to period 25. One observes that the optimal policy involves committing to the creation of an output boom once the natural rate again becomes positive, and hence to the creation of future inflation. Such a commitment stimulates aggregate demand and reduces deflationary pressures while the economy remains in the "liquidity trap", through each of several channels. As Krugman (1998) points out, creating the expectation of future inflation can lower real interest rates, even when the nominal interest rate cannot be reduced. In the context of Krugman's model, it might seem that this requires that inflation be promised quite quickly (by the following "period"). Our fully intertemporal model shows how even the expectation of later inflation — nominal interest rates are not expected to rise to offset it — can stimulate current demand, since in our model current spending decisions depend on real interest-rate expectations far in the future. For the same reason, the expectation that nominal interest rates will be kept low later, when the central bank might otherwise have raised them, will also stimulate spending while the zero bound still binds. And finally, the expectation of higher future income should stimulate current spending, in accordance with the permanent income hypothesis. In addition, prices are less likely to fall, even given the current level of real activity, insofar as future inflation is expected. This reduces the distortions created by deflation itself.

On the other hand, these gains from the change in expectations during the "trap" can be achieved (given rational expectations on the part of the private sector) only if the central bank is expected to actually pursue the inflationary policy after the natural rate returns to its normal level. This will in turn create distortions then, which limits the extent to which this tool is used under an optimal policy. Hence some contraction of output and some deflation occur during the period that the natural rate is negative, even under the optimal policy commitment. It is also worth noting that while the optimal policy involves commitment to a higher price level in the future, the price level will ultimately be stabilized. This is in sharp contrast to a constant positive inflation target that would imply an ever-increasing price level.

Figure 4 shows the corresponding state-contingent nominal interest rate under the optimal commitment, and contrasts it to the evolution of the nominal interest rate under a zero inflation target. To increase inflation expectations in the trap, the central bank commits to keeping the nominal interest rates at zero after the natural rate of interest becomes positive again. In contrast, if the central bank targets zero inflation, it raises the nominal interest rate as soon as the natural rate of interest becomes positive again. The optimal commitment is an example of history-dependent policy, in which the central bank commits to raise the interest rates slowly at the time the natural rate becomes positive in order to affect expectations when the zero bound is binding.

The nature of the additional history-dependence of the optimal policy may perhaps be more easily seen if we consider the evolution of inflation, output and interest rates under a single possible realization of the random fundamentals. Figure 5 compares the equilibrium evolution of all three variables, both under the zero inflation target and under optimal policy, in the case that the natural rate of interest is negative for 15 quarters ($t = 0$ through 14), though it is not known until quarter 15 that the natural rate will return to its normal level in that quarter. Under the optimal policy, the nominal interest rate is kept at zero for five more quarters ($t = 15$ through 19), whereas it immediately returns to its long-run steady-state level in quarter 15 under the forward-looking policy. The consequence of the anticipation of policy of this kind is that both the contraction of real activity and the deflation that occur under the strict inflation target are largely avoided, as shown in the second and third panels of the figure.

3 Implementing Optimal Policy

We turn now to the question of how policy should be conducted in order to bring about the optimal equilibrium characterized in the previous section. The question of the implementation of optimal policy remains a non-trivial one, even after the optimal state-contingent evolutions of all variables have been identified, for in general the solution obtained for the optimal state-contingent path of the policy instrument (i.e. the short-term nominal interest rate) does *not* represent in itself a useful description of a policy rule.[25] For example, in the context of the present model, a commitment to a state-contingent nominal interest-rate path, even when fully credible, does not imply determinate rational-expectations equilibrium paths for inflation and output; it is instead necessary for the central bank to be committed (and understood to be committed) to a particular way of responding to deviations of inflation and the output gap from their desired evolution. Another problem is that a complete description of the optimal state-contingent interest-rate path is unlikely to be feasible. In the previous section, we showed that it is possible to characterize (at least numerically) the optimal state-contingent interest-rate path in the case of one very particular kind of stochastic process for the natural rate of interest. But a solution of this kind allowing for all the possible states of belief about the probabilities of various future evolutions of the natural rate (and disturbances to the aggregate-supply relation as well) would be difficult to write down, let alone to explain to the public.

Here we show that optimal policy can nonetheless be implemented through commitment to a *policy rule* that specifies the central bank's short-run targets at each point in time as a (fairly simple) function of what has occurred prior to that date.

How can the optimal policy be implemented? One may be tempted to believe that our suggested policy is not entirely realistic or operational. Figures 3 and 4, for example, indicate that the optimal policy involves a complicated state contingent plan for the nominal interest rate, that may be hard to communicate to the public. Furthermore, it may appear that it depends on a knowledge of a special statistical process for the natural rate of interest, that is in practice hard to estimate. Our discussion of the fixed inflation target suggest that the effectiveness of increasing inflation expectation to close the output gap depends on the difference between the announced inflation target and the natural rate of interest. It may, therefore, seem crucial to estimate the natural rate of interest to implement the optimal policy. Below we show the striking result that the optimal policy rule can be implemented without any estimate or knowledge of the statistical process for the natural rate of interest. This is an example of a robustly optimal direct policy rule of the kind discussed in Giannoni and Woodford (2002) for the case of a general class of linear-quadratic policy problems. An interesting feature of the present example is that we show how to construct an robustly optimal rule in the same spirit, in a case where not all of the relevant constraints are linear (owing to the fact that the zero bound binds at some times and not at others).

3.1 An Optimal Targeting Rule

To implement the rule proposed here, the central bank need only observe the price level and the output gap. The rule suggested replicates exactly the history dependence discussed in the last section. The rule is implemented as follows.

[i] In each and every period, there is a predetermined price-level target p_t^*. The Central

Bank chooses the interest rate i_t to achieve the target relation

$$\tilde{p}_t = p_t^* \tag{32}$$

if this is possible; if it is not possible, even by lowering the nominal interest rate to zero, then $i_t = 0$. Here $\tilde{p}_t$ is an output-gap adjusted price index,[26] defined by

$$\tilde{p}_t \equiv p_t + \frac{\lambda}{\kappa} x_t.$$

[ii] The target for the next period is then determined as

$$p_{t+1}^* = p_t^* + \beta^{-1}(1 + \kappa\sigma)\Delta_t - \beta^{-1}\Delta_{t-1} \tag{33}$$

where Δ_t is the target shortfall in period t

$$\Delta_t \equiv p_t^* - \tilde{p}_t. \tag{34}$$

It can be verified that this rule does indeed achieve the optimal commitment solution. If the price-level target is not reached, because of the zero bound, the central bank increases its target for the next period. This, in turn, increases inflation expectations further in the trap, which is exactly what is needed to reduce the real interest rate.

Figure 6 shows how the price-level target p_t^* would evolve over time, depending on the number of periods for which the natural rate of interest remains negative, in the same numerical experiment as in Figure 3. (Here the solid lines show the evolution of the gap-adjusted price level $\tilde{p}_t$, while the dashed lines show the evolution of p_t^*.) One observes that the target price level is ratcheted steadily higher during the period in which the natural rate remains negative, as the actual price level continues to fall below the target by an increasing amount. Once the natural rate of interest becomes positive again, the degree to which the gap-adjusted price level undershoots the target begins to shrink, although the target often continues to be undershot (as the zero bound continues to bind) for several more quarters. (How long this is true depends on how high the target price level has risen relative to the actual index; it will be higher the longer the time for which the natural rate has been negative.) As the degree of undershooting begins to shrink, the price-level target begins to fall again, as a result of the dynamics specified by (33). This hastens the date at which the target can actually be hit with a non-negative interest rate. Once the target ceases to be undershot, it no longer changes, and the central bank targets and achieves a new constant value for the gap-adjusted price level $\tilde{p}_t$, one slightly higher than the target in place before the disturbance occurred.

Note that this approach to implementing optimal policy gives an answer to the question whether there is any point in announcing an inflation target (or price-level target) if one knows that it is extremely unlikely that in the short run it can be achieved, owing to the fact that the zero bound is likely to continue to bind. The answer here is yes. The central bank wishes to make the private sector aware of its commitment to the time-varying price-level target described by (32) – (34), since eventually it *will* be able to hit the target, and the anticipation of that fact (i.e. of the level that the price level will eventually reach, as a result of the policies that the bank will follow after the natural rate of interest again becomes positive) while the natural rate is still negative is important in mitigating the distortions caused by the zero bound. The fact that the target is not hit immediately should not create doubts about the meaningfulness of central-bank announcements regarding its target, if it is explained that the bank is committed to hitting the target *if* this is possible at a non-negative interest rate, so that at each point in time, either the target will be attained or a zero-interest-rate policy will be followed. The existence of the target is relevant even when it is not being attained, as it allows the private sector to judge how close the central bank is to a situation in which it would feel justified in abandoning the zero-interest-rate policy; hence the current gap between the actual and target price level should shape private-sector expectations of the time for which interest rates are likely to remain low.[27]

Would the private sector have any reason to believe that the central bank was serious about the price-level target, if each period all that is observed is a zero nominal interest rate and yet another target shortfall? The best way of making a rule credible is for the central bank to conduct policy over time in a way that demonstrates its commitment. Ideally, the central bank's commitment to the price-level targeting framework would be demonstrated before the zero bound came to bind (at which time the central bank would have frequent opportunities to show that the target did determine its behavior). The rule proposed above is one that would be equally optimal under normal circumstances as in the case of the relatively unusual kind of disturbance that causes the natural rate of interest to be substantially negative.

To understand how the rule works out of the trap it is useful to note that when the nominal interest rate is positive, $\Delta_t = 0$ at all times. The central bank, therefore, should demonstrate a commitment to subsequently undo overshoots and undershoots of the price-level target. In this case, deflation that occurs when the economy finds itself in a liquidity trap should create expectations of future inflation, as mandated by optimal policy. The additional term Δ_t implies that when the zero bound is binding, the central bank should raise its long-run price-level target even further, thus increasing inflation expectations even more.

It may be wondered why we discuss our proposal in terms of a (gap-adjusted) price-level target, rather than an inflation target. In fact, we could equivalently describe the policy in terms of a time-varying target for the gap-adjusted inflation rate $\tilde{\pi}_t \equiv \tilde{p}_t - \tilde{p}_{t-1}$. The reason that we prefer to describe the rule as a price-level targeting rule is that the essence of the rule is easily described in those terms. As we show below, a *fixed* target for the gap-adjusted price level would actually represent quite a good approximation to optimal policy, whereas a fixed inflation target would not come close, as it would fail to allow for any of the history-dependence of policy that is necessary to mitigate the distortions resulting from the zero bound.

3.2 A Simpler Proposal

One may argue that an unappealing aspect of the rule suggested above is that it involves the term Δ_t, which determines the change in the price-level target, and is only non-zero when the zero bound is binding. Suppose that the central bank's commitment to a policy rule can only become credible over time through repeated demonstrations of its commitment to act in accordance with it. In that case, the part of the rule that involves the adjustment of the target in response to target shortfalls when the zero bound binds might not come to be understood well by the private sector for a very long time, since the occasions on which the zero bound binds will presumably be relatively infrequent.

Fortunately, most of the benefits that can be achieved in principle through a credible commitment to the optimal targeting rule can be achieved through commitment to a much simpler rule, which would not involve any special provisos that are invoked only in the event of a liquidity trap. Let us consider the following simpler rule,

$$p_t + \frac{\lambda}{\kappa} x_t = p^*, \tag{35}$$

where now the target for the gap-adjusted price level is fixed at all times. The advantage of this rule, although not fully optimal when the zero bound is binding, is that it may be more easily communicated to the public. Note that the simple rule is fully optimal in the absence of the zero bound. In fact, even if the zero bound occasionally binds, this rule results in distortions only a bit more severe than those associated with the fully optimal policy.

Figure 7 and 8 compares the result for these two rules. The dotted line shows the equilibrium under the constant price-level target rule in (35) whereas the solid line shows the fully optimal rule in (32)-(34). As the figures show, the constant price-level targeting rule results in state-contingent responses of output and inflation that are very close to those under the optimal commitment, even if under this rule the price level falls farther during the period while the zero bound binds, and only asymptotically returns from below to the level that it had prior to the disturbance. Table 1 shows that most of the welfare gain achieved by the optimal policy, relative to what can be achieved by a purely forward-looking policy such as a strict inflation target, is already achieved by the simple rule. The table reports the value of expected discounted losses (28), conditional on the occurrence of the disturbance in period zero, under the three policies shown in Figure 2, the optimal policy characterized in Figure 3, and under the constant price-level targeting rule. Both of the latter two history-dependent policies are vastly superior to any of the strict inflation targets. While it is true that losses remain twice as large under the simple rule as under the optimal rule, we are referring to fairly small losses at this point.

Table 1

Policy	*Loss* (percent)
Strict inflation target, $\pi^* = 0$	100
Strict inflation target, $\pi^* = 0$	24.1
Strict inflation target, $\pi^* = 0$	32
Constant prive-level target	0.0725
Optimal rule	0.036

As with the fully optimal rule, no estimate of the natural rate of interest is needed to implement the constant price-level targeting rule. At first, it may seem puzzling that a constant price-level targeting rule does well, since no account is taken of the size of the disturbance to the natural rate of interest. This is because a price-level target commits the central bank to undo any deflation by subsequent inflation; a larger disturbance, that creates a larger initial deflation, automatically creates greater inflation expectations in response. Thus there is an "automatic stabilizer" built into the price-level target, that is lacking under a strict inflation targeting regime.[28]

A proper communication strategy for the central bank about its objectives and targets when outside the trap is of crucial importance for this policy rule to be successful. To see this, consider a rule that is equivalent to (35) when the zero bound is not binding. Taking the difference of (35) we obtain:

$$\pi_t + \frac{\lambda}{\kappa}(x_t - x_{t-1}) = 0 \tag{36}$$

Although this rule results in an identical equilibrium to the constant price-level targeting rule when the zero bound is not binding, the result is dramatically different when the zero bound is binding. This is because this rule implies that the inflation rate is proportional to the negative of the growth rate of the output gap. Thus it mandates deflation when there is growth in the output gap. This implies that the central bank will deflate once out of a liquidity trap since this is a period of output growth. This is exactly opposite to what is optimal as we have observed above. Thus the outcome under this rule is even worse than a strict zero inflation target, even if this rule replicates the price level targeting rule when out of the trap. What this underlines is that it is not enough to replicate the equilibrium behavior that correspond to (35) at normal times to induce the correct set of expectations when the zero bound is binding. It is crucial to communicate to the public that the government is committed to a long-run price-level target. This commitment is exactly what creates the desired inflation expectations when the zero bound is binding.

3.3 Should a Central Bank "Keep Powder in the Keg?"

Thus far we have only considered alternative policies that might be followed from the date at which the natural rate of interest unexpectedly falls to a negative value, causing the zero bound to bind. A question of considerable current interest in countries like the U.S., however, is how policy should be affected by the *anticipation* that the zero bound might well bind before long, even if this is not yet the case. Some commentators have argued that in such circumstances the Fed should be cautious about lowering interest rates all the way to zero too soon, in order to "save its ammunition" for future emergencies. This suggests that the anticipation that the zero bound could bind in the future should lead to tighter policy than would otherwise be justified given current conditions. Others argue alternatively that policy should instead be more inflationary than one might otherwise prefer, in order to reduce the probability that a further negative shock can result in a situation where the zero bound binds.

Our above characterization of the optimal targeting rule can shed light on this debate. Recall that the rule (32) – (34) describes optimal policy *regardless*) of the assumed stochastic process for the natural rate of interest, and not only in the case of the particular two-state Markov process assumed in Figure 3. In particular, the same rule is optimal in the case that information is received indicating the likelihood of the natural rate of interest becoming negative before this actually occurs. How should the conduct of policy be affected by that news? Under the optimal targeting rule, the optimal target for $\tilde{p}_t$ is *unaffected* by such expectations, as long as a situation has not yet been reached in which the zero bound binds, since it is only target shortfalls that have already occurred that can justify a change in the target value p_t^*. Thus an increased assessment of the likelihood of a binding zero bound over the coming year or two would *not* be a reason for increasing the price-level target (or the implied target rate of inflation).[29]

On the other hand, the evolution of inflation, output and interest rates will be affected by this news, even in the absence of any immediate change in the central bank's price level-target owing to the effect on forward-looking private-sector spending and pricing decisions. The anticipation of a coming state in which the natural rate of interest will be negative, and actual interest rates will not be able to fall as much, owing to the zero bound, will reduce both desired real expenditure (at unchanged short-term interest rates) and desired price increases, as a result of the anticipated negative output gaps and price declines in the future. This change in the behavior of the private sector's outlook will require a change in the way that the central bank must conduct policy in order to hit its *unchanged* target for the gap-adjusted price level, likely in the direction of a pre-emptive loosening of policy.

This is illustrated by the numerical experiment shown in Figure 9. Here we suppose that in quarter zero it is learned (by both the central bank and the private sector) that the natural rate of interest will fall to the level of -2 percent per annum only in period 4. It is known that the natural rate will remain at its normal level, +4 percent per annum until then; after the drop, it will return to the normal level with a probability of 0.1 each quarter, as in the case considered earlier. We now consider the character of optimal policy from period zero onward, given this information. Figure 9 again shows the optimal state-contingent paths of inflation and output in the case that the disturbance to the natural rate, when it arrives lasts for one quarter, two quarters, and so on.

We observe that under the optimal policy commitment, prices begin to decline mildly as soon as the news of the coming disturbance is received. The central bank is nonetheless able to avoid undershooting its target for $\tilde{p}_t$ at first, by stimulating an increase in real activity sufficient to justify the mild deflation. (Given the shift to pessimism on the part of the private sector, this is the policy dictated by the targeting rule, given that even a mild immediate increase in real activity is insufficient to prevent price declines, owing to the anticipated decline in real demand when the disturbance hits.) By quarter 3, this is no longer possible, and the central bank undershoots its target for $\tilde{p}_t$ (as both prices and output decline), even though the nominal interest rate is at zero. Thus optimal policy involves driving the nominal interest rate to zero *even before* the natural rate of interest has turned negative, when that development can already be anticipated for the near future. The fact that the zero bound binds even before the natural rate of interest becomes negative means that the price-level target is higher than it otherwise would have been at the time that the disturbance to the natural rate arrives. As a result, the deflation and output gaps during the period in which the natural rate is negative are less severe than in the case in which the disturbance is unanticipated. In this scenario, optimal policy is somewhat more inflationary after the disturbance occurs than in the case considered in Figure 3, for in this case the optimal policy commitment takes into account the contractionary effects in periods *before* the disturbance takes effect of anticipations that the disturbance will result in price-level and output declines. The fact that optimal policy after the disturbance occurs is different in this case, despite the fact that the disturbance has exactly the same effects as before from quarter 4 onward, is another illustration of the history-dependence of optimal policy.

4 Preventing a Self-Fulfilling Deflationary Trap

In our analysis thus far, we have assumed that the real disturbance results in a negative natural rate of interest only temporarily. We have therefore supposed that price-level stabilization will eventually be consistent with positive nominal interest rates, and accordingly that a time will foreseeably be reached at which it is possible for the central bank to create inflation by keeping short-term nominal rates at a low (but non-negative) level. Some may ask, however, if it is not possible for the zero bound to bind *forever* in equilibrium, not because of a permanently negative natural rate, but simply because deflation continues to be (correctly) expected indefinitely. If so, it might seem that the central bank's commitment to a non-decreasing price-level target would be irrelevant; the actual price level would fall further and further short of the target, but because of the binding zero bound, there would never be anything the central bank could do about this.

In the model presented in section 2, a self-fulfilling permanent deflation is indeed consistent with both the Euler equation (2) for aggregate expenditure, the money-demand relation (3) and the pricing relations (7) – (9). Suppose that from some date τ onward, all disturbances $\xi_t = 0$ with certainty, so that the natural rate of interest is expected to take the constant value $\bar{r} = \beta^{-1} - 1 > 0$, as in the scenarios considered in section 3. Then possible paths for inflation, output, and interest rates consistent with each of the relations just listed in all periods $t \geq \tau$ is given by

$$i_t = 0,$$

$$P_t/P_{t-1} = \beta < 1,$$

$$p_t^*/P_t = \tilde{p}^* \equiv \left(\frac{1-\alpha\beta^{\theta-1}}{1-\alpha}\right)^{\frac{1}{1-\theta}} < 1,$$

$$Y_t = \tilde{Y}$$

for all $t \geq \tau$, where $\tilde{Y} < \bar{Y}$ is implicitly defined by the relation

$$\Pi_1(\tilde{p}^*, \tilde{p}^*, 1; \tilde{Y}, \bar{m}(\tilde{Y}; 0), 0) = 0.$$

Note that this deflationary path is consistent with monetary policy as long as real balances satisfy $M_t/P_t \geq \bar{m}(\tilde{Y}; 0)$ each period; faster growth of the money supply does nothing to prevent consistency of this path with the requirement that money supply equal money demand each period.

There remains, however, one further requirement for equilibrium in the model of section 2, the transversality condition (6), or equivalently the requirement that households exhaust their intertemporal budget constraints. Whether the deflationary path is consistent with this condition as well depends, properly speaking, on the specification of fiscal policy: it is a matter of whether the government budget results in contraction of the nominal value of total government liabilities D_t at a sufficient rate asymptotically. Under some assumptions about the character of fiscal policy, such as the "Ricardian" fiscal policy rule assumed by Benhabib *et al.*, the nominal value of government liabilities will necessarily contract along with the price level, so that (6) is also satisfied, and the processes described above will indeed represent a rational-expectations equilibrium. In such a case, then, a commitment to the price-level targeting rule proposed in the previous section will be equally consistent with more than one equilibrium: if people expect the optimal price-level process characterized earlier, then that will indeed be an equilibrium, but if they expect perpetual deflation, this will be an equilibrium as well.

We can, however, exclude this outcome through a suitable commitment with regard to the asymptotic evolution of total government liabilities. Essentially, there needs to be a commitment to policies that ensure that the nominal value of government liabilities cannot contract at the rate required for satisfaction of the transversality condition despite perpetual deflation. One example of a commitment that would suffice is a commitment to a balanced-budget policy of the kind analyzed by Schmitt-Grohé and Uribe (2000). These authors show that self-fulfilling deflations are not possible under commitment to a Taylor rule, together with the balanced-budget fiscal commitment. The key to their result is that the fiscal rule includes a commitment *not to allow budget surpluses* any more than budget deficits would be allowed; hence it is not possible for the nominal value of government liabilities to contract, even when the price level falls exponentially forever.

The credibility of this sort of fiscal commitment might be doubted, and so it is worth mentioning that another way of maintaining a floor on the asymptotic nominal value of total government liabilities is through a commitment *not to contract the monetary base*, together with a commitment of the government to maintain a non-negative asymptotic present value of the public debt. In particular, suppose that the central bank commits itself to follow a base-supply rule of the form

$$M_t = P_t^* \bar{m}(Y_t; \xi_t) \tag{37}$$

in each period when the zero bound binds (i.e. when it is not possible to hit the price-level target with a positive nominal interest rate), where

$$P_t^* \equiv \exp\left\{p_t^* - \frac{\lambda}{\kappa}x_t\right\}$$

is the current price-level target implied by the adjusted price-level target p_t^*. When the zero bound does not bind, the monetary base is whatever level is demanded at the nominal interest rate required to hit the price-level target. This is a rule in the same spirit as (11), specifying a particular level of excess supply of base money in the case that the zero bound binds, but letting the monetary base be endogenously determined by the central bank's other targets at all other times. Equation (37) is a more complicated formula than is necessary to make our point, but it has the advantage of making the monetary base a continuous function of other aggregate state variables at the point where the zero bound just ceases to bind.

This particular form of commitment has the advantage that it may be considered less problematic for the central bank to commit itself to maintain a particular nominal value for its liabilities than for the Treasury to do so. It can also be justified as a commitment that is entirely consistent with the central bank's commitment to the price-level targeting rule; even when the target cannot be hit, the central bank supplies the quantity of money *that would be demanded if the price level were at the target level.* Doing so — refusing to contract the monetary base even under circumstances of deflation — is a way of signalling to the public that the bank is serious about its intention to see the price level restored to the target level.

If we then assume a fiscal commitment that guarantees that

$$\lim_{T\to\infty} E_t Q_{t,T} B_T = 0, \tag{38}$$

i.e. that the government will asymptotically be neither creditor nor debtor, the transversality condition (6) reduces to

$$\lim_{T\to\infty} \beta^T E_t[u_c(Y_T, M_T/P_T; \xi_T) M_T/P_T] = 0. \tag{39}$$

In the case of the base-supply rule (37), this condition is violated in the candidate equilibrium described above, since the price-level and output paths specified would imply that

$$\begin{aligned} \beta^T E_t[u_c(Y_T, M_T/P_T; \xi_T) M_T/P_T] &= \beta^\tau u_c(\tilde{Y}, \bar{m}(\tilde{Y}; 0); 0) \bar{m}(\tilde{Y}; 0) P_T^*/P_\tau \\ &\geq \beta^\tau u_c(\tilde{Y}, \bar{m}(\tilde{Y}; 0); 0) \bar{m}(\tilde{Y}; 0) P_\tau^*/P_\tau, \end{aligned}$$

where the last inequality makes use of the fact that under the price-level targeting rule, $\{p_t^*\}$ is a non-decreasing series. Note that the final expression on the right-hand side is independent of T, for all dates $T \geq \tau$. Hence the series is bounded away from zero, and condition (39) is violated.

Thus a commitment of this kind can exclude the possibility of a self-fulfilling deflation of the sort described above as a possible rational-expectations equilibrium. It follows that there is a possible role for "quantitative easing" — understood to mean supply of base money beyond the minimum quantity required for consistency with the zero nominal interest rate — as an element of an optimal policy commitment. A commitment to supply base money in proportion to the *target* price level, and not the actual current price level, in a period in which the zero bound prevents the central bank from hitting its price-level target, can be desirable both as a way of ruling out self-fulfilling deflations and as a way of signalling the central bank's continuing commitment to the price-level target, even though it is temporarily unable to hit it.

Note that this result does not contradict the irrelevance proposition of section 2, for we have here made a different assumption about the nature of the fiscal commitment than the one made in section 2. Condition (38) implies that the evolution of total nominal government liabilities will not be independent of the central bank's target for the monetary base. As a consequence, the neutrality proposition of section 2 no longer holds. The import of that proposition is that expansion of the monetary base when the economy is in a liquidity trap is necessarily pointless; rather, it is that any effect of such action must depend either on changing expectations regarding future interest-rate policy *or* on changing expectations regarding the future evolution of total nominal government liabilities. The present discussion has illustrated circumstances under which expansion of the monetary base — or at any rate, a commitment not to contract it — could serve both of these ends.

Nonetheless, the present discussion does *not* support the view that the central bank should be able to hit its price-level target at all times, simply by flooding the economy with as much base money as is required to prevent the price level from falling below the target at any time. Our analysis in section 3 still describes all of the possible paths for the price level consistent with rational-expectations equilibrium, and we have seen that even if the central bank were able to choose the expectations that the private sector should have (as long as it were willing to act in accordance with them), the zero bound would prevent it from being able to fully stabilize inflation and the output gap. Furthermore, the degree of base expansion during a "liquidity trap" called for by rule (37) is quite modest. The monetary base will be gradually raised, if the zero bound continues to bind, as the price-level target is ratcheted up to steadily higher levels. But our calibrated example above indicates that this would typically involve only quite a modest increase in the monetary base, even in the case of a "liquidity trap" that lasts for several years. There would be no obvious benefit to the kind of rapid expansion of the monetary base actually tried in Japan over the past two years. An expansion of the monetary base of this kind is evidently *not* justified by any intentions regarding the future price level, and hence regarding the size of the monetary base once Japan exits from the "trap." But an injection of base money that is expected to be removed again once the zero bound ceases to bind should have little effect on spending or pricing behavior, as shown in section 2.

5 Further Aspects of the Management of Expectations

In section 2, we argued that neither expansion of the monetary base as such nor purchases of particular types of assets through open-market purchases should have any effect on either inflation or real activity, *except* to the extent that such actions might result in changes in expectations regarding future interest-rate policy (or possibly expectations regarding the asymptotic behavior of total nominal government liabilities, and hence the question of whether the transversality condition should be satisfied). Because of this, we were able, in sections 3 and 4, to characterize the optimal policy commitment without any reference to the use of such instruments of policy; a consideration of the different possible joint paths of interest rates, inflation and output that would be consistent with rational-expectations equilibrium sufficed to allow us to determine the best possible equilibrium that one could hope to arrange, and to characterize it in terms of the interest-rate policy that one should wish for the private sector to expect.

However, this does not mean that other aspects of policy — beyond a mere announcement of the rule according to which the central bank wishes to be understood to be committed in setting future interest-rate policy — cannot matter. They may matter insofar as certain kinds of present actions may help to *signal* what the bank's intentions regarding future policy are, or may make it more *credible* that the central bank will indeed carry out these intentions. A full analysis of the ways in which policy actions may be justified as helping to steer expectations is beyond the scope of this article, and in any event the question is one that has as much to do with psychology and effective communication as with economic analysis. Nonetheless, we offer a few remarks here about the kinds of policies that might contribute to the creation of desirable expectations.

5.1 Demonstrating Resolve

One way in which current actions may help to create desirable expectations regarding future policy is by being seen to be consistent with the same principles that the central bank wishes the private sector to understand will guide its policy in the future. We have already mentioned one example of this, when we remarked that one way to convince the private sector that the central bank will follow the optimal price-level targeting rule following a period in which the zero bound has been hit is by following this rule *before* such a situation arises.

Our discussion in the previous section provides a further example. Adjustment of the supply of base money during the period in which the zero bound binds so as to keep the monetary base proportional to the *target* price level rather than the actual current price level can be helpful, even though it is irrelevant as far as interest-rate control is concerned, as a way of making visible to the private sector the central bank's belief about whether the price level ought properly to be (and hence, the quantity of base money that the economy ought to need). By making the existence of the price-level target more salient, such an action can help to create the expectations regarding future interest-rate policy that are necessary in order to mitigate the distortions created by the binding zero bound.

As a further example, Clouse *at al.* (2003) argue that open-market operations may be stimulative, even when the zero bound has been reached, because they "demonstrate resolve" to keep the nominal interest rate at zero for a longer time than would otherwise be expected. Here it should be remarked that an expansion of the monetary base when the zero bound is binding *need not* be interpreted in this way. Consider, for example, a central bank with a constant zero inflation target, as discussed in section 2.2. When the zero bound binds, such a bank is unable to hit its inflation target, and should exhibit frustration with this state of affairs. If some within the bank believe that it should always be possible to hit the target with sufficiently vigorous monetary expansion, one might well observe substantial growth in the monetary base at a time when the inflation target is being undershot. Nonetheless, this would not imply any commitment to looser policy subsequently; such a central bank would never intentionally allow the monetary base to be higher than that required to hit the inflation target, in a period in which it is possible to hit it. The result should be the equilibrium evolution shown in Figure 2, and no effect of the "quantitative easing" that occurs while the zero bound binds. This shows that it matters what the private sector understands to be the *principle* that motivates "quantitative easing", and not simply the size of the increase in the monetary base that occurs.

Similarly, open-market purchases of long-term Treasuries when short rates are at zero, as advocated by Bernanke (2002) and Cecchetti (2003), among others, may well have a stimulative effect even if portfolio-balance effects are quantitatively unimportant. We have argued in sections 2 and 3 that it is desirable for the central bank to commit itself under such circumstances to maintain low short-term rates even after the natural rate of interest rises again. The level of long rates can provide an indicator of the extent to which the markets actually believe in such a commitment. If a central bank's judgment is that long rates are remaining higher than they should be under the optimal equilibrium owing to private-sector skepticism about whether the history-dependent interest-rate policy will actually be followed, then a willingness to buy long bonds from the private sector at a price which it regards as more appropriate is one of way of demonstrating publicly that it expects to carry out its commitment regarding future interest-rate policy. Given that the private sector is likely to be uncertain about the nature of the central bank's commitment (in the case of imperfect credibility), and that it can reasonably assume that the central bank knows more about its own degree of resolve than others do, action by the central bank that is consistent with a belief on its own part that it will keep short rates low in the future is likely to shift private beliefs in the same direction. If so, open-market purchases of long bonds could lower long-term interest rates, stimulate the economy immediately, and bring the economy closer to the optimal rational-expectations equilibrium. Note, however, that the effect follows, not from the purchases themselves, but from the way in which they are interpreted. In order for them to be interpreted as indicating a particular kind of commitment with regard to future policy, it is important that the central bank have *itself* formulated such an intention, and that it speak about it to the public, so that its open-market purchases will be seen in this light.

Similar remarks apply to the proposals by McCallum (2000) and Svensson (2001) that purchases of foreign exchange be used to stimulate the economy through devaluation of the exchange rate.[30] Under the optimal policy commitment described in section 2, a decline in the natural rate of interest should be accompanied by depreciation of the exchange rate, both because nominal interest rates fall (and are expected to remain low for some time) and because the expected long-run price level (and hence the expected long-run nominal exchange rate) should increase. It follows that the extent to which the exchange rate depreciates can provide an indicator of the extent to which the markets believe that the central bank is committed to such the optimal policy; and if the depreciation is insufficient, purchases of foreign exchange by the central bank provide one way for it to demonstrate its own confidence in its policy intentions. Again, the effect in question is not a mechanical consequence of the bank's purchases, but instead depends on their interpretation.[31]

5.2 Providing Incentives to Improve Credibility

A related but somewhat distinct argument is that actions at the zero bound may help to render the central bank's commitment to an optimal policy more credible, by providing the bank with a *motive* to behave in the future in the way that it would currently wish that people would expect it to behave. Here we briefly discuss how policy actions that are possible while the economy remains in a "liquidity trap" may be helpful in this regard. Our perspective is not so much that the central bank is in need of a "commitment technology" because it will itself be unable to resist the temptation to break its commitments later in the absence of such a constraint, as that it may well be in need of a way of making its commitment *visible* to the private sector. Taking actions now that imply that the central bank will be disadvantaged later if it were to deviate from the policy to which it wishes to commit itself can serve this purpose.

To consider what kind of current actions provide useful incentives, it is helpful to analyze (Markov) equilibrium under the assumption that policy is conducted by a discretionary optimizer, unable to commit its future actions at all, as in Eggertsson (2003a, b). Let us first consider what a Markov equilibrium under discretionary optimization would be like, in the case that the only policy instrument is the choice each period of a short-term nominal interest rate, and the objective of the central bank is the minimization of the loss function (28). As shown in section 3, if credible commitment of future interest-rate policy is possible, this problem has a solution in which the zero bound does not result in too serious a distortion, though it does bind.

Under discretion, however, the outcome will be much inferior. Note that discretionary policy (under the assumption of Markov equilibrium in the dynamic policy game) is an example of a purely forward-looking policy. It then follows from our argument in section 3 that the equilibrium outcome will correspond to the kind of equilibrium discussed there in the case of a strict inflation target. More specifically, it is obvious that the equilibrium is the same as under a strict inflation target $\pi^* = 0$, since this is the inflation rate that will be chosen by the discretionary optimizer once the natural rate is again at its steady-state level. (From that point onward, a policy of zero inflation clearly minimizes the remaining terms in the discounted loss function.)

As shown in Figure 2, an expectation by the private sector that the central bank will behave in this fashion results in a deep and prolonged contraction of economic activity and a sustained deflation, in the case that the natural rate of interest remains negative for several quarters. We have also seen that these effects could largely be avoided, even in the absence of other policy instruments, if the central bank were able to credibly commit itself to a history-dependent monetary policy in later periods. Thus, in the kind of situation considered here, there is a *deflationary bias* to discretionary monetary policy, although, at its root, the problem is again the one identified in the classic analysis of Kydland and Prescott (1977). Let us now consider instead the extent to which the outcome could be improved, even in a Markov equilibrium with discretionary optimization, by changing the nature of the policy game.

One example of a current policy action, available even when the zero bound binds, that can help to shift expectations regarding future policy in a desirable way is for the government to cut taxes and issue additional nominal debt, as discussed in Eggertsson (2003a). Alternatively, the tax cut can be financed by money creation — for when the zero bound binds, there is no difference between expanding the monetary base and issuing additional short-term Treasury debt at a zero interest rate. This is essentially the kind of policy imagined when people speak of a "helicopter drop" of additional money on the economy; but it is the *fiscal* consequence of such an action with which we are here concerned.

Of course, if the objective of the central bank in setting monetary policy remains as assumed above, this will make no difference to the discretionary equilibrium — the optimal policy once the natural rate of interest becomes positive again will once more appear to be the immediate pursuit of a strict zero inflation target. However, if the central bank also cares about reducing the social costs of increased taxation — whether due to collection costs or other distortions — as it ought if it really takes social welfare into account, the result is different. As shown in Eggertsson (2003a), the tax cut will then increase inflation expectations, even if the government cannot commit to future policy.

It may be asked why, if such an incentive exists, Japan continues to suffer deflation, given the growth during the 1990s in Japanese government debt. One possible answer is that although gross nominal debt over GDP is 140 percent in Japan today, this does not reflect the true inflation incentives of the government. The ratio of gross national debt to GDP *overestimates* the inflation incentives of the government, because a substantial portion of Treasury debt is held by other governmental institutions.[32] *Net* government debt is only 67 percent of GDP, and as a result inflation incentives may not be much greater in Japan than in a number of other countries.

An even more likely reason for the continued low expectations of inflation in Japan at present, despite the current size of the nominal public debt, is skepticism as to whether the central bank can be expected to care about reducing the burden of the public debt when determining future monetary policy. The Bank of Japan may not be believed by the public to have such an objective; the expressed resistance of the Bank to suggestions that it increase its purchases of Japanese government bonds, on the ground that this could encourage a lack of fiscal discipline, certainly suggests that reducing the burden of government finance is not among its highest priorities. As Eggertsson (2003a) stresses, in order for fiscal policy to be effective as a means of increasing inflationary expectations, fiscal and monetary policy must be coordinated to maximize social welfare. The consequences of a narrow concern with inflation stabilization on the part of the central bank, together with in ability to credibly commit future monetary policy, can be dire, even from the point of view of the bank's own stabilization objectives.

Another instrument that may be used to change expectations regarding future monetary policy is open-market purchases of real assets or foreign exchange. An open-market purchase of real assets (say, real estate) can be thought of as another way of increasing nominal government liabilities, which should affect inflation incentives in much the same way as deficit spending, as discussed in Eggertsson (2003a). The alternative approach has the advantage of not worsening the overall fiscal position of the government — a current concern in Japan, owing to the size of the existing gross debt — while still increasing the fiscal incentive for inflation. A further advantage of this approach is that it need not depend on a perceived central-bank interest in reducing the burden of the public debt. Since the (nominal) capital gains from inflation accrue to the central bank itself under this policy, the central bank may be perceived to have an incentive to inflate simply on the ground that it cares about *its own* balance sheet, for example on the ground that a strong balance sheet will help to ensure its independence. (One can easily argue that under a rational scheme of cooperation between the central bank and the government, the central bank should *not* choose policy on the basis of concerns about its balance sheet — but under such an ideal regime, it should choose monetary policy with a view to reduction of the burden of the public debt, among other goals.)

The incentive effects of open-market operations in foreign exchange are even simpler, as shown by Eggertsson (2003b). Open-market purchases of foreign assets give the central bank an incentive to inflate in the future in order to obtain capital gains at the expense of foreigners. These will be valuable if it cares either about its own balance sheet or about reducing the burden of the public debt, as in the case of real asset purchases. However, capital gains on foreign exchange as a result of depreciation of the domestic currency will be valuable *even* in the case that the central bank does not care about its balance sheet (for example, because it cooperates perfectly with the Treasury) and yet does not care about the burden of the debt either (for example, because non-distorting sources of revenue are available to the Treasury). For capital gains at the expense of foreigners would allow an increase in domestic spending (by either the government or the private sector), and this must be valued by a central bank that acts in the national interest.

Under rational expectations, of course, no such capital gains are realized on average. Still, the purchase of foreign assets can work as a commitment device, because reneging on its inflation commitment would cause capital losses if the government holds foreign assets. Purchases of foreign assets are thus a way of committing the government to looser monetary policy in the future. This creates a reason for purchases of foreign exchange to cause a devaluation (which will also stimulate current demand), even without any assumption of a deviation from interest-rate parity, of the kind relied upon by authors such as McCallum (2000) in recommending devaluation for Japan.

Clouse *et al.* (2003) argue that open-market purchases of long-term Treasuries should also change expectations in a way that results in immediate stimulus. The argument is that if the central bank were not to follow through on its commitment to keep short rates low for a period of time, it should suffer a capital loss on the long bonds that it purchased at a price that made sense only on the assumption that it *would* keep interest rates low. Similarly, Tinsley (1999) has proposed for a policy that would create this kind of incentive even more directly, namely, the sale by the Fed of options to obtain federal funds at a future date at a certain price, on which the Fed would then stand to lose money if it did not keep the funds rate at the rate to which it had previously committed itself.

While these proposals should also help to reinforce the credibility of the kind of policy commitment associated with the optimal equilibrium (characterized in section 2), they have at least one important disadvantage relative to purchases of real assets or of foreign exchange. This is that they only provide the central bank an incentive to maintain low nominal interest rates for a certain period of time; they do not provide it with an incentive to ensure that the price level eventually rises to a higher level, and so they may do little to counter private-sector expectations that nominal interest rates will remain low for years — but because goods prices are going to continue to fall, not because the central bank is committed to eventual reflation. This is arguably the kind of expectations that have now taken root in Japan, where even ten-year bond yields are already well below one percent, though prices continue to fall and economic activity remains anemic. Creation of a perception that the central bank has an incentive to continue trying to raise the price level, and not to be content as long as nominal interest rates remain low, may be more a successful way of creating the sort of expectations associated with the optimal equilibrium.

6 Conclusion

We have argued that the key to dealing with a situation in which monetary policy is constrained by the zero lower bound on short-term nominal interest rates is the skillful management of expectations regarding the future conduct of policy. By "management of expectations" we do not mean that the central bank should imagine that with sufficient guile it can lead the private sector to believe whatever if wishes it to, independently of what it actually does; we have instead assumed that there is no point in trying to get the private sector to expect something that it does not itself intend to bring about. But we do contend that it is highly desirable for a central bank to be able to commit itself in advantage to a course of action that is desirable due to the benefits that flow from its being anticipated, and then to work to make this commitment credible to the private sector.

In the context of a simple optimizing model of the monetary transmission mechanism, we have shown a purely forward-looking approach to policy — under which there is therefore no possibility of committing future policy to respond to conditions at an earlier date — can lead to quite bad outcomes in the event of a temporary decline in the natural rate of interest, regardless of the kind of policy that is pursued at the time of the disturbance. We have also characterized optimal policy, under the assumption that credible commitment is possible, and shown that it involves a commitment to eventually bring the general level of prices back up to a level even higher than it would have had if the disturbance had never occurred. Finally, we have described a type of history-dependent price-level targeting rule with the property that a commitment to base interest-rate policy on this rule determines the optimal equilibrium, and that the same form of targeting rule continues to describe optimal policy regardless of which of a very large number of types of disturbances may affect the economy.

Given the role of private-sector anticipation of history-dependent policy in making possible a desirable outcome, it is important for central banks to develop effective methods of signaling their policy commitments to the private sector. An essential precondition for this, certainly, is for the central bank itself to clearly understand the kind of history-dependent behavior to which it should be seen to be committed, so that it can communicate its thinking on the matter and act consistently with the principles that it wishes the private sector to understand. Simply conducting policy in accordance with a rule may not suffice in itself to bring about an optimal, or nearly optimal, equilibrium; but it is the place to start.

Notes

[1]We do not here explore the possibility of relaxing the constraint by taxing money balances, as originally proposed by Gesell (1929) and Keynes (1936), and more recently by Buiter and Panigirtzoglou (1999) and Goodfriend (2000). While this represents a solution to the problem in theory, there are substantial practical difficulties with such a proposal, not least the political opposition that such an institutional change would be likely to generate. Our consideration of the optimal policy problem also abstracts from the availability of fiscal instruments such as the time-varying tax policy recommended by Feldstein (2002). We agree with Feldstein that there is a particularly good case for state-contingent fiscal policy as a way of dealing with a liquidity trap, even if fiscal policy is not a very useful tool for stabilization policy more generally. Nonetheless, we consider here only the problem of the proper conduct of monetary policy, taking as given the structure of tax distortions. As long as one does not think that state-contingent fiscal policy can (or will) be used to eliminate even temporary declines in the natural rate of interest below zero, the problem for monetary policy that we consider here remains relevant.

[2]In the simple model presented here, this occurs solely as a result of intertemporal substitution in private expenditure. But there are a number of reasons to expect long rates, rather than short rates, to be the critical determinant of aggregate demand. For example, in an open-economy model, the real exchange rate becomes an important determinant of aggregate demand. But the real exchange rate should be closely linked to a very long domestic real rate of return (or alternatively, to the expected future path of short rates) as a result of interest-rate parity, together with an anchor for the expected long-term real exchange rate (coming, for example, from long-run purchasing-power parity).

[3]See Kimura *et al.* (2002) for discussion of this policy, as well as an expression of doubts about its effectiveness.

[4]We shall not introduce fractional-reserve banking into our model. Technically, M_t refers to the monetary base, and we represent households as obtaining liquidity services from holding this base, either directly or through intermediaries (not modelled).

[5]We use this approach to modelling the transactions demand for money because of its familiarity. As shown in Woodford (2003, appendix section A.16), a cash-in-advance model leads to equilibrium conditions of essentially the same general form, and the neutrality result that we present below would hold in essentially identical form were we to model the transactions demand for money after the fashion of Lucas and Stokey (1987).

[6]For simplicity, we here abstract from government purchases of goods. Our equilibrium conditions directly extend to the case of exogenous government purchases, as shown in Woodford (2003, chap. 4).

[7]In the case that $i_t = 0$, $L(Y_t, 0; \xi_t)$ is defined as the minimum level of real balances that would satisfy the first-order condition, so that the function L is continuous.

[8]We might alternatively assume specialization across households in the type of labor supplied; in the presence of perfect sharing of labor income risk across households, household decisions regarding consumption and labor supply would all be as assumed here.

[9]In equilibrium, all firms in an industry charge the same price at any time. But we must define profits for an individual supplier i in the case of contemplated deviations from the equilibrium price.

[10]It is important to note that the specification of monetary and fiscal policy in the particular way that we propose here is not intended to suggest that either monetary or fiscal policy *must* be expected to

be conducted according to rules of the sort assumed here. Indeed, in later sections of this paper, we recommend policy commitments on the part of both monetary and fiscal authorities that *do not* conform to the assumptions made in this section. The point is to define what we mean by the qualification that open-market operations are irrelevant if they do not change expected future monetary or fiscal policy. In order to make sense of such a statement, we must define what it would mean for these policies to be specified in a way that prevents them from being affected by past open-market operations. The specific classes of policy rules discussed here show that our concept of "unchanged policy" is not only logically possible, but that it could correspond to a policy commitment of a fairly familiar sort, one that would represent a commitment to "sound policy" in the views of some.

[11]For example, security j in period $t-1$ is a one-period riskless nominal bond if $b_t(j)$ and $F_t(\cdot, j)$ are zero in all states, while $a_t(j) > 0$ is the same in all states. Security j is instead a one-period real (or indexed) bond if $a_t(j)$ and $F_t(\cdot, j)$ are zero, while $b_t(j) > 0$ is the same in all states. It is a two-period riskless nominal pure discount bond if instead $a_t(j)$ and $b_t(j)$ are zero, $F_t(i, j) = 0$ for all $i \neq k$, $F_t(k, j) > 0$ is the same in all states, and security k in period t is a one-period riskless nominal bond.

[12]We might, of course, allow for other types of fiscal decisions from which we abstract here — government purchases, tax incentives, and so on — some of which may be quite relevant to dealing with a "liquidity trap." But our concern here is solely with the question of what can be achieved by monetary policy; we introduce a minimal specification of fiscal policy only for the sake of closing our general-equilibrium model, and in order to allow discussion of the fiscal implications of possible actions by the central bank.

[13]See, Clouse *et al.* (2003) and Orphanides (2003).

[14]Our general-equilibrium analysis is in the spirit of the irrelevance proposition for open-market operations of Wallace (1981). Wallace's analysis is often supposed to be of little practical relevance for actual monetary policy because his model is one in which money serves only as a store of value, so that it is not possible for there to be an equilibrium in which money is dominated in rate of return by short-term Treasury securities, something that is routinely observed. However, in the case of open-market operations that are conducted at the zero bound, the liquidity services provided by money balances at the margin have fallen to zero, so that an analysis of the kind proposed by Wallace is correct.

[15]Okun (1963) and Modigliani and Sutch (1967) are important early discussions that reached this con-

clusion. Meulendyke (1998) summarizes the literature, and finds that the predominant view is that the effect was minimal.

[16]Examples of studies finding either no effects or only quantitatively unimportant ones include Mogigliani and Sutch (1967), Frankel (1985), Agell and Persson (1992), Wallace and Warner (1996), and Hess (1999). Roley (1982) and Friedman (1992) find somewhat larger effects.

[17]Cecchetti (2003) similarly argues that it should be possible for the Fed to independently affect long-bond yields if it is determined to do so, given that it can print money without limit to buy additional long-term Treasuries if necessary.

[18]This explains the apparent difference between our result and the one obtained by Auerbach and Obstfeld (2003) in a similar model. These authors assume explicitly that an increase in the money supply while the zero bound binds carries with it the implication of a permanently higher money supply, and also that there exists a future date at which the zero bound ceases to bind, so that the higher money supply will imply a different interest-rate policy at that later date. Clouse *et al.* (2003) also stress that maintenance of the higher money supply until a date at which the zero bound would not otherwise bind represents one straightforward channel through which open markets operations while the zero bound is binding could have a stimulative effect, though they discuss other possible channels as well.

[19]Clouse *et al.* (2003) argue that this is one important channel through which open-market operations can be effective.

[20]As we shall see, it is easier to explain the nature of the optimal commitment if it is described as a history-dependent price-level target.

[21]See Woodford (2003, chapter 7) for more detailed discussion of this point. The fact that zero inflation is optimal, rather than mild deflation, depends on our abstracting from transactions frictions, as discussed further in footnote xx below. As shown by Woodford, a long-run inflation target of zero is optimal in this model, even when the steady-state output level associated with zero inflation is suboptimal, owing to market power.

[22]In our numerical analysis, we interpret periods as quarters, and assume coefficient values of $\sigma = 0.5$, $\kappa = 0.02$, and $\beta = 0.99$. The assumed value of the discount factor implies a long-run real rate of interest of $\bar{r}$ equal to four percent per annum, as noted in the text. The assumed value of κ is consistent with the

empirical estimate of Rotemberg and Woodford (1997). The assumed value of σ represents a relatively low degree of interest-sensitivity of aggregate expenditure. We prefer to bias our assumptions in the direction of only a modest effect of interest rates on the timing of expenditure, so as not to exaggerate the size of the output contraction that is predicted to result from an inability to lower interest rates when the zero bound binds. As Figure 2 shows, even for this value of σ, the output contraction that results from a slightly negative value of the natural rate of interest is quite substantial.

[23]See Woodford (2003, chapter 6) for details. This approximation applies in the case that we abstract from monetary frictions as assumed in this section. If transactions frictions are instead non-negligible, the loss function should include an additional term proportional to This would indicate welfare gains from keeping nominal interest rates as close as possible to the zero bound (or, more generally, the lower bound i^m). Nonetheless, because of the stickiness of prices, it would not be optimal for interest rates to be at zero at all times, as implied by the flexible-price model discussed by Uhlig (2000). The optimal inflation rate in the absence of shocks would be slightly negative, rather than zero as in the "cashless" model considered in this section; but it would not be so low that the zero bound would be reached, except in the event of temporary declines in the natural rate of interest, as in the analysis here.

Note also that (28) implies that the optimal output gap is zero. More generally, there should be an output-gap stabilization objective of the form $(x_t - x*)^2$; the utility-based loss function involves $x* = 0$ only if one assumes the existence of an output or employment subsidy that offsets the distortion due to the market power of firms. However, the value of $x*$ does not affect the optimal state-contingent paths derived in this section and shown in figures 3 and 4, nor the formulas given in section 3 for the optimal targeting rule.

[24]Jung *et al.* (2001) discuss the solution of these equations only for the case in which the number of periods for which the natural rate of interest will be negative is known with certainty at the time that the disturbance occurs. Here we show how the system can be solved in the case of a stochastic process for the natural rate of a particular kind.

[25]For further discussion in a more general context, see Woodford (2003, chapter 7).

[26]On the desirability of a target for this index in the case that the zero bound does not bind, see Woodford (2003, chapter 7). This would correspond to a nominal GDP target in the case that $\lambda = \kappa$,

and that the natural rate of output follows a deterministic trend. However, the utility-based loss function derived in Woodford (2003, chapter 6) involves $\lambda = \kappa/\theta$, where $\theta > 1$ is the elasticity of demand faced by the suppliers of differentiated goods, so that the optimal weight on output is considerably less than under a nominal GDP target. Furthermore, the welfare-relevant output gap is unlikely to correspond too closely to deviations of real GDP from a deterministic trend.

[27]An interesting feature of the optimal rule is that it involves history-dependence that cannot be summarized solely by the past history of short-term nominal interest rates; if the nominal interest rate has fallen to zero in the recent past, it matters to what it extent the zero bound has prevented the central bank from pursuing as stimulative a policy as it otherwise would have. In this respect, the optimal policy rule derived here is similar to the rules advocated by Reifschneider and Williams (1999), under which the interest-rate operating at each point in time should depend on how low the central bank *would have* lowered interest rates in the past had the zero bound not prevented this.

[28]Wolman (2003) also stresses this advantage of rules that incorporate a price-level target over rules that only respond to the inflation rate, such as a conventional Taylor rule.

[29]This particular aspect of our conclusions, however, is likely to depend on a relatively special feature of our model, namely, the fact that our target variables (inflation and the output gao) are both purely forward-looking variables: their equilibrium values at any point in time depend (in our simple model) only on the economy's exogenous state and the expected conduct of policy from the current period onward. There are a variety of reasons why a more realistic model may well imply that these variables are functions of lagged endogenous variables as well, and hence of past policy. In such a case, the optimal target criterion will be at least somewhat forward-looking, as discussed in Giannoni and Woodford (2003).

[30]Svensson's proposal includes a target path for the price level, which the exchange-rate policy is used to (eventually) achieve, and in this respect is similar to the policy advocated here. However, Svensson's discussion of the usefulness of intervention in the market for foreign exchange does not emphasize the role of such interventions as a signal regarding future policy.

[31]The numerical analysis by Coenen and Wieland (2003) finds that an exchange-rate policy can be quite effective in creating stimulus when the zero bound is binding. But what is actually shown is that a rational-expectations equilibrium exists in which the exchange rate depreciates and deflation is halted;

these effects could be viewed as resulting from a credible commitment to a target path for the price level, similar to the one discussed in section 3, and not requiring any intervention in the foreign exchange market at all.

[32]Government institutions such as Social Security, Postal Savings, Postal Life Insurance and the Trust Fund Bureau hold a large part of this nominal debt. If the part of the public debt that is held by these institutions is subtracted from the total value of gross government debt it turns out that the "net" government debt over output is only 51 percent. The important thing to notice is that most of the government institutions that hold the government nominal debt have *real liabilities.* For example, Social Security (that holds roughly 25% of the nominal debt held by the government itself) pays Japanese pensions and medical expenses. Those pensions are *indexed to the CPI.* If inflation increases, the real value of Social Security assets will decrease but the real value of most its liabilities remain unchanged. Thus the Ministry of Finance would eventually have to step in to make up for any loss in the value of Social Security assets if the government is to keep its pension program unchanged. Therefore, the gains of reducing the real value of outstanding debt is partly offset by a decrease in the real value of the assets of government institutions such as Social Security.

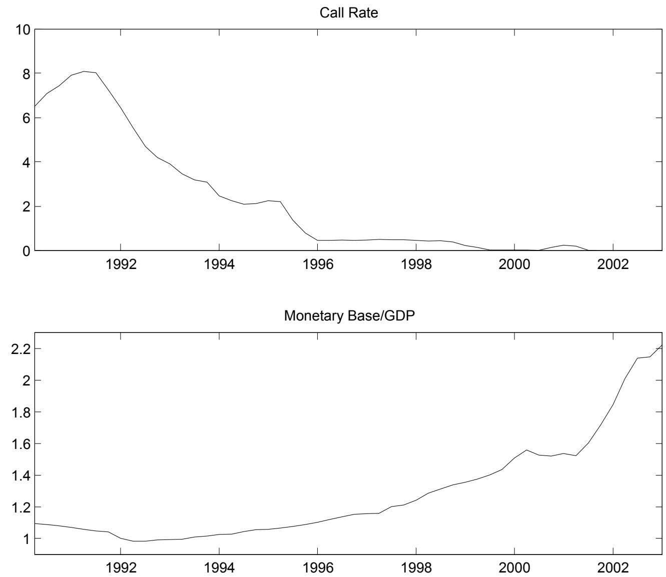

Figure 1: Evolution of the call rate on uncollateralized overnight loans in Japan, and the Japanese monetary base relative to GDP [1992 = 1.0].

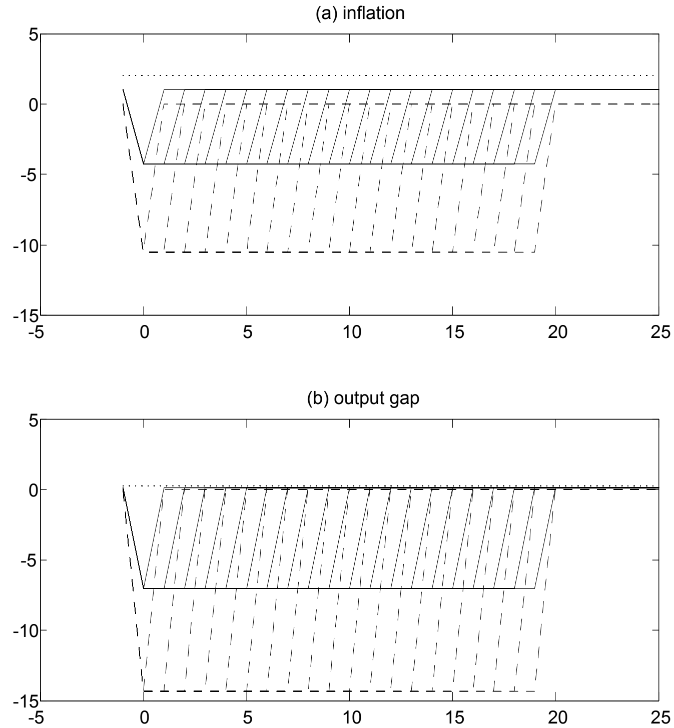

Figure 2: Dynamics of inflation and the output gap under strict inflation targeting, for three alternative inflation targets.

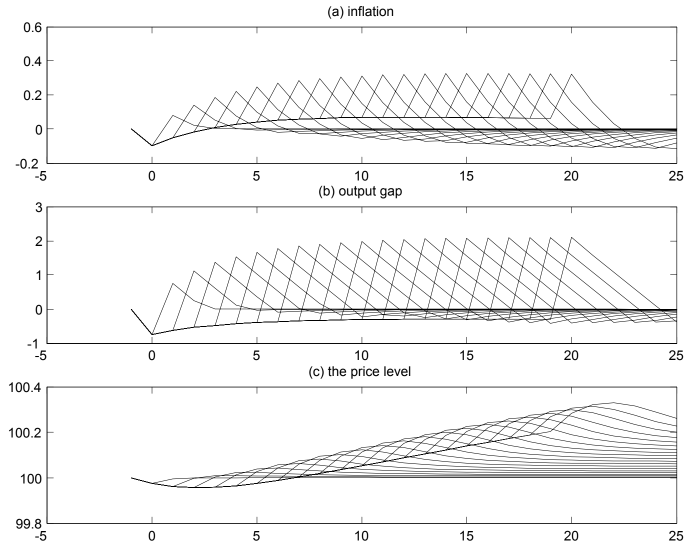

Figure 3: Dynamics of the output gap and inflation under an optimal policy commitment.

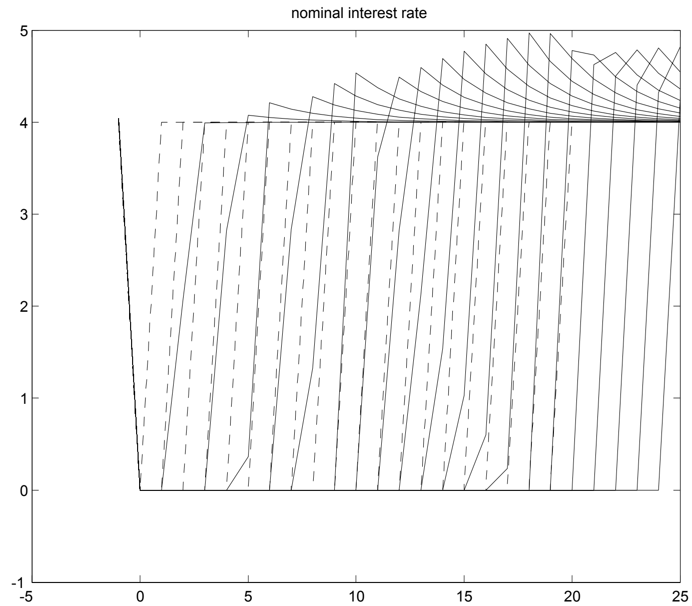

Figure 4: The associated state-contingent path of the short-term nominal interest rate, under the policy shown in Figure 3 [solid line], and under the zero inflation target shown in Figure 2 [dashed line].

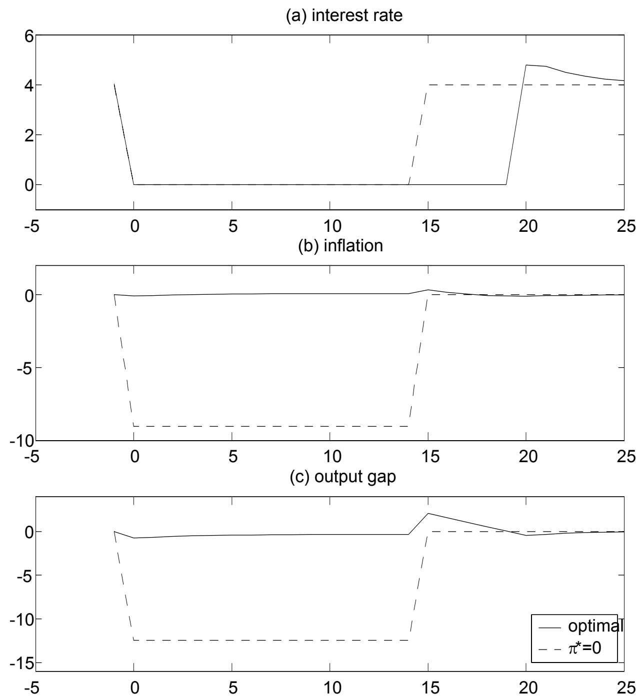

Figure 5: Comparison of the state-contingent paths under the two policies compared in Figure 4, in the case that the natural rate of interest is negative for 15 quarters.

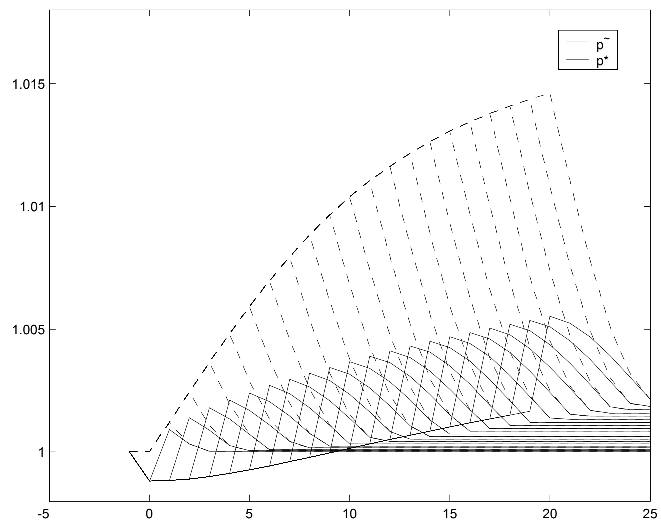

Figure 6: Responses of the price-level target and the gap-adjusted price level to a shock to the natural rate of interest.

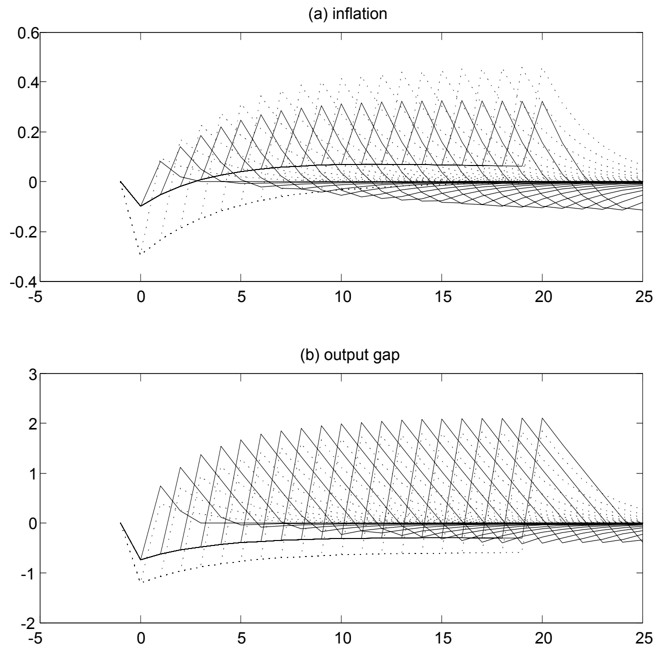

Figure 7: State-contingent paths of inflation and the output gap under the optimal targeting rule [solid lines] and under the simple rule [dotted lines].

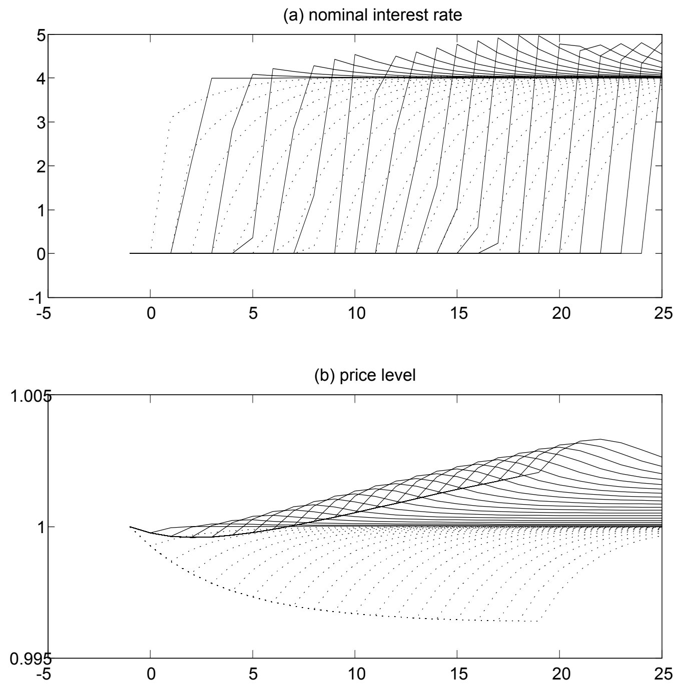

Figure 8: State-contingent paths of the nominal interest rate and the price level under the same two policies as in Figure 7.

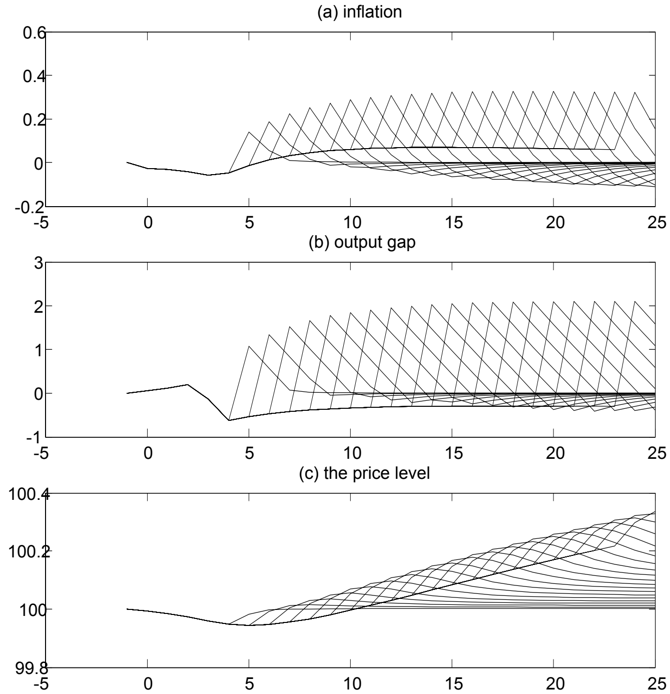

Figure 9: State-contingent paths of inflation and output under optimal policy, when the decline in the natural rate of interest can be anticipated four quarters in advance.

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A Appendix: The Numerical Solution Method

Here we illustrate a solution method for the optimal commitment solution discussed in section 2.3. This same method can also be applied, with appropriate modification of each of the steps, to find the solution in the case that the central bank commits to a constant price level target rule or to a constant inflation target. We assume that the natural rate of interest becomes unexpectedly negative in period 0 and the reverts back to normal with probability α_t in every period t. Our numerical work assumes that there is a final date S in which the natural rate becomes positive with probability one (this date can be arbitrarily far into the future).

The solution takes the form:

$$
\begin{aligned}
i_t &= 0 \qquad \forall \qquad if \qquad 0 \leq t < \tau + k \\
i_t &> 0 \qquad \forall \qquad if \qquad t \geq \tau + k
\end{aligned}
$$

It follows that:

$$
\begin{aligned}
E_t x_{t+1} - x_t + \sigma(E_t \pi_{t+1} + r_t^n) &= 0 \qquad if \quad t < \tau + k \\
\phi_{1t} &= 0 \qquad if \quad t \geq \tau
\end{aligned}
$$

Here τ is he stochastic date at which the natural rate of interest returns to steady state. We assume that τ can take any value between 1 and the terminal date S that can be arbitrarily far into the future. The number $\tau + k_\tau$ is the period in which the zero bound stops being binding in the contingency when the natural rate of interest becomes positive in period τ. Note that the value of k_τ can depend on the value of τ. We will first show the solution for the problem as if we knew the sequence $\{k_\tau\}_{\tau=1}^{S}$. We then describe a numerical method to find the sequence $\{k_\tau\}_{\tau=1}^{S}$.

A.0.1 The solution for $t \geq \tau + k_\tau$

The system can be written in the form:

$$\begin{bmatrix} E_t Z_{t+1} \\ \\ P_t \end{bmatrix} = M \begin{bmatrix} Z_t \\ \\ P_{t-1} \end{bmatrix}$$

If there are two eigenvalues of the matrix M outside the unit circle this system has a unique bounded solution of the form:

$$P_t = \Omega^0 P_{t-1} \tag{40}$$

$$Z_t = \Lambda^0 P_{t-1} \tag{41}$$

A.0.2 The solution for $\tau \leq t < \tau + k$

Again this is a perfect foresight solution but with the zero bound binding. The solution satisfies the equations:

$$\pi_t = \kappa x_t + \beta \pi_{t+1}$$

$$x_t = \sigma(r_t^n + \pi_{t+1}) + x_{t+1} \tag{42}$$

$$\pi_t + \phi_{2t} - \phi_{2t-1} - \beta^{-1}\sigma\phi_{1t-1} = 0$$

$$\lambda_x x_t + \phi_{1t} - \beta^{-1}\phi_{1t-1} - \kappa\phi_{2t} = 0$$

The system can be written as:

$$\begin{bmatrix} P_t \\ \\ Z_t \end{bmatrix} = \begin{bmatrix} A & B \\ \\ C & D \end{bmatrix} \begin{bmatrix} P_{t-1} \\ \\ Z_{t+1} \end{bmatrix} + \begin{bmatrix} M \\ \\ V \end{bmatrix}$$

This system has a solution of the form:

$$P_{\tau+j} = \Omega^{k_\tau - j} P_{t-1} + \Phi^{k_\tau - j} \tag{43}$$

$$Z_{\tau+j} = \Lambda^{k_\tau - j} P_{\tau,t-1} + \Theta^{k_\tau - j} \tag{44}$$

where $j = 0, 1, 2, ..., k$. Here $\Omega^{k_\tau - j}$ is the coefficient in the solution when there are $k_\tau - j$ periods until the zero bound stops being binding (i.e. when $j - k_\tau = 0$ the zero bound is not binding anymore and the solution is equivalent to (40)-(41)). We can find the numbers $\Lambda^j, \Omega^j, \Theta^j$ and Φ^j for $j = 2, 3, \ldots.., k$ by solving the equations below using the initial conditions $\Phi^0 = \Theta^0 = 0$ for $j = 0$ and the initial conditions for Λ^j and Ω^j given in (40)-(41):

$$\Omega^j = [I - B\Lambda^{j-1}]^{-1} A$$

$$\Lambda^j = C + D\Lambda^{j-1}\Omega^j$$

$$\Phi^j = (I - B\Lambda^{j-1})^{-1}[B\Theta^{j-1} + M]$$

$$\Theta^j = D\Lambda^{j-1}\Phi^j + D\Theta^{j-1} + V$$

A.0.3 The solution for $t < \tau$

The solution satisfies the following equations:

$$\tilde{\pi}_t = \kappa\tilde{x}_t + \beta\{(1 - \alpha_{t+1})\tilde{\pi}_{t+1} + \alpha_{t+1}(\Lambda_{11}^{k_{t+1}}\tilde{\phi}_{1t} + \Lambda_{12}^{k_{t+1}}\tilde{\phi}_{2t} + \Theta_1^{k_{t+1}})\}$$

$$\tilde{x}_t = \sigma\{r_t^{nL} + (1-\alpha_{t+1})\tilde{\pi}_{t+1} + \alpha_{t+1}(\Lambda_{11}^{k_{t+1}}\tilde{\phi}_{1t} + \Lambda_{12}^{k_{t+1}}\tilde{\phi}_{2t} + \Theta_1^{k_{t+1}})\} +$$

$$\{(1-\alpha_{t+1})\tilde{x}_{t+1} + \alpha_{t+1}(\Lambda_{21}^{k_{t+1}}\tilde{\phi}_{1t} + \Lambda_{22}^{k_{t+1}}\tilde{\phi}_{2t} + \Theta_2^{k_{t+1}})\}$$

$$\tilde{\pi}_t + \tilde{\phi}_{2t} - \tilde{\phi}_{2t-1} - \beta^{-1}\sigma\tilde{\phi}_{1t-1} = 0$$

$$\lambda_x\tilde{x}_t + \tilde{\phi}_{1t} - \beta^{-1}\tilde{\phi}_{1t-1} - \kappa\tilde{\phi}_{2t} = 0$$

Here hat on the variables refers to the value of each variable contingent on that the natural rate of interest is negative. $\Lambda_{ij}^{k_{t+1}}$ is the ijth element of the matrix $\Lambda^{k_{t+1}}$. The value k_{t+1} depends on for how many additional periods the zero bound is binding (recall that here we are solving for the equilibrium assuming that we know the value of the sequence $\{k_\tau\}_{\tau=1}^{S}$). We can write the system as:

$$\begin{bmatrix} \tilde{P}_t \\ \\ \tilde{Z}_t \end{bmatrix} = \begin{bmatrix} A_t & B_t \\ \\ C_t & D_t \end{bmatrix} \begin{bmatrix} \tilde{P}_{t-1} \\ \\ \tilde{Z}_{t+1} \end{bmatrix} + \begin{bmatrix} M_t \\ \\ V_t \end{bmatrix}$$

We can solve this backwards from the date S in which the natural rate returns back to normal with probability one. We can then calculate the path for each variable to date 0. Note that.

$$B_{S-1} = D_{S-1} = 0$$

By recursive substitution we can find a solution of the form:

$$\tilde{P}_t = \Omega_t \tilde{P}_{t-1} + \Phi_t \tag{45}$$

$$\tilde{Z}_t = \Lambda_t \tilde{P}_{t-1} + \Theta_t \tag{46}$$

where the coefficients are time dependent. To find the numbers $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t consider the solution of the system in period $S-1$ when $B_{S-1} = D_{S-1} = 0$. We have:

$$\Omega_{S-1} = A_{S-1}$$

$$\Phi_{S-1} = M_{S-1}$$

$$\Lambda_{S-1} = C_{S-1}$$

$$\Theta_{S-1} = V_{S-1}$$

We can find of numbers $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t for period 0 to $S-2$ by solving the system below (using the initial conditions shown above for $S-1$):

$$\Omega_t = [I - B_t\Lambda_{t+1}]^{-1}A_t$$

$$\Lambda_t = C_t + D_t\Lambda_{t+1}\Omega_t$$

$$\Phi_t = (I - B_t\Lambda_{t+1})^{-1}[B_t\Theta_{t+1} + M_t]$$

$$\Theta_t = D_t\Lambda_{t+1}\Phi_t + D_t\Theta_{t+1} + V_t$$

Using the initial condition $\tilde{P}_{-1} = 0$ we can solve for each of the endogenous variables under the contingency that the trap last to period S by (45) and (46). We then use the solution from (40)-(44)

to solve for each of the variables when the natural rate reverts back to steady state.

A.0.4 Solving for $\{k_\tau\}_{t=0}^{\infty}$

A simple way to find the value for $\{k_\tau\}_{\tau=1}^{\infty}$ is to first assume that k_τ is the same for all τ and find the k so that the zero bound is never violated. Suppose that the system has converged at t=25 (i.e. the response of each of the variables is the same). Then we can move to 24 and see if $k_\tau = 4$ for $\tau = 1, 2, ...24$ is a solution that never violates the zero bound. If not move to 23 and try the same thing and so on. For preparing this paper we wrote a routine in MATLAB that applied this method to find the optimal solution and verified that the results satisfied all the necessary conditions.

Chapter 2:

The Deflation Bias and Committing to being Irresponsible

Abstract

I model deflation, at zero nominal interest rate, in a microfounded general equilibrium model. I show that deflation can be analyzed as a credibility problem if the government has only one policy instrument, i.e. increasing money supply by open market operations in short-term bonds, and cannot commit to future policies. I propose several policies to solve the credibility problem. They involve printing money or issuing nominal debt and either 1) cutting taxes, 2) buying real assets such as stocks, or 3) purchasing foreign exchange. The government credibly "commits to being irresponsible" by using these policy instruments. It commits to higher money supply in the future so that the private sector expects inflation instead of deflation. This is optimal since it curbs deflation and increases output by lowering the real rate of return.

Can the government lose control over the price level so that no matter how much money it prints, it has no effect on inflation or output? Ever since Keynes' General Theory this question has been hotly debated. Keynes answered yes, Friedman and the monetarists said no. Keynes argued increasing money supply has no effect at low nominal interest rates. This is what he referred to as the liquidity trap. The zero short-term nominal interest rate in Japan today, together with the lowest short-term interest rate in the US in 45 years, make this old question interesting again. The Bank of Japan (BOJ) has nearly doubled the monetary base over the past 5 years, yet the economy still suffers deflation, and growth is stagnant. Was Keynes right? Is increasing money supply ineffective when the interest rate is zero? This paper revisits this question using a microfounded intertemporal general equilibrium model and assuming rational expectations. Both views are supported under different assumptions about policy expectations. Expectations about future policy are crucial, because they determine long-term interest rates. Even if short-term interest rates are binding, increasing money supply by open market operations in certain assets can stimulate demand by changing expectations about future short-term interest rates, thus reducing long-term interest rates.

The paper has three key results. The first is that monetary and fiscal policy have no effect in a liquidity trap *if* expectations about future money supply are independent of past policy decisions, and certain restrictions on fiscal policy apply. This is shown in a standard New Keynesian general equilibrium model widely used in the literature. The message is not that monetary and fiscal policy are irrelevant. Rather, the point is that monetary and fiscal policy have their largest impact in a liquidity trap through expectations. This indicates that the old fashion IS-LM model is a blind alley. That model assumes that expectations are exogenous. In contrast, expectations are at the heart of this paper.

I assume that expectations are rational. The government maximizes social welfare and I analyze two different equilibria. First I assume that the government is able to commit to future policy. This is the commitment equilibrium. Then I assume that the government is unable to commit to any future policy apart from paying back the nominal value of its debt. This is the Markov equilibrium. The optimal commitment is to commit to low future interest rates, modest inflation and an output boom once the exogenous shocks subside as in Eggertsson and Woodford (2003). This reduces the real rate of return in a liquidity trap and increases demand. In a Markov equilibrium, however, this commitment may not be feasible.

The second key result of the paper is that in a Markov equilibrium, deflation can be modelled as a credibility problem. This problem arises if the government has only one policy instrument, i.e. open market operations in government bonds, and is faced with temporary shocks that make the zero bound binding. Under these conditions there is excessive deflation if the government cannot commit to future policy. This is the deflation bias of discretionary policy. This theory of deflation, derived from the analysis of a Markov equilibrium, is in sharp contrast to conventional wisdom about deflation in Japan today (or, for that matter, US during the Great Depression). The conventional wisdom blames deflation on policy mistakes by the central bank or bad policy rules (see e.g. Friedman and Schwartz (1963), Krugman (1998), Buiter (2003), Bernanke (2000) and Benabib et al (2002)).[1] Deflation in this paper, however, is not attributed to an inept central bank or bad policy rules. It is a direct consequence of the central bank's *policy constraints* and inability to commit to the optimal policy when faced with negative demand shocks. This result, however, does not absolve the government of responsibility for deflation. Rather, it identifies the possible policy constraints that result in inefficient deflation in equilibrium (without resorting to an irrational policy maker). I identify two sources of inefficient deflation of equal importance. The first is the inability of the government to commit. The second is that open market operations in short-term government bonds is the only policy instrument. The central question of the paper, therefore, is how the government can use additional policy instruments to fight deflation even if it cannot commit to future policy.

The third key result of the paper is that in a Markov equilibrium the government can eliminate deflation by deficit spending. Deficit spending eliminates deflation for the following reason: If the government cuts taxes and increases nominal debt, and taxation is costly, inflation expectations increase (i.e. the private sector expects higher money supply in the future). Inflation expectation increase because higher nominal debt gives the government an incentive to inflate to reduce the real value of the debt. To eliminate deflation the government simply cuts taxes until the private sector expects inflation instead of deflation. At zero nominal interest rates higher inflation expectations reduce the real rate of return, and thereby raise aggregate demand and the price level. The two main assumption behind this result is that (i) there is some cost of taxation which makes this policy credible and that (ii) monetary and fiscal policy are coordinated.[2]

Deficit spending has exactly the same effect as the government following Friedman's famous suggestion to "drop money from helicopters" to increase inflation. At zero nominal interest rates money and bonds are perfect substitutes. They are one and the same: A government issued piece of paper that carries no interest but has nominal value. It does not matter, therefore, if the government drops money from helicopters or issues government bonds. Friedman's proposal thus increases the price level through the same mechanism as deficit spending This result, however, is not a vindication of the quantity theory of money. Dropping money from helicopters does *not* increase prices in a Markov equilibrium because it increases the *current* money supply. It creates inflation by increasing government debt which is defined as the sum of money and bonds. In a Markov equilibrium *it is government debt that determines the price level in a liquidity trap* because it determines expectations about *future* money supply.

The key mechanism that increases inflation expectation in this paper, and thus eliminating deflation, is government nominal debt. The government, however, can increase its debt in several ways. Cutting taxes or dropping money from helicopters are only two examples. The government can also increase debt by printing money (or issuing nominal bonds) and buy private assets, such as stocks, or foreign exchange. In a Markov equilibrium these operations increase prices and output *because they change the inflation incentive of the government by increasing government debt* (money+bonds). Hence, when the short-term nominal interest rate is zero, open market operations in real assets and/or foreign exchange increase prices through the same mechanism as deficit spending in a Markov equilibrium. This channel of monetary policy does not rely on the portfolio effect of buying real assets or foreign exchange. This paper thus compliments Meltzer's (1999) and McCallum (1999) arguments for foreign exchange interventions that rely on the portfolio channel.[3]

Deflationary pressures in this paper are due to temporary exogenous real shocks that shift aggregate demand.[4] The paper, therefore, does not address the origin of the deflationary shocks during the Great Depression in the US or in Japan today. These deflationary shocks are most likely due to a host of factors, including the stock market crash and banking problems. I take these deflationary pressures as given and ask: How can the government eliminate deflation by monetary and fiscal policy even if the zero bound is binding and it cannot commit to future policy? There is no doubt that there are several other policy challenges for a government that faces large negative shocks, and various structural problems, as in Japan.[5] Stabilizing the price level (and reducing real rates) by choosing the optimal mix of monetary and fiscal policy, however, is an obvious starting point and does not preclude other policy measures and/or structural reforms.

I study this model, and some extensions, in a companion paper with explicit reference to the current situation in Japan and some historical episodes (the Great Depression in particular). That paper also demonstrates that deficit spending may have little or no effect if the central bank is "goal independent". It follows that monetary and fiscal policy need to be coordinated for deficit spending to be effective, an assumption that is maintained in this paper.

1 The Model

Here I outline a simple sticky prices general equilibrium model and define the set of feasible equilibrium allocations. This prepares the grounds for the next section, which considers whether "quantitative easing" – a policy currently in effect at the Bank of Japan – and/or deficit spending have any effect on the feasible set of equilibrium allocations.

1.1 The private sector

1.1.1 Households

The representative household that maximizes expected utility over the infinite horizon:

$$E_t \sum_{T=t}^{\infty} \beta^T U_T = E_t \left\{ \sum_{T=t}^{\infty} \beta^T [u(C_T, \frac{M_T}{P_T}, \xi_T) + g(G_T, \xi_T) - \int_0^1 v(h_T(i), \xi_T) di] \right\} \quad (1)$$

where C_t is a Dixit-Stiglitz aggregate of consumption of each of a continuum of differentiated goods,

$$C_t \equiv [\int_0^1 c_t(i)^{\frac{\theta}{\theta-1}}]^{\frac{\theta-1}{\theta}}$$

with elasticity of substituting equal to $\theta > 1$, G_t is is a Dixit-Stiglitz aggregate of government consumption, ξ_t is a vector of exogenous shocks, M_t is end-of-period money balances, P_t is the Dixit-Stiglitz price index,

$$P_t \equiv [\int_0^1 p_t(i)^{1-\theta}]^{\frac{1}{1-\theta}}$$

and $h_t(i)$ is quantity supplied of labor of type i. $u(.)$ is concave and strictly increasing in C_t for any possible value of ξ. The utility of holding real money balances is increasing in $\frac{M_t}{P_t}$ for any possible value of ξ up to a satiation point at some finite level of real money balances as in Friedman (1969).[6] $g(.)$ is the utility of government consumption and is concave and strictly increasing in G_t for any possible value of ξ. $v(.)$ is the disutility of supplying labor of type i and is increasing and convex in $h_t(i)$ for any possible value of ξ. E_t denotes mathematical expectation conditional on information available in period t. ξ_t is a vector of r exogenous shocks. The vector of shocks ξ_t follows a stochastic process as described below.[7]

A1 (i) $pr(\xi_{t+j}|\xi_t) = pr(\xi_{t+j}|\xi_t, \xi_{t-1}, ...)$ for $j \geq 1$ where $pr(.)$ is the conditional probability density function of ξ_{t+j}. (ii) All uncertainly is resolved before a finite date K that can be arbitrarily high.

For simplicity I assume complete financial markets and no limit on borrowing against future income. As a consequence, a household faces an intertemporal budget constraint of the form:

$$E_t \sum_{T=t}^{\infty} Q_{t,T}[P_T C_T + \frac{i_T - i^m}{1+i_T} M_T] \leq W_t + E_t \sum_{T=t}^{\infty} Q_{t,T}[\int_0^1 Z_T(i)di + \int_0^1 n_T(j)h_T(j)dj - P_T T_T] \tag{2}$$

looking forward from any period t. Here $Q_{t,T}$ is the stochastic discount factor that financial markets use to value random nominal income at date T in monetary units at date t; i_t is the riskless nominal interest rate on one-period obligations purchased in period t, i^m is the nominal interest rate paid on money balances held at the end of period t, W_t is the beginning of period nominal wealth at time t (note that its composition is determined at time $t-1$ so that it is equal to the sum of monetary holdings from period $t-1$ and the (possibly stochastic) return on non-monetary assets), $Z_t(i)$ is the time t nominal profit of firm i, $n_t(i)$ is the nominal wage rate for labor of type i, T_t is net real tax collections by the government. The problem of the household is: at every time t the household takes W_t and $\{Q_{t,T}, n_T(i), P_T, T_T, Z_T(i), \xi_T; T \geq t\}$ as exogenously given and maximizes (1) subject to (2) by choice of $\{M_T, h_T(i), C_T; T \geq t\}$.

1.1.2 Firms

The production function of the representative firm that produces good i is:

$$y_t(i) = f(h_t(i), \xi_t) \tag{3}$$

where f is an increasing concave function for any ξ. I abstract from capital dynamics. As in Rotemberg (1983), firms face a cost of price changes given by the function $d(\frac{p_t(i)}{p_{t-1}(i)})$.[8] Price variations have a welfare cost that is separate from the cost of expected inflation due to real money balances in utility. I show that the key results of the paper do not depend on this cost being particularly large, indeed they hold even if the cost of price changes is arbitrarily small. The Dixit-Stiglitz preferences of the household imply a demand function for the product of firm i given by

$$y_t(i) = Y_t(\frac{p_t(i)}{P_t})^{-\theta}$$

The firm maximizes

$$E_t \sum_{T=t}^{\infty} Q_{t,T} Z_T(i) \tag{4}$$

where

$$Q_{t,T} = \beta^{T-t} \frac{u_c(C_T, \frac{M_T}{P_T}, \xi_T)}{u_c(C_t, \frac{M_t}{P_t}, \xi_t)} \frac{P_t}{P_T} \tag{5}$$

I can write firms period profits as:

$$Z_t(i) = (1+s) Y_t P_t^{\theta} p_t(i)^{1-\theta} - n_t(i) f^{-1}(Y_t P_t^{\theta} p_t^{-\theta}) - P_t d(\frac{p_t(i)}{p_{t-1}(i)}) \tag{6}$$

where s is an exogenously given production subsidy that I introduce for computational convenience (for reasons described later sections).[9] The problem of the firm is: at every time t the firm takes $\{n_T(i), Q_{t,T}, P_T, Y_T, C_T, \frac{M_T}{P_T}, \xi_T; T \geq t\}$ as exogenously given and maximizes (4) by choice of $\{p_T(i); T \geq t\}$.

1.1.3 Private Sector Equilibrium Conditions: AS, IS and LM Equations

In this subsection I show the necessary conditions for equilibrium that stem from the maximization problems of the private sector. These conditions must hold for *any* government policy. The first order conditions of the household maximization imply an Euler equation of the form:

$$\frac{1}{1+i_t} = E_t\{\frac{\beta u_c(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}, \xi_{t+1})}{u_c(C_t, \frac{M_t}{P_t}, \xi_t)} \frac{P_t}{P_{t+1}}\} \tag{7}$$

where i_t is the nominal interest rate on a one period riskless bond. This equation is often referred to as the IS equation. Optimal money holding implies:

$$\frac{u_{\frac{M}{P}}(C_t, \frac{M_t}{P_t}, \xi_t)}{u_c(C_t, \xi_t)} = \frac{i_t - i^m}{1+i_t} \tag{8}$$

This equation defines money demand or what is often referred as the "LM" equation. Utility is weakly increasing in real money balances. Utility does not increase further at some finite level of real money balances. The left hand side of (8) is therefore weakly positive. Thus there is bound on the short-term nominal interest rate given by:

$$i_t \geq i^m \tag{9}$$

In most economic discussions it is assumed that the interest paid on the monetary base is zero so that (9) becomes $i_t \geq 0$.[10]

The optimal consumption plan of the representative household must also satisfy the transversality condition

$$\lim_{T\to\infty} E_t(Q_{t,T}\frac{W_T}{P_t}) = 0 \tag{10}$$

to ensure that the household exhausts its intertemporal budget constraint. I assume that workers are wage takers so that households optimal choice of labor supplied of type j

satisfies

$$n_t(j) = \frac{P_t v_h(h_t(j);\xi_t)}{u_c(C_t, \frac{M_t}{P_t}, \xi_t)} \tag{11}$$

I restrict my attention to a symmetric equilibria where all firms charge the same price and produce the same level of output so that

$$p_t(i) = p_t(j) = P_t; \quad y_t(i) = y_t(j) = Y_t; \quad n_t(i) = n_t(j) = n_t; \quad h_t(i) = h_t(j) = h_t \quad \text{for} \quad \forall\ j, i \tag{12}$$

Given the wage demanded by households I can derive the aggregate supply function from the first order conditions of the representative firm, assuming competitive labor market so that each firm takes its wage as given. I obtain the equilibrium condition often referred to as the AS or the "New Keynesian" Phillips curve:

$$\begin{aligned} &\theta Y_t[\frac{\theta-1}{\theta}(1+s)u_c(C_t, \frac{M_t}{P_t}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] + u_c(C_t, \frac{M_t}{P_t}, \xi_t)\frac{P_t}{P_{t-1}}d'(\frac{P_t}{P_{t-1}}) \\ &- E_t \beta u_c(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}, \xi_{t+1})\frac{P_{t+1}}{P_t}d'(\frac{P_{t+1}}{P_t}) = 0 \end{aligned} \tag{13}$$

where for notational simplicity I have defined the function:

$$\tilde{v}(y_t(i), \xi_t) \equiv v(f^{-1}(y_t(i)), \xi_t) \tag{14}$$

1.2 The Government

There is an output cost of taxation (e.g. due to tax collection costs as in Barro (1979)) captured by the function $s(T_t)$.[11] For every dollar collected in taxes $s\,(T_t)$ units of output are waisted without contributing anything to utility. Government real spending is then given by:

$$F_t = G_t + s(T_t) \tag{15}$$

I could also define cost of taxation as one that would result from distortionary taxes on income or consumption and obtain similar results.[12]

I assume a representative household so that in a symmetric equilibrium, all nominal claims held are issued by the government. It follows that the government flow budget constraint is

$$B_t + M_t = W_t + P_t(F_t - T_t) \tag{16}$$

where B_t is the end-of-period nominal value of bonds issued by the government. Finally, market clearing implies that aggregate demand satisfies:

$$Y_t = C_t + d(\frac{P_t}{P_{t-1}}) + F_t \tag{17}$$

I now define the set of possible equilibria that are consistent with the private sector equilibrium conditions and the technological constraints on government policy.

Definition 1 A *Private Sector Equilibrium (PSE) is a collection of stochastic processes*

$\{P_t, Y_t, W_{t+1}, B_t, M_t, i_t, F_t, T_t, Q_t, Z_{t,} G_t, C_t, n_t, h_t, \xi_t\}$ *for* $t \geq t_0$ *that satisfy equations (2)-(17) for each* $t \geq t_0$*, given* W_{t_0}*,* P_{t_0-1} *and the exogenous stochastic process* $\{\xi_t\}$ *that satisfies A1 for* $t \geq t_0$.

Having defined feasible sets of equilibrium allocations, it is now meaningful to consider how government policies affect the feasible set of outcomes in the model.

2 Equilibrium with exogenous policy expectations

According to Keynes (1936) famous analysis, monetary policy loses its power when the short term nominal interest rate is zero. Others argue, most notably Friedman and

Schwartz (1963) and the monetarist, that a monetary expansion increases aggregate demand even under such circumstances, and this is what lies behind the "quantitative easing" policy of the BOJ since 2001.

One of Keynes better known suggestions is to increase demand in a liquidity trap by government deficit spending. Many have raised doubts recently about the importance of this channel, pointing to Japan's mountains of nominal debt, citing the Ricardian equivalence, i.e. the principle that any decrease in government savings should be offset by an increase in private savings (to pay for higher future taxes). Yet another group of economists argue that the Ricardian equivalence fails if deficit spending is financed by money creation (see e.g. Buiter (2003) and Bernanake (2000,2003)).

Here I consider whether or not "quantitative easing" and deficit spending are separate policy tools in the explicit intertemporal general equilibrium model laid out in the last section. The key result is that neither "quantitative easing" nor deficit spending have any effect on the feasible set of equilibrium allocations *if* expectations about future money supply remain unchanged – or alternatively – expectations about future interest rate policy remain unchanged. Furthermore, this result is unchanged if these two operations are used together, hence our analysis does not support the proposition that "money financed deficit spending" increases demand independently of the expectation channel. This result is an extension of the Eggertsson and Woodford (2003) irrelevance result, extended to include fiscal policy.

I do not contend that deficit spending and/or quantitative easing are irrelevant in a liquidity trap. Rather, my the point is that the main effect of these policies is best illustrated by analyzing how they change expectations about future policy, in particular expectations about future money supply. As we shall see the exact effect of these policy measures depends on assumptions about how future monetary and fiscal policies are conducted when the zero bound is not binding.

2.1 The irrelevance of monetary and fiscal policy when policy expectations are exogenous

Here I characterize policy that allows for the possibility that the government increases money supply by "quantitative easing" when the zero bound is binding and/or engages in deficit spending. The money supply is determined by a policy function:

$$M_t = M(s_t, \xi_t) I_t \tag{18}$$

where s_t is a vector that may include any of the endogenous variables *that are determined at time* t (note that as a consequence s_t cannot include W_t that is predetermined at time t). The multiplicative factor I_t satisfies the conditions

$$I_t = 1 \text{ if } i_t > 0 \quad \text{otherwise} \tag{19}$$

$$I_t = \psi(s_t, \xi_t) \geq 1. \tag{20}$$

The rule (18) is a fairly general specification of policy (since I assume that M_t is a function of all the endogenous variables). It could for example include simple Taylor type rules, monetary targeting, and any policy that does not depend on the past values of any of the endogenous variables.[13] Following Eggertsson and Woodford (2003) I define the multiplicative factor $I_t = \psi(s_t, \xi_t)$ when the zero bound is binding. Under this policy regime a policy of "quantitative easing" is represented by a value of the function ψ that is positive.

Note that I assume that the functions M and ψ are only functions of the endogenous variables and the shocks at time t. This separates the direct effect of a quantitative easing from the effect of a policy that influences expectation about future money supply. I impose the restriction on the policy rule (18) that:

$$M_t \geq M^*. \tag{21}$$

This restriction says the nominal value of the monetary base can never be smaller than some finite number M^*. This number can be arbitrarily small, so I do not view this as a very restrictive (or unrealistic) assumption since I am not modelling any technological innovation in the payment technology (think of M^* as being equal to one cent!). I assume, for simplicity, that the central bank does quantitative easing by buying government bonds, but the model can be extended to allow for the possibility of buying a range of other long or short term financial assets (see Eggertsson and Woodford (2003)). I also assume that the government only issues one period riskless nominal bonds so that B_t in equation (16) refers to a one period riskless nominal debt. Fiscal policy is defined by a function for real government spending:

$$F_t = F \tag{22}$$

and a policy function for deficit spending

$$T_t = T(s_t, \xi_t) \tag{23}$$

I assume that real government spending F_t is constant at all times in order to focus on deficit spending which is defined by the function $T(.)$ that specifies the evolution of taxes. Debt is issued at the end of period t is then defined by the consolidated government budget constraint (16) and the policy specifications (18)-(23). Finally I assume that the government is neither a debtor or a creditor asymtotically so that

$$\lim_{T\to\infty} E_t Q_{t,T} B_T = 0 \tag{24}$$

This is a fairly weak condition on the debt accumulation of the government policy stating that asymtotically it cannot accumulate real debt at a higher rate than the real rate of interest.[14] I can now obtain the following irrelevance result for monetary and fiscal policy

Proposition 1 *The set of paths $\{P_t, Y_t, i_t, F_t, Q_t, Z_{t,}G_t, C_t, n_t, h_t, \xi_t\}$ consistent with a PSE and the monetary and fiscal policy regimes (18)-(24) is independent of the specification of the functions $\psi(.)$ and $T(.)$.*

The proof of this proposition is fairly simple, and the formal details are provided in the Technical Appendix. I show that I can write all the equilibrium conditions for a PSE in a way that does not involve the functions T or ψ. First, I use market clearing to show that the intertemporal budget constraint of the household can be written without reference to either function. This relies on the Ricardian properties of the model. Second, I show that (10) is satisfied regardless of the specification of these functions using the two restrictions we imposed on policy given by (21) and (24). Finally I show that I can write the remaining conditions without any reference to the function $\psi(.)$, following the proof by Eggertsson and Woodford (2003).

2.2 Discussion

Proposition 1 says that a policy of quantitative easing and/or deficit spending has no effect on the set of feasible equilibrium allocations that are consistent with the policy regimes I specified. It may seem that this result contradicts Keynes' view that deficit spending is an effective tool to escape the liquidity trap. It may also seem to contradict the monetarist view (see e.g. Friedman and Schwartz) that increasing the money supply is effective at low interest rates. But this would only be true if one took a narrow view of these schools of thought like Hicks (1933) does in his ground breaking paper "Keynes and the Classics". Hicks develops a static version of the General Theory and contrasts it to the monetarist view assuming that expectation are exogenous constants. This is the IS-LM model. My analysis, however, indicates is that it is the intertemporal elements of the liquidity trap that are crucial to understand the effects of different policy actions, namely their effect on expectations (to be fair to Hick he was very explicit that he was abstracting from expectation and recognized this was a major issues). Both Keynes (1936) and many monetarists (e.g. Friedman and Schwartz (1963)) discussed the importance of expectations in some detail in their work. Trying to evaluate the theories of "Keynes and the Classics" in a static model is therefore not going to resolve the debate.

My result is that deficit spending has no effect on whether a given deflationary path represents an equilibrium if it does not change expectations about future policy. But as we shall see in later sections (when analyzing a Markov equilibrium) deficit spending can be very effective to *change expectations*. Thus the irrelevance result still leaves an important role for deficit spending, namely, it can be useful to change expectations. My result that quantitative easing is ineffective also relies on constant policy expectations. But as we shall also see (when analyzing a Markov equilibrium) quantitative easing changes expectation if the money printed is used to buy some private asset. Thus the irrelevance result also leaves an important role for quantitative easing through the expectation channel. Thus by modelling expectations explicitly, I believe my result neither contradicts Friedman and Schwartz' interpretation of the "Classics" , i.e. the Quantity Theory of Money, nor Keynes' General Theory. On the contrary, it may serve to integrate the two through modelling the expectation channel.

Proposition 1 may also seem to contradict the claims of Bernanke (2003) and Buiter (2003). Both authors indicate that money financed tax cuts increase demand. Buiter, for example, writes that "base money-financed tax cuts or transfer payments – the mundane version of Friedman's helicopter drop of money – will always boost aggregate demand." But what Buiter implicitly has in mind, is tax cuts permanently increasing the money supply. Thus a tax cut today, in his model, increases expectations about future money supply. Thus my proposition does not disprove Buiter's or Bernanke's claims since I assume that money supply in the future is set without any reference to past policy actions. The propositions, therefore, clarifies that tax cuts will only increase demand to the extent that they change beliefs about future money supply. The higher demand equilibrium that Buiter analyses, therefore, does not depend on the tax cut itself, only on expectations about future money supply. A similar comment applies to Auerbach and Obstfeld's (2003) result. They argue that open-market operations will increase aggregate demand. But their assumption is that open-market operations increase expectations about future money supply. It is that belief that matters and not the open market operation itself.[15]

3 Equilibrium with Endogenous Policy Expectations

The main lesson from the last section is that expectations about future monetary and fiscal policy are crucial. Deficit spending and quantitative easing have no effect if they do not change expectations about future policy. But does deficit spending have no effect on expectations under reasonable assumptions about how these expectation are formed? Suppose, for example, that the government prints unlimited amounts of money and drops it from helicopters, distributes it by tax cuts, or prints money and buys unlimited amounts of some private asset. Would this not alter expectations about future money supply? To answer this question I need an explicit model of how the government sets policy in the future. To do this I assume that the government sets monetary and fiscal policy optimally at all future dates. By optimal, I mean that the government maximizes social welfare that is given by the utility of the representative agent. I analyze equilibrium under two assumptions about policy formulation. Under the first assumption, which I call the commitment equilibrium, the government can commit to future policy in order to influence the equilibrium outcome by choosing future policy actions (at all different states of the world). Rational expectations require that these commitments are fulfilled in equilibrium. Under the second assumption, the government cannot commit to future policy. In this case the government maximizes social welfare under discretion in every period, disregarding any past policy actions, except insofar as they have affected the endogenous state of the economy at that date (defined more precisely below). Thus the government can only choose its current policy instruments, it cannot directly influence future governments actions. This is what I call the Markov equilibrium. In the Markov

Equilibrium, following Lucas and Stockey (1983) and a large literature that followed, I assume that the government is capable of issuing one period riskless nominal debt and committing to paying it back with certainty. In this sense, even under discretion, the government is capable of limited commitment.

3.1 Recursive representation

To analyze the commitment and Markov equilibrium it is useful to rewrite the model in a recursive form so that I can identify the endogenous state variables at each date. When the government can only issue one period nominal debt I can write the total nominal claims of the government (which in equilibrium are equal to the total nominal wealth of the representative household) as:

$$W_{t+1} = (1 + i_t)B_t + (1 + i^m)M_t$$

Substituting this into (16) and defining the variables $w_t \equiv \frac{W_{t+1}}{P_t}$, $m_t \equiv \frac{M_t}{P_{t-1}}$ and $\Pi_t = \frac{P_t}{P_{t-1}}$ I can write the government budget constraint as:

$$w_t = (1 + i_t)(w_{t-1}\Pi_t^{-1} + (F - T_t) - \frac{i_t - i^m}{1 + i_t} m_t \Pi_t^{-1}) \tag{25}$$

Note that I use the time subscript t on w_t (even if it denotes the real claims on the government at the beginning of time $t + 1$) to emphasize that this variable is determined at time t. I assume that $F_t = F$ so that real government spending is an exogenous constant at all times. In Eggertsson (2004) I treat F_t as a choice variable. Instead of the restrictions (21) and (24) I imposed in the last section on government policies, I impose a borrowing limit on the government that rules out Ponzi schemes:

$$u_c w_t \leq \bar{w} < \infty \tag{26}$$

where $\bar{w}$ is an arbitrarily high finite number. Here I have used (17) to substitute for consumption in the utility function. This condition can be justified by the fact the government can never borrow more than the equivalence of the expected discounted value of its maximum tax base (e.g. discounted future value of all future output).[16] It is easy to show that this limit ensures that the transversality condition of the representative household is satisfied at all times.

The treasury's policy instruments is taxation, T_t, that determines the end-of-period government debt which is equal to $B_t + M_t$. The central bank determines how the end-of-period debt is split between bonds and money by open market operations. Thus the central banks policy instrument is $M_{t.}$ Note that since P_{t-1} is determined in the previous period, I may think of $m_t \equiv \frac{M_t}{P_{t-1}}$ as the instrument of monetary policy.

It is useful to note that I can reduce the number of equations that are necessary and sufficient for a private sector equilibrium substantially from those listed in Definition 1. First, note that the equations that determine $\{Q_t,\ Z_t, G_t, C_t, n_t, h_t\}$ are redundant, i.e. each of them is only useful to determine one particular variable but has no effect on the any of the other variables. Thus I can define necessary and sufficient condition for a private sector equilibrium without specifying the stochastic process for $\{Q_t, Z_t, G_t, C_t, n_t, h_t\}$ and do not need to consider equations (3), (5), (6), (11), (15) and I use (17) to substitute out for C_t in the remaining conditions. Furthermore, condition (26) ensures that the transversality condition of the representative household is satisfied at all times so I do not need to include (10) in the list of necessary and sufficient conditions.

It is useful to define the expectation variable

$$f_t^e \equiv E_t u_c(Y_{t+1} - d(\Pi_{t+1}) - F, m_{t+1}\Pi_{t+1}^{-1}, \xi_{t+1})\Pi_{t+1}^{-1} \tag{27}$$

as the part of the nominal interest rates that is determined by the expectations of the private sector formed at time t. The IS equation can then be written as

$$1 + i_t = \frac{u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)}{\beta f_t^e} \tag{28}$$

Similarly it is useful to define the expectation variable

$$S_t^e \equiv E_t u_c(Y_{t+1} - d(\Pi_{t+1}) - F, m_{t+1}\Pi_{t+1}^{-1}, \xi_{t+1})\Pi_{t+1} d'(\Pi_{t+1}) \tag{29}$$

The AS equation can now be written as:

$$\theta Y_t[\frac{\theta-1}{\theta}(1+s)u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] + u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)\Pi_t d'(\Pi_t) - \beta S_t^e = 0 \tag{30}$$

Finally the money demand equation (8) can be written in terms of m_t and Π_t as

$$\frac{u_m(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)\Pi_t^{-1}}{u_c(Y_t - d(\Pi_t) - F, \xi_t)} = \frac{i_t - i^m}{1 + i_t} \tag{31}$$

The next two propositions are useful to characterize equilibrium outcomes. Proposition 2 follows directly from our discussion above:

Proposition 2 *A necessary and sufficient condition for a PSE at each time $t \geq t_0$ is that the variables $(\Pi_t, Y_t, w_t, m_t, i_t, T_t)$ satisfy: (i) conditions (9), (25),(26), (28), (30), (31) given w_{t-1} and the expectations f_t^e and S_t^e. (ii) in each period $t \geq t_0$, expectations are rational so that f_t^e is given by (27) and S_t^e by (29).*

Proposition 3 *The possible PSE equilibrium defined by the necessary and sufficient conditions for any date $t \geq t_0$ onwards depend only on w_{t-1} and ξ_t.*

The second proposition follows from observing that w_{t-1} is the only endogenous variable that enters with a lag in the necessary and sufficient conditions in (i) of Proposition 2 and using the assumption that ξ_t is Markovian (i.e. using A1) so that the conditional probability distribution of ξ_t for $t > t_0$ only depends on ξ_{t_0}. It follows from this proposition that (w_{t-1}, ξ_t) are the only state variables at time t that directly affect the PSE. I may economize on notation by introducing vector notation. I define vectors

$$\Lambda_t \equiv \begin{bmatrix} \Pi_t & Y_t & m_t & i_t & T_t \end{bmatrix}^T, \text{ and } e_t \equiv \begin{bmatrix} f_t^e \\ S_t^e \end{bmatrix}.$$

Since Proposition 3 indicates that w_t is the only relevant endogenous state variable, I prefer not to include it in either vector but keep track of it separately. It simplifies notation a bit to write the utility function as the function of Λ_t i.e. I define the function $U : \mathbb{R}^{5+r} \to \mathbb{R}$

$$U_t = U(\Lambda_t, \xi_t)$$

using (15) to solve for G_t as a function of F and T_t, along with (12) and (14) to solve for $h_t(i)$ as a function of Y_t.

3.2 The Commitment Equilibrium

Definition 2 *The optimal commitment solution at date $t \geq t_0$ is the PSE that maximizes the utility of the representative household given w_{t_0-1} and ξ_{t_0}.*

The equilibrium can be characterized by using a Lagrangian method fairly standard in the literature. The solution is shown in the Technical Appendix.

3.3 The Markov equilibrium

Here I consider an equilibrium that occurs when policy is conducted under discretion so that the government is unable to commit to any future actions To do this I solve for a Markov equilibrium (it is formally defined by Maskin and Tirole (2001)) that has been extensively applied in the monetary literature. The basic idea behind this equilibrium concept is to define a minimum set of state variables that directly affect market conditions and assume that the strategies of the government and the private sector expectations depend only on this minimum state. Proposition 3 indicates that a Markov equilibrium requires that the variables (Λ_t, w_t) only depend on (w_{t-1}, ξ_t), since this is the minimum set of state variables that affect the PSE.

The timing of events in the game is as follows: At the beginning of each period t, w_{t-1} is a predetermined state variable. At the beginning of the period, the vector of exogenous disturbances ξ_t is realized and observed by the private sector and the government. The monetary and fiscal authorities choose policy for period t given the state and the private sector forms expectations e_t. Note that I assume that the private sector may condition its expectation at time t on w_t, i.e. it observes the policy actions of the government in that period so that Λ_t and e_t are jointly determined. This is important because w_t is the relevant endogenous state variable at date $t+1$. Since the state in this game is captured by (w_{t-1}, ξ_t) a Markov equilibrium requires that there exist policy functions $\bar{\Pi}_t(.), \bar{Y}_t(.), \bar{m}_t(.), \bar{\imath}_t(.), \bar{T}_t(.)$, that I denote by the vector valued function $\bar{\Lambda}_t(.)$

and a function $\bar{w}_t(.)$, such that each period:[17]

$$\begin{bmatrix} \Lambda_t \\ \\ w_t \end{bmatrix} \equiv \begin{bmatrix} \bar{\Lambda}_t(w_{t-1},\xi_t) \\ \\ \bar{w}_t(w_{t-1},\xi_t) \end{bmatrix} \tag{32}$$

Note that the function $\bar{\Lambda}_t(.)$ and $\bar{w}_t(.)$ will also define a set of functions of (w_{t-1}, ξ_t) for $(Q_t,\ Z_t, G_t, C_t, n_t, h_t)$ by the redundant equations from Definition 1. Using $\bar{\Lambda}_t(.)$ I may also use (27) and (29) to define a function $\bar{e}_t(.)$ so so that

$$e_t = \begin{bmatrix} f_t^e \\ \\ S_t^e \end{bmatrix} = \begin{bmatrix} \bar{f}_t^e(w_t,\xi_t) \\ \\ \bar{S}_t^e(w_t,\xi_t) \end{bmatrix} = \bar{e}_t(w_t,\xi_t) \tag{33}$$

Rational expectations imply that the function $\bar{e}_t$ satisfies

$$\bar{e}_t(w_t,\xi_t) = \begin{bmatrix} E_t u_c(\bar{C}_t(w_t,\xi_{t+1}), \bar{m}_t(w_t,\xi_{t+1})\bar{\Pi}_t(w_t,\xi_{t+1})^{-1};\xi_{t+1})\bar{\Pi}_t(w_t,\xi_{t+1})^{-1} \\ \\ E_t u_c(\bar{C}_t(w_t,\xi_{t+1}), \bar{m}_t(w_t,\xi_{t+1})\bar{\Pi}_t(w_t,\xi_{t+1})^{-1};\xi_{t+1})\bar{\Pi}_t(w_t,\xi_{t+1})d'(\bar{\Pi}_t(w_t,\xi_{t+1})) \end{bmatrix} \tag{34}$$

I define a value function $J_t(w_{t-1}, \xi_t)$ as the expected discounted value of the utility of the representative household, looking forward from period t, given the evolution of the endogenous variable from period t onwards that is determined by $\bar{\Lambda}_t(.)$ and $\{\xi_t\}$. Thus I define:

$$J_t(w_{t-1},\xi_t) \equiv E_t \left\{ \sum_{T=t}^{\infty} \beta^T [U(\bar{\Lambda}_T(w_{T-1},\xi_T),\xi_T] \right\} \tag{35}$$

The optimizing problem of the government is as follows. Given w_{t-1} and ξ_t, the government chooses the values for (Λ_t, w_t) (by its choice of the policy instruments m_t and T_t) to maximize the utility of the representative household subject to the constraints in Proposition 1 and (33). Thus its problem can be written as:

$$\max_{m_t,w_t} [U(\Lambda_t,\xi_t) + \beta E_t J(w_t,\xi_{t+1})] \tag{36}$$

s.t. (9), (25),(26), (28), (30), (31) and (33)

I can now define a Markov equilibrium.

Definition 2 *A Markov equilibrium is a collection of functions* $\bar{\Lambda}_t(.), \bar{w}_t(), J_t(.), \bar{e}_t(.)$, *such that (i) given the function* $J_t(w_{t-1}, \xi_t)$ *and the vector function* $\bar{e}_t(w_t, \xi_t)$ *the solution to the policy maker's optimization problem (36) is given by* $\Lambda_t = \bar{\Lambda}_t(w_{t-1}, \xi_t)$ *and* $w_t = \bar{w}_t(w_{t-1}, \xi_t)$ *for each possible state* $(w_{t-1,}\xi_t)$ *(ii) given the vector function* $\bar{\Lambda}_t(w_{t-1}, \xi_t)$ and $\bar{w}_t(w_{t-1}, \xi_t)$ *then* $e_t = \bar{e}_t(w_t, \xi_t)$ *is formed under rational expectations (see equation (34)). (iii) given the vector function* $\bar{\Lambda}_t(w_{t-1}, \xi_t)$ *and* $\bar{w}_t(w_{t-1}, \xi_t)$ *the function* $J_t(w_{t-1}, \xi_t)$ *satisfies (35).*

I will only look for a Markov equilibrium in which the functions $\bar{\Lambda}_t(.)$, $J_t(.)$, $\bar{e}_t(.)$ are continuous and have well defined derivatives. Then the value function satisfies the Bellman equation:

$$J_t(w_{t-1}, \xi_t) = \max_{m_t, w_t} [U(\Lambda_t, \xi_t) + E_t \beta J_t(w_t, \xi_{t+1})] \tag{37}$$

s.t. (9), (25),(26), (28), (30), (31) and (33).

The solution can now be characterized by using a Lagrangian method for the maximization problem on the right hand side of (37). In addition, the solution satisfies an envelope conditions. The Lagrangian, associated with the appropriate first order condition, and the envelope condition, are shown in the Technical Appendix.

3.4 Approximation method

I define a steady state as a solution in the absence of shocks in which each of the variables $(\Pi_t, Y_t, m_t, i_t, T_t, w_t, f_t^e, S_t^e) = (\Pi, Y, m, i, T, w, f^e, S^e)$ are constants. Following Woodford

(2003), I define a steady state where monetary frictions are trivial. To do this I parameterize the utility function by the technology parameter $\bar{m}$ so that as $\bar{m}$ is reduced the household will demand ever lower real money balances. I denote the policy instrument as $\tilde{m}_t \equiv \frac{m_t}{\bar{m}}$ and it is still meaningful to discuss the evolution of the nominal stock of money even as $\bar{m} \to 0$ (see Technical Appendix for details). Furthermore I assume, following Woodford (2003), that the steady state is fully efficient so that $1 + s = \frac{\theta-1}{\theta}$. Finally I suppose that in steady state $i^m = 1/\beta - 1$. To summarize:

A2 Steady state assumptions. (i) $\bar{m} \to 0$, (ii) $1 + s = \frac{\theta-1}{\theta}$ (iii) $i^m = 1/\beta - 1$

Using A2 I prove in the Technical Appendix the existence of a steady state for both the commitment and the Markov solution given by $(\Pi, Y, \frac{m}{\bar{m}}, i, T, w, f^e, S^e) = (1, \bar{Y}, \tilde{m}, \frac{1}{\beta} - 1, \bar{F}, 0, u_c(\bar{Y} - \bar{F}), 0)$ and show the equations the values $\bar{Y}$, $\bar{F}$ and $\tilde{m}$ satisfy. Furthermore I discuss how the state for the Markov equilibrium relates to the results in Dedola (2002), King and Wolman (2003), Albanesi et al (2003) and Klein et al (2003). I then show that solution can be approximated around this steady state and that the resulting solution, which is locally unique, is accurate to the order $o(||\xi, \bar{\delta}||)$ where $\bar{\delta} \equiv \frac{i-i^m}{1+i}$. A complication is introduced by the presence of the interest rate bound inequality and I discuss how I treat this problem in the Technical Appendix. A further complication arises because in the Markov equilibrium the expectation functions $\bar{e}_t(.)$ are in general unknown. I illustrate a simple way of approximate these functions in Proposition 7.

4 The Deflation Bias

In the last section I showed how an equilibrium with endogenous policy expectations can be defined and approximated. I now analyze the approximate equilibrium and show that deflation can be modeled as a credibility problem. The point of this section is not to absolve the government of responsibility for deflation. Rather, the point is to identify the policy constraints that result in inefficient deflation. The policy constraint in this section, apart from the governments inability to commit to future policy, is the assumption that government spending and taxes are constant. Money supply, by open market operations in short-term government bonds, is the governments only policy instrument. This is equivalent to assuming that the interest rate is the only policy instrument. In the next section I relax this assumption. An appealing interpretation of the results is that they apply if the central bank does not coordinate its action with the treasury, i.e. if the central bank is "goal independent". This interpretation is discussed further in a companion paper Eggertsson (2004).

The assumption about the policy instruments of the government in this section is as follows:

A3 *Limited instruments: Open market operations in government bonds, i.e.* $\tilde{m}_t$, *is the only policy instrument. Fiscal policy is constant so that* $w_t = 0$ *and* $T_t = F$ *at all times*

To gain insights it is useful to consider the linear approximation of the private sector equilibrium constraints. The AS equation (13) is to the first order

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{38}$$

where $\kappa \equiv \theta\frac{(\sigma^{-1}+\lambda_2)}{d''}$. Here $\pi_t \equiv \Pi_t - 1$ is the inflation rate, $x_t \equiv \frac{Y_t - Y_t^n}{Y_t^n}$ is the output gap, i.e. it is the percentage deviation of output from the natural rate of output. The natural rate of output is the output that would be produced if prices where completely flexible, i.e. it is the output that solves the equation[18]

$$v_y(Y_t^n, \xi_t) = \frac{\theta - 1}{\theta}(1+s)u_c(Y_t^n, \xi_t). \tag{39}$$

This "Phillips curve" has become close to standard in the literature. In a linear approximation of the equilibrium the IS equation is given by:

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n) \tag{40}$$

where $\sigma \equiv -\frac{u_{cc}Y}{u_c}$ and r_t^n is the natural rate of interest, i.e. the real interest rate that is consistent with the natural rate of output and is only a function of the exogenous shocks. The exact form of r_t^n is shown in the Technical Appendix and it summarizes all the disturbances that appear in the linearized private sector equilibrium conditions.[19] I first show that if the natural rate of interest is positive at all times, and A2 and A3 hold, the commitment and the Markov solution are identical and the zero bound is never binding. To be precise, the assumption on the natural rate of interest is:

A4 $r_t^n \in [i^m, S]$ *at all times where S is a finite number greater than i^m.*

Assuming this restriction on the natural rate of interest I can proof the following proposition.

Proposition 4 ***Markov and the commitment equivalence.*** *If A2,A3(i),A3(ii) and A4 then the following must hold at least locally to the steady state and for S close enough* *to i^m: There is a unique bounded Markov and commitment solution given by $i_t = r_t^n \geq i^m$ and $\pi_t = x_t = 0$. The equilibrium is accurate up to an error of the order $o(||\xi, \bar{\delta}||^2)$*

Proof: See Technical Appendix

The intuition for this result is straight forward and can be understood by inspecting the linear approximation of the IS and AS conditions in addition to a second order expansion of the representative household utility (but the household utility is the objective of the government). When fiscal policy is held constant, the utility of the representative household, to the second order, is equal to:[20]

$$U_t = -[\pi_t^2 + \frac{\kappa}{\theta}(x_t - x^*)^2] + o(||\xi, \bar{\delta}, 1 + s - \frac{\theta}{\theta - 1}||^3) + t.i.p. \tag{41}$$

where $x^* = (\omega + \sigma^{-1})^{-1}(1 - \frac{\theta-1}{\theta}(1+s))$ and $t.i.p$ is terms independent of policy. In A2(ii) I assume that $(1 + s) = \frac{\theta}{\theta-1}$ an therefore $x^* = 0$. One can then observe by the IS and the AS equation that the government can completely stabilize the loss function at zero inflation and zero output gap in an equilibrium where $i_t = r_t^n$ at all times. Since this policy maximizes the government's objective at all times, there is no incentive for the government to deviate. Therefore the government's ability to commit has no effect on the equilibrium outcome, which is the intuition behind the formal proof of Proposition 4 in the Technical Appendix.

Proposition 4 only applies when $x^* = 0$ as in A2. When $x^* > 0$, the commitment and Markov solutions differ because of the classic inflation bias (stemming from monopoly powers of the firms) as first demonstrated by Kydland and Prescott (1977). I will now show that even when $x^* = 0$, the commitment and Markov solutions may also differ because of shocks that render the zero bound binding and which in turn trigger temporary excessive deflation in the Markov equilibrium. This new dynamic inconsistency problem is the deflation bias. I assume that $x^* > 0$ in the next subsection and show the connection between the inflation and the deflation bias.

The deflation bias can be derived by a simple assumption about the natural rate of interest r_t^n(recall that all the shocks that change the private sector equilibrium constraints can be captured by the natural rate of interest). Here I assume that the natural rate of interest becomes unexpectedly lower than i^m (e.g. negative) in period 0 and then reverts back to a positive steady state in every subsequent period with some probability. At the time r_t^n reverts back to steady state, a stochastic date denoted τ, it stays there forever. Assuming that all uncertainty is resolved before a finite date K simplifies the proofs. This is not a very restrictive assumptions since K may be arbitrarily high. To be more precise I assume:

A5 *$r_t^n = r_L^n < i^m$ at $t = 0$ and $r_t^n = r_{ss}^n = \frac{1}{\beta} - 1$ at all $0 < t < K$ with probability α if $r_{t-1}^n = r_L^n$ and probability 1 if $r_{t-1}^n = r_{ss}^n$ at all $t > 0$. The stochastic date when r_t^n* reverts to r_{ss}^n is denoted τ. *There is an arbitrarily large number K so that $r_t^n = r_{ss}^n$ with probability 1 for all $t \geq K$* and thus $\tau \leq K$.

The commitment and the Markov solutions derived in Proposition 4 are not feasible if A5 holds because the solution in Proposition 4 requires that $i_t = r_t^n$ at all times. If the natural rate of interest is temporarily below i^m, as in A5, this would imply a nominal interest rate below the bound i^m for the equilibrium to be achieved. How does the solution change when the natural rate of interest is below i^m (for example negative)?

Consider first the commitment solution. A simple numerical example is useful. Suppose that in period 0 the natural rate of interest is unexpectedly negative so that $r_L^n = -2\%$ and then reverts back to steady state of $r_{ss}^n = 2\%$ with 10 percent probability in each period (taken to be a quarter here). The calibration parameters I use are the same as in Eggertsson and Woodford (2003) (see details in the The Technical Appendix). Figure 1 shows (solid lines) the solution for inflation, the output gap, and the interest rate using the approximation method described in the Technical Appendix. The first line in the first panel shows inflation when the natural rate of interest reverts to the steady state in period 1, the second if it returns back in period 2 and so on.[21] The central bank offsets a low natural rate of interest by lowering the interest rate correspondingly. But when the natural rate of interest is negative this is not feasible. To offset the shock the government commits to inflation and a temporary boom *in the future*, i.e. once the natural rate of interest returns to normal, and keeping the nominal interest rate low for a substantial period. Furthermore (see figure 6 and 7) the optimal commitment implies a higher price level in the future and a higher money supply. The expectations of future inflation and output boom are beneficial when $r_L^n < 0$ because they offset the negative demand effect of the shock. To see this consider the IS equation (40). Even if the nominal interest rate cannot fall below 0 in period t, the real rate of return (i.e. $i_t - E_t\pi_{t+1}$) is what is relevant for aggregate demand and it can still be lowered by increasing inflation expectations. This is captured by the second element of the right hand side of equation (40). Furthermore, a commitment to a temporary boom, i.e. higher $E_t x_{t+1}$, also stimulates demand by the permanent income hypothesis. This is represented by the first term on the right hand side of equation (40).

Bank of Japan officials have objected to an inflation target on the grounds that it is not be "credible" since they cannot lower the nominal interest rate to manifest their intentions. The optimal commitment depends on manipulating expectations and one should consider the extent to which this policy commitment is credible, i.e. if the government has an incentive to deviate from the optimal plan. Consider now the Markov equilibrium. For the case $K \to \infty$ it can be shown to yield the simple closed form solution:[22]

$$x_t = \frac{1-\beta(1-\alpha)}{\alpha(1-\beta(1-\alpha))-\sigma\kappa(1-\alpha)}\sigma r_L^n \text{ if } r_t^n = r_L^n \text{ and } x_t = 0 \text{ otherwise}$$
$$\pi_t = \frac{1}{\alpha(1-\beta(1-\alpha))-\sigma\kappa(1-\alpha)}\kappa\sigma r_L^n \text{ if } r_t^n = r_L^n \text{ and } \pi_t = 0 \text{ otherwise}$$

The solution is shown in figure 2 for the calibrated example. It shows excessive deflation. The 90 percent chance of the natural rate of interest remaining negative for the next quarter creates the expectation of future deflation and a continued negative output gap, which creates even further deflation. Even if the central bank lowers the short-term nominal interest rate to zero, the real rate of return is positive, because the private sector expects deflation. In contrast to the optimal commitment, the Markov solution mandates zero inflation and zero output gap as soon as the natural rate of interest is positive. Thus the government cannot commit to a higher future price level as the optimal commitment implies and this lack of commitment is the main culprit for deflation. This is the deflation bias of discretionary policy.

Proposition 5 ***The deflation bias***. *If A2(i), A2(ii), A3 and A4 then the following must hold at least locally to the steady state. The Markov equilibrium for $t \geq \tau$ is given by $\pi_t = x_t = 0$ and the result is excessive deflation and output gap for $t < \tau$ relative to a policy that implies $\pi_\tau > 0$ and $x_\tau > 0$ and $i_t = 0$ when $t \leq \tau$. The equilibrium is accurate* *up to an error of the order* $o(||\xi,\bar{\delta}||^2)$

Proof: See Technical Appendix

What is the logic behind the deflation bias? Consider one realization of the shock from the numerical example. Figure 3 shows the commitment and the Markov solution for $\tau = 15$. The optimal commitment is to keep the nominal interest rate low for a substantial period of time after the natural rate becomes positive resulting in $x^C_{\tau=15} > 0$ and $\pi^C_{\tau=15} > 0$. If the government is discretionary, however, this type of commitment is not credible. In period 15, once the natural rate becomes positive again, the government raises the nominal interest rate to steady state, thus achieving zero inflation and zero output gap from period 15 onward. The result of this policy, however, is excessive deflation in period 0 to 14. Why does the government choose this suboptimal policy if it cannot commit? Consider the objectives of the government (recall that I assume that $x^* = 0$). Once the natural rate of interest has become positive again, at time $t = 15$, the optimal policy is to set the nominal interest rate at the steady state from then on since this policy will result in zero output gap and zero inflation *at that time onwards* — thus the Markov policy is maximizing the objectives (41) from period 15 onwards. The government, therefore, has an incentive to renege on the optimal commitment because the optimal commitment results in a temporary boom and inflation in period 15 and thus implies higher utility losses in period 15 onwards relative to the Markov solution. In rational expectation, however, the private sector understands the government's incentives. If the government is unable to commit the result is excessive deflation and an output gap in period 0 to 14 when the zero bound is binding. The deflation bias is not an artifact of the numerical values assumed in the example. Proposition 5 is proofed analytically in the Technical Appendix without the cost of changing prices being above any critical value. Thus it remains true even if the cost of changing prices is made arbitrarily small, as long as it is not exactly zero.[23]

In the Markov solution any increase in the monetary base at zero interest rate will *always* be expected to be reversed. This can help explain why BOJ aggressive increase in the monetary base has had little effect. It cannot credibly promise higher *future* money supply – the private sector expects the BOJ to contract as soon as there is any sign of inflation. It is a credibility problem of a *rational* central bank that *cannot commit to future policy.* Krugman (1998) recognizes a commitment problem at zero interest rate. He assumes that the government follows a monetary policy targeting rule so that $M_t = M^*$. He then shows that if expectation about future money supply are fixed at M^*, increasing money supply at time t has no effect at zero interest rate. Krugman calls this "the inverse of the usual credibility problem." The key to effective policy, according to Krugman, is to commit to higher money supply in the future (as is verified by our numerical example), i.e. to "commit to being irresponsible". My result illustrates that this problem is not isolated to a government that is expected to follow a monetary targeting rule. The problem arises for a government that maximizes social welfare and has only one policy instrument but is unable to commit to *not re-optimize in the future disregarding past decisions.* This is of practical importance. According to my solution, inefficient deflation is consistent with a rational government, as long as it is unable to commit to future policy. It may, therefore, be hard for it to change expectations for a government that has little credibility. In contrast, Krugman's government is committed to some monetary targeting policy rule that is suboptimal. It may, therefore, seem that it is easy to change policy expectations and that the only problem is only to find the optimal policy. This result, however, indicates that more may be required.

4.1 Extension: The inflation bias vs the deflation bias

The government's inability to commit in this model results in chronic inflation if $x^* > 0$. It is easy to show that if the zero bound is never binding (e.g under A3) inflation is given by

$$\pi_t = \bar{\pi} = \frac{1-\beta}{1-\beta+\theta\kappa} x^* > 0 \tag{42}$$

which is inefficient. This is the inflation bias of discretionary policy shown by Kydland and Prescott (1977) and Barro and Gordon (1983). It implies that the equilibrium nominal interest rate is given by

$$i_t = r_t^n + \bar{\pi}$$

Thus when there is an inflation bias in the economy, denoted by $\bar{\pi}$, a necessary condition for avoiding the interest rate bound is $r_t^n + \bar{\pi} \geq i^m$. If the natural rate of interest is low enough, however, there is a deflation bias. The government's inability to commit to a higher inflation rate than $\bar{\pi}$ results in excessive deflation. To summarize:

Proposition 6 ***The inflation bias vs the deflation bias.*** *If A2(i), A3, A5 and $0 \leq s < \frac{1}{\theta-1}$ then $\pi_t = \frac{\kappa}{1-\beta}\bar{x} = \bar{\pi}$ for $t \geq \tau$ and there is excessive deflation and an output gap in period $t < \tau$ if $r_L^n < i^m - \bar{\pi}$ relative to a policy that implies $\pi_\tau > \bar{\pi}$ and $x_\tau > \bar{x}$ and $i_t = 0$ when $t < \tau$. Here $\bar{\pi}$ is a solution to the equation $\bar{\pi} = \frac{1-\beta}{1-\beta+\theta\kappa}x^* \geq 0$. The equilibrium is accurate up to an error of the order $o(||\xi, \bar{\delta}, 1+s-\frac{\theta}{\theta-1}||^2)$.*

Proof: See Technical Appendix

Figure 4 shows the solution for inflation and the output gap for different values of x^*. Note that according to equation (42) a different value of x^* translates into different inflation targets for the government in a Markov equilibrium . The figure shows values of x^* that corresponds to 1%, 2% and 4% inflation targets respectively (I may vary this number by assuming different values for s in the expression for x^*). I assume A5 but the natural rate of interest is -4% in the low state and reverts back to steady state with 10 percent probability in each period. Note that only when the inflation bias corresponds to $\bar{\pi} = 4\%$ is there no deflation bias. If $\bar{\pi} < -r_L^n = 4\%$, the result is excessive deflation. The picture also illustrates, and this is the lesson of Proposition 6, that the deflation bias is a problem even in an economy with an average inflation bias, as long as the negative shock is large enough. The higher the average inflation bias, however, the larger the shock required for the deflation bias to be problematic. What is a realistic inflation bias in an industrial economy? If I use the same calibration values as the figures above (see Computational Appendix) the implied inflation bias is 0.75 percent inflation per year. If the model is applied to Japan, this is indeed quite consistent with average inflation rates during the 80's and early 90's (before deflationary pressures emerged). The inflation bias, therefore, is relatively low and a deflationary bias is a considerable concern. I think it is fairly realistic to assume a low inflation bias for Japan. Throughout the 80's an early 90's, for example, there was virtually no unemployment, and the government had a small incentive to inflate, consistent with that x^* close to zero. The assumption that $x^* = 0$, therefore, does not seem grossly at odds with the evidence for Japan, and as argued by Rogoff (2003) the great disinflation in the world indicates that the inflation bias may be small (and shrinking) throughout the rest of the world.

Two aspects of a liquidity trap render the deflation bias a particularly acute problem, and possibly a more serious than the inflation bias First, announcing a higher inflation target in a liquidity trap involves no direct policy action - since the short-term nominal interest rate is at zero it cannot be lowered any further. The central bank has, therefore, no obvious means to demonstrate its desire for inflation. Thus announcing an inflation target in a liquidity trap may be less credible then under normal circumstances when the central bank can take direct actions to show its commitment. Second, unfavorable shocks create the deflation bias. It may be hard for the central bank to acquire any reputation for dealing shocks if they are infrequent – which is presumably the case with shocks that make the zero bound binding given the few historical examples of the liquidity trap. To make matters worse, optimal policy in a liquidity trap involves committing to inflation. In an era of price stability the optimal policy under commitment is fundamentally different from what has been observed in the past.

5 Committing the Being Irresponsible

Last section demonstrated that deflation can be modelled as a credibility problem if the government is unable to commit to future policy and it's only instrument is open market operations. This section illustrates how the result changes if the government can use fiscal policy as an additional policy instrument. I first explore if deficit spending increases demand. When the government coordinates fiscal and monetary policy it can commit to future inflation and low nominal interest rate by cutting taxes and issuing nominal debt. I then use the result to interpret the effect of open market operations in a large spectrum of private assets, such as foreign exchange or real assets.

The assumption about monetary and fiscal policy is:

A6 *Coordinated fiscal and monetary policy instruments: Open market operations in government bonds, i.e.* $\tilde{m}_t$*, and deficit spending,* $B_t - T_t$*, are the instruments of policy.*

Using this assumption I can proof the following proposition.

Proposition 7 ***Committing to being irresponsible.*** *If A2, A5 and A6 then there is a solution at date* $t \geq \tau$ *for each of the endogenous variables given by* $\Lambda_t = \Lambda^1 w_{t-1}$*, and* $w_t = w^1 w_{t-1}$ *where* Λ^1 *and* w^1 *are constants. For a given value of* w^1 *there is a unique solution for* Λ^1*. The coefficient* w^1 *is a number that solves equation (146) in the The Technical Appendix. The solution for inflation is* $\pi_t = \pi^1 w_{t-1}$ *and the government can use deficit spending to increase inflation expectations when* $\pi^1 \neq 0$*, curbing deflation and the output gap in period* $t < \tau$*. The equilibrium is accurate up to an error of the order* $o(||\xi, \bar{\delta}||^2)$

I prove this proposition in the Appendix. The solution shows that nominal debt effectively commits the government to inflation even if it is discretionary. It is instructive to write out the algebraic expression for the inflation coefficient in the solution. I show in the Appendix that at $t \geq \tau$ the solution for inflation is

$$\pi_t = \pi^1 w_{t-1} \text{ where } \pi^1 = \frac{s' g_G}{d'' u_c} \beta^{-1} + \phi_4^1 \tag{43}$$

The government can reduce the real value of its debt (and future interest payments) by either increasing taxes or inflation. Since both inflation and taxes are costly, it chooses a combination of the two. The presence of debt creates inflation through two channels in our model: (1) If the government has outstanding *nominal* debt it has incentives to create inflation to reduce the real value of the debt. This incentive is captured by the term $\frac{s' g_G}{d'' u_c}\beta^{-1}$ in equation (43). The marginal cost of taxation is $s' g_G$ and the marginal cost of inflation is $d'' u_c$ (2) If the government issues debt at time t, it has incentives to lower the real rate of return its pays on the debt it rolls over to time $t+1$. This incentive also translates into higher inflation.[24] This incentive is reflected in the value of the coefficient ϕ_4^1 which is the coefficient in the solution for the Lagrangian multiplier on the AS equation i.e. $\phi_{4t} = \phi_4^1 w_{t-1}$. This coefficient reflects the value of relaxing the aggregate supply constraint, which can be beneficial because of the reduction in the real interest rate paid on debt associated with higher output; i.e. the government has an incentive to create a boom (by lowering the real rate of interest) to lower the service on the debt it rolls over to the next period.

As I showed in the previous section, committing to future inflation and an output boom is exactly what is mandated by the optimal commitment. Using the same numerical example as in previous section, figures 5 and 6 show that it is optimal for a discretionary government to issue debt when the zero bound is binding. This effectively commits it to future inflation and an output boom once the zero bound is no longer binding.[25] By cutting taxes and issuing debt in a liquidity trap the government curbs deflation and increases output to nearly the optimal commitment level. Figure 5also shows that the nominal interest rate stays below the steady state after the natural rate of interest returns to normal and rises only slowly.

The Markov solution is still not fully optimal since it does not replicate the commitment solution perfectly. Table 1 shows welfare under three policy regimes. Welfare is evaluated by utility of the representative household. The first regime, R1, is a government that can fully commit to future policy and uses both monetary and fiscal policy to achieve its objective. The second, R2, is a government that cannot commit to future policy but uses both monetary and fiscal policy to maximize utility. The third regime, R3, is a government that is unable to commit to future policy and has only one policy instrument, i.e. open market operations in short-term government bonds. This table shows that the government's ability to use debt as a commitment device nearly eliminates all the costs of discretion. The interpretation of this utility index is that under R1 the representative household would pay 0.02 percent of its steady state quarterly consumption (forever) to avoid moving to regime R2. Thus the number 0.02 reflects that value of commitment if the government can coordinate monetary and fiscal policy. In contrast the loss in utility to move from R1 to R3 is very large or 13.48 percent of quarterly consumption.[26]

Table 1

Policy regime	Utility in cons. eq. units
R1	100
R2	99.98
R3	86.52

Proposition 7, figures 5 and 6, and Table 1 summarize the central results of this paper. Even if the government cannot commit it can stabilize the price level in a liquidity trap. A simple way of increasing inflation expectations is coordinating fiscal and monetary policy and running budget deficits, which in turn increases output and prices. The channel is simple. Budget deficits generate nominal debt. Nominal debt, in turn, makes a higher inflation target credible because the real value of the debt increases if the government reneges on the target. Higher inflation expectations lower the real rate of interest and thus stimulate aggregate demand. This policy involves direct actions by the government which can be useful to communicate the policy (a criticism that is sometimes raised about the commitment policy is that it does not require any actions, only announcements about future intentions, see e.g. Friedman (2003)). The government can announce an inflation target and proceed to increase budget deficits until the target is reached.

Discussion To contrast the commitment and the discretion solutions, it is useful to consider the evolution of the price level. Figure 7 shows the evolution of the price level under the three policy regimes reported in Table 1. The optimal solution (i.e. R1) is to commit to a higher future price level as can be seen in panel a of figure 7, although the extent to which the price level increases is small. If the government is unable to commit, however, this policy is not credible. A dramatic decline in the price level occurs under monetary discretion (i.e. R3) as shown in panel b. The price level declines by 35 percent, for example, if the natural rate of interest becomes positive in period 15 this is the case I showed in figure 3). Panel c of figure 2 shows the large price decline can be avoided if the government uses fiscal policy to "commit to being irresponsible" (i.e. R2). This commitment involves increasing the price level once the natural rate becomes positive. When the natural rate of interest reverts to steady state in period 15, for example, the long run price level falls by less than 1 percent, compared to 35 percent decline under monetary discretion (R3).

It is worth considering the evolution of money supply in these different equilibria.[27] Figure 8 shows the long run nominal stock of money under each of the three policy regimes discussed above. In the figure I show the future level of the nominal stock of money in the case when the natural rate of interest reverts back to steady state in periods 3, 6, 9,12 and 15. The figure shows the level of money supply under each policy once the price level has converged back to its new steady state (so I do not need to make any assumptions here about the interest rate elasticity or output elasticity of money demand.[28] I assume that the value of the money supply is 1 before the shocks hit the economy. The figure illustrates that the optimal commitment (R1) involves committing to a nominal money supply in the future that is only marginally larger than before the shock. In contrast the monetary discretion (R3) involves a considerable contraction in the nominal monetary base. The government will accommodate any deflation at $t < \tau$ by contracting the monetary base as soon as the natural rate of interest becomes positive again,.in order to prevent inflation at $t \geq \tau$. Under a monetary and fiscal discretion regime (R2) aggressive deficit spending allows the government to credibly commit to a higher money supply, thus suppressing deflationary expectations. As a result the government achieves an equilibrium outcome that is close to the commitment solution, as illustrated in the welfare evaluation above and shown in figures 5 and 6.

An obvious question that arises if this model is applied to Japan. The gross national debt is currently over 130 percent of GDP. Why has the high level of outstanding debt in Japan failed to increase inflation expectations? There are at least two possible explanations of this. First, a large part of Japans debt is held by public institution and therefore not creating any inflation incentive. A better measure of the actual inflation incentive is net government debt. Net debt government debt as a fraction of GDP is not as high in Japan, about 70 percent, and only slightly above the G7 average. The other explanation (see Eggertsson (2004)) is that the Bank of Japan (BOJ) does not internalize the inflation incentive of outstanding government debt, i.e. that it has an objective that is more narrow than social welfare (that paper proofs that if the objective of BOJ is given by $\pi_t^2 + \lambda x_t^2$ deficit spending has no effect because it does not change the future incentive of the bank to inflate). Eggertsson (2004) argues that this indicates that there may be benefits of monetary and fiscal coordination, as suggested by Bernanke (2003), and verified by our welfare evaluation, and maintains that such cooperation may only need to be temporary to be effective.

5.1 Extension: Dropping money from helicopters and open market operations in foreign exchange as a commitment device

The model can be extended to analyze non-standard open market operations such as the purchase of foreign exchange and other private assets, or even more exotically, dropping money from helicopters. Here I discuss how these extensions enrich the results (an earlier version of this paper works out the details analytically – see Eggertsson (2003)).

Friedman suggests that the government can always control the price level by increasing money the supply, even in a liquidity trap. According to Friedman's famous *reductio ad absurdum* argument, if the government wants to increase the price level it can simply "drop money from helicopters." Eventually this should increase the price level – liquidity trap or not. Bernanke (2000) revisits this proposal and suggests that Japanese government should make "money-financed transfers to domestic households–the real-life equivalent of that hoary thought experiment, the "helicopter drop" of newly printed money." This analysis supports Friedman and Bernanke's suggestions. The analysis suggests, however, that it is the increase in government liabilities (money+bonds), rather than the increase in the money supply that has this effect. Since money and bonds are equivalent in a liquidity trap dropping money from helicopters is exactly equivalent to issuing nominal bonds. If the treasury and the central bank coordinate policy the effect of dropping money from helicopters will have exactly the same effect as deficit spending. Thus this paper's model can be interpreted as establishing a "fiscal theory" of dropping money from helicopters.

The model can also be extended to consider the effects of the government buying foreign exchange (or any other private assets). It is often suggested that the central bank can depreciate the exchange rate and stimulate spending by buying foreign exchange (and similar arguments are sometimes raised about some other private assets and their corresponding price). Due to the interest rate parity (and similar asset pricing equations for other private assets), however, buying foreign exchange should have no effect on the exchange rate unless it changes expectations about future policy (since the interest rate parity says that the exchange rate should depend on current and expected interest rate differentials). Will such operations have any effect on expectations about future policy? Open market operations in foreign exchange (or any other private asset) would lead to a corresponding increase in public debt defined as money plus government bonds. This gives the government an incentives to create inflation *through exactly the same channel as I have explored in this paper and, therefore, leads to a corresponding depreciation in the nominal exchange rate hand-in-hand with the rise in inflation expectations.* An advantage of buying private assets, as opposed to cutting taxes, is that it does not worsen the net fiscal position of the government. It only changes the inflation incentive of the government.

6 Conclusion

My analysis, may offer some insights into the current state of the Japanese economy. The irrelevance proposition presented in the paper implies that "quantitative easing" beyond the size of the monetary base required to keep the call rate at zero may be ineffective if it fails to change expectations about future monetary policy. This may help explain the apparent ineffectiveness of the "quantitative easing" by Bank of Japan (BOJ) since May 2001. The irrelevance proposition can also shed light on the failure of deficit spending to do more to stimulate the Japanese economy and eliminate expectations of deflation. In the model the principle of "Ricardian equivalence" holds. This aspect of the model is plainly an idealization, and one would not expect Ricardian equivalence to hold exactly in a more realistic model. Nonetheless, the essential prediction of such a model does not seem too far off in Japan: decreases in government saving (increases in government borrowing) have offset increases in private savings resulting in no discernible increase in aggregate demand.

The analysis of the Markov equilibrium indicates that if the BOJ has limited credibility open market operations in short-term government bonds may be insufficient to fight deflation even if the goal is to change expectations. Coordinating the interest rate with other policy instruments, however, can be effective. Government deficits combined with appropriate interest rate policy can be used to stimulate aggregate demand by changing expectations from being deflationary to inflationary. If future monetary policy takes account of the distortions resulting from high taxes, a higher nominal public debt results in more inflationary monetary policy. This does not, however, seem to match the expectations of many observers regarding the likely behavior of the BOJ. In particular, the public may not believe that the BOJ will care much about reducing the burden of public debt when determining future monetary policy, given some statements by BOJ officials. This implies that cooperation between the Ministry of Finance (MOF) and the BOJ may be useful to increase inflation expectations, as suggested by Bernanke (2003), and discussed in more detail in a companion paper (see Eggertsson (2004)).

Notes

[1]There is a large literture that discusses optimal monetray policy rules when the zero bound is binding. Contributions include Summers (1991), Fuhrer and Madigan (1997), Woodford and Rotemberg (1997), Wolman (1999), Reifschneider and Williams (2000) and references there in. Since monetary policy rules arguably become credible over time these contributions can be viewed as illustration of how to *avoid* a liquidity trap rather than a prescription of how to escape them which is the focus here.

[2]The Fiscal Theory of the Price Level (FTPL) popularized by Leeper (1991), Sims (1994) and Woodford (1994,1996) also stresses that fiscal policy can influence the price level. What separates this analysis from the FTPL (and the seminal contribution of Sargent and Wallace (1981)) is that in my setting fiscal policy only affects the price level because it changes the *inflation incentive* of the government. In contrast, according to the FTPL fiscal policy affects the price level because it is *assumed* that the monetary authority commits to a (possibly suboptimal) interest rate rule and fiscal policy is modelled as a (possibly suboptimal) exogenous path of real government surpluses. Under these assumptions innovations in real government surpluses can influence the price level, since the prices may have to move for the government budget constraint to be satisfied. In my setting, however, the government budget constraint is a *constraint* on the policy choices of the government.

[3]The argument in the paper is also complimentary to Svensson's (2001) "foolproof"' way of escaping the liquidity trap by foreign exchange intervention. I show explicitly how foreign exchange rate intervention increase inflation expectation even if the government cannot commit to future policy and maximizes social welfare.

[4]In contrast to Benabib et al (2002) where deflation is due to selffulfilling deflationary spirals.

[5]See for example Caballero et al (2003) that argue that banking problems are at the heart of the Japaneese recession.

[6]The idea is that real money balances enter the utility because they facilitate transactions. At some finite level of real money balances, e.g. when the representative household holds enough cash to pay for all consumption purchases in that period, holding more real money balances will not facilitate transaction any further and thereby add nothing to utility. This is at the "satiation" point of real money balances. We assume that there is no storage cost of holding money so increasing money holding can never reduce utility directly through $u(.)$. A satiation level in real money balances is also implied by several cash-in-advance models such as Lucas and Stokey (1987).

[7]Assumption A1 (i) is the Markov property. This assumption is not very restrictive since the vector ξ_t can be augmented by lagged values of a particular shock. Assumption A1 (ii) is added for tractability. Since K can be arbitrarily high it is not very restrictive.

[8]I assume that $d'(\Pi) > 0$ if $\Pi > 1$ and $d'(\Pi) < 0$ if $\Pi < 1$. Thus both inflation and deflation are costly. $d(1) = 0$ so that the optimal inflation rate is zero (consistent with the interepretation that this represent a cost of changing prices). Finally, $d'(1) = 0$ so that in the neighborhood of the zero inflation the cost of price changes is of second order.

[9]I introduce it so that I can calibrate an inflationary bias that is independent of the other structural parameters, and this allows me to define a steady state at the fully efficient equilibrium allocation. I abstract from any tax costs that the financing of this subsidy may create.

[10]The intuition for this bound is simple. There is no storage cost of holding money in the model and money can be held as an asset. It follows that i_t cannot be a negative number. No one would lend 100 dollars if he or she would get less than 100 dollars in return.

[11]The function s(T) is assumed to be differentiable with derivatives $s'(T) > 0$ and $s''(T) > 0$ for $T > 0$.

[12]The specification used here, however, focuses the analysis on the channel of fiscal policy that I am interested in. This is because for a constant F_t the level of taxes has no effect on the private sector equilibrium conditions (see equations above) but only affect the equilibrium by reducing the utility of the households (because a higher tax costs mean lower government consumption G_t). This allows me to isolate the effect current tax cuts will have on expectation about future monetary and fiscal policy, abstracting

away from any effect on relative prices that those tax cuts may have. This is the key reason that I can obtain Propostion 1 in the next section even if taxation is costly. There is no doubt that tax policy can change relative prices and that these effects may be important. Those effects, however, are quite separate from the main focus of this paper.There is work in progress by Eggertsson and Woodford that considers how taxes that change relative prices can be used to affect the equilibrium allocations. That work considers labor and consumption taxes.

[13]The Taylor rule is a member of this family in the following sense. TheTaylor rule is

$$i_t = \phi_\pi \Pi_t + \phi_y Y_t$$

The money demand equation (8) defines the the interest rate as a function of the monetary base, inflation and output. This relation may then be used to infer the money supply rule that would result in an indentical equilibrium outcome as a Taylor rule and would be a member of the rules we consider above.

[14]One plausible sufficient condition that would guarantee that (24) must always hold is to assume that the private sector would never hold more government debt that correpondes to expected future discounted level of some maximum tax level – that would be a sum of the maximum seignorage revenues and some technology constraint on taxation.

[15]An obvious criticism of the irrelevance result for fiscal policy in Proposition 1 is that it relies on Ricardian equivalence. This aspect of the model is unlikely to hold exactly in actual economies. If taxes effect relative prices, for example if I consider income or consumption taxes, changes in taxation change demand in a way that is independent of expectations about future policy. Similarly, if some households have finite-life horizons and no bequest motive, current taxing decisions affect their wealth and thus aggregate demand in a way that is also independent of expectation about future policy.This is a point developed by Ireland (2001) who show that in an overlapping generation model wealth transfers increase demand at zero nominal interest rate (this of course would also be true at positive interest rate). The assumption of Ricardian equivalence is not applied here, however, to downplay the importance of these additional policy channels. Rather, it is made to focus the attention on how fiscal policy may change policy expectations. That exercise is most clearly defined by specifying taxes so that they can *only* affect the equilibrium through expectations about future policy. Furthermore, since our model indicates that expectations about

future monetary policy have large effects in equilibrium, my conjecture is that this channel is of first order in a liquidity trap and thus a good place to start.

[16]Since this constraint is never binding in equilibrium and $\bar{w}$ can be any arbitrarily high number for the results to be obtained, I do not model in detail the endogenous value of the debt limit.

[17]Note that if the conditional expectation of ξ_{t+1} at time t does not depend on calender time, these functions will be time invariant and one may drop the subscript t.

[18]Note that this definition of the natural rate of output is different from the efficient level of output which is obtained if $(1+s) = \frac{\theta}{\theta-1}$ and prices are flexible. Also note that I allow for both s and i^m to be different from A2 so that the AS and the IS equation are accurate to the order $o(||\xi, \bar{\delta}, 1+s-\frac{\theta}{\theta-1}||^2)$.

[19]Although note that other shocks may change the government objectives, e.g. through the utility of government consumption, and that I abstract from stochastic variations in markups.

[20]Here I have expanded this equation around the steady state discussed in section (3.4) and allowed for stochastic variations in ξ and also assumed that s and i^m may be deviate from the steady state I expand around. Derivation is available upon request.

[21]The numerical solution reported here is exactly the same as the one shown by Eggertsson and Woodford (2003) in a model that is similar but has Calvo prices (instead of the quadratic adjustment costs I assume here). Their solution also differs in that they compute the optimal policy in a linear quadratic framework. As our numerical solution illustrates, however, the results for the commitment equilibrium are identical. Jung et al (2001) also derive the commitment equilibrium in a linear quadratic framework but assume a deterministic process for the natural rate.

[22]Note that to ensure that the solution is bounded I need to assume that α satisfies the inequalities $\beta\alpha^2 + (1+\sigma\kappa-\beta)\alpha - \sigma\kappa > 0$ and $0 < \alpha < 1$. If this condition is not satisfied the solution explodes and a linear approximation of the IS and the AS equation is not valid for shocks of any order of magnitude. Thus I would need to use other nonlinear solution methods to solve for the equilibrium if the value of α does not satisfy these bounds. Here I simply assume parameters so that these two inequalities are satisfied and a linear approximation of the IS and AS is feasible and the solution is accurate of order $o(||\xi, \bar{\delta}||^2)$ (see Technical Appendix).

[23]It is easiest to see this for a special case of A5. If $\alpha = 1$ the natural rate of interest is positive with

probability 1 in period 1. Then Proposition 6 indicates that the solution in period 1 onwards is given by $\pi_t = x_t = 0$ for $t \geq 1$. The IS indicates that in period 0 the output gap is $x_0 = \sigma r_t^n$. Note that the output gap in period 0 is independent of the cost of changing prices since neither r_t^n nor σ are a function of the cost of price changes. This is because the output gap only depends on the difference between the current interest rate and the natural rate of interest and expectations about future inflation and output gap, and the latter are zero in period 1 onwards. The AS equation, however, indicates that the deflation in period 0 is going to depend on the cost of changing prices, i.e. $\pi_0 = \kappa x_0$. The lower the cost of changing prices the higher is $\kappa = \frac{\theta}{d''}(\sigma^{-1} + \omega)$ which indicates that there will be more deflation, the lower the cost of price changes (since x_0 is given by the IS equation which does not depend on d"). The intuition for this is that the lower the cost of price changes, the more prices need to adjust for the equation $x_0 = \sigma r_t^n$ to be satisfied. Thus the deflation bias is worse – in terms of actual fall in the price level – the lower the cost of changing prices. This basic intuition will also carry through to the stochastic case.

[24]Obstfeld (1991,1997) analyses a flexible price model with real debt (as opposed to nominal as in our model) but seignorage revenues due to money creation. He obtains a solution similar to mine (i.e. debt in his model creates inflation but is paid down over time). Calvo and Guidotti (1992) similarly illustrate a flexible price model that has a similar solution. The influence of debt on inflation these authors illustrate is closely related to the first channel we discuss above. The second channel we show, however, is not present in these papers since they assume flexible prices.

[25]In general there are more than one solution for w^1 in equation 146. In the numerical examples I have done, however, all but one of the values that satisfy this equation are explosive and imply that by equation (146) that the value of γ_{2t} is negative once the debt limit of the government is reached. This in turn, violates the inequality constraint of this mulitplier, imlying that an explosive solution does not solve the first order conditions of the governments maximization problem. It can be proofed in a simplified version of the model that there is always a unique solution w^1 that solves the model and that it implies that debt converges back to steady state. For this version of the model, however, an analytic proof is not available, but in all the calibrated examples that I have explored this is indeed the case.

[26]Here I normalize the utility flow by transforming the utility stream (which is the future discounted stream of utility from private and public consumption – in all states of the world – minus the flow from

the disutility of working) into a stream of a constant private consumption endowment.

[27]I have assumed that monetary frictions are very small, but as I discuss in the Technical Appendix money demand is still well defined so that it remains meaningful to discuss the growth rate of money supply (even if the real monetary base relative to output is very small). The money demand equation defines the evolution for real money balances in the equilibrium, i.e the variables $\tilde{m}_t$ which is normalized by the transaction technology parameter, and the growth rate of money supply can then be inferred from equation (66) in the Technical Appendix. I can then calculate the money supply for each of the different equilibria.

[28]It is not very instructive to consider the evolution of the nominal stock in the transition periods because the large movement in the nominal interest rate cause large swings in the nominal stock of money).

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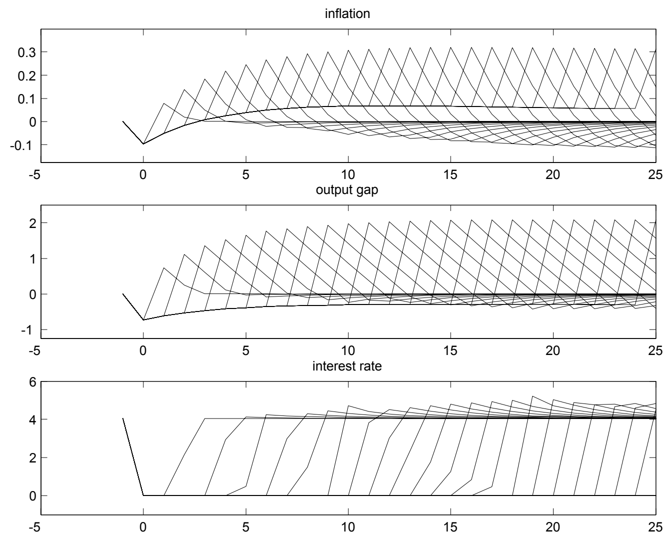

Figure 1: Inflation, the output gap, and the short-term nominal interest rate under optimal policy committment when the goverment can only use open market operations as its policy instrument. Each line represent the response of inflation, the output gap or the nominal interest rate when the natural rate of interest returns to its steady-state value in that period.

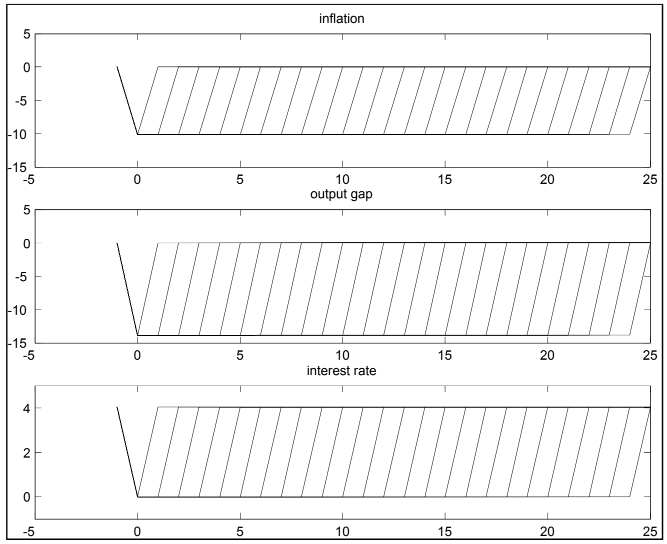

Figure 2: Inflation, the output gap, and the short-term nominal interest rate in a Markov equilibrium under discretion when the goverment can only use open market operations as its policy instrument. Each line represent the response of inflation, the output gap or the nominal interest rate when the natural rate of interest returns to its steady-state value in that period.

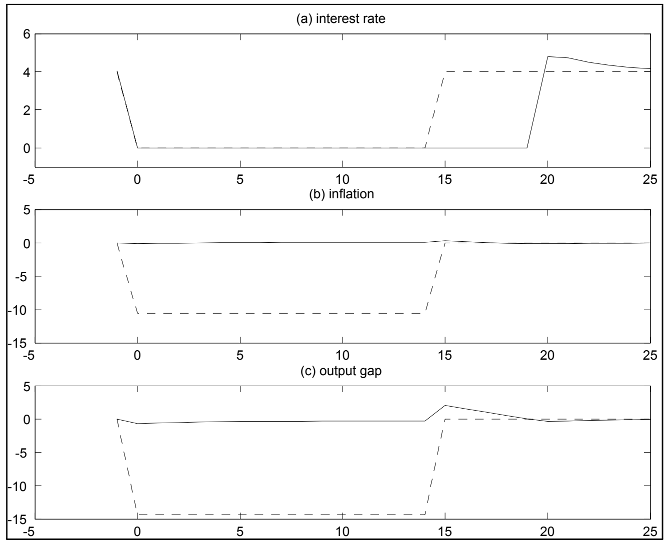

Figure 3: Response of the nominal interest rate, inflation and the output gap to a shocks that lasts for 15 quarters.

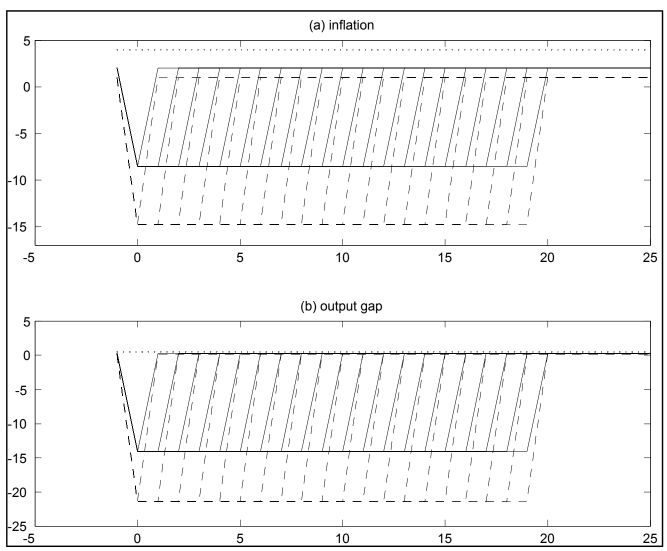

Figure 4: Inflation and the output gap under different assumption about steady state inflation bias when the natural rate of interest is temporarily -4 percent. The dotted lines correpond to a 4 percent steady state inflation bias, the solid line 2 percent and the dashed line 1 percent.

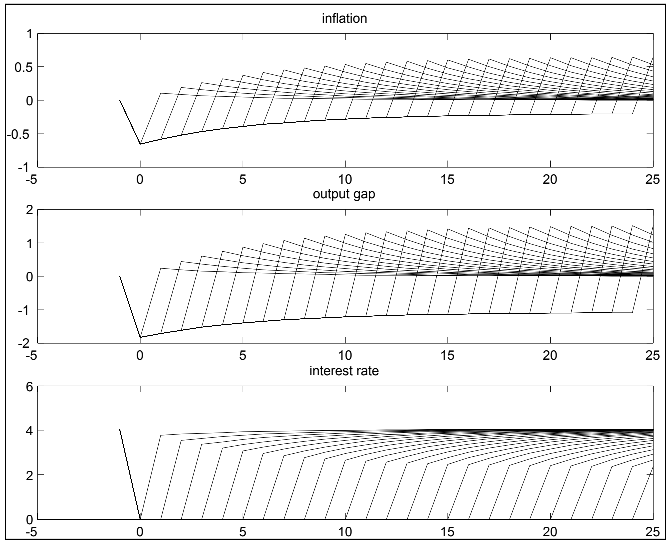

Figure 5: Inflation and output gap in a Markov equilibrium under discretion, when the government can use both monetary and fiscal policy to respond to a negative natural rate of interest.

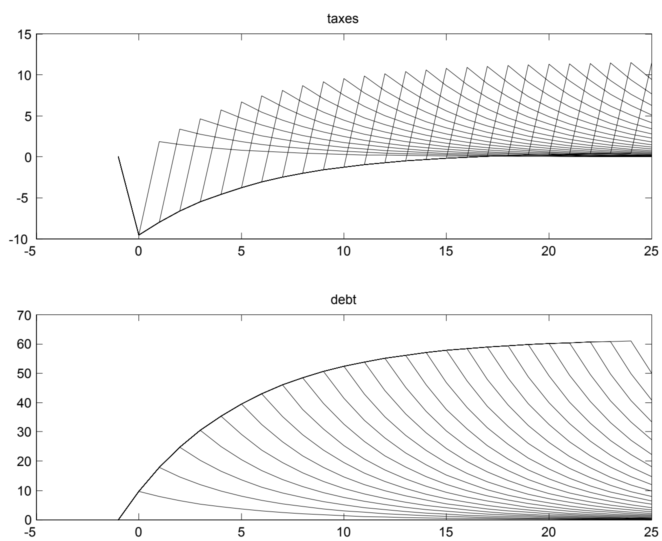

Figure 6: Taxes and debt in a Markov equilibrium under discretion, when the government can use both monetary and fiscal policy to respond to a negative natural rate of interest.

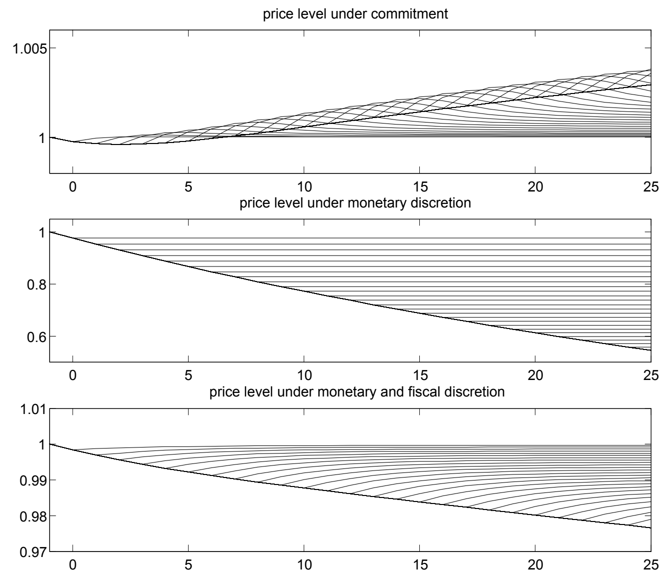

Figure 7: The evolution of the price level under different assumptions about policy.

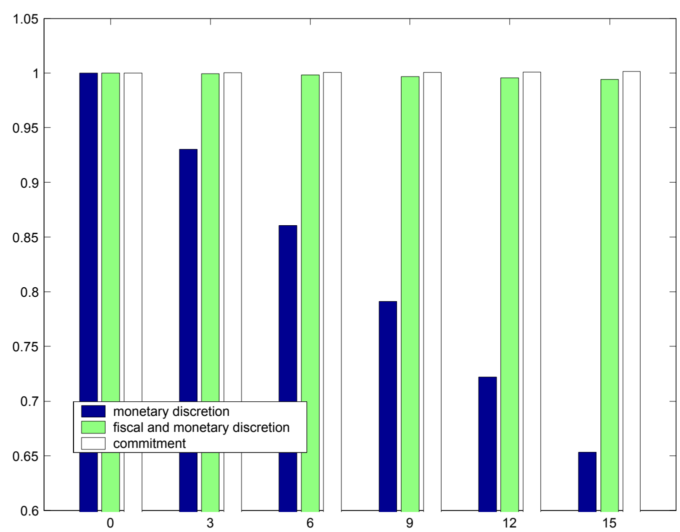

Figure 8: Long run nominal stock of money under different contingencies for the natural rate of interest.

A Technical Appendix

This Technical Appendix contains the numerical solution methods used and some further details for the proofs, for readers interested in the technical details. The appendix is not intended for publication so that it contains quite extensive details to facilitate the verification of the results. Some of this material is also contained in the Technical Appendix of a companion paper Eggertsson (2004) and the computation method shown in section (A.2.5) is also applied (with appropriate modifications) in Eggertsson and Woodford (2003).

A.1 Explicit first order conditions for the commitment and Markov solution

This section shows the non-linear first order conditions of the governments maximization problem in the Markov and the commitment equilibrium.

A.1.1 Commitment FOC

In its explicit form the commitment Lagrangian is

$$L_t = E_{t_0} \sum_{t=t_0}^{\infty} \beta^t [u(Y_t - d(\Pi_t) - F, m_t \Pi_t^{-1}, \xi_t)) + g(F - s(T_t), \xi_t) - \tilde{v}(Y_t)$$

$$+ \phi_{1t}(\frac{u_m(Y_t - d(\Pi_t) - F, m_t \Pi_t^{-1}, \xi_t)\Pi_t^{-1}}{u_c(Y_t - d(\Pi_t) - F, m_t \Pi_t^{-1}, \xi_t)} - \frac{i_t - i^m}{1 + i_t})$$

$$+ \phi_{2t}(w_t - (1 + i_t)\Pi_t^{-1} w_{t-1} - (1 + i_t)F + (1 + i_t)T_t + (i_t - i^m) m_t \Pi_t^{-1})$$

$$+ \phi_{3t}(\beta f_t^e - \frac{u_c(Y_t - d(\Pi_t) - F, m_t \Pi_t^{-1}, \xi_t)}{1 + i_t})$$

$$+ \phi_{4t}(\theta Y_t [\frac{\theta - 1}{\theta}(1 + s) u_c(Y_t - d(\Pi_t) - F, m_t \Pi_t^{-1}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)]$$

$$+ u_c(Y_t - d(\Pi_t) - F, m_t \Pi_t^{-1}, \xi_t)\Pi_t d'(\Pi_t) - \beta S_t^e)$$

$$+ \psi_{1t}(f_t^e - u_c(Y_{t+1} - d(\Pi_{t+1}) - F, m_{t+1}\Pi_{t+1}^{-1}, \xi_{t+1})\Pi_{t+1}^{-1})$$

$$+ \psi_{2t}(S_t^e - u_c(Y_{t+1} - d(\Pi_{t+1}) - F, m_{t+1}\Pi_{t+1}^{-1}, \xi_{t+1})\Pi_{t+1} d'(\Pi_{t+1})) + \gamma_{1t}(i_t - i^m) + \gamma_{2t}(\bar{w} - w_t)]$$

FOC (all the derivatives should be equated to zero) at all dates $t \geq 1$.

$$\frac{\delta L_s}{\delta \Pi_t} = -u_c d'(\Pi_t) - u_m m_t \Pi_t^{-2} \tag{44}$$

$$+\phi_{1t}[-\frac{u_{mc} d' \Pi_t^{-1}}{u_c} - \frac{u_{mm} m_t \Pi_t^{-2}}{u_c} - \frac{u_m \Pi_t^{-2}}{u_c} + \frac{u_m u_{cc} d' \Pi_t^{-1}}{u_c^2} + \frac{u_m u_{cm} m_t \Pi_t^{-2}}{u_c}]$$

$$+\phi_{2t}[(1+i_t) w_{t-1} \Pi_t^{-2} - (i_t - i^m) m_t \Pi_t^{-2}] + \phi_{3t}[\frac{u_{cc} d'}{1+i_t} + \frac{u_{cm} m_t \Pi_t^{-2}}{(1+i_t)}]$$

$$+\phi_{4t}[-Y_t(\theta-1)(1+s)(u_{cc} d' + m_t \Pi_t^{-2} u_{cm}) - u_{cc} \Pi_t d'^2 - u_{cm} m_t \Pi_t^{-1} d' + u_c \Pi_t d'' + u_c d']$$

$$+\beta^{-1} \psi_{1t-1}[u_{cc} d' \Pi_t + u_{cm} m_t \Pi_t^{-1} + u_c \Pi_t^{-2}]$$

$$+\beta^{-1} \psi_{2t-1}[u_{cc} d'^2 \Pi_t + u_{cm} d' m_t \Pi_t^{-1} - u_c d' - u_c d'' \Pi_t]$$

$$\frac{\delta L_s}{\delta Y_t} = u_c - \tilde{v}_y + \phi_{1t}[\frac{u_{mc} \Pi_t^{-1}}{u_c} - \frac{u_m \Pi_t^{-1}}{u_c^2}] - \phi_{3t} \frac{u_{cc}}{1+i_t} \tag{45}$$

$$+\phi_{4t}[\theta(\frac{\theta-1}{\theta}(1+s) u_c - \tilde{v}_y) + \theta Y_t(\frac{\theta-1}{\theta}(1+s) u_{cc} - \tilde{v}_{yy}) + u_{cc} \Pi_t d']$$

$$-\beta^{-1} \psi_{1t-1} u_{cc} \Pi_t - \beta^{-1} \psi_{2t-1} u_{cc} d' \Pi_t$$

$$\frac{\delta L_s}{\delta i_t} = -\phi_{1t} \frac{1+i^m}{(1+i_t)^2} + \phi_{2t}(m_t \Pi_t^{-1} + T_t - w_{t-1} \Pi_t - F) + \phi_{3t} \frac{u_c}{(1+i_t)^2} + \gamma_{1t} \tag{46}$$

$$\frac{\delta L_s}{\delta m_t} = u_m \Pi_t^{-1} + \phi_{1t}[\frac{u_{mm}}{u_c} - \frac{u_m}{u_c^2} u_{cm} \Pi_t^{-1}]\Pi_t^{-1} + \phi_{2t}(i_t - i^m)\Pi_t^{-1} - \phi_{3t}\frac{u_{cm}}{1+i_t}\Pi_t^{-1} \quad (47)$$

$$-\phi_{4t}[Y_t(\theta - 1)(1+s)u_{cm}\Pi_t^{-1} - u_{cm}d'] - \psi_{1t-1}u_{cm}\Pi_t^{-2} - \psi_{2t-1}u_{cm}d'$$

$$\frac{\delta L_s}{\delta T_t} = -g_G s'(T_t) + \phi_{2t}(1+i_t) \quad (48)$$

$$\frac{\delta L_s}{\delta w_t} = \phi_{2t} - \beta E_t \phi_{2t+1}(1+i_{t+1})\Pi_{t+1}^{-1} - \gamma_{2t} \quad (49)$$

$$\frac{\delta L_s}{\delta f_t^e} = \beta\phi_{3t} + \psi_{1t} \quad (50)$$

$$\frac{\delta L_s}{\delta S_t^e} = -\beta\phi_{4t} + \psi_{2t} \quad (51)$$

The complementary slackness conditions are:

$$\gamma_{1t} \geq 0, \ i_t \geq i^m, \quad \gamma_{1t}(i_t - i^m) = 0 \quad (52)$$

$$\gamma_{2t} \geq 0, \quad \bar{w} - w_t \geq 0, \quad \gamma_{2t}(\ \bar{w} - w_t) = 0 \quad (53)$$

For date $t = 0$ the same condition apply if $\psi_{1t-1} = \psi_{2t-1} = 0$.

A.1.2 Markov equilibrium FOC

Markov equilibrium period Lagrangian:

$$L_t = u(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)) + g(F - s(T_t), \xi_t) - \tilde{v}(Y_t) + E_t\beta J(w_t, \xi_{t+1})$$

$$+ \phi_{1t}(\frac{u_m(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)\Pi_t^{-1}}{u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)} - \frac{i_t - i^m}{1 + i_t})$$

$$+ \phi_{2t}(w_t - (1 + i_t)\Pi_t^{-1}w_{t-1} - (1 + i_t)F + (1 + i_t)T_t + (i_t - i^m)m_t\Pi_t^{-1})+$$

$$+ \phi_{3t}(\beta f_t^e - \frac{u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)}{1 + i_t})$$

$$+ \phi_{4t}(\theta Y_t[\frac{\theta - 1}{\theta}(1 + s)u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)]$$

$$+ u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)\Pi_t d'(\Pi_t) - \beta S_t^e)$$

$$+ \psi_{1t}(f_t^e - \bar{f}^e(w_t, \xi_t)) + \psi_{2t}(S_t^e - \bar{S}^e(w_t, \xi_t)) + \gamma_{1t}(i_t - i^m) + \gamma_{2t}(\bar{w} - w_t)$$

FOC (all the derivatives should be equated to zero)

$$\frac{\delta L_s}{\delta \Pi_t} = -u_c d'(\Pi_t) - u_m m_t \Pi_t^{-2} \tag{54}$$

$$+\phi_{1t}[-\frac{u_{mc} d' \Pi_t^{-1}}{u_c} - \frac{u_{mm} m_t \Pi_t^{-2}}{u_c} - \frac{u_m \Pi_t^{-2}}{u_c} + \frac{u_m u_{cc} d' \Pi_t^{-1}}{u_c^2} + \frac{u_m u_{cm} m_t \Pi_t^{-2}}{u_c}]$$

$$+[\phi_{2t}(1+i_t) w_{t-1} \Pi_t^{-2} - (i_t - i^m) m_t \Pi_t^{-2}] + \phi_{3t}[\frac{u_{cc} d'}{1+i_t} + \frac{u_{cm} m_t \Pi_t^{-2}}{(1+i_t)}]$$

$$+\phi_{4t}[-Y_t(\theta-1)(1+s)(u_{cc} d' + m_t \Pi_t^{-2} u_{cm}) - u_{cc} \Pi_t d'^2 - u_{cm} m_t \Pi_t^{-1} d' + u_c \Pi_t d'' + u_c d']$$

$$\frac{\delta L_s}{\delta Y_t} = u_c - \tilde{v}_y + \phi_{1t}[\frac{u_{mc}}{u_c} - \frac{u_m}{u_c^2}]\Pi_t^{-1} - \phi_{3t} \frac{u_{cc}}{1+i_t} \tag{55}$$

$$+\phi_{4t}[\theta(\frac{\theta-1}{\theta}(1+s)u_c - \tilde{v}_y) + \theta Y_t(\frac{\theta-1}{\theta}(1+s)u_{cc} - \tilde{v}_{yy}) + u_{cc} \Pi_t d']$$

$$\frac{\delta L_s}{\delta i_t} = -\phi_{1t} \frac{1+i^m}{(1+i_t)^2} + \phi_{2t}(m_t \Pi_t^{-1} + T_t - w_{t-1} \Pi_t^{-1} - F) + \phi_{3t} \frac{u_c}{(1+i_t)^2} + \gamma_{1t} \tag{56}$$

$$\frac{\delta L_s}{\delta m_t} = u_m \Pi_t^{-1} + \phi_{1t}[\frac{u_{mm}}{u_c} - \frac{u_m}{u_c^2} u_{cm} \Pi_t^{-1}]\Pi_t^{-1} + \phi_{2t}(i_t - i^m)\Pi_t^{-1} - \phi_{3t} \frac{u_{cm}}{1+i_t} \Pi_t^{-1} \tag{57}$$

$$-\phi_{4t}[Y_t(\theta-1)(1+s) u_{cm} \Pi_t^{-1} - u_{cm} d']$$

$$\frac{\delta L_s}{\delta T_t} = -g_G s'(T_t) + \phi_{2t}(1+i_t) \tag{58}$$

$$\frac{\delta L_s}{\delta w_t} = \beta E_t J_w(w_t, \xi_{t+1}) - \psi_{1t} f_w^e - \psi_{2t} S_w^e + \phi_{2t} - \gamma_{2t} \tag{59}$$

$$\frac{\delta L_s}{\delta f_t^e} = \beta\phi_{3t} + \psi_{1t} \tag{60}$$

$$\frac{\delta L_s}{\delta S_t^e} = -\beta\phi_{4t} + \psi_{2t} \tag{61}$$

The complementary slackness conditions are:

$$\gamma_{1t} \geq 0,\ i_t \geq i^m, \quad \gamma_{1t}(i_t - i^m) = 0 \tag{62}$$

$$\gamma_{2t} \geq 0, \quad \bar{w} - w_t \geq 0, \quad \gamma_{2t}(\ \bar{w} - w_t) = 0 \tag{63}$$

The optimal plan under discretion also satisfies an envelope condition:

$$J_w(w_{t-1}, \xi_t) = -\phi_{2t}(1 + i_t)\Pi_t^{-1} \tag{64}$$

A.2 Approximation Method

This section show the approximation method used to approximate the commitment and Markov Equilibrium.

A.2.1 Equilibrium in the absence of seigniorage revenues

As discussed in the text it simplifies the discussion to assume that the equilibrium base money is small, i.e. that m_t is a small number (see Woodford (2003), chapter 2, for a detailed treatment). I discuss in the footnote some reasons for why I conjecture that this abstraction has no significant effect.[29]

To analyze an equilibrium with a small monetary base I parameterize the utility function by the parameter $\bar{m}$ and assume that the preferences are of the form:

$$u(C_t, m_t\Pi_t^{-1}, \xi_t) = \tilde{u}(C_t, \xi_t) + \chi(\frac{m_t}{\bar{m}}\Pi_t^{-1}C_t^{-1}, \xi_t) \tag{65}$$

As the parameter $\bar{m}$ approaches zero the equilibrium value of m_t approaches zero as well. At the same time it is possible for the value of u_m to be a nontrivial positive number, so that money demand is well defined and the government's control over the short-term nominal interest rate is still well defined (see discussion in the proofs of Propositions 8 and 9 in the Appendix). I can define $\tilde{m}_t = \frac{m_t}{\bar{m}}$ as the policy instrument of the government, and this quantity can be positive even as $\bar{m}$ and m_t approach zero. Note that even as the real monetary base approaches the cashless limit the growth rate of the nominal stock of money associated with different equilibria is still well defined. I can then still discuss the implied path of money supply for different policy options. To see this note that

$$\frac{\tilde{m}_t}{\tilde{m}_{t-1}} = \frac{\frac{M_t}{P_{t-1}\bar{m}}}{\frac{M_{t-1}}{P_{t-2}\bar{m}}} = \frac{M_t}{M_{t-1}}\Pi_{t-1}^{-1} \tag{66}$$

which is independent of the size of $\bar{m}$. For a given equilibrium path of inflation and $\tilde{m}_t$ I can infer the growth rate of the nominal stock of money that is required to implement this equilibrium by the money demand equation. Since much of the discussion of the zero bound is phrased in terms of the implied path of money supply, I will devote some space to discuss how money supply adjusts in different equilibria in the paper. By assuming $\bar{m} \to 0$ I only abstract from the effect this adjustment has on the marginal utility of consumption and seigniorage revenues, both of which would be trivial in a realistic calibration (see footnote 29).

A.2.2 Steady state discussion and relation to literature on Markov Equilibrium

I define a steady state as a solution in the absence of shocks were each of the variables $(\Pi_t, Y_t, m_t, i_t, T_t, w_t, f_t^e, S_t^e) = (\Pi, Y, m, i, T, w, f^e, S^e)$ are constants. In general a steady-state of a Markov equilibrium is non-trivial to compute, as emphasized by Klein et al (2003). This is because each of the steady state variables depend on the mapping between the endogenous state (i.e. debt) and the unknown functions $J(.)$ and $\bar{e}(.)$, so that one needs to know the derivative of these functions with respect to the endogenous policy state variable to calculate the steady state. Klein et al suggest an approximation method by which one may approximate this steady state numerically by using perturbation methods. Here I take a different approach. Below I show that a steady state may be calculated under assumptions that are fairly common in the monetary literature (i.e. A2), without any further assumptions about the unknown functions $J(.)$ and $e(.)$.

Proposition 8 *If $\xi = 0$ at all times and A2(i)-(iii) hold there is a commitment equilibrium steady* *state that is given by* $i = 1/\beta - 1$, $w = S^e = \phi_1 = \phi_3 = \phi_4 = \psi_1 = \psi_2 = \gamma_1 = \gamma_2 = 0$, $\Pi = 1$, $\phi_2 = g_G(\bar{F} - s(\bar{F}))s'(\bar{F})$, $f^e = u_c(\bar{Y}), F = \bar{F} = G = T + s(T)$ *and* $Y = \bar{Y}$ *where* $\bar{Y}$ *is the unique solution to the equation* $u_c(Y - F) = v_y(Y)$

Proposition 9 *If* $\xi = 0$ *at all times and A2(i)-(iii) hold there is a Markov equilibrium steady state that is given by* $i = 1/\beta - 1$, $w = S^e = \phi_1 = \phi_3 = \phi_4 = \psi_1 = \psi_2 = \gamma_1 = \gamma_2 = 0$, $\Pi = 1$, $\phi_2 = g_G(\bar{F} - s(\bar{F}))s'(\bar{F})$, $f^e = u_c(\bar{Y}), F = \bar{F} = G = T + s(T)$ *and* $Y = \bar{Y}$ *where* $\bar{Y}$ *is the unique solution to the equation* $u_c(Y - F) = v_y(Y)$.

To proof these propositions I look at the algebraic expressions of the first order conditions of the government maximization problem. The proof is in section (A.4) of this Appendix. A noteworthy feature of the proof is that the mapping between the endogenous state and the functions $J(.)$ and $e(.)$ does not matter (i.e. the derivatives of these functions cancel out). The reason is that the Lagrangian multipliers associated with the expectation functions are zero in steady state and I may use the envelope condition to substitute for the derivative of the value function. The intuition for why these Lagrange multipliers are zero in equilibrium is simple. At the steady state the distortions associated with monopolistic competition are zero (because of A2 (ii) in the text). This implies that there is no gain of increasing output from steady state. In the steady the real debt is zero and according to assumption (i) seigniorage revenues are zero as well. This implies that even if there is cost of taxation in the steady state, increasing inflation does not reduce taxes. It follows that all the Lagrangian multipliers are zero in the steady state apart from the one on the government budget constraint. That multiplier, i.e. ϕ_2, is positive because there are steady state tax costs. Hence it would be beneficial (in terms of utility) to relax this constraint.

Proposition 8 and 9 give a convenient point to approximate around because the commitment and Markov solution are identical in this steady state. In the text, I relax both assumption A2(ii) and A3(iii) and investigate the behavior of the model local to this steady state. A major convenience of using A2 is that I can proof all of the key propositions in the coming sections analytically but do not need to rely on numerical simulation except to graph up the solutions.

There is by now a rich literature studying the question whether there can be multiple Markov equilibria in monetary models that are similar in many respects to the one I have described here (see e.g. Albanesi et al (2003), Dedola (2002) and King and Wolman (2003)). I do not proof the global uniqueness of the steady state in Proposition 9 but show that it is locally unique.[30] I conjecture, however, that the steady state is globally unique under A2.[31] But even if I would have written the model so that it had more than one steady state, the one studied here would still be the one of principal interest as discussed in the footnote.[32]

A.2.3 Approximate system and order of accuracy

The conditions that characterize equilibrium, in both the Markov and the commitment solution, are given by the constraints of the model and the first order conditions of the governments problem. A linearization of this system is complicated by the Kuhn-Tucker inequalities (52) and (53). I look for a solution in which the bound on government debt is never binding, and then verify that this bound is never binding in the equilibrium I calculate. Under this conjectured the solution to the inequalities (52) and (53) can be simplified into two cases:

$$\text{Case 1} : \gamma_t^1 = 0 \;\; \text{if } i_t > i^m \tag{67}$$

$$\text{Case 2} : i_t = i^m \;\; \text{otherwise} \tag{68}$$

Thus in both Case 1 and 2 I have equalities characterizing equilibrium. In the case of commitment, for example, these equations are (9), (25),(26), (28), (30), (31), (27), (29). and (44)-(51) and either (67) when $i_t > i^m$ or (68) otherwise. Under the condition A1(i) and A1(ii) but $i^m < \frac{1}{\beta} - 1$ then $i_t > i^m$ and Case 1 applies in the absence of shocks. In the knife edge case when $i^m = \frac{1}{\beta} - 1$,however, the equations that solve the two cases (in the absence of shocks) are identical since then both $\gamma_{1t} = 0$ and $i_t = i^m$. Thus both Case 1 and Case 2 have the same steady state in the knife edge case $i_t = i^m$. If I linearize around this steady state (which I show exists in Proposition 8 and 9) I obtain a solution that is accurate up to a residual $(||\xi||^2)$ for both Case 1 and Case 2. As a result I have one set of linear equations when the bound is binding, and another set of equations when it is not. The challenge, then, is to find a solution method that, for a given stochastic process for $\{\xi_t\}$, finds in which states of the world the interest rate bound is binding and the equilibrium has to satisfy the linear equations of Case 1, and in which states of the world it is not binding and the equilibrium has to satisfy the linear equations in Case 2. Since each of these solution are accurate to a residual $(||\xi||^2)$ the solutions can be made arbitrarily accurate by reducing the amplitude of the shocks. The next subsection show a solution method, assuming as simple process for the natural rate of interest, that numerically calculates when Case 1 applies and when Case 2 applies..

Note that I may also consider solutions when i^m is below the steady state nominal interest rate. A linear approximation of the equations around the steady state in Proposition 8 and 9 is still valid if the opportunity cost of holding money, i.e. $\bar{\delta} \equiv (i - i^m)/(1+i)$, is small enough. Specifically, the result will be exact up to a residual of order $(||\xi, \bar{\delta}||^2)$. In the numerical example below I suppose that $i^m = 0$ (see Eggertsson and Woodford (2003) for further discussion about the accuracy of this approach when the zero bound is binding). A non-trival complication of approximating the

Markov equilibrium is that I do not know the unknown expectation functions $\bar{e}(.)$. I illustrate a simple way of matching coefficients to approximate this function in the proof of Propositions 7.

A.2.4 Linearized solution

I here linearize the first order conditions and the constraints around the steady state in Propositions 8 and 9). I assume the form of the utility discussed in section A.2.1. I allow for deviations in the vector of shocks ξ_t, the production subsidy s (the latter deviation is used in Proposition 6) and in i^m (which I assume is zero) so that the equations are accurate of order $o(||\xi, \bar{\bar{\delta}}, 1 + s - \frac{\theta}{\theta-1}||^2)$. I abstract from the effect of the shocks on the disutility of labor. Here I use the notation $dz_t = z_t - z_{ss}$

The economic constraints under both commitment and discretion are:

$$\bar{u}_c d'' d\Pi_t + \theta(\bar{u}_{cc} - \bar{v}_{yy})dY_t + (\theta - 1)\bar{u}_c ds + \theta\bar{u}_{c\xi}d\xi_t - \bar{u}_c d''\beta E_t d\Pi_{t+1} = 0 \tag{69}$$

$$\bar{u}_{cc}dY_t + \bar{u}_{c\xi}d\xi_t - \beta\bar{u}_{cc}E_t dY_{t+1} - \beta\bar{u}_{c\xi}E_t d\xi_{t+1} - \beta\bar{u}_c di_t + \beta\bar{u}_c E_t d\Pi_{t+1} = 0 \tag{70}$$

$$dw_t - \frac{1}{\beta}dw_{t-1} + \frac{1}{\beta}dT_t = 0 \tag{71}$$

$$dS^e_t - \bar{u}_c d'' E_t d\Pi_{t+1} = 0 \tag{72}$$

$$df_t^e + \bar{u}_c E_t d\Pi_{t+1} - \bar{u}_{cc} E_t dY_{t+1} - \bar{u}_{c\xi} E_t \xi_{t+1} = 0 \tag{73}$$

The equation determining the natural rate of output is:

$$(v_{yy} - u_{cc}) dY_t^n + (v_{y\xi} - u_{c\xi}) d\xi_t - \frac{(\theta - 1)}{\theta} u_c ds = 0 \tag{74}$$

The equation determining the natural rate of interest is:

$$\beta E_t(\bar{u}_{cc} dY_{t+1}^n - \bar{u}_{c\xi} E_t d\xi_{t+1}) - (\bar{u}_{cc} dY_t^n - \bar{u}_{c\xi} d\xi_t) + \beta \bar{u}_{cc} dr_t^n = 0 \tag{75}$$

Note that the real money balances deflated by $\bar{m}$, i.e. $\tilde{m}_t$, are well defined in the cashless limit so that equation 66 is

$$d\tilde{m}_t - d\tilde{m}_{t-1} - d\frac{M_t}{M_{t-1}} + d\pi_{t-1} = 0$$

and money demand is approximated by

$$\frac{\bar{\chi}_{mm}}{u_c} d\tilde{m}_t - \frac{\bar{\chi}_{mm}}{u_c} \tilde{m} d\Pi_t - \frac{\bar{\chi}_{mm}}{u_c} \tilde{m} dY_t - \beta di_t + \beta di^m = 0$$

The Kuhn Tucker conditions imply that

Case 1 when $i_t > i^m$

$$d\gamma_{1t} = 0 \tag{76}$$

Case 2 when $i_t = i^m$

$$di_t = 0 \tag{77}$$

I look for a solution in which case the debt limit is never binding so that $d\gamma_{2t} = 0$ at all times and verify that this is satisfied in equilibrium. The linearized FOC in a commitment equilibrium are:

$$-d''\bar{u}_c d\Pi_t + \bar{\phi}_2 \beta^{-1} dw_{t-1} + d''\bar{u}_c d\phi_{4t} - \bar{u}_c d\phi_{3t-1} - d''\bar{u}_c d\phi_{4t-1} = 0 \tag{78}$$

$$(\bar{u}_{cc} - \bar{v}_{yy})dY_t + \bar{u}_{c\xi} d\xi_t - \bar{v}_{y\xi} d\xi_t - \bar{u}_{cc}\beta d\phi_{3t} + \theta(\bar{u}_{cc} - \bar{v}_{yy})d\phi_{4t} + \bar{u}_{cc} d\phi_{3t-1} = 0 \tag{79}$$

$$\bar{\phi}_2 dT_t - \bar{\phi}_2 dw_{t-1} + \bar{u}_c \beta^2 d\phi_{3t} + d\gamma_{1t} = 0 \tag{80}$$

$$\bar{g}_{GG}(s')^2 dT_t - \bar{g}_G s'' dT_t - \bar{g}_{G\xi} d\xi_t + \beta^{-1} d\phi_{2t} + \bar{\phi}_2 di_t = 0 \tag{81}$$

$$d\phi_{2t} - E_t d\phi_{2t+1} - \beta\bar{\phi}_2 E_t di_{t+1} + \bar{\phi}_2 E_t d\Pi_{t+1} - d\gamma_{2t} = 0 \tag{82}$$

Linearized FOC in a Markov Equilibrium

$$-d''\bar{u}_c d\Pi_t + \bar{\phi}_2\beta^{-1}dw_{t-1} + d''\bar{u}_c d\phi_{4t} = 0 \tag{83}$$

$$(\bar{u}_{cc} - \bar{v}_{yy})dY_t + \bar{u}_{c\xi}d\xi_t - \bar{v}_{y\xi}d\xi_t - \bar{u}_{cc}\beta d\phi_{3t} + \theta(\bar{u}_{cc} - \bar{v}_{yy})d\phi_{4t} = 0 \tag{84}$$

$$\bar{\phi}_2 dT_t - \bar{\phi}_2 dw_{t-1} + \bar{u}_c\beta^2 d\phi_{3t} + d\gamma_{1t} = 0 \tag{85}$$

$$\bar{g}_{GG}(s')^2 dT_t - \bar{g}_G s'' dT_t - \bar{g}_{G\xi}d\xi_t + \beta^{-1}d\phi_{2t} + \bar{\phi}_2 di_t = 0 \tag{86}$$

$$d\phi_{2t} - E_t d\phi_{2t+1} - \beta\bar{\phi}_2 E_t di_{t+1} + \bar{\phi}_2 E_t d\Pi_{t+1} + \beta f_w d\phi_{3t} - \beta S_w d\phi_{4t} - d\gamma_{2t} = 0 \tag{87}$$

Note that the first order condition with respect to m_t does not play any role in the cashless limit so that it is omitted above.

A.2.5 Computational method

Here I illustrate a solution method for the optimal commitment solution. This method can also be applied, with appropriate modification of each of the steps, to find the Markov solution. I assume shocks so that the natural rate of interest becomes unexpectedly negative in period 0 and the reverts back to normal with probability α_t in every period t as in A5 (one may use (74) and (75) to find what a given negative number for the natural rate of interest implies for the underlying exogenous shocks). I assume that there is a final date K in which the natural rate becomes positive with probability one (this date can be arbitrarily far into the future).

The solution takes the form:

$$\begin{array}{lclllll} \text{Case 2 } i_t & = & 0 & \forall & t & 0 \leq t < \tau + k \\ \text{Case 1 } i_t & > & 0 & \forall & t & t \geq \tau + k \end{array}$$

Here τ is he stochastic date at which the natural rate of interest returns to steady state. I assume that τ can take any value between 1 and the terminal date K that can be arbitrarily far into the future. The number $\tau + k_\tau$ is the period in which the zero bound stops being binding in the contingency when the natural rate of interest becomes positive in period τ. Note that the value of k_τ can depend on the value of τ. I first show the solution for the problem as if I knew the sequence $\{k_\tau\}_{\tau=1}^{S}$. I then describe a numerical method to find the sequence $\{k_\tau\}_{\tau=1}^{S}$.

The solution for $t \geq \tau + k_\tau$ The system of linearized equations (78)-(82) (and (83)-(87) in the case of Markov), (69)-(73), and (76) can be written in the form:

$$\begin{bmatrix} E_t Z_{t+1} \\ \\ P_t \end{bmatrix} = M \begin{bmatrix} Z_t \\ \\ P_{t-1} \end{bmatrix}$$

where $Z_t \equiv \begin{bmatrix} \Lambda_t & e_t & \phi_t \end{bmatrix}^T$ and $P_t \equiv \begin{bmatrix} w_t & e_t & \psi_t & \gamma_t^1 \end{bmatrix}^T$. If there are eleven eigenvalues of the matrix M outside the unit circle this system has a unique bounded solution of the form:

$$P_t = \Omega^0 P_{t-1} \tag{88}$$

$$Z_t = \Lambda^0 P_{t-1} \tag{89}$$

The solution for $\tau \leq t < \tau + k$ Again this is a perfect foresight solution but with the zero bound binding. The solution now satisfies the equations (78)-(82) (and (83)-(87) in the case of

Markov), (69)-(73) but (77) instead of (76).The system can be written on the form:

$$\begin{bmatrix} P_t \\ \\ Z_t \end{bmatrix} = \begin{bmatrix} A & B \\ \\ C & D \end{bmatrix} \begin{bmatrix} P_{t-1} \\ \\ Z_{t+1} \end{bmatrix} + \begin{bmatrix} M \\ \\ V \end{bmatrix}$$

This system has a solution of the form:

$$P_{\tau+j} = \Omega^{k_\tau - j} P_{\tau+j-1} + \Phi^{k_\tau - j} \tag{90}$$

$$Z_{\tau+j} = \Lambda^{k_\tau - j} P_{\tau+j-1} + \Theta^{k_\tau - j} \tag{91}$$

where $j = 0, 1, 2, ..., k$. Here $\Omega^{k_\tau - j}$ is the coefficient in the solution when there are $k_\tau - j$ periods until the zero bound stops being binding (i.e. when $j - k_\tau = 0$ the zero bound is not binding anymore and the solution is equivalent to (88)-(89)). We can find the numbers $\Lambda^j, \Omega^j, \Theta^j$ and Φ^j for $j = 1, 2, 3, ..., k$ by solving the equations below using the initial conditions $\Phi^0 = \Theta^0 = 0$ for $j = 0$ and the initial conditions for Λ^j and Ω^j given in (88)-(89):

$$\Omega^j = [I - B\Lambda^{j-1}]^{-1} A$$

$$\Lambda^j = C + D\Lambda^{j-1}\Omega^j$$

$$\Phi^j = (I - B\Lambda^{j-1})^{-1}[B\Theta^{j-1} + M]$$

$$\Theta^j = D\Lambda^{j-1}\Phi^j + D\Theta^{j-1} + V$$

The solution for $t < \tau$ The solution satisfies (78)-(82) (and (83)-(87) in the case of Markov), (69)-(73), and (77). Note that each of the expectation variables can be written as $\tilde{x}_t = E_t x_{t+1} = \alpha_{t+1}\tilde{x}_{t+1} + (1 - \alpha_{t+1})x_{t+1}$ where α_{t+1} is the probability that the natural rate of interest becomes positive in period $t + 1$. Here hat on the variables refers to the value of each variable contingent on that the natural rate of interest is negative. I may now use the solution for Z_{t+1} in 91 to substitute for Z_{t+1}, i.e. the value of each variable contingent on that the natural rate becomes positive again in terms of the hatted variables. The value of x_{t+1}, for example, can be written as $x_{t+1} = \Lambda_{21}^{k_{t+1}}\tilde{\phi}_{1t} + \Lambda_{22}^{k_{t+1}}\tilde{\phi}_{2t} + \Theta_2^{k_{t+1}}$ where $\Lambda_{ij}^{k_{t+1}}$ is the ijth element of the matrix $\Lambda^{k_{t+1}}$ and the value k_{t+1} depends on the number of additional periods that the zero bound is binding (recall that

I am solving the equilibrium on the assumption that I know the value of the sequence $\{k_\tau\}_{\tau=1}^{S})$.

Hence I can write the system as:

$$\begin{bmatrix} \tilde{P}_t \\ \\ \tilde{Z}_t \end{bmatrix} = \begin{bmatrix} A_t & B_t \\ \\ C_t & D_t \end{bmatrix} \begin{bmatrix} \tilde{P}_{t-1} \\ \\ \tilde{Z}_{t+1} \end{bmatrix} + \begin{bmatrix} M_t \\ \\ V_t \end{bmatrix}$$

I can solve this backwards from the date K in which the natural rate returns back to normal with probability one. I can then calculate the path for each variable to date 0. Note that.

$$B_{K-1} = D_{K-1} = 0$$

By recursive substitution I can find a solution of the form:

$$\tilde{P}_t = \Omega_t \tilde{P}_{t-1} + \Phi_t \tag{92}$$

$$\tilde{Z}_t = \Lambda_t \tilde{P}_{t-1} + \Theta_t \tag{93}$$

where the coefficients are time dependent. To find the numbers $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t consider the solution of the system in period $K-1$ when $B_{K-1} = D_{K-1} = 0$. I have:

$$\Omega_{K-1} = A_{K-1}$$

$$\Phi_{K-1} = M_{K-1}$$

$$\Lambda_{K-1} = C_{K-1}$$

$$\Theta_{K-1} = V_{K-1}$$

I can find of numbers $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t for period 0 to $K-2$ by solving the system below (using the initial conditions shown above for $S-1$):

$$\Omega_t = [I - B_t \Lambda_{t+1}]^{-1} A_t$$

$$\Lambda_t = C_t + D_t \Lambda_{t+1} \Omega_t$$

$$\Phi_t = (I - B_t \Lambda_{t+1})^{-1} [B_t \Theta_{t+1} + M_t]$$

$$\Theta_t = D_t \Lambda_{t+1} \Phi_t + D_t \Theta_{t+1} + V_t$$

Using the initial condition $\tilde{P}_{-1} = 0$ I can solve for each of the endogenous variables under the contingency that the trap last to period K by (92) and (93). This initial condition corresponds to the assumption that the system is at its steady state at time $t-1$ and the initial shock is unexpected. I then use the solution from (88)-(91) to solve for each of the variables when the natural rate reverts back to steady state.

Solving for $\{k_\tau\}_{t=0}^{\infty}$ A simple way to find the value for $\{k_\tau\}_{\tau=1}^{\infty}$ is to first assume that k_τ is the same for all τ and find the k so that the zero bound is never violated. Suppose that the system has converged at $t = 25$ (i.e. the response of each of the variables is the same). Then I can move to 24 and see if $k_\tau = 4$ for $\tau = 1, 2, ...24$ is a solution that never violates the zero bound. If not move to 23 and try the same thing and so on. For preparing this paper I wrote a routine in MATLAB that applied this method to find the optimal solution and verified that the results satisfied all the necessary conditions.

A.3 Calibration parameters

In the numerical examples I assume the following functional forms for preferences and technology:

$$u(C,\xi) = \frac{C^{1-\sigma^{-1}}\bar{C}^{\sigma^{-1}}}{1-\sigma^{-1}}$$

where $\bar{C}$ is a preference shock assumed to be 1 in steady state.

$$g(G,\xi) = g_1 \frac{G^{1-\sigma^{-1}}\bar{G}^{\sigma^{-1}}}{1-\sigma^{-1}}$$

where $\bar{G}$ is a preference shock assumed to be 1 in steady state

$$v(H,\xi) = \frac{\lambda_1}{1+\omega} H^{1+\omega} \bar{H}^{-\lambda_2}$$

where $\bar{H}$ is a preference shock assumed to be 1 in steady state

$$y = Ah^{\epsilon}$$

where A is a technology shock assumed to be 1 in steady state. I may substitute the production function into the disutility of working to obtain (assuming A=1):

$$\tilde{v}(Y,\xi_t) = \frac{\lambda_1}{1+\lambda_2} Y^{1+\lambda_2} \bar{H}^{-\omega}$$

When calibrating the shocks that generate the temporarily negative natural rate of interest I assume that it is the shock $\bar{C}$ that is driving the natural rate of interest negative (as opposed to A) since otherwise a negative natural rate of interest would be associated with a higher natural rate of output which does not seem to be the most economically interesting case. I assume that the shock $\bar{G}$ is such that the F_t would be constant in the absence of the zero bound, in order to keep the optimal size of the government (in absence of the zero bound) constant (see Eggertsson (2004) for details)). The cost of price adjustment is assumed to take the form:

$$d(\Pi) = d_1 \Pi^2$$

The cost of taxes is assumed to take to form:

$$s(T) = s_1 T^2$$

Aggregate demand implies$Y = C + F = C + G + s(F)$. I normalize $Y = 1$ in steady state and assume that the share of the government in production is $F = 0.3$. Tax collection as a share of government spending is assumed to be $\gamma = 5\%$ of government spending. This implies

$$0.1 = \frac{s(F)}{F} = s_1 F$$

so that $s_1 = \frac{\gamma}{F}$. The result for the inflation and output gap response are not very sensitive to varying γ under either commitment or discretion. The size of the public debt issued in the Markov equilibrium, however, crucially depends on this variable. In particular if γ is reduced the size of the debt issued rises substantially. For example if $\gamma = 0.5\%$ the public debt issued is about ten times bigger than reported in the figure in the paper. I assume that government spending are set at their optimal level in steady state giving me the relationship (see Eggertsson 2004 for details on how this is derived)

$$g_2 = \frac{u_c}{g_G - s' g_G} = \frac{C^{-\sigma^{-1}}}{G^{-\sigma^{-1}}(1-s')} = (\frac{G}{C})^{\sigma^{-1}} \frac{1}{1-s'} = (\frac{G}{C})^{\sigma^{-1}} \frac{1}{1-2s_1 F}$$

The IS equation and the AS equation are

$$x_t = E_t x_{t+1} - \tilde{\sigma}(i_t - E_t \pi_{t+1} - r_t^n)$$

$$\pi_t = k\pi_t + \beta E_t \pi_{t+1}$$

I assume, as Eggertsson and Woodford (2003), that the interest rate elasticity, $\tilde{\sigma}$, is 0.5. The relationship between σ and $\tilde{\sigma}$ is

$$\sigma = \tilde{\sigma}\frac{Y}{C}$$

I assume that κ is 0.02 as in Eggertsson and Woodford (2003). The relationship between κ and the other parameters of the model is$\kappa = \theta\frac{(\tilde{\sigma}^{-1}+\lambda_2)}{d''}$. I scale hours worked so that $Y = 1$ in steady state which implies

$$v_y = \lambda_1$$

Since $u_c = \tilde{v}_y$ in steady state I have that

$$\theta = 7.87$$

Finally I assume that $\theta = 7.89$ as in Rotemberg and Woodford and that $\lambda_2 = 2$. The calibration value for the parameters are summarized in the table below:

Table 2

σ	0.71
g_1	0.33
λ_1	1.65
λ_2	2
d_1	787
s_1	0.17
θ	7.87

A.4 Proofs

A.4.1 Proof of Proposition 1:

I proof this proposition by showing that all the necessary and sufficient conditions for a PSE listed in Definition 1 (i.e. equation (3)-(17)) can be written without any reference to either T_t or ψ_t.

1. I first show that the equilibrium conditions can be written without any references to the

function $T(.)$. Since only one period bonds are issued I can write $W_{t+1} = (1 + i_t)B_t + (1 + i^m)M_t$ and equation (16) becomes

$$\frac{1}{1+i_t} W_{t+1} = W_t + P_t F - P_t T_t - \frac{i_t - i^m}{1+i_t} M_t \tag{94}$$

which defines W_{t+1} as a function of T_t and M_t so that I must show that I can write all the necessary condition for equilibrium without any reference to $W(.)$ as well. Using equation (94) and (10) I can write

$$W_t - E_t \sum_{T=t}^{\infty} Q_{t,T}(P_T T_T - \frac{i_T - i^m}{1+i_T} M_T) = E_t \sum_{T=t}^{\infty} Q_{t,T} P_T F$$

Furthermore I can use the expression for firms profits and the requirement of symmetric equilibrium to yield:

$$E_t \sum_{T=t}^{\infty} Q_{t,T}[\int_0^1 Z_T(i)di + \int_0^1 n_T(j)h_T(j)dj = E_t \sum_{T=t}^{\infty} Q_{t,T} P_t Y_t$$

Using the last two equation I can write the intertemporal budgets constraint (25) as:

$$E_t \sum_{T=t}^{\infty} Q_{t,T} P_T C_T \leq E_t \sum_{T=t}^{\infty} Q_{t,T}[P_T Y_T - P_T F] \tag{95}$$

Thus this constraint can be written without any reference to the functions $T(.)$ or $W(.)$. The other condition that need to be satisfied regardless of the specification of $T(.)$ is equation (10). Using $W_{t+1} = (1 + i_t)B_t + (1 + i^m)M_t$ and (24) this condition can be simplified to yield:

$$\lim_{T\to\infty} \beta^T E_t[u_c(Y_T - d(\Pi_T) - F_T, M_T/P_T; \xi_T)M_T/P_T] = 0 \tag{96}$$

which neither depends on $T(.)$ or $W(.)$.

2. I now show that all the constraint of required for a private sector equilibrium can be expressed independently of the specification of $\psi(.)$. I first consider equation (96). If the nominal interest rate is never binding asymtotically M_t will not depend on $\psi(.)$ according (18). The specification of $\psi(.)$ could be important if the zero bound is asymtotically binding. Assuming A1 the equilibrium is deterministic at all dates $t \geq K$. It is easy to show that for the zero bound to be asymtotically binding I must have $\Pi_t = \frac{P_t}{P_{t-1}} = \beta$ and $Y_t = \bar{Y}$. Then I can write, for all $t \geq K$

(i.e. all dates after the uncertainty is resolved) $P_t = \beta^{t-K} P_K$. Then (96) becomes

$$\lim_{T\to\infty} \beta^K [u_c(Y_T - d(\Pi_T) - F_T, M_T/P_T; \xi_T)\frac{M_T}{P_K} = 0$$

This condition is only satisfied if $M_T \to 0$. But this would violate (21) and thus an asymtotic deflationary equilibrium is not consistent with my specification of fiscal and monetary policy. It follows that the specification of $\psi(.)$ has no effect on whether or not (96) is satisfied since I have just shown that the zero bound cannot be asymtotically binding under the monetary and fiscal regime specified. What remains to be shown is that all the other equilibrium conditions can be written without any reference to the function $\psi(.)$. That part of the proof follows exactly the same steps as the proof of Proposition 1 in Eggertsson and Woodford (2003).

A.4.2 Proof of Propositions 4 and 5

In the assumption made in the proposition I assume the cashless limit and the form of the utility given by (65) so that

$$u(C_t, m_t\Pi_t^{-1}, \xi_t) = \tilde{u}(C_t, \xi_t) + \chi(\frac{m_t}{\bar{m}}\Pi_t^{-1}C_t^{-1}, \xi_t) \tag{97}$$

The partial derivatives with respect to each variable are given by

$$u_c = \tilde{u}_c - \chi' \frac{m}{\bar{m}} C^{-2} \Pi^{-1} \tag{98}$$

$$u_m = \frac{\chi'}{\bar{m}} C^{-1} \Pi^{-1} \tag{99}$$

$$u_{mm} = \frac{\chi''}{\bar{m}^2} C^{-2} \Pi^{-2} < 0 \tag{100}$$

$$u_{cm} = -\chi'' \frac{m}{\bar{m}^2} C^{-3} \Pi^{-2} - \frac{\chi'}{\bar{m}} C^{-2} \Pi^{-1} \tag{101}$$

As $\bar{m}- > 0$ I assume that for $\tilde{m} = \frac{m}{\bar{m}} > 0$ I have

$$lim_{\bar{m} \to 0} \frac{\chi\prime}{\bar{m}} \equiv \bar{\chi}' \geq 0 \tag{102}$$

$$lim_{\bar{m} \to 0} \frac{\chi\prime\prime}{\bar{m}^2} \equiv \bar{\chi}'' > 0 \tag{103}$$

This implies that there is a well defined money demand function, even as money held in equilibrium

approaches zero, given by

$$\frac{\bar{\chi}\prime(\tilde{m} C_t^{-1} \Pi_t^{-1}, \xi_t) C_t^{-1} \Pi_t^{-1}}{\bar{u}_c(C_t, \xi_t)} = \frac{i_t - i^m}{1 + i_t}$$

so that $\bar{\chi}' = 0$ when $i_t = i_t^m$. From the assumptions (102)-(103) it follows that:

$$lim_{\bar{m}\to 0}\chi\prime = 0$$

$$lim_{\bar{m}\to 0}\chi\prime\prime = 0$$

Then the derivatives u_c and u_{cm} in the cashless limit are:

$$\lim_{\bar{m}\to 0} u_c = \tilde{u}_c$$

and

$$\lim_{\bar{m}\to 0} u_{cm} = \lim_{\bar{m}\to 0}[-\bar{m}\frac{\chi\prime\prime}{\bar{m}^2}\frac{m}{\bar{m}}C^{-3}\Pi^{-1} - \frac{\chi\prime}{\bar{m}}C^{-2}] = -\bar{\chi}\prime C^{-2}$$

Hence in a steady state in which $\bar{m} \to 0$ and $i_t = i^m$ I have that $\bar{\chi}\prime = 0$ so that at the steady state

$$\lim_{\bar{m}\to 0} u_{cm} = 0. \tag{104}$$

Note that this does not imply that the satiation point of holding real balances is independent of consumption. To see this note that the satiation point of real money balances is is given by some finite number $S^* = \frac{m}{\bar{m}}Y$ which implies that $\chi(S \geq S^*) = \tilde{v}(S^*)$. The value of the satiation point as $\bar{m} \to 0$ is:

$$lim_{\bar{m}\to 0} S^* \equiv \bar{S} = \tilde{m}C$$

The value of this number still depends on C even as $\bar{m} \to 0$ and even if $u_{cm} = 0$ at the satiation point.

I now show that the steady state stated in Proposition 3 and 4 satisfies all the first order conditions and the constraints. The steady state candidate solution in both proposition is:

$$i = \frac{1}{\beta} - 1, w = \phi_1 = \phi_3 = \phi_4 = \psi_1 = \psi_2 = \gamma_1 = \gamma_2 = 0, \Pi = 1, \phi_2 = g_G s',\ T = F \tag{105}$$

Note that (105) and the functional assumption about d (see footnote 5) imply that:

$$d' = 0 \tag{106}$$

Let us first consider the constraints. In the steady state the AS equation is

$$\theta Y[\frac{\theta-1}{\theta}(1+s)u_c - \tilde{v}_y] - u_c\Pi d'(\Pi) + \beta u_c\Pi d'(\Pi) = 0$$

Since by (106) d'=0, and according to assumption (ii) of the propositions $\frac{\theta-1}{\theta}(1+s) = 1$ the AS equation is only satisfied in the candidate solution if

$$u_c = v_y \tag{107}$$

Evaluated in the candidate solution the IS equation is:

$$\frac{1}{1+i} = \frac{\beta u_c}{u_c}\Pi^{-1} = \beta$$

which is always satisfied at because it simply states that $i = 1 - 1/\beta$ which is consistent with the steady state I propose in the propositions and assumption (iii). The budget constraint is:

$$w - (1+i)\Pi^{-1}w - (1+i)F + (1+i)T + (1+i)\bar{m}\tilde{m}\Pi_t^{-1} = 0$$

which is also always satisfied in our candidate solution since it states that $F = T$, $w = 0$ and

$\bar{m} \to 0$. The money demand equation indicates that the candidate solutions is satisfied if

$$u_m = \Pi u_c \frac{i - i^m}{1 + i} = 0 \tag{108}$$

By (27) and (29) the expectation variables in steady state are

$$S^e = u_c \Pi d'$$

$$f^e = u_c \Pi$$

Since $\Pi = 1$ and $d' = 0$ by (106) these equations are satisfied in the candidate solution. Finally both the inequalities (9) and (26) are satisfied since $\bar{w} > w = 0$ in the candidate solution and $i = i^m$.

I now show that the first order conditions, i.e. the commitment and the Markov equilibrium first order conditions, that are given by (44)-(53) and (54)-(64) respectively, are also consistent with the steady state suggested. I first show the commitment equilibrium. The proof for the Markov equilibrium will follow along the same lines.

Commitment equilibrium steady state Let us start with (44). It is

$$-u_c d' - u_m \bar{m}\tilde{m}\Pi^{-2} + \phi_1[-\frac{u_{mc}d'\Pi^{-1}}{u_c} - \frac{u_{mm}\bar{m}\tilde{m}\Pi^{-2}}{u_c} - \frac{u_m\Pi^{-2}}{u_c} + \frac{u_m u_{cc} d'\Pi^{-1}}{u_c^2} + \frac{u_m u_{cm} m \Pi^{-2}}{u_c}]$$

$$+[\phi_2(1+i)w\Pi^{-2} - (i-i^m)\bar{m}\tilde{m}\Pi^{-2}] + \phi_3[\frac{u_{cc}d'}{1+i} + \frac{u_{cm}\bar{m}\tilde{m}\Pi^{-2}}{(1+i)}]$$

$$+\phi_4[-Y(\theta-1)(1+s)(u_{cc}d' + \bar{m}\tilde{m}\Pi^{-2}u_{cm}) - u_{cc}\Pi d'^2 - u_{cm}\bar{m}\tilde{m}\Pi^{-1}d' + u_c\Pi d'' + u_c d']$$

$$+\beta^{-1}\psi_1[u_{cc}d'\Pi + u_{cm}\bar{m}\tilde{m}\Pi^{-1} + u_c\Pi^{-2}] + \beta^{-1}\psi_2[u_{cc}d'^2\Pi + u_{cm}d'\bar{m}\tilde{m}\Pi^{-1} - u_c d' - u_c d''\Pi] = 0$$

By (106) and (108) the first two terms are zero. The constraints that are multiplied by ϕ_1, ϕ_3, ϕ_4, ψ_1 and ψ_2 are also zero because each of these variables are zero in our candidate solution (105). Finally, the term that is multiplied by ϕ_2 (which is positive) is also zero because $w = 0$ in our candidate solution (105) and so is $i - i^m$. Thus I have shown that the candidate solution (105) satisfies (44).

Let us now turn to (45). It is

$$
\begin{aligned}
& u_c - \tilde{v}_y + \phi_1[\frac{u_{mc}\Pi^{-1}}{u_c} - \frac{u_m\Pi^{-1}}{u_c^2}] - \phi_3\frac{u_{cc}}{1+i} \\
& +\phi_4[\theta(\frac{\theta-1}{\theta}(1+s)u_c - \tilde{v}_y) + \theta Y(\frac{\theta-1}{\theta}(1+s)u_{cc} - \tilde{v}_{yy}) + u_{cc}\Pi d'] \\
& -\psi_1\beta^{-1}u_{cc}\Pi - \psi_2\beta^{-1}u_{cc}d'\Pi \\
= \quad & 0
\end{aligned}
$$

The first two terms $u_c - v_y$ are equal to zero by (107). The next terms are also all zero because they are multiplied by the terms ϕ_1, ϕ_3, ϕ_4, ψ_1 and ψ_2 which are all zero in our candidate solution (105). Hence this equation is also satisfied in our candidate solution. Let us then consider (46). It is:

$$
-\phi_1\frac{1+i^m}{(1+i)^2} + \phi_2(\bar{m}\tilde{m} + T - w\Pi^{-1} - F) + \phi_3\frac{u_c}{(1+i)^2} + \gamma_1 = 0
$$

Again this equation is satisfied in our candidate solution because $\phi_1 = \phi_3 = w = 0$, $F = T$ and

$\bar{m} \to 0$ in the candidate solution. Conditions (47) in steady state is:

$$\bar{m}\tilde{m}u_m\Pi^{-1} + \phi_1[\frac{u_{mm}}{u_c}\Pi^{-1} - \frac{u_m}{u_c^2}u_{cm}\Pi^{-2}] + \phi_2(i - i^m)\Pi^{-1} - \phi_3\frac{u_{cm}}{1+i}\Pi^{-1} \tag{109}$$

$$-\phi_4[Y(\theta - 1)(1+s)u_{cm}\Pi^{-1} - u_{cm}d'] - \psi_1 u_{cm}\Pi^{-2} - \psi_2 u_{cm}d' = 0$$

The first term is zero by (108). All the other terms are also zero because ϕ_1, ϕ_3, ϕ_4, ψ_1 and ψ_2 are all zero in our candidate solution (105). Finally $i = i^m$ in our candidate solution so that the third term is zero as well. Condition (48) in steady state is:

$$-g_G s'(T) + \phi_2(1+i) = 0 \tag{110}$$

which is satisfied in the candidate solution. Condition (49) is

$$\phi_2 - \beta\phi_2(1+i)\Pi^{-1} - \gamma_2 = 0$$

This condition is also satisfied in our candidate solution because $\gamma_2 = 0$ and $(1+i)\Pi^{-1} = \frac{1}{\beta}$.

Conditions (50) and (51) are:

$$\beta\phi_3 + \psi_1 = 0 \tag{111}$$

$$\beta\phi_4 + \psi_2 = 0 \tag{112}$$

Since $\phi_3 = \phi_4 = \psi_1 = \psi_2 = 0$ in our candidate solution, these conditions are also satisfied.

Finally our candidate solution (105) indicates that (52) and (53) are also satisfied in steady state.

I have now showed that our candidate solution satisfies all necessary and sufficient conditions for an equilibrium and Proposition 3 is thus proofed.

Markov equilibrium steady state Let us now turn to the Markov equilibrium. The first order conditions in steady state are

$$-u_c d' - u_m \bar{m}\tilde{m}\Pi^{-2} \tag{113}$$

$$+\phi_1[-\frac{u_{mc}d'\Pi^{-1}}{u_c} - \frac{u_{mm}\bar{m}\tilde{m}\Pi^{-2}}{u_c} - \frac{u_m\Pi^{-2}}{u_c} + \frac{u_m u_{cc}d'\Pi^{-1}}{u_c^2} + \frac{u_m u_{cm} m\Pi^{-2}}{u_c}] \tag{114}$$

$$+[\phi_2(1+i)w\Pi^{-2} - (i-i^m)\bar{m}\tilde{m}\Pi^{-2}] + \phi_3[\frac{u_{cc}d'}{1+i} + \frac{u_{cm}\bar{m}\tilde{m}\Pi^{-2}}{(1+i)}]$$

$$+\phi_4[-Y(\theta-1)(1+s)(u_{cc}d' + \bar{m}\tilde{m}\Pi^{-2}u_{cm}) - u_{cc}\Pi d'^2 - u_{cm}\bar{m}\tilde{m}\Pi^{-1}d' + u_c\Pi d'' + u_c d']$$

$$+\beta^{-1}\psi_1[u_{cc}d'\Pi + u_{cm}\bar{m}\tilde{m}\Pi^{-1} + u_c\Pi^{-2}] + \beta^{-1}\psi_2[u_{cc}d'^2\Pi + u_{cm}d'\bar{m}\tilde{m}\Pi^{-1} - u_c d' - u_c d''\Pi] = 0$$

$$u_c - \tilde{v}_y + \phi_1[\frac{u_{mc}}{u_c} - \frac{u_m}{u_c^2}] - \phi_3 \frac{u_{cc}}{1+i} + \phi_4[\theta(\frac{\theta-1}{\theta}(1+s)u_c - \tilde{v}_y) - \theta Y(\frac{\theta-1}{\theta}(1+s)u_{cc} - \tilde{v}_{yy}) - u_{cc}\Pi d'] = 0 \tag{115}$$

$$-\phi_1 \frac{1+i^m}{(1+i)^2} + \phi_2(\bar{m}\tilde{m} + T - w\Pi^{-1} - F) + \phi_3 \frac{u_c}{(1+i)^2} + \gamma_1 = 0 \tag{116}$$

$$u_m \Pi^{-1} + \phi_1[\frac{u_{mm}}{u_c} - \frac{u_m}{u_c^2} u_{cm}\Pi^{-1}] + \phi_2(i - i^m)\bar{m}\tilde{m} - \phi_3 \frac{u_{cm}}{1+i}\Pi^{-1} - \phi_4[Y(\theta-1)(1+s)u_{cm}\Pi^{-1} - u_{cm}d'] = 0 \tag{117}$$

$$-g_G s'(T) + \phi_2(1+i) = 0 \tag{118}$$

$$\beta J_w - \psi_1 \beta f_w^e - \psi_2 \beta S_w^e + \phi_2 - \gamma_2 = 0 \tag{119}$$

$$\beta\phi_3 + \psi_1 = 0 \tag{120}$$

$$\beta\phi_4 + \psi_2 = 0 \tag{121}$$

$$J_w = -\phi_2(1+i)\Pi^{-1} \tag{122}$$

Condition (113)-(118) and (120)-(121) are the same as in the commitment equilibrium, apart from the presence of ψ_1 and ψ_2 in the equations corresponding to (113) and (115). Since $\psi_1 = \psi_2 = 0$ in the candidate solution this does not change our previous proof. Thus, exactly the same arguments as I made to show that the candidate solution (105) satisfied the first order conditions in the commitment equilibrium can be used in the Markov equilibrium for equations (113)-(118) and (120)-(121). The crucial difference between the first order conditions in the Markov and the commitment equilibrium is in equation (119). This equation involves three unknown functions, J_w, f_w^e and S_w^e.I can use (122) to substitute for J_w in (119) obtaining

$$-\beta\phi_2(1+i)\Pi^{-1} - \psi_1\beta f_w^e - \psi_2\beta S_w^e + \phi_2 - \gamma_2 = 0 \tag{123}$$

In general I cannot know if this equation is satisfied without making further assumption about f_w^e and S_w^e. But note that in our candidate solution $\psi_1 = \psi_2 = 0$. Thus the terms involving these two derivatives in this equation are zero. Since $\gamma_2 = 0$, this equation is satisfied if $(1+i)\Pi^{-1} = 1/\beta$. This is indeed the case in our candidate solution. Thus I have shown that all the necessary and sufficient conditions of a Markov equilibrium are satisfied by our candidate solution (105). QED

A.4.3 Proof of Proposition 6

In this equilibrium there is only one policy instrument so that $dT_t = dw_t = 0$ and I may ignore the linearized first order conditions (81), (82) for commitment and (86) and (87) in the Markov equilibrium. The remaining FOC along with the constraint (69), (70) and (76) determine the equilibrium.

1. I first consider the commitment case. Equation (81) indicates that $\phi_{3t} = 0$. Then I can write (78) and (79) in terms of inflation and output gap as (using (74) to solve it in terms of the output gap):

$$\pi_t - \phi_{4t} + \phi_{4t-1} = 0$$

$$x_t + \theta\phi_{4t} = 0$$

Substituting these two equations into the AS equation (38) combined with (74) I can write the solution in terms of a second order difference equation:

$$\beta E_t x_{t+1} - (1 + \beta + \kappa\theta) x_t + x_{t-1} \tag{124}$$

The characteristic polynomial

$$\beta\mu^2 - (1 + \beta + \kappa\theta)\mu + 1 = 0$$

has two real roots

$$0 < \mu_1 < 1 < \beta^{-1} < \mu_2 = (\beta\mu_1)^{-1}$$

and it follows that (124) has an unique bounded solution $x_t = 0$ consistent with the the initial condition that $x_{-1} = 0$. Substituting this solution into (38) I can verify that the unique bounded solution for inflation is $\pi_t = 0$.

2. In the case of the Markov solution equation (86) indicates that $\phi_{3t} = 0$. Then I can write (83) and (84) so that (using (74) to solve it in terms of the output gap):

$$-\pi_t + \phi_{4t} = 0 \tag{125}$$

$$x_t + \theta\phi_{4t} = 0 \tag{126}$$

I can substitute these equations into the AS (38) together with (74) and write the solution in terms of the difference equation:

$$(1 + \theta\kappa)x_t - \beta E_t x_{t+1} = 0 \tag{127}$$

This equation has a unique bounded solution $x_t = 0$ and it follows that the unique bounded solution for inflation is $\pi_t = 0$.

A.4.4 Proof of Proposition 7

1. The first part of the proposition is that $\pi_t = x_t = 0$ for $t \geq \tau$. The proof for this follows directly from the second part of the proof for Proposition 6 since for $t \geq \tau$ there are no shocks and the Markov equilibrium is the one given in that Proposition. To see this note that the first order condition for $t \geq \tau$ are again given by (125) and (126) and I can again write the difference equation (127). Since this equation involves no history dependence (i.e. initial conditions do not matter) it follows once again that the unique bounded solution when $t \geq \tau$ is $x_t^M = \pi_t^M = 0$.

2. The second part of the proposition is that the Markov solution results in excessive deflation and output gap in period $0 < t < \tau$ relative to a policy that implies $\pi_\tau^C > 0$ and $x_\tau^C > 0$. I proof this by first showing that this must hold true for $\tau = K$ and then show that this implies it must hold for any $\tau < K$. Note first that our solution for the Markov equilibrium at any date $t \geq \tau$ implies that

$$\pi_\tau^C - \pi_\tau^M > 0 \tag{128}$$

$$x_\tau^C - x_\tau^M > 0 \tag{129}$$

The IS and AS equation implies that in the Markov equilibrium at date $K-1$ is

$$\tilde{x}_{K-1}^M = \sigma \tilde{r}_{K-1}^n$$

$$\tilde{\pi}_{K-1}^M = \kappa \tilde{x}_{K-1}^M$$

where I denote each of the variables by a hat to state that it is their value conditional on the natural rate of interest being negative at that time. Compared to a solution that implies that

$x_K^C > 0$ and $\pi_K^C > 0$ I can use the AS and the IS equations to write the inequalities:

$$\tilde{x}_{K-1}^C - \tilde{x}_{K-1}^M = x_K^C + \sigma\pi_K^C > 0$$

and

$$\tilde{\pi}_{K-1}^C - \tilde{\pi}_{K-1}^M = \kappa(\tilde{x}_{K-1}^M - \tilde{x}_{K-1}^C) + \beta\pi_K^C > 0$$

Using these two equation I can use the IS and AS equations at time $K-2$, (128)-(129), and the assumption about the natural rate of interest to write:

$$\tilde{x}_{K-2}^C - \tilde{x}_{K-2}^M = \alpha[(x_{K-1}^C - x_{K-1}^M) + \sigma(\pi_{K-1}^C - \pi_{K-1})] + (1-\alpha)[(\tilde{x}_{K-1}^C - \tilde{x}_{K-1}^M) + \sigma(\tilde{\pi}_{K-1}^C - \tilde{\pi}_{K-1})] > 0 \tag{130}$$

$$\tilde{\pi}_{K-2}^M - \tilde{\pi}_{K-2}^C = \kappa(\tilde{x}_{K-2}^M - \tilde{x}_{K-2}^M) + \alpha\beta\alpha(\pi_{K-1}^C - \pi_{K-1}) + (1-\alpha)\beta(\tilde{\pi}_{K-1}^C - \tilde{\pi}_{K-1}) > 0 \tag{131}$$

I can similarly solve the system backwards and write equation (130) and (131) for $K-2, K-3, ..., 0$ using at each time t the solution for $t+1$ and thereby the proposition is proofed.

A.4.5 Proof of Proposition 8

1. I first proof the that the solution for $t \geq \tau$ in the Markov solution is given by the one solution stated in the proposition. In the case of the Markov solution equation (86) indicates that $\phi_{3t} = 0$. When s is away from $\frac{\theta}{\theta-1}$ I can write (83) and (84) so that:

$$-\pi_t + \phi_{4t} = 0 \tag{132}$$

$$(x_t - x^*) + \theta\phi_{4t} = 0 \tag{133}$$

where $x^* = (\omega + \sigma^{-1})^{-1}(1 - \frac{\theta-1}{\theta}(1+s))$. These two equation imply that $\pi_t = -\theta^{-1}(x_t - x^*)$.I can substitute this into the AS (38) equation and write the solution in terms of the difference equation:

$$(1 + \theta\kappa)x_t - \beta E_t x_{t+1} = (1 - \beta)x^* \tag{134}$$

This equation has a unique bounded solution given by $x_t = \frac{1-\beta}{1-\beta+\theta\kappa}x^*$ and it follows that the unique bounded solution for inflation is $\pi_t = \frac{\kappa}{1-\beta+\theta\kappa}x^*$.

2. The second part of the proposition follows exactly the same steps as the second part of Proposition 7.

A.4.6 Proof of Proposition 9

At time $t \geq \tau$ the system is deterministic. In this case the functions $\Lambda_t = \bar{\Lambda}_t(w_{t-1}, \xi)$ and $\mathrm{w}_t = \bar{\mathrm{w}}(\mathrm{w}_{t-1}, \xi)$are independent of the calender time. Then I can approximate these functions to yield $w_t = w^1 w_{t-1}$ and $d\Lambda_t = \Lambda^1 w_{t-1}$, where the first element of the vector $d\Lambda_t$ is $d\pi_t = \pi^1 w_{t-1}$, the second $dY_t = Y^1 w_{t-1}$ and so on and $\mathrm{w}_t = w^1 w_{t-1}$ where the vector Λ^1 and the number w^1 are some unknown constants. To find the value of each of these coefficients I substitute this solution into the system (69)-(73) and (83)-(87) and match coefficients. For example equation (69) implies that

$$\bar{u}_c d'' \pi^1 w_{t-1} + \theta(\bar{u}_{cc} - \bar{v}_{yy}) Y^1 w_{t-1} - \bar{u}_c d'' \beta \pi^1 w^1 w_{t-1} = 0 \tag{135}$$

where I have substituted for $d\pi_t = \pi^1 w_{t-1}$ and for $d\pi_{t+1} = \pi^1 w_t = \pi^1 w^1 w_{t-1}$. Note that I assume that $t \geq \tau$ so that there is perfect foresight and I may ignore the expectation symbol. This equation implies that the coefficients π^1, y^1 and w^1 must satisfy the equation:

$$\bar{u}_c d'' \pi^1 + \theta(\bar{u}_{cc} - \bar{v}_{yy}) Y^1 - \bar{u}_c d'' \beta \pi^1 w^1 = 0 \tag{136}$$

I may similarly substitute the solution into each of the equation (69)-(73) and (83)-(87) to obtain a system of equation that the coefficients must satisfy:

$$\bar{u}_c d'' \pi^1 + \theta(\bar{u}_{cc} - \bar{v}_{yy}) Y^1 - \bar{u}_c d'' \beta \pi^1 w^1 = 0 \tag{137}$$

$$\bar{u}_{cc} Y^1 - \beta \bar{u}_{cc} Y^1 w^1 - \beta \bar{u}_c i^1 + \beta \bar{u}_c \pi^1 w^1 = 0 \tag{138}$$

$$w^1 - \frac{1}{\beta} + \frac{1}{\beta} T^1 = 0 \tag{139}$$

$$S^1 - \bar{u}_c d'' \pi^1 w^1 = 0 \tag{140}$$

$$f^1 + \bar{u}_c \pi^1 w^1 - \bar{u}_{cc} Y^1 w^1 = 0 \tag{141}$$

$$-d_{\mathrm{J}} \bar{u}_c \pi^1 + \frac{s' \bar{g}_G}{\beta} + d'' \bar{u}_c \phi_4^1 = 0 \tag{142}$$

$$(\bar{u}_{cc} - \bar{v}_{yy}) Y^1 - \bar{u}_{cc} \beta \phi_3^1 + \theta(\bar{u}_{cc} - \bar{v}_{yy}) \phi_4^1 = 0 \tag{143}$$

$$s' \bar{g}_G T^1 - s' \bar{g}_G + \bar{u}_c \beta^2 \phi_3^1 = 0 \tag{144}$$

$$\bar{g}_{GG}(s')^2T^1 - \bar{g}_G s'' T^1 + \beta^{-1}\phi_2^1 + \bar{g}_G s' i^1 = 0 \tag{145}$$

$$\phi_2^1 - \phi_2^1 w^1 - \beta\bar{g}_G s' i^1 w^1 + \bar{g}_G s' \pi^1 w^1 + \beta f^1 \phi_3^1 - \beta S^1 \phi_4^1 = 0 \tag{146}$$

There are 10 unknown coefficients in this system i.e. $\pi^1, Y^1, i^1, S^1, f^1, T^1, \phi_2^1, \phi_3^1, \phi_4^1, w^1$. For a given value of w^1, (137)-(145) is a linear system of 9 equations with 9 unknowns, and thus there is a unique value given for each of the coefficients as long as the system is non-singular (which can be verified to be the case for standard functional forms for the utility and technology functions).

Notes

[29] First, as shown by Woodford (2003), for a realistic calibration parameters, this abstraction has trivial effect on the AS and the IS equation under normal circustances. Furthermore, at zero nominal interest rate, increasing money balances further *does nothing* to facilitate transactions since consumer are already satiated in liquidity. This was one of the key insights of Eggertsson and Woodford (2003), which showed that at zero nominal interest rate increasing money supply has no effect if expectations about future money supply do not change. It is thus of even *less* interest to consider this additional channel for monetary policy at zero nominal interest rates than if the short-term nominal interest rate was positive. Second, assuming m_t is a very small number is likely to change the government budget constraint very little in a realistic calibration. By assuming the cashless limit I am assuming no seignorage revenues so that the term $\frac{i_t - i^m}{1+i_t} m_t \Pi_t^{-1}$ in the budget constraint has no effect on the equilibrium. Given the low level of seignorage revenues in industrialized countries (see King and Plosser (1985) I do not think this is a bad assumption. Furthermore, in the case the bound on the interest rate is binding, this term is zero, making it of even *less* interest when the zero bound is binding than under normal circumstances.

[30]See Woodford (2003) Appendix A3 for definition and discussion of local uniqueness in stochastic general equilibrium models of this kind.

[31]The reason for this conjecture is that in this model, as opposed to Albanesi et al and Dedola work, I assume in A2 that there are no monetary frictions. The source of the multiple equilibria in those papers, however, is the payment technology they assume. The key difference between the present model and that of King and Wolman, on the other hand, is that they assume that some firms set prices at different points in time. I assume a representative firm, thus abstacting from the main channel they emphasize in generating multiple equilibria. Finally the present model is different from all the papers cited above in that I introduce nominal debt as a state variable. Even if the model I have illustrated above would be augmented to incorporate additional elements such as montary frictions and staggering prices, I conjecture that the steady state would remain unique due to the ability of the government to use nominal debt to change its future inflation incentive. That is, however, a topic for future reasearch and there is work in progress by Eggertsson and Swanson that studies this question.

[32]Even if I had written a model in which the equilibria proofed above is not the unique global equilibria the one I illustrate here would still be the one of principal interest. Furthermore a local analysis would still be useful. The reason is twofold. First, the equilibria analyzed is identical to the commitment equilibrium (in the absence of shocks) and is thus a natural candidate for investigation. But even more importantly the work of Albanesi et al (2002) indicates that if there are non-trivial monetary frictions there are in general only two steady states.There are also two steady states in King and Wolman's model. (In Dedola's model there are three steady states, but the same point applies.) The first is a low inflation equilibria (analogues to the one in Proposition 1) and the other is a high inflation equilibria which they calibrate to be associated with double digit inflation. In the high inflation equilibria, however, the zero bound is very unlikely ever to be binding as a result of real shocks of the type I consider in this paper (since in this equilibria the nominal interest rate is very high as I will show in the next section). And it is the distortions created by the zero bound that are the central focus of this paper, and thus even if the model had a high inflation steady state, that equilibria would be of little interest in the context of the zero bound.

Chapter 3:

Monetary and Fiscal Coordination in a Liquidity Trap

Abstract

This paper analyses the effects of fiscal policy when monetary policy is frustrated by the zero bound. I solve a stochastic general equilibrium model with sticky prices assuming the government cannot commit to future policy. Real government spending increases demand by increasing public consumption. Deficit spending increases demand by generating inflation expectations. An increase in inflation expectations, at zero nominal interest rate, reduces the real rate of return, thereby stimulating demand. When fiscal and monetary policy are coordinated, deficit is more effective than real government spending in a calibrated model. When the central bank is "goal independent" real government spending is still effective but deficit spending is not.

1 Introduction

"It is important to recognize that the role of an independent central bank is different in inflationary and deflationary environments. In the face of inflation, which is often associated with excessive monetization of government debt, the virtue of an independent central bank is its ability to say "no" to the government. With protracted deflation, however, excessive monetary creation is unlikely to be the problem, and a more cooperative stance on the part of the central bank may be called for. Under the current circumstances, greater cooperation for a time between the Bank of Japan and the fiscal authorities is in no way inconsistent with the independence of the central bank, any more than cooperation between two independent nations in pursuit of a common objective is inconsistent with the principle of national sovereignty."

- Ben Bernanke, Governor of the Board of Governors of the Federal Reserve of the US, before the Japan Society of Monetary Economics, Tokyo, Japan, May 31, 2003.

"Coordinate, Coordinate

If monetary policy lacks sufficient power on its own to end deflation, the solution is not to give up but to try a coordinated monetary and fiscal stimulus."

– The Economist, June 2003, editorial on Japan fiscal and monetary policy

The conventional wisdom about monetary and fiscal policy is as follows[1]: "The first line of defence against an economic slump is monetary policy: the ability of the central bank – the Federal Reserve, the European Central Bank, the Bank of Japan – to cut interest rate. Lower real interest rates persuade businesses and consumers to borrow and spend, which creates new jobs, which encourages people to spend more, and so on. Since the 1930's this strategy has worked. Specifically interest rate cuts have pulled the US out of each of its big recession in the past 30 years – in 1975, 1982 and 1991. The second line of defence is fiscal policy: If cutting interest rates isn't enough to support the economy, the government can pump up demand by cutting taxes or its own spending. The conventional wisdom among economic analysts is that fiscal policy is not necessary to deal with most recessions, that interest-policy is enough. But the possibility of fiscal action always stands in reserve."

When the central bank has cut the short-term nominal interest rate to zero the second line of defence may be needed. Many economist believe it was wartime government spending that finally pulled the US out of the Great Depression, a period in which the short-term nominal interest rate had been close to zero for several years. Recent events in Japan also raise questions about the effectiveness of the second line of defense. The Bank of Japan (BOJ) cut the short-term nominal interest rate to zero in 1998 and since then the budget deficit has ballooned with the gross public debt exceeding 130 percent of GDP today (although whether cyclically adjusted real government spending has been increased or not is debatable, see e.g. Kuttner and Posen (2001)). Yet deflation persists and unemployment is at a historic high. Is standard fiscal and monetary insufficient to curb deflation and increase demand? Should we overturn the conventional Keynesian wisdom? This paper addresses these questions from a theoretical perspective by analyzing a stochastic general equilibrium model with sticky prices. I analyze two different fiscal policy options. The first is increasing real government spending, i.e. raising government consumption (holding the budget balanced). The second is increasing deficit spending, i.e. cutting taxes and accumulating debt (holding real government spending constant). The central conclusion is that either deficit spending or real government spending can be used to eliminate deflation and increase demand when the short-term nominal interest rate is zero. Of the two options I find deficit spending is more effective, both in terms of reducing deflation/increasing output in equilibrium, and improving aggregate welfare. This conclusion may seem to vindicate the conventional wisdom. There is, however, at least two non-conventional twists. First, deficit spending is only effective if fiscal and monetary policy are coordinated. If the central banks objectives are different from social welfare (e.g. a narrow inflation target) – this what I call a goal independent central bank – deficit spending has no effect. Second, real government spending does not only work through current spending. It also works through expectation about future spending when the zero bound is binding. Indeed, expectations about future spending are much more important than current spending increases, contrary to the old fashion IS-LM model where expectations are fixed.

It is worth stating that I do not view fiscal policy as the only way to stimulate prices and output when the zero bound is binding. More unconventional policies may be effective. The central bank can, for example, print money and buy real assets, such as stocks and real estate, or foreign exchange.[2] Under certain conditions this may indeed increase demand as discussed in Eggertsson (2004)[3]. But there are at least two reasons for paying special attention to fiscal policy. First, it is a common view that active fiscal policy ended the Great Depression in the US and enabled Japan to avoid the Great Depression in the early 30's. It is important to understand the reasons for its past successes versus its current failure in Japan. Second, fiscal policy may have several advantages over other unconventional options. It does not, for example, require the central bank to purchase large amounts of private property. Neither does it rely on unilateral actions in the foreign exchange markets that may cause political difficulties and cause negative reactions from trading partners. Deficit spending, i.e. tax cuts and debt accumulation, simply involves moving government nominal wealth from the government to the public. Similarly increasing real government spending simply involves the government buying real goods and services. These are among the most basic policy measures in the governments arsenal and have a long history in economic thought.

I analyze fiscal policy in a stochastic general equilibrium model assuming rational expectations. I assume sticky prices to obtain a "New Keynesian" Phillips curve. The zero bound is binding due to temporary demand shocks that make the natural rate of interest – i.e. the real interest rate consistent with zero output gap – temporarily negative. Under monetary and fiscal policy coordination, I assume that the government, i.e. the treasury and the central bank, maximizes social welfare given by the utility of a representative household. Following Stokey and Lucas (1983), I assume that the government can commit to pay back the face value of debt issued. I assume that it cannot commit to any other future policy action. These assumptions have several advantages. The first is that it allows us to use modern game theory to analyze a Markov equilibrium in an infinite game between the private sector and the government (as defined by Maskin and Tirole (2001)). A Markov equilibrium is subgame perfect, so that the government has no incentive to deviate from it's policy. A common criticism of policy proposals, e.g. for the BOJ, is that they are not credible. Since no one has an incentive to deviate in a Markov equilibrium the policies analyzed are, by construction, fully credible. The second advantage of assuming no commitment is that it gives a rigorous theory of expectations. As emphasized by Eggertsson and Woodford (2003), expectations about future policy are crucial to understanding the effect of different policy alternatives. Analyzing a Markov equilibrium provides a clear theory of how expectations about future policy are formed: Agents are rational so they anticipate future actions of the government. The government's future policy actions, on the other hand, are determined by its *incentives* from that period onwards. The third advantage of assuming no commitment is that it is rare for a central bank or a treasury to announce future policies that cannot be reversed in the light of new circumstances (apart from paying back debt issued!). Furthermore, since most governments are elected for short periods of time future regimes may not regard their predecessors announcements as binding.[4]

Two lines of research have emerged on the zero bound. The first attributes a binding zero bound to a suboptimal policy rule and views the liquidity trap as an example of a self-fulfilling "bad equilibrium" that is not driven by real shocks. The solution is for the government to commit to a different policy rule that eliminates the self-fulfilling "bad" equilibria (leading examples of this approach include Benhabib et al (2002) and Buiter (2003)). The other line of research attributes deflation and the zero bound to an inefficient policy response to real disturbances. In this case the zero bound can either be binding because of an inefficient policy rule (see Eggertsson and Woodford (2003)) or because of the governments inability to commit to future policy (see Eggertsson (2004)). This paper follows the second line of research so that the zero bound is binding due temporary real disturbances and the resulting equilibrium may be suboptimal due to the governments policy constraints and inability to commit to future policy. As emphasized by Krugman (1998) and Eggertsson and Woodford (2003) the optimal policy is to commit to higher future inflation when the zero bound stops being binding, but as shown by Eggertsson (2004) this policy may not be credible if the government has no explicit commitment mechanism. In this paper, as in Eggertsson (2004), deficit spending is mainly useful because it helps the government to solve this commitment problem. Real government spending is mainly effective because it reduces the potency of negative shocks by increasing aggregate spending when the zero bound is binding. Rogoff (1998) and Gertler (2003) also point out that government spending can help fight deflation by counteracting negative shocks. The result I obtain here is consistent with their arguments. Although the Markov solution is inferior to the solution if the government can commit to future policy, an estimate of the utility of the representative household shows that the difference is small if monetary and fiscal policy are coordinated In the absence of coordination, in contrast, the difference can be very big.

A body of literature has emerged in recent years emphasizing the connection between the price level and fiscal policy. This literature is often referred to as the Fiscal Theory of the Price Level (FTPL) (see e.g. Leeper (1992), Sims (1994) and Woodford (1996) and Sargent and Wallace (1981) for an early contribution). A key difference between the approach in this paper and the FTPL is the way the government is modelled. Papers applying the FTPL often model the central bank as committing to an (possibly suboptimal) interest rate feedback rule and fiscal policy is modelled as an (possibly suboptimal) exogenous path of real government surpluses (typically abstracting away from any variations in real government spending). Under these assumptions innovations in real government surpluses may influence the price level since prices may have to move for the government budget constraint to be satisfied (because any changes in the policy choices of the government are ruled out by assumption, i.e. by the assumed policy commitments of the government). In contrast, in my setting, fiscal policy can only affect the price level because it changes the government future inflation incentive or because real government spending directly increases demand. The government budget constraint is simply a constraint on public policy choices (just as any technology constraint or private sector equilibrium condition) and it only influences the price level to the extent that it affects the government's inflation and through those incentives actual policy *choices*.[5]

2 The Model

Here I outline a simple sticky prices general equilibrium model and define the set of feasible equilibrium allocations that are consistent with the private sector maximization problems and the technology constraints the government faces.

2.1 The private sector

2.1.1 Households

I assume that there is a representative household that maximizes expected utility over the infinite horizon:

$$E_t \sum_{T=t}^{\infty} \beta^T U_T = E_t \left\{ \sum_{T=t}^{\infty} \beta^T [u(C_T, \frac{M_T}{P_T}, \xi_T) + g(G_T, \xi_T) - \int_0^1 v(h_T(i), \xi_T) di] \right\} \quad (1)$$

where C_t is a Dixit-Stiglitz aggregate of consumption of each of a continuum of differentiated goods

$$C_t \equiv [\int_0^1 c_t(i)^{\frac{\theta}{\theta-1}}]^{\frac{\theta-1}{\theta}}$$

with elasticity of substituting equal to $\theta > 1$, G_t is is a Dixit-Stiglitz aggregate of government consumption, ξ_t is a vector of exogenous shocks, M_t is end-of-period money balances, P_t is the Dixit-Stiglitz price index,

$$P_t \equiv [\int_0^1 p_t(i)^{1-\theta}]^{\frac{1}{1-\theta}}$$

and $h_t(i)$ is quantity supplied of labor of type i. $u(.)$ is assumed to be concave and strictly increasing in C_t for any possible value of ξ. The utility of holding real money balances is assumed to be increasing in $\frac{M_t}{P_t}$ for any possible value of ξ up to a satiation point at some finite level of real money balances as in Friedman (1969).[6] $g(.)$ is the utility of government consumption and is assumed to be concave and strictly increasing in G_t for any possible value of ξ. $v(.)$ is the disutility of supplying labor of type i and is assumed to be an increasing and convex in $h_t(i)$ for any possible value of ξ. E_t denotes mathematical expectation conditional on information available in period t. ξ_t is a vector of r exogenous shocks. I assume that ξ_t follows a Markov process so that:[7]

A1 *(i)* $pr(\xi_{t+j}|\xi_t) = pr(\xi_{t+j}|\xi_t, \xi_{t-1}, ...)$ *for* $j \geq 1$ *where* $pr(.)$ *is the conditional probability density function of* ξ_{t+j}.

For simplicity I assume complete financial markets and no limit on borrowing against future income. As a consequence, a household faces an intertemporal budget constraint of the form:

$$E_t \sum_{T=t}^{\infty} Q_{t,T}[P_T C_T + \frac{i_T - i^m}{1+i_T} M_T] \leq W_t + E_t \sum_{T=t}^{\infty} Q_{t,T}[\int_0^1 Z_T(i)di + \int_0^1 n_T(j)h_T(j)dj - P_T T_T] \tag{2}$$

looking forward from any period t. Here $Q_{t,T}$ is the stochastic discount factor that financial markets use to value random nominal income at date T in monetary units at date t; i_t is the riskless nominal interest rate on one-period obligations purchased in period t, i^m is the nominal interest rate paid on money balances held at the end of period t, W_t is the beginning of period nominal wealth at time t (note that its composition is determined at time $t-1$ so that it is equal to the sum of monetary holdings from period $t-1$ and return on non-monetary assets), $Z_t(i)$ is the time t nominal profit of firm i, $n_t(i)$ is the nominal wage rate for labor of type i, T_t is net real tax collections by the government. The problem of the household is: at every time t the household takes W_t and $\{Q_{t,T}, n_T(i), P_T, T_T, Z_T(i), \xi_T; T \geq t\}$ as exogenously given and maximizes (1) subject to (2) by choice of $\{M_T, h_T(i), C_T; T \geq t\}$.

2.1.2 Firms

The production function of the representative firm that produces good i is:

$$y_t(i) = f(h_t(i), \xi_t) \tag{3}$$

where f is an increasing concave function for any ξ and ξ is again the vector of shocks defined above (that may include productivity shocks). I abstract from capital dynamics. As Rotemberg (1983), I assume that firms face a cost of price changes given by the function $d(\frac{p_t(i)}{p_{t-1}(i)})$[8] but I can derive exactly the same result assuming that firms adjust their prices at stochastic intervals as assumed by Calvo (1983).[9] Price variations have a welfare cost that is separate from the cost of expected inflation due to real money balances in utility. The Dixit-Stiglitz preferences of the household imply a demand function for the product of firm i given by

$$y_t(i) = Y_t(\frac{p_t(i)}{P_t})^{-\theta}$$

The firm maximizes

$$E_t \sum_{T=t}^{\infty} Q_{t,T} Z_T(i) \tag{4}$$

where

$$Q_{t,T} = \beta^{T-t} \frac{u_c(C_T, \frac{M_T}{P_T}, \xi_T)}{u_c(C_t, \frac{M_t}{P_t}, \xi_t)} \frac{P_t}{P_T} \tag{5}$$

I can write firms period profits as:

$$Z_t(i) = (1+s)Y_t P_t^{\theta} p_t(i)^{1-\theta} - n_t(i) f^{-1}(Y_t P_t^{\theta} p_t^{-\theta}) - P_t d(\frac{p_t(i)}{p_{t-1}(i)}) \tag{6}$$

where s is an exogenously given production subsidy that I introduce for algebraic convenience (for reasons described below).[10] The problem of the firm is: at every time t the firm takes $\{n_T(i), Q_{t,T}, P_T, Y_T, C_T, \frac{M_T}{P_T}, \xi_T; T \geq t\}$ as exogenously given and maximizes (4) by choice of $\{p_T(i); T \geq t\}$.

2.1.3 Private Sector Equilibrium Conditions: AS, IS and LM Equations

This subsection illustrates the necessary conditions for equilibrium that stem from the maximization problems of the private sector. These conditions must hold for *any* government policy. The first order conditions of the household maximization imply an Euler equation of the form:

$$\frac{1}{1+i_t} = E_t\{\frac{\beta u_c(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}, \xi_{t+1})}{u_c(C_t, \frac{M_t}{P_t}, \xi_t)} \frac{P_t}{P_{t+1}}\} \tag{7}$$

where i_t is the nominal interest rate on a one period riskless bond. This equation is often referred to as the IS equation. Optimal money holding implies:

$$\frac{u_{\frac{M}{P}}(C_t, \frac{M_t}{P_t}, \xi_t)}{u_c(C_t, \frac{M_t}{P_t}, \xi_t)} = \frac{i_t - i^m}{1 + i_t} \tag{8}$$

This equation defines money demand and is often referred as the "LM" equation. Utility is increasing in real money balances. At some finite level of real money balances, further holdings of money add nothing to utility so that $u_{\frac{M}{P}} = 0$. The left hand side of (8) is therefore weakly positive. Thus there is bound on the short-term nominal interest rate given by:

$$i_t \geq i^m \tag{9}$$

In most economic discussion it is assumed that the interest paid on the monetary base is zero so that (9) becomes $i_t \geq 0$.[11]

The optimal consumption plan of the representative household must also satisfy the transversality condition[12]

$$\lim_{T\to\infty} E_t Q_{t,T} \frac{W_T}{P_t} = 0 \tag{10}$$

to ensure that the household exhausts its intertemporal budget constraint. I assume that workers are wage takers so that the households optimal choice of labor supplied of type j satisfies

$$n_t(j) = \frac{P_t v_h(h_t(j); \xi_t)}{u_c(C_t, \frac{M_t}{P_t}, \xi_t)} \tag{11}$$

I restrict my attention to a symmetric equilibria where all firms charge the same price and produce the same level of output so that

$$p_t(i) = p_t(j) = P_t; \quad y_t(i) = y_t(j) = Y_t; \quad n_t(i) = n_t(j) = n_t; \quad h_t(i) = h_t(j) = h_t \quad \text{for} \quad \forall\, j, i \tag{12}$$

Given the wage demanded by households I can derive the aggregate supply function from the first order conditions of the representative firm, assuming competitive labor market so that each firm takes its wage as given. I obtain the equilibrium condition often referred to as the AS or the "New Keynesian" Phillips curve:

$$\theta Y_t[\frac{\theta-1}{\theta}(1+s)u_c(C_t,\frac{M_t}{P_t},\xi_t)-\tilde{v}_y(Y_t,\xi_t)]+u_c(C_t,\frac{M_t}{P_t},\xi_t)\frac{P_t}{P_{t-1}}d'(\frac{P_t}{P_{t-1}}) \tag{13}$$
$$-E_t\beta u_c(C_{t+1},\frac{M_{t+1}}{P_{t+1}},\xi_{t+1})\frac{P_{t+1}}{P_t}d'(\frac{P_{t+1}}{P_t})=0$$

where for notational simplicity I have defined the function:

$$\tilde{v}(y_t(i),\xi_t)\equiv v(f^{-1}(y_t(i)),\xi_t) \tag{14}$$

2.2 The Government

There is an output cost of taxation (e.g. due to tax collection costs as in Barro (1979)) captured by the function $s(T_t)$.[13] For every dollar collected in taxes $s\,(T_t)$ units of output are waisted without contributing anything to utility. Government real spending is then given by:

$$F_t=G_t+s(T_t) \tag{15}$$

I could also define the tax cost that would result from distortionary taxes on income or consumption and obtain similar results.[14] I assume a representative household so that in a symmetric equilibrium, all nominal claims held are issued by the government. It follows that the government flow budget constraint is:

$$B_t+M_t=W_t+P_t(F_t-T_t) \tag{16}$$

where B_t is the end-of-period nominal value of bonds issued by the government. Having defined both private and public spending I can verify that market clearing implies that aggregate demand satisfies:

$$Y_t = C_t + d(\frac{P_t}{P_{t-1}}) + F_t \tag{17}$$

I now define the set of possible equilibria that are consistent with the private sector equilibrium conditions and the technological constraints on government policy.

Definition 1 *Private Sector Equilibrium (PSE) is a collection of stochastic processes*

$\{P_t, Y_t, W_{t+1}, B_t, M_t, i_t, F_t, T_t, Q_t, Z_{t,} G_t, C_t, n_t, h_t, \xi_t\}$ *for* $t \geq t_0$ *that satisfy equations (2)-(17) for each* $t \geq t_0$*, given* W_{t_0} *and* $P_{t_{0-1}}$ *and the exogenous stochastic process* $\{\xi_t\}$ *that satisfies A1 for* $t \geq t_0$*.*

2.3 Recursive representation

It is useful to rewrite the model in a recursive form so that I can identify the endogenous state variables at each date. When the government only issues one period nominal debt I can write the total nominal claims of the government (which in equilibrium are equal to the total nominal wealth of the representative household) as:

$$W_{t+1} = (1 + i_t)B_t + (1 + i^m)M_t$$

Substituting this into (16) and defining the variables $w_t \equiv \frac{W_{t+1}}{P_t}$, $m_t \equiv \frac{M_t}{P_{t-1}}$ and $\Pi_t = \frac{P_t}{P_{t-1}}$ I can write the government budget constraint as:

$$w_t = (1 + i_t)(w_{t-1}\Pi_t^{-1} + (F - T_t) - \frac{i_t - i^m}{1 + i_t} m_t \Pi_t^{-1}) \tag{18}$$

Note that I use the time subscript t on w_t (even if it denotes the real claims on the government at the beginning of time $t + 1$) to emphasize that this variable is determined at time t. I impose a borrowing limit on the government that rules out Ponzi schemes:

$$u_c w_t \leq \bar{w} < \infty \tag{19}$$

where $\bar{w}$ is an arbitrarily high finite number. This condition can be justified by that the government can never borrow more than the equivalent of the expected discounted value of its maximum tax base (e.g. discounted future value of all future output).[15] It is easy to show that this limit ensures that the representative household's transversality condition is satisfied at all times.

The treasury's policy instruments is taxation, T_t, that determines the end-of-period government debt which is equal to $B_t + M_t$, and real government spending F_t. The central bank determines how the end-of-period debt is split between bonds and money by open market operations. Thus the central bank's policy instrument is $M_{t.}$ Note that since P_{t-1} is determined in the previous period I may think of $m_t \equiv \frac{M_t}{P_{t-1}}$ as the instrument of monetary policy.

It is useful to note that I can reduce the number of equations that are necessary and sufficient for a private sector equilibrium substantially from those listed in Definition 1. First, note that the equations that determine $\{Q_t,\ Z_t, G_t, C_t, n_t, h_t\}$ are redundant, i.e. each of them is only useful to determine one particular variable but has no effect on the any of the other variables. Thus I can define the necessary and sufficient condition for a private sector equilibrium without specifying the stochastic process for $\{Q_t,\ Z_t, G_t, C_t, n_t, h_t\}$ and do not need to consider equations (3), (5), (6), (11), (15) and I use (17) to substitute out for C_t in the remaining conditions. Furthermore condition (19) ensures that the transversality condition of the representative household is satisfied at all times so I do not need to include (10) in the list of necessary and sufficient conditions.

It is useful to define the expectation variable

$$f_t^e \equiv E_t u_c(Y_{t+1} - d(\Pi_{t+1}) - F, m_{t+1}\Pi_{t+1}^{-1}, \xi_{t+1})\Pi_{t+1}^{-1} \tag{20}$$

as the part of the nominal interest rates that is determined by the expectations of the private sector formed at time t. Here I have used (17) to substitute for consumption in the utility function. The IS equation can then be written as:

$$1 + i_t = \frac{u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)}{\beta f_t^e} \tag{21}$$

Similarly it is useful to define the expectation variable

$$S_t^e \equiv E_t u_c(Y_{t+1} - d(\Pi_{t+1}) - F, m_{t+1}\Pi_{t+1}^{-1}, \xi_{t+1})\Pi_{t+1} d'(\Pi_{t+1}) \tag{22}$$

The AS equation can now be written as:

$$\theta Y_t[\frac{\theta - 1}{\theta}(1+s)u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] + u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)\Pi_t d'(\Pi_t) - \beta S_t^e = 0 \tag{23}$$

Finally the money demand equation (8) can be written in terms of m_t and Π_t as

$$\frac{u_m(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)\Pi_t^{-1}}{u_c(Y_t - d(\Pi_t) - F, \xi_t)} = \frac{i_t - i^m}{1 + i_t} \tag{24}$$

The next two propositions are useful to characterize equilibrium outcomes. Proposition 1 follows directly from our discussion above:

Proposition 1 *A necessary and sufficient condition for the set of variables* $(\Pi_t, Y_t, F_t, w_t, m_t, i_t, T_t)$ *in a PSE at each time* $t \geq t_0$ *is that they satisfy : (i) conditions (9), (18),(19), (21), (23) and (24) given* w_{t-1} *and the expectations* f_t^e *and* S_t^e*. (ii) in each period* $t \geq t_0$*, expectations are rational so that* f_t^e *is given by (20) and* S_t^e *by (22).*

Proposition 2 *The possible PSE equilibrium for the variables* $(\Pi_t, Y_t, F_t, w_t, m_t, i_t, T_t)$ *defined by the necessary and sufficient conditions for any date* $t \geq t_0$ *onwards depends only on* w_{t-1} *and* ξ_t.

The second proposition follows from observing that w_{t-1} is the only endogenous variable that enters with a lag in the necessary conditions specified in (i) of Proposition 1 and using the assumption that ξ_t is Markovian (i.e. using A1) so that the conditional probability distribution of ξ_t for $t > t_0$ only depends on ξ_{t_0}. It follows from this proposition (w_{t-1}, ξ_t) are the only state variables at any time t that directly affects the PSE.

2.4 Policy Objectives and Policy Games

To define equilibrium I need to specify policy objectives for the government, i.e. the treasury and the central bank. Throughout this paper I assume that the treasury maximizes social welfare, which is given by the utility of the representative household. Furthermore, following Lucas and Stokey (1983), I assume that the treasury can commit to paying the face value of debt issues which is assumed to be issued in nominal terms. The treasury cannot commit to any other future policy action and I only consider Markovian strategies that will be more precisely defined in the next section. Where as fiscal policy maximizes social welfare at all times, I consider monetary policy under two institutional arrangement. Under the first arrangement, which I call coordination, monetary and fiscal policy are coordinated to maximize social welfare. I define the maximization problem in the next two sections when I define the Markov equilibrium.

A2 Coordinated Fiscal and Monetary Policy. *The government, i.e. the treasury and* *the central bank, determine F_t, T_t and m_t to maximize the utility of the representative household.*

Under the second institutional arrangement, I assume that monetary policy is delegated to satisfy goals that are different social welfare. The is what Svensson (2000) calls a flexible inflation target and I refer to as a "goal independent" central bank. In this case the central bank seeks to minimize the criterion $L_t = [\pi_t^2 + \lambda_x x_t^2]$ where x_t is the output gap, defined as the percentage difference between actual output, Y_t, and the natural rate of output, Y_t^n, i.e. $x_t \equiv Y_t/Y_t^n - 1$. The natural rate of output is the output that would be produced if prices where completely flexible, i.e. it is the output that solves the equation

$$v_y(Y_t^n, \xi_t) = \frac{\theta - 1}{\theta}(1 + s)u_c(Y_t^n, \xi_t). \tag{25}$$

There is a long tradition in the literature of assuming that this loss function describes the the behavior of independent central banks. Under the flexible inflation target the central bank minimizes its loss function and the treasury sets taxes and real spending to maximize social welfare.

A3 Goal Independent Central Bank. *The central bank sets m_t to maximize $U_t = -E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda_x x_t^2]$. The treasury sets T_t and F_t to maximize the utility of the representative household.*

The motivation for A2 is that in several industrial countries monetary policy has been separated from fiscal policy and given to independent central bankers. It is common practice to give the central bank a fairly narrow mandate such as aiming for "price stability" and protecting employment. In very few cases does the central bank mandate pay any attention to fiscal variables. Indeed the move towards central bank independence has often involved explicitly excluding fiscal policy consideration from the bank's mandate. In the case of Japan, for example, the Diet explicitly forbade the BOJ from underwriting government bonds after the experience of hyperinflation in World War II. Similarly the Federal Reserve's role in government finances was substantially reduced in the 1950s. I argue later in the paper that these institutional reforms may make some sense under normal circumstances (especially when inflation is a problem). They can, however, limit the effectiveness of fiscal and monetary policy when the economy is plagued by deflation. I argue that cooperation (at least temporary) between the treasury and the central bank, as defined in A2, may be useful to fight deflation. A2 captures precisely what I mean by cooperation or what I call coordination of policy, i.e. it simply states that the central bank and treasury both set their instruments to maximize social welfare. Note that A3, i.e. the goal independent central bank, is consistent with Rogoff's (1985) conservative central banker and is also consistent with Dixit and Lambertini (2003) institutional framework, but the latter authors also assume that the treasury maximizes social welfare but the central bank has more narrow goals.[16]

I should be careful to note, however, that coordination does not necessarily mean that central bank independence is reduced if one thinks of "independence" as meaning the ability of the central bank to set its own policy instruments. Indeed, as Bernanke (2003) argues, cooperation between the central bank and the treasury need not to imply the elimination of the central bank's independence "any more than cooperation between two independent nations in pursuit of a common objective is inconsistent with the principle of national sovereignty." Bernanke interpretation of "cooperation" as a "pursuit of common objective" is consistent with A2 where this common objective is simply social welfare. Thus although I will refer to A2 as referring to "goal independence", in practice a move towards coordination of policy would not need to imply that the instrumental independence of the central bank would be reduced. Indeed, no particular institutional changes are needed, the central bank itself only need to make the fiscal health of the government one of its policy concerns.

3 Markov Equilibrium under Coordinated Monetary and Fiscal Policy

3.1 Defining a Markov Equilibrium under Coordination

In this section I define a Markov equilibrium under A2, i.e. the condition that monetary and fiscal policy are coordinated. I defer to section 4 to discuss the case when the central bank is goal independent. A Markov equilibrium is formally defined by Maskin and Tirole (2001) and has been extensively applied in the monetary literature. The basic idea behind this equilibrium concept is to restrict attention to equilibria that only depend on the minimum set of variables that directly affect market conditions.

The timing of events in the game is as follows: At the beginning of each period t, w_{t-1} is a predetermined state variable. At the beginning of the period, the vector of exogenous disturbances ξ_t is realized and observed by the private sector and the government. The monetary and fiscal authorities choose policy for period t given the state and the private sector forms expectations e_t. Note that I assume that the private sector may condition its expectation at time t on w_t, i.e. it observes the policy actions of the government in that period so that Λ_t and e_t are jointly determined. This is important because w_t is the relevant endogenous state variable at date $t+1$. The set of possible values (Λ_t, w_t) that can be achieved by the policy decisions of the government are those that satisfy the equations given in Propositions 2 given the values of w_{t-1}, ξ_t and the expectation f_t^e and S_t^e.

I may economize on notation by introducing vector notation. I define vectors

$$\Lambda_t \equiv \begin{bmatrix} \Pi_t & Y_t & i_t & m_t & F_t & T_t \end{bmatrix}^T, \text{ and } e_t \equiv \begin{bmatrix} f_t^e \\ S_t^e \end{bmatrix}.$$

Since Proposition 3 indicates that w_t is the only endogenous state variable I prefer not to include it in either vector but keep track of it separately. Proposition 3 indicates that a Markov equilibrium requires that the variables (Λ_t, w_t) only depend on (w_{t-1}, ξ_t), since this are the minimum set of state variables that affect the private sector equilibrium. Thus, in a Markov equilibrium, there must exist policy functions $\bar{\Pi}(.), \bar{Y}(.), \bar{\imath}(.), \bar{m}(.), \bar{F}(.), \bar{T}(.), \bar{w}(.)$ that I denote by the vector valued function $\bar{\Lambda}(.)$ and the function $\bar{w}(.)$ such that each period:

$$\begin{matrix} \Lambda_t & & \bar{\Lambda}(w_{t-1},\xi_t) \\ & \equiv & \\ w_t & & \bar{w}(w_{t-1},\xi_t) \end{matrix} \tag{26}$$

Note that the functions $\bar{\Lambda}(.)$ and $\bar{w}(.)$ also defines a set of functions of (w_{t-1}, ξ_t) for $(Q_t, Z_t, G_t, C_t, n_t, h_t)$ by the redundant equations from Definition 1. Using $\bar{\Lambda}(.)$ I may also use (20) and (22) to define a function $\bar{e}(.)$ so so that

$$e_t = \begin{bmatrix} f_t^e \\ S_t^e \end{bmatrix} = \begin{bmatrix} \bar{f}^e(w_t, \xi_t) \\ \bar{S}^e(w_t, \xi_t) \end{bmatrix} = \bar{e}(w_t,\xi_t) \tag{27}$$

Rational expectations imply that the function $\bar{e}$ satisfies:

$$\bar{e}(w_t,\xi_t) = \begin{bmatrix} E_t u_c(\bar{C}(w_t,\xi_{t+1}), \bar{m}(w_t,\xi_{t+1})\bar{\Pi}(w_t,\xi_{t+1})^{-1};\xi_{t+1})\bar{\Pi}(w_t,\xi_{t+1})^{-1} \\ E_t u_c(\bar{C}(w_t,\xi_{t+1}), \bar{m}(w_t,\xi_{t+1})\bar{\Pi}(w_t,\xi_{t+1})^{-1};\xi_{t+1})\bar{\Pi}(w_t,\xi_{t+1})d'(\bar{\Pi}(w_t,\xi_{t+1})) \end{bmatrix} \tag{28}$$

To economize on notation I can write the utility function as the function $U : \mathbb{R}^{7+r} \to \mathbb{R}$

$$U_t = U(\Lambda_t, \xi_t)$$

using (15) to solve for G_t as a function of F and T_t, along with (12) and (14) to solve for $h_t(i)$ as a function of Y_t. I define a value function $J(w_{t-1},\xi_t)$ as the expected discounted value of the utility of the representative household, looking forward from period t, given the evolution of the endogenous variable from period t onwards that is determined by $\Lambda(.)$ and $\{\xi_t\}$. Thus I define:

$$J(w_{t-1},\xi_t) \equiv E_t \left\{ \sum_{T=t}^{\infty} \beta^T [U(\bar{\Lambda}(w_{T-1},\xi_T),\xi_T] \right\} \tag{29}$$

The optimizing problem of the government is as follows. Given w_{t-1} and ξ_t the government chooses the values for (Λ_t, w_t) (by its choice of the policy instruments m_t, F_t, w_t and T_t) to maximize the utility of the representative household subject to the constraints in Proposition. Thus its problem can be written as:

$$\max_{m_t,F_t,T_t,w_t} [U(\Lambda_t,\xi_t) + \beta E_t J(w_t,\xi_{t+1})] \tag{30}$$

s.t. (9), (18),(19), (21), (23), (24) and (27).

I can now define a Markov Equilibrium under coordination

Definition 2 *A Markov Equilibrium under coordination is a collection of functions* $\bar{\Lambda}(.), \bar{w}(.), J(.), \bar{e}(.)$, *such that (i) given the function* $J(w_{t-1},\xi_t)$ *and the vector function* $\bar{e}(w_t,\xi_t)$ *the solution to the policy maker's optimization problem (30) is given by* $\Lambda_t = \bar{\Lambda}(w_{t-1},\xi_t)$

and $\bar{w}(.)$ for each possible state (w_{t-1},ξ_t) (ii) given the vector function $\bar{\Lambda}(w_{t-1},\xi_t)$ and $\bar{w}(w_{t-1},\xi_t)$ then $e_t = \bar{e}(w_t,\xi_t)$ is formed under rational expectations (see equation (28)). (iii) given the vector function $\bar{\Lambda}(w_{t-1},\xi_t)$ and $\bar{w}(w_{t-1},\xi_t)$ the function $J(w_{t-1},\xi_t)$ satisfies (29).

I will only look for a Markov equilibrium in which the functions $\bar{\Lambda}(.), \bar{w}(),\ J(.),\ \bar{e}(.)$ are continuous and have well defined derivatives. The value function satisfies the Bellman equation:

$$J(w_{t-1},\xi_t) = \max_{m_t,F_t,T_t,w_t} [U(\Lambda_t,\xi_t) + E_t \beta J(w_t,\xi_{t+1})] \tag{31}$$

s.t. (9), (18),(19), (21), (23), (24) and (27).

The solution can now be characterized by using a Lagrangian method for the maximization problem on the right hand side of (31). In addition the solution satisfies an envelope conditions. The Lagrangian, associated with the appropriate first order condition, and the envelope condition, are shown in the Technical Appendix.

3.2 Approximation method

I define a steady state as a solution in the absence of shocks were each of the variables $(\Pi_t, Y_t, m_t, i_t, T_t, w_t, f_t^e, S_t^e) = (\Pi, Y, m, i, T, w, f^e, S^e)$ are constants. I define a steady state in a cashless limit at the efficient equilibrium allocation so that (see Technical Appendix for further discussion):

A4 *Steady state assumptions. (i)* $\bar{m} \to 0$, *(ii)* $1 + s = \frac{\theta - 1}{\theta}$ *(iii)* $i^m = 1/\beta - 1$

A4 (ii) implies that there is no inflation bias in steady state. In Eggertsson (2004) I relax this assumption and illustrate that the basic issues addressed here (i.e. inefficient deflation) is still a problem, provided that the shocks that the economy is subject to (that I define in A5) are correspondingly larger. I can now proof the existence of a steady state under A4 of the form

Using A2 I proof in the Technical Appendix I that there exist a steady state for both the commitment and the Markov solution given by $(\Pi, Y, \frac{m}{\tilde{m}}, i, F, T, w, f^e, S^e) = (1, \bar{Y}, \tilde{m}, \frac{1}{\beta} - 1, \bar{F}, \bar{T}, 0, u_c(\bar{Y} - \bar{F}), 0)$ and give the equations the values $\bar{T}$, $\bar{F}$, $\bar{T}$ and $\tilde{m}$ must satisfy. I discuss how this result relates to the work of Albanesi et al (2002), Dedola (2002) and King and Wolman (2003) in the Technical Appendix. The solution can now be approximated by a linearization around this steady state, keeping explicit track of the Kuhn-Tucker conditions. The resulting equilibrium is accurate to the order $o(||\xi||^2)$. A complication is introduced by the presence of the inequality constraints due to the Kuhn-Tucker conditions and I apply a solution method discussed in the Technical Appendix to solve this problem. As discussed in the Technical Appendix the approximate solution is also valid for $i^m = 0$ which I assume in the following sections, and the resulting solution is accurate to the order $o(||\xi, \delta||^2)$ where $\delta \equiv \frac{i - i^m}{1+i}$. A further complication arises because of the expectation function $\bar{e}(w_t, \xi_t)$ is unknown. The method to approximate this function is shown in the Technical Appendix, where I also discuss how my solution method relates to Klein et al (2003). Matlab codes were written to implement these solution method discussed in the Technical Appendix.[17]

3.3 Results under coordination

Here I show the optimal policy in a Markov equilibrium under coordination. To identify the power of real government spending vs deficit spending I analyze the result in three steps. I first show the result when deficit and real government spending are constrained at zero. I then introduce one of these two instruments at a time and illustrate how the solution changes. I find that either deficit or real government spending can substantially improve the equilibrium outcome.

3.3.1 Pushing on a string

I first consider optimal monetary policy assuming real spending, taxes and debt are held constant. That is, I assume that

$$F_t = \bar{F},\ T_t = \bar{F} = \bar{T} \text{ and } w_t = 0. \tag{32}$$

This assumption is convenient because it allows me to obtain a benchmark scenario in order to isolate the effect of deficit spending and real government spending by relaxing these constraints one at a time. Also, as I will show, under this assumption the result is remarkably simple and can be expressed analytically and thus give a clear intuition for the key problem that leads to excessive deflation. Note that equations (32) simply imposes additional conditions on the private sector equilibrium that the government faces. Thus I can substitute these conditions into the vector valued function (**??**) and then my definition of a Markov equilibrium is the same as in Definition 1 (even though in this case ξ_t is now the only relevant state variable since w_t is constant at zero).

To gain insights into the solution in an approximate equilibrium, it is useful to consider the linear approximation of the private sector equilibrium constraints. The AS equation is:

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \qquad \text{AS} \tag{33}$$

where $\kappa \equiv \theta \frac{(\sigma^{-1}+\lambda_2)}{d''}$. Here $\pi_t \equiv \Pi_t - 1$ is the inflation rate and is the output gap. The "Phillips curve" in (33) has become close to standard in the literature. In a linear approximation of the equilibrium the IS equation is given by:

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n) \qquad \text{IS} \tag{34}$$

where $\sigma \equiv -\frac{u_{cc}Y}{u_c}$ and

$$r_t^n = \frac{1-\beta}{\beta} + \frac{\sigma^{-1}\omega}{\sigma^{-1}+\omega}(g_t - E_t g_{t+1}) + \frac{\sigma^{-1}\omega F}{\sigma^{-1}+\omega}(F_t - E_t F_{t+1}) \tag{35}$$

is a linear approximation of the natural rate of interest, i.e. the real interest rate that is consistent with the natural rate of output. In this expression $\omega = \frac{v_y}{v_{yy}\bar{Y}}$ and $g_t \equiv -\frac{u_{c\xi}}{\bar{Y}u_{cc}}\xi_t$ is a linear combination of all the shocks in the model.

As in Eggertsson (2003) and Eggertsson and Woodford (2003) I limit my attention to stochastic shocks that make the natural rate of interest temporarily negative. I denote the part of the natural rate of interest that is exogenous in my model (i.e. the natural rate of interest if government spending are held constant) as r_t^{nF}. The following assumption allows for a simple characterization of the equilibrium when the zero bound is binding.

A5 $r_t^{nF} = r_L^n < i^m$ *at* $t = 0$ *and* $r_t^{nF} = r_{ss}^n = \frac{1}{\beta} - 1$ *at all* $0 < t < K$ *with probability* α *if* $r_{t-1}^{nF} = r_L^n$ *and probability 1 if* $r_{t-1}^n = r_{ss}^n$ *at all* $t > 0$. *There is an arbitrarily large number* K *so that* $r_t^n = r_{ss}^n$ *with probability 1 for all* $t \geq K$..

According to this assumption the natural rate of interest becomes temporarily negative in period 0 and reverts back to steady state with certain probability in the following periods. In the limit as $K \to \infty$ the natural rate reverts back with a fixed probability α in all remaining periods so that the expected duration of the shock is $\frac{1}{\alpha}$. As shown in

Eggertsson (2003) the first best allocation would be achieved if the government could set $i_t = r_t^n$ at all times. In this case the government can achieve $x_t = 0$ and $\pi_t = 0$ at all times. This maximizes the utility of the representative consumer because output is at the natural rate of output at all times and inflation is zero (and as shown by Eggertsson (2003) the utility of the representative household in this model can be approximated by quadratic deviation of each of the these variables from zero under A4). This solution however, cannot be attained if r_t^n is lower than 0, since this implies a nominal interest rate that violates the zero bound.

I now consider the solution under A5. Note first that for all $t \geq K$ then $\pi_t = x_t = 0$ (this is formally proofed in Eggertsson (2003)). This can be easily seen by noting that the objectives of the government, under the restriction imposed in (32), can be approximated by the quadratic objectives $-\pi_t^2 - \lambda_x x_t^2$ in each period. Thus once the natural rate of interest becomes positive (i.e. for all $t \geq K$) those objectives can be minimized in each period from then on by $\pi_t = x_t = 0$. Since the government is Markovian it will immediately achieve this equilibrium, even if the optimal commitment solution may involve a different outcome as I discuss further below. I first consider the most simple case when $K = 1$. In this case the first best allocation cannot be achieved in period zero and the zero bound will be binding. Since I know how the the solution looks like in period $t \geq 1$ I can write $E_0\pi_1 = E_0 x_1 = 0$ and then observe from (33) and (34) that since $i_0 = 0$ the solution the takes the form:[18]

$$x_0 = \sigma r_L^n < 0$$

$$\pi_0 = k\sigma r_L^n < 0$$

This solution illustrates that the presence of the zero bound creates deflation and output gap if the natural rate of interest is negative. What if the natural rate can be negative for more than one period? Consider first the case $K = 2$. In this case the natural rate of interest can either be r_L (with prob. $1 - \alpha$) or r_{ss} (with prob. α) in period 1. If $r_1^n = r_L$ the solution is the same as above in period 1. If $r_1^n = r_{ss}^n$ then $x_1 = \pi_1 = 0$. Then one observes from (34) that the solution in period 0 is:

$$x_0 = E_0 x_1 - \sigma(i_t - E_t \pi_{t+1} - r_t^n) = (1 - \alpha)\sigma r_L^n + \sigma\kappa(1 - \alpha)\sigma r_L^n + \sigma r_L^n < \sigma r_L^n < 0 \quad (36)$$

Note that this expression indicates that the output gap is larger if the private sector puts a positive probability of the zero bound to be binding for more than one period. This is due to the first two term in the right hand side of (36). The logic is simple: The expectation of lower output in period 1 (the first term) reduces demand by the permanent income hypothesis. The expectation of future deflation (the second term) increases the real rate of return thus depression demand. These two forces, that come about through expectation about future slump, have significant effect on demand in period 0 . One can similarly use (33) to solve for the deflation in period 0.

Equation (36) indicates that expectation about future slumps can make the current slump even worse. I can similarly solve for inflation and output by the same backward induction for the case when K is arbitrarily high. In the limit as $K \to \infty$ it is easy to show that the solution is:

$$x_t = \frac{1 - \beta(1 - \alpha)}{\alpha(1 - \beta(1 - \alpha)) - \sigma\kappa(1 - \alpha)} \sigma r_L^n \text{ if } r_t^n = r_L^n \text{ and } x_t = 0 \text{ otherwise}$$

$$\pi_t = \frac{1}{\alpha(1 - \beta(1 - \alpha)) - \sigma\kappa(1 - \alpha)} \kappa\sigma r_L^n \text{ if } r_t^n = r_L^n \text{ and } \pi_t = 0 \text{ otherwise}$$

Note that to ensure that the solution is bounded I need to assume that α satisfies the inequalities $\beta\alpha^2 + (1 + \sigma\kappa - \beta)\alpha - \sigma\kappa > 0$ and $0 < \alpha < 1$. If this condition is not satisfied the solution explodes and a linear approximation of the IS and the AS equation is not valid for shocks of any order of magnitude. Thus I would need to use other nonlinear solution methods to solve for the equilibrium if the value of α does not satisfy these bounds. Here I simply assume parameters so that these two inequalities are satisfied and a linear approximation of the IS and AS is feasible and the solution is accurate of order $o(||\xi, \delta||^2)$ (see Technical Appendix). This solution illustrates that the associated output gap and deflation can be substantial if the natural rate of interest is expected to stay negative for a long time. In particular, the higher probability of the natural rate of interest staying low for long, the more negative the output gap and the deflation. Thus even if the natural rate of interest is only modestly negative, the effect can be dramatic, if it is expected to stay there for an extended period. It follows that small shocks can have very bad consequences when the zero bound is binding and especially if one assumes, as I do in condition (32), that fiscal policy cannot be used to fight the problem.

Figure 1 shows the stage-contingent path of output gap and inflation for a numerical example. In the figure we assume that in period 0 that the natural rate of interest becomes -2 percent per annum and then reverts back to the steady-state value of $+4$ percent per annum with a probability 0.1 each quarter. Thus the natural rate of interest is expected to be negative for 10 quarters on average at the time the shock occurs. The numerical values assumed for this exercise are the same as in Eggertsson (2003) and Eggertsson and Woodford (2003) (see the Technical Appendix for the nonlinear solution method and the numerical values assumed). The first line shows the equilibrium if the natural rate of interest returns back to steady state in period 1, the next line if it returns in period 2, and so on. The inability of the central bank to set a negative nominal interest rate results in roughly 15 percent output gap and 10 percent deflation. Expectations of future slumps make the outcome much worse than in the case the trap lasts for only a single period. Since there is a 90 percent chance of the natural rate of interest remaining negative next quarter, expectations of future deflation and negative output gap create even further deflation.

Open market operations, i.e. printing money and buying government bonds, does nothing to increase either output or prices. As stressed by Eggertsson (2004), when the zero bound is binding the private sector will regard any increase in the money supply as temporary because the government has an incentive to contract the money supply to it's previous level once deflationary pressures have subsided. This can explain why BOJ has more than doubled the monetary base in recent years without any apparent effect on prices or inflation expectations. Note that if the government could commit to permanently increasing the money supply this would indeed increase inflation expectation and stimulate demand – which is optimal. As I have shown in this section, however, this commitment is not feasible in a Markov equilibrium under the constraints imposed in (32).[19]

The Power of Real Government Spending under Coordination In this section I explore the power of real government spending to close the output gap and curb deflation when monetary and fiscal policy are coordinated. To focus on the effects of real government spending I assume that the budget is balanced at all times so that $F_t = T_t$ and then relax this assumption in the next section. To be precise I assume

$$F_t = T_t \text{ and } w_t = 0 \tag{37}$$

If the zero bound is never binding, the governments maximization problem (30) implies a FOC condition that equates marginal utility of spending to its marginal cost

$$u_c(Y_t - d(\pi_t) - F_t, \xi_t) + g_G(F - s(F_t), \xi_t)s'(F_t) = g_G(F - s(F_t), \xi_t) \tag{38}$$

This condition says that the marginal utility of increasing government spending (the left hand side) should be equal to the marginal cost (the right hand side). Note that the marginal cost of increasing government spending is the sum of private consumption forgone by additional spending and the cost of taxation due to the higher tax rates. The first order condition (52) in the Technical Appendix indicates that the only reason the treasury may deviate from this rule is if the zero bound is binding. The zero bound gives the treasury a reason to use fiscal spending for stabilization purposes.

Variation in the optimal size of the government, i.e. the value of F_t, depends on how the marginal utility of private and public consumption shifts with the vector of shocks ξ_t. For simplicity I assume that these shocks shift $u_c(., \xi)$ and $g_G(., \xi)$ so that the optimal size of the government, in the absence of the zero bound, is constant over time so that there is a unique value $F_t = \bar{F}$ that solves (38). This assumption is useful for interpreting the results below because it implies that all variation in fiscal spending away from $\bar{F}$ are due to the zero bound.

To understand the importance of real spending when the zero bound is binding let us again do the simple experiment I conducted in the last section: Suppose the natural rate of interest is unexpectedly negative in period 0 and reverts back to the steady state with a fixed probability in every period. Figure 2 shows the same numerical experiment as in the last section, but now the treasury can increase fiscal spending to eliminate deflation. I use the approximation method shown in the Technical Appendix to solve the model numerically. Figure 2 indicates that the treasury increases government spending by 4 percent (as a fraction of GDP) when the zero bound is binding. This eliminates about 80 percent of the deflation and similarly substantially reduces the output gap. This large effect of small government spending may appear to resurrect a large "multiplier" of government real spending that was found in many old fashion Keynesian models. As I discuss below the term "multiplier", however, is quite misleading in a general equilibrium model of this sort.

The Keynesian Channel vs the RBC channel of government spending Through what channels does government spending increase output when the zero bound is binding? Government spending works through two separate channels. Real spending increases the natural level of output through the first. This channel has been extensively documented in the RBC literature (see e.g. Baxter and King (1993) and references there in). In the context of our model, just as in Baxter and King, the natural rate of output increases if government expenditures increase. A first order approximation of the natural rate of output (the output that would be produced if prices are flexible) yields:

$$Y_t^n = \frac{\sigma^{-1}}{\omega + \sigma^{-1}} g_t + \frac{\sigma^{-1}}{\omega + \sigma^{-1}} F_t \tag{39}$$

Thus the model predicts that an increase in fiscal spending increases the natural rate of output. This increase is due to an increase in the willingness of people to work. Higher government spending increases the marginal utility of consumption (for given level of employment) which in turn induces people to work more to equate the marginal utility of private consumption and the disutility of working.

Government spending influences output in the model thought another channel. I call this the *Keynesian channel* of government spending. The Keynesian channel only works if prices are sticky, i.e. if the real rate can be different from the natural rate of interest (which is the real interest rate if prices are perfectly flexible). To see the Keynesian channel note that by equation (72) an increase in government spending (holding everything else constant) increases the natural rate of interest. Then if the nominal interest rate is held fixed and expectations about future inflation are held constant, a wedge opens between the real interest rate and the natural rate of interest. By the IS equation (holding expectation about future output gap constant) a positive wedge between $r_t = i_t - E_t\pi_{t+1}$ and r_t^n stimulates demand. This is the Keynesian channel for government spending. In the next paragraph, I make this statement more precise in order to compare the effects of the two channels.

I now do the following thought experiment: Suppose the central bank in period t and successive government agencies follow optimal strategies. What is the marginal effect of the treasury increasing F_t above its steady state? I can calculate this marginal effect by substituting $x_t = \frac{Y_t - Y_t^n}{\bar{Y}}$ into to IS equation and taking a partial derivative with respect to F_t. This yields:

$$\frac{\partial Y_t}{\partial F_t} = \frac{\partial Y_t^n}{\partial F_t} - \sigma(\frac{\partial i_t}{\partial F_t} - \frac{\partial r_t^n}{\partial F_t}) \tag{40}$$

where the derivative with respect to π_{t+1} and x_{t+1} is zero because these variables are determined by successive government (since there is no state variable in the game under condition (37) it follows that $\frac{\partial x_{t+1}}{\partial F_t} = \frac{\partial \pi_{t+1}}{\partial F_t} = 0$). The first term of the derivative in (40) is $\frac{\partial Y_t^n}{\partial F_t} = \frac{\sigma^{-1}}{\omega+\sigma^{-1}}$. This is the RBC channel for fiscal policy. The second term of this derivative is $-\sigma(\frac{\partial i_t}{\partial F_t} - \frac{\partial r_t^n}{\partial F_t})$. This is the Keynesian channel of real government spending. Note that if the zero bound is not binding and the central bank maximized social welfare under condition (37) then $i_t = r_t^n$ at all times and this remains true regardless of the value of F_t. It follows that the Keynesian channel is zero in the absence of the zero bound: The central bank offsets any increase/decrease in the natural rate of interest. In contrast, if the natural rate of interest is negative and the zero bound is binding, it is easy to verify that $\frac{\partial i_t}{\partial F_t} = 0$. In this case (by equation (72)) the value of the second derivative is $-\sigma(\frac{\partial i_t}{\partial F_t} - \frac{\partial r_t^n}{\partial F_t}) = \frac{\omega}{\sigma^{-1}+\omega}$. [20] In sum, then, the marginal effect of increasing government spending on output is $\frac{\sigma^{-1}}{\omega+\sigma^{-1}} + \frac{\omega}{\sigma^{-1}+\omega} = 1$. This is exactly what Krugman (1998) notes. He argues that real government spending is not very effective in fighting deflation because the "multiplier" is small – only 1! Incidentally this number is also equal to the "balanced budget" multiplier in the old fashion IS-LM model.

The large effect of government real spending in general equilibrium One aspect of figure 2 that may be surprising is that only 4 percent of government spending in each period (when the zero bound is binding) eliminates about 80 percent of the output gap and the deflation. The may be particularly surprising given the small value of the partial derivative discussed in the last paragraph, i.e. the small "multiplier". This large effect of small government spending is due to the expectation channel. As I discuss in the last section, the main cause of the large decline in output and prices is the expectation of a future slump and deflation. Consider the outcome from the perspective of period 0. If the private sector expect even only a small increase in future government spending when the zero bound is binding, deflation expectation are changed in all periods when the zero bound is binding; thus having a large effect on spending in period 0. This illustrates that an analysis of partial derivatives – of the type I discussed in the last section – is very misleading to understand the general equilibrium effect of real government spending in a liquidity trap.

3.3.2 The Power of Deficit Spending under Coordination

In this subsection I explore the ability of deficit spending to close the output gap and curb deflation when the zero bound is binding. Deficit spending is the difference between real spending and current taxes i.e. $d_t = F_t - T_t$. To contrast the power of deficit spending to real government spending I assume that the latter is constant i.e.

$$F_t = \bar{F}$$

When government uses deficit spending, the value of the real debt becomes a state variable. This allows the government to change deflationary expectations into inflationary ones by increasing nominal debt. This is exactly what is needed when the zero bound is binding. To see this consider the IS equation. This equation illustrates that the output gap depends on an expected future path or real interest rate, i.e. $i_t - E_t\pi_{t+1}$. Even if demand cannot be increased by lowering the nominal interest rate, it can still be increased by raising inflation expectations. This is not possible if the only instrument of monetary policy is open market operations because even if the central bank has an incentive to promise future inflation when zero bound is binding, it has an incentive renege on this promise on once deflationary pressures have subsided (since there is cost of inflation in the model). Thus a discretionary central bank cannot increase inflation expectations when the zero bound is binding and the result is excessive deflation. This is what Eggertsson (2004) calls the deflation bias of discretionary policy. When monetary and fiscal policy are coordinated, however, the government can credibly commit to future inflation by increasing government debt. This is exactly why deficit spending is effective when the zero bound is binding, it increases inflation expectations.

The channel is simple. Budget deficits generate nominal debt. Nominal debt in turn makes a higher inflation target in the future credible because the real value of the debt increases if the government reneges on the target. Higher debt is undesirable for the government if there are some tax distortions. Higher inflation expectations lower the real rate of interest and thus stimulate aggregate demand. This channel can be critical when there are large deflationary shocks since under these circumstances monetary policy can be frustrated by the zero bound on the short term nominal interest rate.

Figure 3 and 4 shows the equilibrium when the central bank uses deficit spending (see Appendix A for the numerical values assumed). To solve the model I use the approximation method described in the Technical Appendix). As can be seen by this figure the ability of the government to use deficit spending to raise inflation expectations substantially improves the equilibrium. Deficit spending eliminates 93 to 99 percent of the deflation and 85 to 90 percent of the output gap. The price of this improvement during the trap, is an increase in inflation once out.

3.3.3 The relative importance of deficit and real government spending under coordination

How important is deficit spending versus real spending in equilibrium when the government has access to both instruments? Figures 5 and 6 compare the equilibrium under the two policies derived in the last two sections with the optimal policy if the government can use both deficit and real spending. These figures show the same numerical experiment as was done in past section but to reduce the number of lines shown in the graph I only report the path for each variable in the case the natural rate of interest returns back to steady state in period 0,4,7 and so on (thus not graphing the contingencies in between to avoid cluttering the pictures). As can be seen by the figure the government will use both real and government spending in a liquidity trap. Of the two instruments deficit spending is more effective, at least in terms of eliminating deflation and the output gap when the zero bound is binding. The figure indicates that if deficit spending is the only policy instrument, about 93-99 percent of the deflation is eliminated when the zero bound is binding compared to about 80 percent if the government can only use real spending. Similarly deficit spending eliminates about 85-90 percent of the output gap compared to 80 percent if the government can only use real government spending.

An even more instructive measure of the effectiveness of each policy instruments is the utility of the representative household under the different policy regimes. Table 1 lists the welfare under the three policy regimes described above and compares them with the optimal policy if the government could commit to future policy (the Ramsey/Commitment solution). The commitment equilibrium is the fully efficient allocation (it is solved in Eggertsson (2004)) and is thus the best the government could ever hope to achieve. The table expresses utility in terms of consumption equivalence units. This measure expresses the expected utility flow in units of a constant consumption endowment. The table shows that if the government coordinates monetary and fiscal policy and uses both real and deficit spending as policy instruments, the value of commitment is only 0.005 percent per year in terms of a constant consumption endowment stream. Note that deficit spending discretion (i.e. if the government is unable to commit but can use deficit spending as a commitment device) yields higher utility than if the government can use only real government spending discretion. This indicates that of these two instrument deficit spending is more important to improve economic welfare. It may be useful to transform the losses of these policies in net present value. The losses associated with full discretion (i.e. a coordination use of both real and deficit spending) relative to full commitment is then equal to 0.5 percent of the consumption endowment stream, i.e. the representative household would be ready to forgo 0.5 percent of it quarterly consumption under commitment to have the commitment solution rather than the full discretion one. A similar number for deficit spending discretion is 3.8 percent of quarterly consumption and for real government spending discretion it is 14.5 percent of quarterly consumption. The utility losses of the government being unable to use deficit spending are thus non-trivial. If the government cannot commit and is unable to use either real or deficit spending the utility losses are truly colossal or 13.5 percent of quarterly GDP than translates into a present value of 3 years of consumption.

Table 1

	Consumption Equivalence units per quarter
Commitment Equilibrium	100
Full Discretion	99.9950
Deficit Spending Discretion	99.9621
Real Spending Discretion	99.8544
Constrained Discretion	86.5082

One interesting aspect of deficit spending versus real spending that is worth noting (see figure 6) is the different time path of these policy variables. While the real spending solution involves a permanent increase in real spending during all periods in which the zero bound is binding, deficit spending is only temporarily high. Deficit spending is thus more consistent with the old Keynesian idea that a quick jolt of spending can "jump start" the economy. The reason is that it is the level of government debt that is the important state variable, because it increases inflation expectation. Only temporary deficit spending is needed to permanently increase government debt. In contrast, stimulating demand by real government spending requires a sustained increase in government spending in all periods in which the zero bound is binding.

4 The Markov Equilibrium when the Central Bank is Goal Independent

In the preceding section I assumed that monetary and fiscal policy are coordinated to maximize social welfare. This assumption may be questionable. In many countries the central bank has been assigned more narrow goals than social welfare. This institutional framework was made precise in A3, which is what I called a "goal independent" central bank. I now show how the results change if I assume that the central bank has goals as assumed in A3. The main conclusion is that the power of real spending is unchanged but that deficit spending has no effect.

4.1 Defining a Markov Equilibrium when the central bank is goal independent

The timing of events in the game is as follows: At the beginning of each period t, w_{t-1} is a predetermined state variable. At the beginning of the period, the vector of exogenous disturbances ξ_t is realized and observed by the private sector, the treasury and the central bank. The monetary and fiscal simultaneously choose policy at time t given the state. The private sector forms expectations e_t and I assume that the private sector may condition its expectation at time t on w_t, as in the previous section. The policy function of the treasury can then be written as:

$$Tr_t = \begin{bmatrix} F_t \\ T_t \\ w_t \end{bmatrix} = \begin{bmatrix} \bar{F}(w_{t-1}, \xi_t) \\ \bar{T}(w_{t-1}, \xi_t) \\ \bar{w}(w_{t-1}, \xi_t) \end{bmatrix} = \bar{T}r(w_{t-1}, \xi_t) \tag{41}$$

and the policy function of the central bank as:

$$m_t = \bar{m}(w_{t-1}, \xi_t) \tag{42}$$

This implies that in equilibrium I can once again write a function $\Lambda_t = \bar{\Lambda}(w_{t-1}, \xi_t)$ and define a function $\bar{e}(.)$ of the form (27). I define the value functions for the treasury, J^{Tr}, and the central bank,J^{Cb}, as:

$$J^{Tr}(w_{t-1}, \xi_t) \equiv E_t \left\{ \sum_{T=t}^{\infty} \beta^T [U(\bar{\Lambda}(w_{T-1}, \xi_T), \xi_T] \right\} \tag{43}$$

$$J^{Cb}(w_{t-1}, \xi_t) \equiv -E_t \left\{ \sum_{T=t}^{\infty} \beta^T [(\bar{\Pi}(w_{t-1}, \xi_t) - 1)^2 + \lambda(\frac{\bar{Y}(w_{t-1}, \xi_t)}{Y_t^n} - 1)^2] \right\} \tag{44}$$

Given $\bar{m}(.)$, w_{t-1} and ξ_t the treasury maximizes the utility of the representative household subject to the constraints in Proposition 1 summarized by (**??**) and (**??**). Thus its problem can be written as:

$$\max_{F_t, T_t, w_t} [U(\Lambda_t, \xi_t) + \beta E_t J^{Tr}(w_t, \xi_{t+1})] \tag{45}$$

s.t. (9), (18),(19), (21), (23), (24), (27) and (42).

Given $\bar{T}r(.)$, w_{t-1} and ξ_t the central bank maximizes its objective subject to the constraints in Proposition 1 summarized by (**??**) and (**??**). Thus its problem can be written as:

$$\max_{m_t} [-(\Pi_t - 1)^2 - \lambda(\frac{Y_t}{Y_t^n} - 1)^2 + \beta E_t J^{Tr}(w_t, \xi_{t+1})] \tag{46}$$

s.t. (9), (18),(19), (21), (23), (24), (27) and (41).

I can now define a Markov Equilibrium when the central bank is goal independent.

Definition 3 *A Markov Equilibrium when the central bank is goal independent is a collection of functions $\bar{\Lambda}(.), \bar{T}r(.), \bar{m}(.), J^{Tr}(.), J^{Cb}, \bar{e}(.)$, such that: (i) Treasury maximization. Given the functions $J^{Tr}(w_{t-1}, \xi_t)$, $\bar{e}(w_t, \xi_t)$ and $\bar{m}(.)$, the solution to the treasury optimization problem (45) is given by $Tr_t = Tr(w_{t-1}, \xi_t)$ for each possible state (w_{t-1},ξ_t). (ii) Central bank maximization. Given the functions $J^{Tr}(w_{t-1}, \xi_t)$ $\bar{e}(w_t, \xi_t)$ and $\bar{T}r(.)$, the solution to the central bank maker's optimization problem*

(30) is given by $m_t = \bar{m}(w_{t-1}, \xi_t)$ *for each possible state* $(w_{t-1,}\xi_t)$*. (iii)* $\bar{m}(.)$ *and* $\bar{T}r(.)$ *are as subset of the vector function* $\bar{\Lambda}(.)$ *and* $\bar{\Lambda}(.)$ *is PSE (iv) given the vector function* $\bar{\Lambda}(w_{t-1}, \xi_t)$ *then* $e_t = \bar{e}(w_t, \xi_t)$ *is formed under rational expectations. (v) given the vector function* $\bar{\Lambda}(w_{t-1}, \xi_t)$ *the functions* $J^{Tr}(w_{t-1}, \xi_t)$ *and* $J^{Cb}(w_{t-1}, \xi_t)$ *satisfy (43) and (44).*

4.2 Real government spending when the central bank is goal independent

I first consider the power of real government spending when the central bank is goal independent. In order to isolate the effect of real government spending I assume that the budget is balanced at all times so that $F_t = T_t$ and

$$w_t = 0 \tag{47}$$

How does the solution look like? It turns out that the solution – at least to first order – does not depend on whether the central bank is goal independent or not. To be more precise:

Proposition 3 *If equation (47) holds at all times then the solutions under A2 and A3 are identical up to an error of order* $o(||\xi||^2)$

Proof: See Technical Appendix.

Proposition 3 indicates that the power of real government spending is not affected by whether or not the central bank is goal independent. The intuition for the proposition is simple. Observe first that the solution when the natural rate of interest becomes positive is the same under either coordination or goal independence because the central bank will target zero inflation and zero output gap at that time (and the treasury will then set $F_t = F$). Consider now the solution when the zero bound is binding. Since monetary policy is constrained by the zero bound at this time, its different objective is irrelevant during this period as long as it implies a zero interest rate. The central bank interest rate policy, therefore, only matters in period $t \geq K$ and I have just argued that its policy will be the same in those periods as under coordination. Turning to the the treasury, according to A2 it is maximizing social welfare, and it follows that the path for government spending will be exactly the same as analyzed in section 3.3.1 during the trap. It follows that the solution is the same under coordination and goal independence if I assume (47).

4.3 Deficit spending and narrow central bank goals

I now turn to the case of deficit spending when the central bank is goal independent I assume that

$$F_t = F \tag{48}$$

to focus on the effect of deficit spending. In contrast to the last section, I find that there is now a dramatic difference in the effectiveness of deficit spending depending on whether the central bank is goal independent. If the central bank is goal independent, as defined in A3, deficit spending has no effect. To see this it is useful to start with a simple proposition. The next proposition illustrates that under goal independence the choice of $\{d_t\}_{t=0}^{\infty}$ places no restriction on the choice set of the central bank.

Proposition 4 *If (48) then the set of variables* $(\Pi_t, Y_t, w_t, m_t, i_t, T_t)$ *that is consistent with the existence of a PSE is independent of the specification of* $\{d_t\}_{t=0}^{\infty}$

The reason for this is very simple. For a given path of F_t Ricardian equivalence holds in the model so that debt does not enter into any of the equilibrium conditions of the private sector as can be seen by equation (21)-(9) and (13). Deficit policy, therefore, places no restrictions on the permissible paths for inflation ,output, interest rates and the other variables specified in the proposition. Under A3 monetary policy is set to minimize $(\Pi_t - 1)^2 + \lambda_x x_t^2$. Government debt or deficits do not enter this objective or the private sector equilibrium constraints as I observed in the last proposition. It follows that if I set $\{F_t\}_{t=0}^{\infty}$ to be exogenously given, deficit spending has no effect on the equilibrium outcome when the central bank is goal independent. The central bank will determine inflation and the output gap without any reference to deficits or debt.[21] The next proposition, then, follows directly from the last proposition and the assumed objective of the central bank in A3.

Proposition 5 *If equation (48) holds at all times deficit spending has no effect on inflation or the output gap when the central bank is goal independent.*

The effect of fiscal policy when coordinated with monetary policy is thus fundamentally different from its effects if the central bank is goal independent. This can be of potential importance in practice. Thus Krugman (2001) raises the question of why deficit spending in Japan has failed to lift Japan out of its current depression while some economists believe that deficit spending helped Japan avoiding the Great Depression and that the WWII deficit spending jolted the US economy out of the Great Depression. One critical difference between deficit spending of that period and now is that the Bank of Japan is independent today unlike during the Great Depression (and in the US the FED and the Treasury cooperated by establishing an interest rate peg in the 40's). This paper thus points towards an important channel of fiscal and monetary policy that may have been at work in Japan in the Great Depression and the US in WWII but is not present in Japan today. When monetary and fiscal policies are coordinated, deficit spending increases inflation expectations, which in turn lowers the real rate of return and stimulates aggregate demand.

5 Coordination in the Great Depression in Japan and the US

Is it straight forward to change expectation by changing the overall goals of a central bank and increasing deficit spending? Are such regime shifts credible? From a theoretical standpoint the answer to this question is unambiguous. Since the cooperation between the treasury and the central bank, as I define it, involves a maximization of social welfare, it is always credible. The main challenge then, is not really whether or not such policy is credible, but how to make it visible and verifiable by the private sector. One way of doing this is for the central bank to announce its intention to support fiscal policy and then buy government bonds. In principle, such policy should have no effect, because money and bonds are perfect substitutes. But if such operations are accompanied by explicit announcements that the bank is attempting to support fiscal policy, for example by announcing that the debt bought by the central bank would not be collected from the treasury when due, this could have large effect on inflation expectations. The effect follows, not from the purchases themselves, but from the way in which they are interpreted. Thus open market operations can be used to *signal* a change in the central bank's objective and a determination to support fiscal policy to end deflation. A key element of such policy, therefore, is for the bank to be transparent about it's policy objectives and how it want to move expectations.

Have regime changes and coordination been effective in the past to curb deflation? There is an interesting historical precedent from Japan for a cooperative solution. During the late 1920's Japan was slipping into a depression. Growth had slowed down considerably, GNP rose by only 0.5 percent in 1929, 1.1 in 1930 and 0.4 percent in 1931. At the same time deflation was crippling the economy. This was registered by several macroeconomic indicators as is illustrated in Table (2). In December 1931 Korekiyo Takahasi was appointed the Finance Minister of Japan. Takahasi took three immediate actions. First, he abolished the gold standard. Secondly, he subordinated monetary policy to fiscal policy by having the BOJ underwrite government bonds. Third, he ran large budget deficits. These actions had dramatic effects as can be seen in Table 3. All the macroeconomic indicators changed in the direction predicted by our model. As the budget deficit increased, GNP rose and deflation was halted. During the same period, interest rates were at a historical low and rates on government bonds were close to zero during the 30's. In addition to the nominal interest rate cuts our model indicates that the other actions taken, i.e. aggressive deficit spending that was financed by underwriting of government bounds, could have had considerable effects on the real rate of return through increasing *expected inflation*. This channel can be of potential importance in explaining the success of these policy measures in Japan in the Great Depression. In 1936 Takahasi was assassinated and the government finances subjugated to military objectives. The following military expansion eventually led to excessive government debt and hyperinflation. Until Takahasi was assassinated, however, the economic policies in Japan during the 1930's were remarkably successful.

The result in Japan stands in sharp contrast with the experience in the US during the same period. Although the nominal interest rate (measured in terms of yields on short term government bonds that should thus have a small risk premium and only 3 month maturity) in the US went down close to zero during the 30's, this failed to generate a sustained increase in output and inflation. It was not until 1942 that the Treasury and the FED implemented a similar arrangement of "cooperation" as in Japan. In 1942 an "interest rate peg" was established. The FED guaranteed a yield on Treasury bills of 0.33%. What followed was massive deficit spending due to WWII. The "cooperation" between the FED and the Treasury was not established as a response to deflationary pressures, as it was in Japan in the 30's. Rather it was a response to the financial needs of the Treasury during the war. The results, however, were similar to those seen in Japan a decade earlier. During the 40's, there was as sustained increase in output and inflation. Needless to say the reasons for the US recovery are too complicated to be captured by our simple model. It is possible, however, that some part of the explanation lies in the decrease in the real rate of interest that resulted from higher inflation expectations. Our model indicates that the cooperation between the FED and the Treasury in 1942 could have rationalized a substantial increase in inflation expectations.

	Change in GNP deflator	Change in CPI	Change in WPI	Change in GNP	Government surplus over GNP
1929	-	-2.3%	-2.8%	0.5%	-1.0%
1930	-	-10.2%	-17.7%	1.1%	2.0%
1931	-12.6%	-11.5%	-15.5%	0.4%	0.4%
1932	3.3%	1.1%	11.0%	4.4%	-3.5%
1933	5.4%	3.1%	14.6%	10.1%	-3.0%
1934	-1.0%	1.4%	2.0%	8.7%	-3.5%
1935	4.1%	2.5%	2.5%	5.4%	-3.3%
1936	3.0%	2.3%	4.2%	2.2%	-2.0%

Table 2: Coordination of Fiscal and Monetary Policy in the Great Depression in Japan.

A topic for further research that carries considerable promise is to study the relative importance of deficit vs real spending in periods in which the government has aggressively increases both. The US during WWII and Japan in the early 1930's are two obvious examples that come to mind. Japan recent experience is another case worth studying in a calibrated model. As I have argued above I doubt that deficit spending has done much do increase inflation expectation in Japan in recent years, given the ongoing deflation and continuing deflationary expectation (that most surveys indicate still remain subdued). But it may well be that increases in real government purchases have been effective in preventing the Japanese slowdown from being even worse. The model I presented showed that in the absence of any increases in real government spending the resulting deflation and output slump would have been even worse than what has been observed in Japan in recent years. The model indicates that the active increases in real government spending that have been observed in Japan in recent years (in a response to the slump) may have played an important role in preventing an even more acute slump (although it is an open question if more should have been done on that front). It should be noted, however, that there is no agreement on how aggressive the Japanese government has been in using real government spending to increase demand. Kuttner and Posen (2001), for example, argue that cyclically adjusted real government spending increases have been modest at best. In addition they have not been implemented on a sustained basis as would be required by the Markov solution shown here (i.e. real government spending should be increased in all states of the world in which the zero bound is binding). This is important, because our model predicts that it is not the current increase in real government spending that is of principal importance, but the expectation that it will also be increased in all future states of the world in which the zero bound is binding. Thus the government needs to announce that it will increase real government spending *until deflationary pressures have subsided* and this is a credible announcement as shown by the analysis of a Markov equilibrium.

6 Conclusion

Inflation has been considered the main threat to monetary stability for several decades. In the aftermath of the double digit inflation of the 70's, there was a movement to separate monetary policy from fiscal policy and vest it in the hands of "independent" central bankers whose primary responsibility was to prevent inflation. This development was reinforced by important contributions on the theoretical level, most notably by Kydland/Prescott (1977) and Barro/Gordon's (1983) illustration of the "inflation bias" of a discretionary government. It is easy to forget that in the aftermath of the Great Depression, when deflation was the norm, the discussion at the political and theoretical level was quite the opposite. Paul Samuelson claimed that the Federal Reserve was "the prisoner of its own independence" during the Great Depression, exaggerating the slump by its inability to fight deflation.[22] Similarly Milton Friedman claimed that "monetary policy is much too serious a matter to be left to the central bankers".[23] This paper shows that in a deflationary situation there may be some benefit to fiscal and monetary coordination. The exact nature of this coordination is certainly an interesting topic of further research. It is worth pointing out that this paper solution suggests that it may only need to be temporary to be effective, as the solution illustrated under coordination converges to the same that would result in the absence of coordination.

One may argue that the central bank could, without any coordination with the treasury, engage in various activities to stimulate the prices and output, such as purchasing foreign exchange or private assets. An independent central bank may use its own balance sheet to achieve a similar commitment to higher future prices as was illustrated for deficit spending under coordination in this paper (i.e. it can increase inflation expectation by open market operations in private assets or foreign exchange). The idea is that an independent central bank is typically very concerned about the value of its balance sheet since it would need to finance any capital losses by either printing money (which may lead inflation than higher than is optimal) or a bailout from the treasury (that may lead to loss of independence).The snag is, however, that if the bank is too concerned about its own balance sheet it may find itself as "the prisoner of its own independence" that prevents it from taking these actions, even if they in principle allow it to commit it to future inflation, much as suggested by Paul Samuelson. The reason is that any asset bought in a non-standard open market operations has uncertain returns, and there are always some states of the world in which the central bank may need to trade-off excessive balance sheet losses to excessive inflation. Thus even if one considers additional policy instruments there may still be an persuasive case for temporary coordination of monetary and fiscal policy.

Notes

[1]The paragraph in the quotation mark is a summary of Krugman's (2001) account of the conventional wisdom.

[2]See e.g. Eggertsson and Woodford (2003) for discussion of alternative policy options. These authors argue that the these policy options are mainly effective if they change expectations.

[3]Although one may argue that it is the fiscal effect of these operations that make them effective, see Eggertsson (2004) for details.

[4]We do not mean to claim that government agencies cannot make any binding commitments under any circumstances. But the assumption about imperfect commitment is particularly appealing when the zero bound is binding. As emphasized by Krugman (1998) (and shown in Eggertsson and Woodford (2003) in fully stochastic dynamic general equilibrium model) when the zero bound is binding, the optimal commitment by the government is to increase inflation expectations. This type of commitment, however, may be unusually hard to achieve in a deflationary environment. One reason is that it requires no actions. Since the short-term nominal interest rate is already at zero the central bank cannot use it's standard policy tool to make this commitment visible to the private sector. The second is that most central banks have required reputation for fighting inflation. Announcing a positive inflation target without direct actions to achieve it, therefore, may not be very effective to change expectations.

[5]Note that this implies that the approach here is not subject to the criticism many have raised against the FTPL such as for example Niepelt (2004).

[6]The idea is that real money balances enter the utility because they facilitate transactions. At some finite level of real money balances, e.g. when the representative household holds enough cash to pay for all consumption purchases in that period, holding more real money balances will not facilitate transaction any further and thereby add nothing to utility. This is at the "satiation" point of real money balances. We assume that there is no storage cost of holding money so increasing money holding can never reduce utility directly through $u(.)$. A satiation level in real money balances is also implied by several cash-in-advance models such as Lucas and Stokey (1987) or Woodford (1998).

[7]Assumption A1 is the Markov property. Since ξ_t is a vector of shocks this assumption is not very restrictive since I can always augment this vector by lagged values of a particular shock.

[8]I assume that $d'(\Pi) > 0$ if $\Pi > 1$ and $d'(\Pi) < 0$ if $\Pi < 1$. Thus both inflation and deflation are costly.

$d(1) = 0$ so that the optimal inflation rate is zero (consistent with the interepretation that this represent a cost of changing prices). Finally, $d'(1) = 0$ so that in the neighborhood of the zero inflation the cost of price changes is of second order.

[9]The reason I do not assume Calvo prices is that it complicates to solution by introducing an additional state variable, i.e. price dispersion. This state variable, however, has only second order effects local to the steady state I approximate around and the resulting equilibrium is to first order exactly the same as derived here.

[10]I introduce it so that I can calibrate an inflationary bias that is independent of the other structural parameters, and this allows me to define a steady state at the fully efficient equilibrium allocation. I abstract from any tax costs that the financing of this subsidy may create.

[11]The intuition for this bound is simple. There is no storage cost of holding money in the model and money can be held as an asset. It follows that i_t cannot be a negative number. No one would lend 100 dollars if he or she would get less than 100 dollars in return.

[12]For a detailed discussion of how this transversality condition is derived see Woodford (2003).

[13]The function $s(T)$ is assumed to be differentiable with $s'(T) > 0$ and $s''(T) > 0$ for $T > 0$.

[14]The specification used here, however, gives very clear result that clarifies the main channel of taxations that I am interested in. This is because for a constant F_t the level of taxes has no effect on the private sector equilibrium conditions (see equations above) but will only affect the equilibrium by reducing the utility of the households because a higher tax costs mean lower government consumption G_t. This allows me to isolate the effect current tax cuts will have on expectation about future monetary and fiscal policy, abstracting away from any effect on relative prices that those tax cuts may have. It is thus they key behind the proposition that deficit spending has no effect when the central bank is "goal independent." There is no doubt the effect of tax policies on relative prices is important, but that issue is quite separate from the main focus of this paper. There is work in progress by Eggertsson and Woodford that considers how taxes that change relative prices can be used to affect the equilibrium allocations.

[15]Since this constraint will never be binding in equilibrium and $\bar{w}$ can be any arbitrarily high number for the results to be obtained I do not model in detail the endogenous value of the debt limit.

[16]There are two key differences between this analysis and Dixit and Lambertini (2003). First, in their

model fiscal policy is a choice is a choice of the optimal subsidy/tax on the private sector thus changing the equilibrium markup of firms. Here I abstract from any effect fiscal policy can have on relative prices and instead focus on deficit spending and real spending as the principal tools of policy (and these policy instruments have no effect on the markup of firms). Second, and perhaps more obviously, their paper does not address the question posed by the zero bound.

[17] Available at www.princeton.edu/~egertsson.

[18] One can see that i_0 must be equal to zero by the first order condition conditions (**??**). See Eggertsson (2003) for details.

[19] This explains an important difference between my result and the one obtained by Auerbach and Obstfeld (2003) who argue that open market operations are effective. They assume that open market operations automatically increase expectations about future money supply. In a Markov equilibrium, however, expectations about future money supply are unaffected by open market operations.

[20] It may be surprising that the value of this derivative due to the Keynesian channel of real government spending, $\frac{\sigma^{-1}}{\sigma^{-1}+\omega}$, does not rely on the degree of price stickiness. After all, the ability of the government to set the real rate of interest above/below the natural rate of interest depends on prices being sticky. The reason for this is that output is completely determined by the IS equation when the zero bound is binding. In this equation expectation are fixes by expectation about the actions of future government. The IS equation does not in any way depend on price stickiness and the same applies therefore this derivative. The price adjustment that must take place to accommodate the change in government spending when the zero bound is binding, however, is highly dependent on the stickiness of prices. This can be seen by the linear approximation of the AS equation. Since the output gap is determined by the IS equation when the zero bound is binding (and expectations are fixed), we can see from this equation that the level of inflation/deflation depends on the value of κ. This coefficient depends on d_J which reflects the cost of adjusting prices.

[21] Note that if the treasury chooses F_t in each period, deficit spending can in principle have effect by influencing the expectations about future spending F_{t+j}.It can be verified, however, that in this model this effect is only of second order.

[22] See Mayer, Thomas (1990) p. 6.

[23]Although he suggested rules to solve the problem rather than coordinated discretion as I do here.

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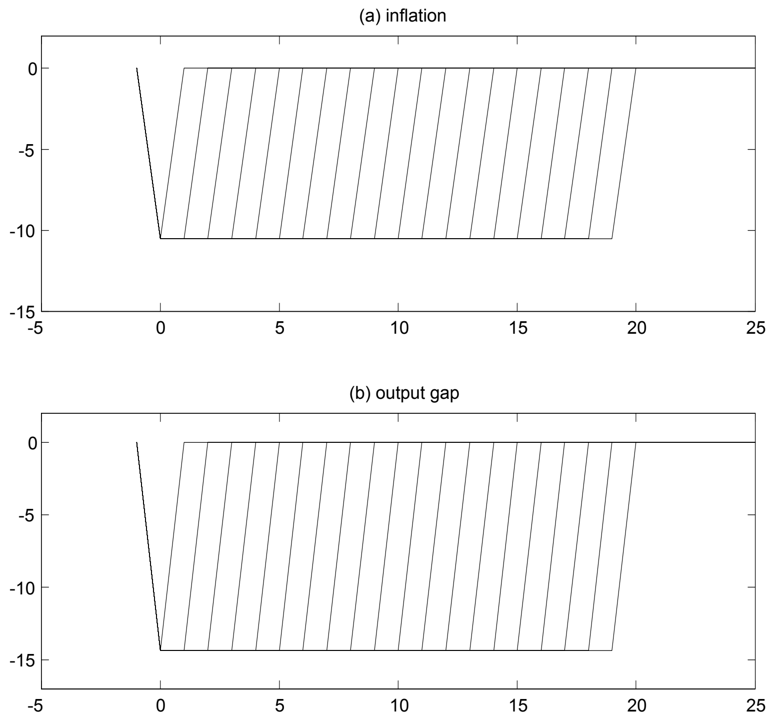

Figure 1: A Markov equilibrium in the absence of active fiscal policy.

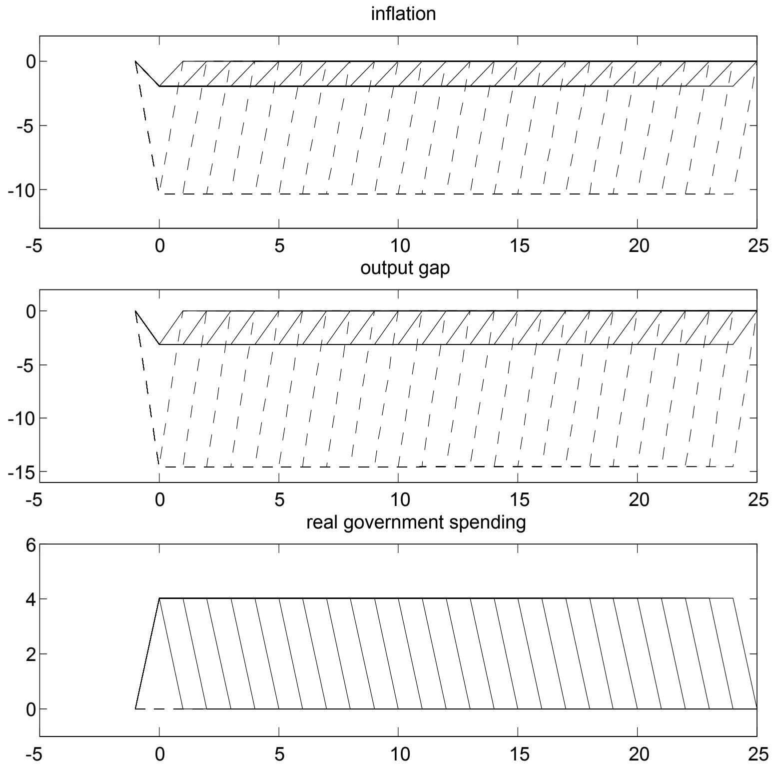

Figure 2: A Markov equilibrium when the government uses discretionary real spending.

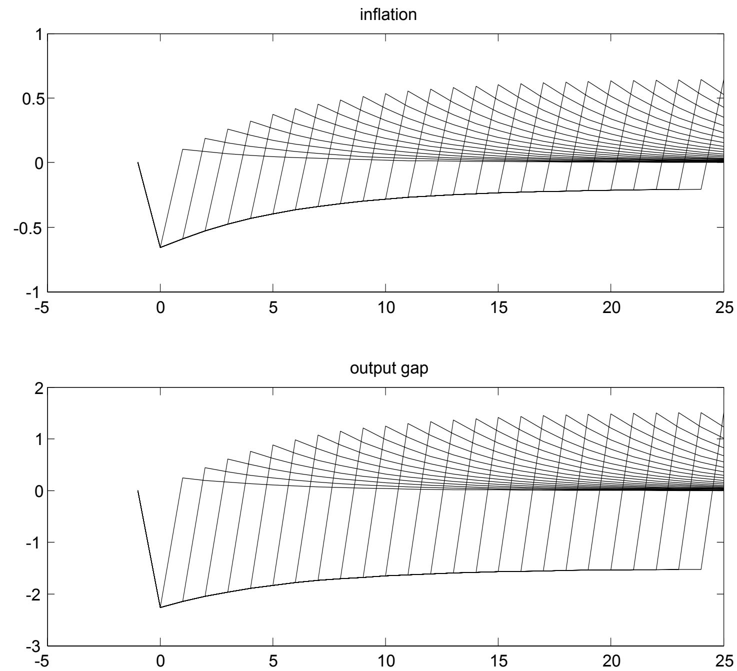

Figure 3: A Markov equilibrium for inflation and the output gap when the government uses discretionary deficit spending.

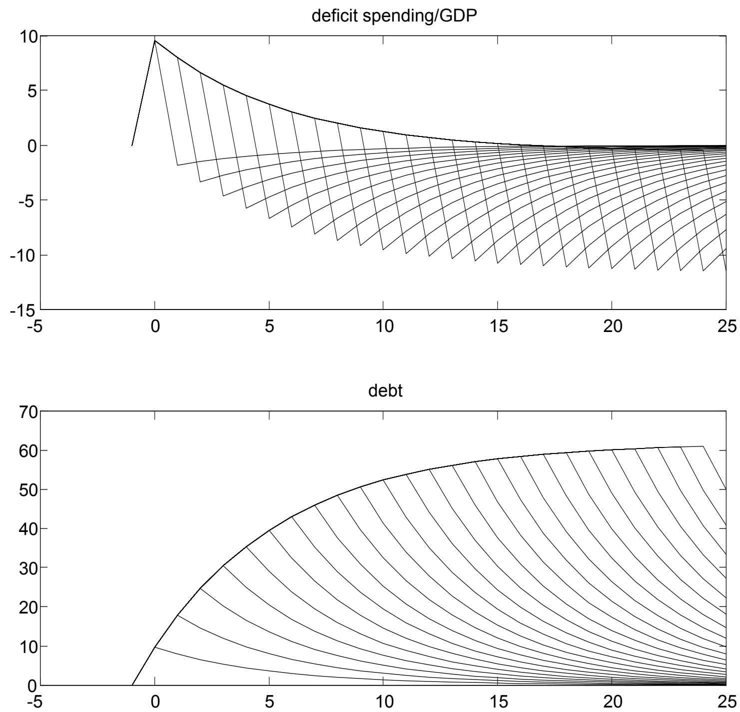

Figure 4: A Markov equilibrium for deficit spending and nominal debt when the government uses discretionary deficit spending.

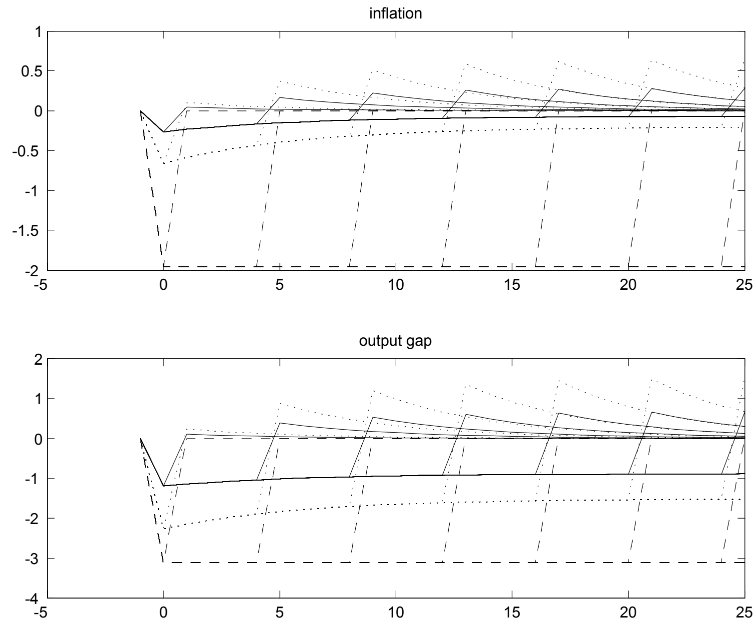

Figure 5: Comparison of inflation and the output gap in Markov equilibrium when the government only utilizes discretionary real spending (dotted line), only uses discretionary deficit spending (dashed line) and when it takes advantage of both (solid line).

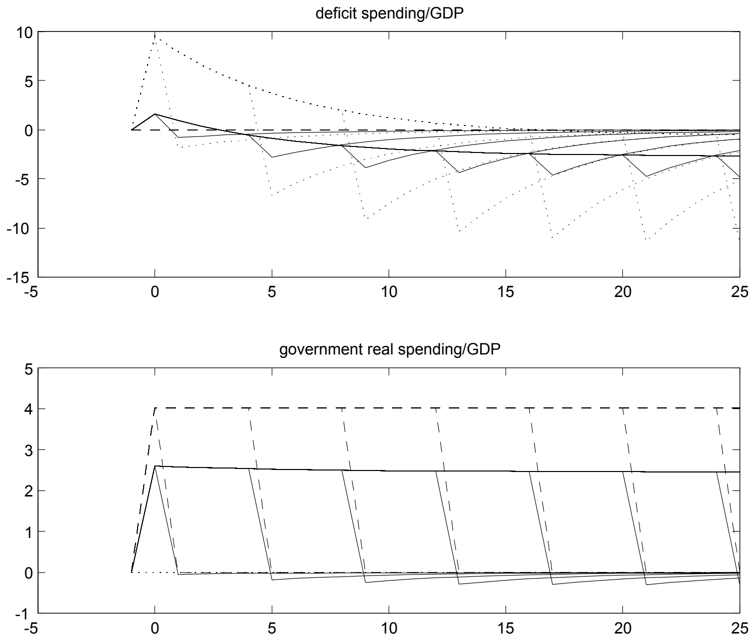

Figure 6: Comparison of deficit spending and debt in Markov equilibrium when the government only utilizes discretionary real spending (dotted line), only uses discretionary deficit spending (dashed line) and when it takes advantage of both (solid line).

A Technical Appendix

This Technical Appendix details the numerical solution methods used and some further details for the proofs, for readers interested in the technical details. Some of this material is also contained in the Technical Appendix of a companion paper Eggertsson (2004) and the computation method shown in section (C.6) is also applied in Eggertsson and Woodford (2003) with appropriate modifications.

B Explicit first order conditions

This section shows the first order conditions of the government maximization problem.

The period Lagrangian is:

$$L_t = u(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)) + g(F_t - s(T_t), \xi_t) - \tilde{v}(Y_t) + E_t\beta J(w_t, \xi_{t+1})$$

$$+ \phi_{1t}(\frac{u_m(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)\Pi_t^{-1}}{u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)} - \frac{i_t - i^m}{1 + i_t})$$

$$+ \phi_{2t}(w_t - (1 + i_t)\Pi_t^{-1}w_{t-1} - (1 + i_t)F_t + (1 + i_t)T_t + (i_t - i^m)m_t\Pi_t^{-1}) +$$

$$+ \phi_{3t}(\beta f_t^e - \frac{u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)}{1 + i_t})$$

$$+ \phi_{4t}(\theta Y_t[\frac{\theta - 1}{\theta}(1 + s)u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)]$$

$$+ u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)\Pi_t d'(\Pi_t) - \beta S_t^e)$$

$$+ \psi_{1t}(f_t^e - \bar{f}^e(w_t, \xi_t)) + \psi_{2t}(S_t^e - \bar{S}^e(w_t, \xi_t)) + \gamma_{1t}(i_t - i^m) + \gamma_{2t}(\bar{w} - w_t)$$

FOC (all the derivative should be equated to zero)

$$\frac{\delta L_t}{\delta \Pi_t} = -u_c d'(\Pi_t) - u_m m_t \Pi_t^{-2} \tag{49}$$

$$+\phi_{1t}[-\frac{u_{mc} d' \Pi_t^{-1}}{u_c} - \frac{u_{mm} m_t \Pi_t^{-2}}{u_c} - \frac{u_m \Pi_t^{-2}}{u_c} + \frac{u_m u_{cc} d' \Pi_t^{-1}}{u_c^2} + \frac{u_m u_{cm} m_t \Pi_t^{-2}}{u_c}] \tag{50}$$

$$+[\phi_{2t}(1+i_t) w_{t-1} \Pi_t^{-2} - (i_t - i^m) m_t \Pi_t^{-2}] + \phi_{3t}[\frac{u_{cc} d'}{1+i_t} + \frac{u_{cm} m_t \Pi_t^{-2}}{(1+i_t)}]$$

$$+\phi_{4t}[-Y_t(\theta-1)(1+s)(u_{cc} d' + m_t \Pi_t^{-2} u_{cm}) - u_{cc} \Pi_t d'^2 - u_{cm} m_t \Pi_t^{-1} d' + u_c \Pi_t d'' + u_c d']$$

$$\frac{\delta L_t}{\delta Y_t} = u_c - \tilde{v}_y + \phi_{1t}[\frac{u_{mc}}{u_c} - \frac{u_m}{u_c^2}]\Pi_t^{-1} - \phi_{3t}\frac{u_{cc}}{1+i_t} + \phi_{4t}[\theta(\frac{\theta-1}{\theta}(1+s)u_c - \tilde{v}_y) + \theta Y_t(\frac{\theta-1}{\theta}(1+s)u_{cc} - \tilde{v}_{yy}) + u_{cc}\Pi_t d'] \tag{51}$$

$$\frac{\delta L_t}{\delta F_t} = -u_c + g_G + \phi_{1t}[-\frac{u_{mc}}{u_c} + \frac{u_m}{u_c^2}]\Pi_t^{-1} + \phi_{3t}\frac{u_{cc}}{1+i_t} - \phi_{4t}[\theta(\frac{\theta-1}{\theta}(1+s)u_c + \theta Y_t(\frac{\theta-1}{\theta}(1+s)u_{cc} + u_{cc}\Pi_t d'] \tag{52}$$

$$\frac{\delta L_t}{\delta i_t} = -\phi_{1t}\frac{1+i^m}{(1+i_t)^2} + \phi_{2t}(m_t \Pi_t^{-1} + T_t - w_{t-1}\Pi_t^{-1} - F) + \phi_{3t}\frac{u_c}{(1+i_t)^2} + \gamma_{1t} \tag{53}$$

$$\frac{\delta L_t}{\delta m_t} = u_m\Pi_t^{-1} + \phi_{1t}[\frac{u_{mm}}{u_c} - \frac{u_m}{u_c^2}u_{cm}\Pi_t^{-1}]\Pi_t^{-1} + \phi_{2t}(i_t - i^m)\Pi_t^{-1} - \phi_{3t}\frac{u_{cm}}{1+i_t}\Pi_t^{-1} - \phi_{4t}[Y_t(\theta-1)(1+s)u_{cm}\Pi_t^{-1} - u_{cm}d'] \tag{54}$$

$$\frac{\delta L_t}{\delta T_t} = -g_G s'(T_t) + \phi_{2t}(1+i_t) \tag{55}$$

$$\frac{\delta L_t}{\delta w_t} = \beta E_t J_w(w_t, \xi_{t+1}) - \psi_{1t} f_w^e - \psi_{2t} S_w^e + \phi_{2t} - \gamma_{2t} \tag{56}$$

$$\frac{\delta L_t}{\delta f_t^e} = \beta\phi_{3t} + \psi_{1t} \tag{57}$$

$$\frac{\delta L_t}{\delta S_t^e} = -\beta\phi_{4t} + \psi_{2t} \tag{58}$$

The complementary slackness conditions are:

$$\gamma_{1t} \geq 0,\ i_t \geq i^m,\quad \gamma_{1t}(i_t - i^m) = 0 \tag{59}$$

$$\gamma_{2t} \geq 0,\quad \bar{w} - w_t \geq 0,\quad \gamma_{2t}(\ \bar{w} - w_t) = 0 \tag{60}$$

The optimal plan under discretion also satisfies an envelope condition:

$$J_w(w_{t-1}, \xi_t) = -\phi_{2t}(1+i_t)\Pi_t^{-1} \tag{61}$$

Necessary and sufficient condition for a Markov equilibrium thus are given by the first order conditions (49) to (61) along with the constraints (8), (18), (21), (23) and the definitions (20) and (22). Note that the first order conditions imply restrictions on the unknown vector function Λ_t and the expectation functions.

C Approximation Method

This section show the approximation method used to approximate the Markov equilibrium.

C.1 Equilibrium in the absence of seigniorage revenues

As discussed in the text it simplifies the discussion to assume that the equilibrium base money small, i.e. that m_t is a small number (see Woodford (2003), chapter 2, for a detailed treatment). This simplifies the algebra and my presentation of the results. I discuss in the footnote some reasons for why I conjecture that this abstraction has no significant effect.[24]

To analyze an equilibrium with a small monetary base I parameterize the utility function by the parameter $\bar{m}$ and assume that the preferences are of the form:

$$u(C_t, m_t\Pi_t^{-1}, \xi_t) = \tilde{u}(C_t, \xi_t) + \chi(\frac{m_t}{\bar{m}}\Pi_t^{-1}C_t^{-1}, \xi_t) \tag{62}$$

As the parameter $\bar{m}$ approaches zero the equilibrium value of m_t approaches zero as well. At the same time it is possible for the value of u_m to be a nontrivial positive number, so that money demand is well defined and the government's control over the short-term nominal interest rate is still well defined (see discussion in the proofs of Propositions **??** and 6 in the Appendix). I can define $\tilde{m}_t = \frac{m_t}{\bar{m}}$ as the policy instrument of the government, and this quantity can be positive even as $\bar{m}$ and m_t approach zero. Note that even as the real monetary base approaches the cashless limit the *growth rate* of the nominal stock of money associated with different equilibria is still well defined. I can then still discuss the implied path of money supply for different policy options. To see this note that

$$\frac{\tilde{m}_t}{\tilde{m}_{t-1}} = \frac{\frac{M_t}{P_{t-1}\bar{m}}}{\frac{M_{t-1}}{P_{t-2}\bar{m}}} = \frac{M_t}{M_{t-1}}\Pi_{t-1}^{-1} \tag{63}$$

which is independent of the size of $\bar{m}$. For a given equilibrium path of inflation and $\tilde{m}_t$ I can infer the growth rate of the nominal stock of money that is required to implement this equilibrium by the money demand equation. By assuming $\bar{m} \to 0$ I only abstract from the effect this adjustment has on the marginal utility of consumption and seigniorage revenues, both of which would be trivial in a realistic calibration (see footnote 24).

C.2 Steady state discussion and relation to literature on Markov Equilibrium

In general a steady-state of a Markov equilibrium is non-trivial to compute, as emphasized by Klein et al (2003). This is because each of the steady state variables depend on the mapping between the endogenous state (i.e. debt) and the unknown functions $J(.)$ and $\bar{e}(.)$, so that one needs to know the derivative of these functions with respect to the endogenous policy state variable to calculate the steady state. Klein et al suggest an approximation method by which one may approximate this steady state numerically by using perturbation methods. In this paper I take a different approach. Proposition (6) shows that a steady state may be calculated under assumptions that are fairly common in the monetary literature, without any further assumptions about the unknown functions $J(.)$ and $e(.)$.

Proposition 6 *If* $\xi = 0$ *at all times and A2(i)-(iii) hold there is a Markov equilibrium steady state that is given by* $i = 1/\beta - 1$, $w = S^e = \phi_1 = \phi_3 = \phi_4 = \psi_1 = \psi_2 = \gamma_1 = \gamma_2 = 0$, $\Pi = 1$, $\phi_2 = g_G(\bar{F} - s(\bar{F}))s'(\bar{F})$, $f^e = u_c(\bar{Y}), F = \bar{F} = G = T + s(T)$ *and* $Y = \bar{Y}$ *where* $\bar{Y}$ *is the unique solution to the equation* $u_c(Y - F) = v_y(Y)$*.*

To proof the proposition about the steady state I look at the algebraic expressions of the first order conditions of the government maximization problem. The proof is in section (D). A noteworthy feature of the proof is that the mapping between the endogenous state and the functions $J(.)$ and $e(.)$ does not matter (i.e. the derivatives of these functions cancel out). The reason is that the Lagrangian multipliers associated with the expectation functions are zero in steady state and I may use the envelope condition to substitute for the derivative of the value function. The intuition for why these Lagrangian multipliers are zero in equilibrium is simple. At the steady state the distortions associated with monopolistic competition are zero (because of A2 (ii)). This implies that there is no gain of increasing output from steady state. In the steady the real debt is zero and according to assumption (i) seigniorage revenues are zero as well. This implies that even if there is cost of taxation in the steady state, increasing inflation does not reduce taxes. It follows that all the Lagrangian multipliers are zero in the steady state apart from the one on the government budget constraint. That multiplier, i.e. ϕ_2, is positive because there are steady state tax costs. Hence it would be beneficial (in terms of utility) to relax this constraint.

There is by now a rich literature studying the question whether there can be multiple Markov equilibria in monetary models that are similar in many respects to the one I have described here (see e.g. Albanesi et al (2003), Dedola (2002) and King and Wolman (2003)). I do not proof the global uniqueness of the steady state in Proposition 6 but show that it is locally unique.[25] I conjecture, however, that the steady state is globally unique under A2.[26] But even if I would have written the model so that it had more than one steady state, the one studied here would still be the one of principal interest as discussed in the footnote.[27]

C.3 Approximate system and order of accuracy

The conditions that characterize equilibrium are given by the constraints of the model and the first order conditions of the governments problem. A linearization of this system is complicated by the Kuhn-Tucker inequalities (**??**) and (**??**). I look for a solution in which the bound on government debt is never binding, and then verify that this bound is never binding in the equilibrium I calculate. Under this conjectured the solution to the inequalities (**??**) and (**??**) can be simplified into two cases:

$$\text{Case } 1 : \gamma_t^1 = 0 \ \text{ if } i_t > i^m \tag{64}$$

$$\text{Case } 2 : i_t = i^m \ \text{ otherwise} \tag{65}$$

Thus in both Case 1 and 2 I have equalities characterizing equilibrium. These equations are (9), (18),(19), (21), (23), (24), (20), (22) and (49)-(59) and either (64) when $i_t > i^m$ or (65) otherwise. Under the condition A1(i) and A1(ii) but $i^m < \frac{1}{\beta} - 1$ then $i_t > i^m$ and Case 1 applies in the absence of shocks. In the knife edge case when $i^m = \frac{1}{\beta} - 1$,however, the equations that solve the two cases (in the absence of shocks) are identical since then both $\gamma_{1t} = 0$ and $i_t = i^m$. Thus both Case 1 and Case 2 have the same steady state in the knife edge case $i_t = i^m$. If I linearize around this steady state (which I show exists in Proposition 6) I obtain a solution that is accurate up to a residual ($||\xi||^2$) for both Case 1 and Case 2. As a result I have one set of linear equations when the bound is binding, and another set of equations when it is not. The challenge, then, is to find a solution method that, for a given stochastic process for $\{\xi_t\}$, finds in which states of the world the interest rate bound is binding and the equilibrium has to satisfy the linear equations of Case 1, and in which states of the world it is not binding and the equilibrium has to satisfy the linear equations in Case 2. Since each of these solution are accurate to a residual ($||\xi||^2$) the solutions can be made arbitrarily accurate by reducing the amplitude of the shocks. The next subsection show a solution method, assuming as simple process for the natural rate of interest, that numerically calculates when Case 1 applies and when Case 2 applies.

Note that I may also consider solutions when i^m is below the steady state nominal interest rate. A linear approximation of the equations around the steady state in Proposition **??** and 6 is still valid if the opportunity cost of holding money, i.e. $\bar{\delta} \equiv (i - i^m)/(1+i)$, is small enough. Specifically, the result will be exact up to a residual of order $(||\xi, \bar{\delta}||^2)$. In the numerical example in the text I suppose that $i^m = 0$ (see Eggertsson and Woodford (2003) for further discussion about the accuracy of this approach when the zero bound is binding). A nontrivial complication of approximating the Markov equilibrium is that I do not know the unknown expectation functions $\bar{e}(.)$. I illustrate a simple way of matching coefficients to approximate this function in section (C.5).

C.4 Linearized solution

I here linearize the first order conditions and the constraints around the steady state in Propositions 6. I assume the form of the utility discussed in section C.1. I allow for deviations in the vector of shocks ξ_t, the production subsidy s (the latter deviation is used in Proposition **??**) and in i^m so that the equations are accurate of order $o(||\xi, \bar{\delta}, 1+s-\frac{\theta}{\theta-1}||^2)$. I abstract from the effect of the shocks on the disutility of labor. Here $dz_t = z_t - z_{ss}$ The economic constraints are:

$$\bar{u}_c d'' d\Pi_t + \theta(\bar{u}_{cc} - \bar{v}_{yy})dY_t + (\theta - 1)\bar{u}_c ds + \theta \bar{u}_{c\xi} d\xi_t - \bar{u}_c d'' \beta E_t d\Pi_{t+1} = 0 \tag{66}$$

$$\bar{u}_{cc} dY_t + \bar{u}_{c\xi} d\xi_t - \beta \bar{u}_{cc} E_t dY_{t+1} - \beta \bar{u}_{c\xi} E_t d\xi_{t+1} - \beta \bar{u}_c di_t + \beta \bar{u}_c E_t d\Pi_{t+1} = 0 \tag{67}$$

$$dw_t - \frac{1}{\beta} dw_{t-1} + \frac{1}{\beta} dT_t = 0 \tag{68}$$

$$dS_t^e - \bar{u}_c d'' E_t d\Pi_{t+1} = 0 \tag{69}$$

$$df_t^e + \bar{u}_c E_t d\Pi_{t+1} - \bar{u}_{cc} E_t dY_{t+1} - \bar{u}_{c\xi} E_t \xi_{t+1} = 0 \tag{70}$$

The equation determining the natural rate of output is:

$$(v_{yy} - u_{cc})dY_t^n + (v_{y\xi} - u_{c\xi})d\xi_t - \frac{(\theta - 1)}{\theta} u_c ds = 0 \tag{71}$$

The equation determining the natural rate of interest is:

$$\beta E_t(\bar{u}_{cc} dY_{t+1}^n - \bar{u}_{c\xi} E_t d\xi_{t+1}) - (\bar{u}_{cc} dY_t^n - \bar{u}_{c\xi} d\xi_t) + \beta \bar{u}_{cc} dr_t^n = 0 \tag{72}$$

Note that the real money balances deflated by $\bar{m}$, i.e. $\tilde{m}_t$, are well defined in the cashless limit so that equation 63 is

$$d\tilde{m}_t - d\tilde{m}_{t-1} - d\frac{M_t}{M_{t-1}} + d\pi_{t-1} = 0$$

and money demand is approximated by

$$\frac{\bar{\chi}_{mm}}{u_c} d\tilde{m}_t - \frac{\bar{\chi}_{mm}}{u_c} \tilde{m} d\Pi_t - \frac{\bar{\chi}_{mm}}{u_c} \tilde{m} dY_t - \beta di_t + \beta di^m = 0$$

The Kuhn Tucker conditions imply that

Case 1 when $i_t > i^m$

$$d\gamma_{1t} = 0 \tag{73}$$

Case 2 when $i_t = i^m$

$$di_t = 0 \tag{74}$$

I look for a solution in which case the debt limit is never binding so that $d\gamma_{2t} = 0$ at all times and verify that this is satisfied in equilibrium. Linearized FOC in a Markov Equilibrium

$$-d''\bar{u}_c d\Pi_t + \bar{\phi}_2\beta^{-1}dw_{t-1} + d''\bar{u}_c d\phi_{4t} = 0 \tag{75}$$

$$(\bar{u}_{cc} - \bar{v}_{yy})dY_t + \bar{u}_{c\xi}d\xi_t - \bar{v}_{y\xi}d\xi_t - \bar{u}_{cc}\beta d\phi_{3t} + \theta(\bar{u}_{cc} - \bar{v}_{yy})d\phi_{4t} = 0 \tag{76}$$

$$FOC\ with \text{ respcect to F}$$

$$\bar{\phi}_2 dT_t - \bar{\phi}_2 dw_{t-1} + \bar{u}_c\beta^2 d\phi_{3t} + d\gamma_{1t} = 0 \tag{77}$$

$$\bar{g}_{GG}(s')^2 dT_t - \bar{g}_G s'' dT_t - \bar{g}_{G\xi}d\xi_t + \beta^{-1}d\phi_{2t} + \bar{\phi}_2 di_t = 0 \tag{78}$$

$$d\phi_{2t} - E_t d\phi_{2t+1} - \beta\bar{\phi}_2 E_t di_{t+1} + \bar{\phi}_2 E_t d\Pi_{t+1} + \beta f_w d\phi_{3t} - \beta S_w d\phi_{4t} - d\gamma_{2t} = 0 \tag{79}$$

Note that the first order condition with respect to m_t does not play any role in the cashless limit so that it is omitted above. Also note that the two derivatives f_w and S_w are in general not known. In the next section I show how these derivatives can be found

C.5 Approximating f_w and S_w

I show how the two derivatives f_w and S_w can be approximated under A5. At time $t \geq \tau$ the system is deterministic. Then I can approximate these functions to yield $w_t = w^1 w_{t-1}$ and $d\Lambda_t = \Lambda^1 w_{t-1}$, where the first element of the vector $d\Lambda_t$ is $d\pi_t = \pi^1 w_{t-1}$, the second $dY_t = Y^1 w_{t-1}$ and so on and $w_t = w^1 w_{t-1}$ where the vector Λ^1 and the number w^1 are some unknown constants. To find the value of each of these coefficients I substitute this solution into the system (66)-(70) and (75)-(79) and match coefficients. For example equation (66) implies that

$$\bar{u}_c d'' \pi^1 w_{t-1} + \theta(\bar{u}_{cc} - \bar{v}_{yy}) Y^1 w_{t-1} - \bar{u}_c d'' \beta \pi^1 w^1 w_{t-1} = 0 \tag{80}$$

where I have substituted for $d\pi_t = \pi^1 w_{t-1}$ and for $d\pi_{t+1} = \pi^1 w_t = \pi^1 w^1 w_{t-1}$. Note that I assume that $t \geq \tau$ so that there is perfect foresight and I may ignore the expectation symbol. This equation implies that the coefficients π^1, y^1 and w^1 must satisfy the equation:

$$\bar{u}_c d'' \pi^1 + \theta(\bar{u}_{cc} - \bar{v}_{yy}) Y^1 - \bar{u}_c d'' \beta \pi^1 w^1 = 0 \tag{81}$$

I may similarly substitute the solution into each of the equation (66)-(70) and (75)-(79) to obtain a system of equation that the coefficients must satisfy:

$$\bar{u}_c d''\pi^1 + \theta(\bar{u}_{cc} - \bar{v}_{yy})Y^1 - \bar{u}_c d''\beta\pi^1 w^1 = 0 \tag{82}$$

$$\bar{u}_{cc}Y^1 - \beta\bar{u}_{cc}Y^1 w^1 - \beta\bar{u}_c i^1 + \beta\bar{u}_c\pi^1 w^1 = 0 \tag{83}$$

$$w^1 - \frac{1}{\beta} + \frac{1}{\beta}T^1 = 0 \tag{84}$$

$$S^1 - \bar{u}_c d''\pi^1 w^1 = 0 \tag{85}$$

$$f^1 + \bar{u}_c\pi^1 w^1 - \bar{u}_{cc}Y^1 w^1 = 0 \tag{86}$$

$$-d\mathrm{J}\bar{u}_c\pi^1 + \frac{s'\bar{g}_G}{\beta} + d''\bar{u}_c\phi_4^1 = 0 \tag{87}$$

$$(\bar{u}_{cc} - \bar{v}_{yy})Y^1 - \bar{u}_{cc}\beta\phi_3^1 + \theta(\bar{u}_{cc} - \bar{v}_{yy})\phi_4^1 = 0 \tag{88}$$

$$s'\bar{g}_G T^1 - s'\bar{g}_G + \bar{u}_c\beta^2\phi_3^1 = 0 \tag{89}$$

$$\bar{g}_{GG}(s')^2 T^1 - \bar{g}_G s'' T^1 + \beta^{-1}\phi_2^1 + \bar{g}_G s' i^1 = 0 \tag{90}$$

$$\phi_2^1 - \phi_2^1 w^1 - \beta\bar{g}_G s' i^1 w^1 + \bar{g}_G s'\pi^1 w^1 + \beta f^1\phi_3^1 - \beta S^1\phi_4^1 = 0 \tag{91}$$

There are 11 unknown coefficients in this system i.e. $\pi^1, Y^1, i^1, F^1, S^1, f^1, T^1, \phi_2^1, \phi_3^1, \phi_4^1, w^1$. For a given value of w^1, (82)-(90) is a linear system of 10 equations with 10 unknowns, and thus there is a unique value given for each of the coefficients as long as the system is non-singular (which can be verified to be the case for standard functional forms for the utility and technology functions). The value of w^1 is in general not unique, but in the calibrated model there is always a unique bounded solution in the examples I have studied (and the unbounded solutions will violate the debt limit). In a simplified version of the model it can be proofed that there is a unique solution for w^1 that satisfies all the necessary conditions, but I have not managed to proof it in this model (see discussion in Eggertsson (2004)).

C.6 Computational method

Here I illustrate a solution method for the optimal commitment solution. This method can also be applied, with appropriate modification of each of the steps, to find the Markov solution. I assume shocks so that the natural rate of interest becomes unexpectedly negative in period 0 and the reverts back to normal with probability α_t in every period t as in A5 (one may use (71) and (72) to find what a given negative number for the natural rate of interest implies for the underlying exogenous shocks). I assume that there is a final date K in which the natural rate becomes positive with probability one (this date can be arbitrarily far into the future).

The solution takes the form:

$$\text{Case 2 } i_t = 0 \quad \forall \quad t \quad 0 \leq t < \tau + k$$

$$\text{Case 1 } i_t > 0 \quad \forall \quad t \quad t \geq \tau + k$$

Here τ is he stochastic date at which the natural rate of interest returns to steady state. I assume that τ can take any value between 1 and the terminal date K that can be arbitrarily far into the future. The number $\tau + k_\tau$ is the period in which the zero bound stops being binding in the contingency when the natural rate of interest becomes positive in period τ. Note that the value of k_τ can depend on the value of τ. I first show the solution for the problem as if I knew the sequence $\{k_\tau\}_{\tau=1}^{S}$. I then describe a numerical method to find the sequence $\{k_\tau\}_{\tau=1}^{S}$.

C.6.1 The solution for $t \geq \tau + k_\tau$

The system of linearized equations (75)-(79), (66)-(70), and (73) can be written in the form:

$$\begin{bmatrix} E_t Z_{t+1} \\ \\ P_t \end{bmatrix} = M \begin{bmatrix} Z_t \\ \\ P_{t-1} \end{bmatrix}$$

where $Z_t \equiv \begin{bmatrix} \Lambda_t & e_t & \phi_t & \psi_t & \gamma_t^1 \end{bmatrix}^T$ and $P_t \equiv w_t$. If there are fifteen eigenvalues of the matrix M outside the unit circle this system has a unique bounded solution of the form:

$$P_t = \Omega^0 P_{t-1} \tag{92}$$

$$Z_t = \Lambda^0 P_{t-1} \tag{93}$$

C.6.2 The solution for $\tau \leq t < \tau + k$

Again this is a perfect foresight solution but with the zero bound binding. The solution now satisfies the equations (75)-(79), (66)-(70) but (74) instead of (73).The system can be written on the form:

$$\begin{bmatrix} P_t \\ \\ Z_t \end{bmatrix} = \begin{bmatrix} A & B \\ \\ C & D \end{bmatrix} \begin{bmatrix} P_{t-1} \\ \\ Z_{t+1} \end{bmatrix} + \begin{bmatrix} M \\ \\ V \end{bmatrix}$$

This system has a solution of the form:

$$P_{\tau+j} = \Omega^{k_\tau - j} P_{\tau+j-1} + \Phi^{k_\tau - j} \tag{94}$$

$$Z_{\tau+j} = \Lambda^{k_\tau - j} P_{\tau+j-1} + \Theta^{k_\tau - j} \tag{95}$$

where $j = 0, 1, 2, ..., k$. Here $\Omega^{k_\tau - j}$ is the coefficient in the solution when there are $k_\tau - j$ periods until the zero bound stops being binding (i.e. when $j - k_\tau = 0$ the zero bound is not binding anymore and the solution is equivalent to (92)-(93)). We can find the numbers $\Lambda^j, \Omega^j, \Theta^j$ and Φ^j for $j = 1, 2, 3, ..., k$ by solving the equations below using the initial conditions $\Phi^0 = \Theta^0 = 0$ for $j = 0$ and the initial conditions for Λ^j and Ω^j given in (92)-(93):

$$\Omega^j = [I - B\Lambda^{j-1}]^{-1} A$$

$$\Lambda^j = C + D\Lambda^{j-1}\Omega^j$$

$$\Phi^j = (I - B\Lambda^{j-1})^{-1}[B\Theta^{j-1} + M]$$

$$\Theta^j = D\Lambda^{j-1}\Phi^j + D\Theta^{j-1} + V$$

C.6.3 The solution for $t < \tau$

The solution satisfies (75)-(79), (66)-(70), and (74). Note that each of the expectation variables can be written as $\tilde{x}_t = E_t x_{t+1} = \alpha_{t+1}\tilde{x}_{t+1} + (1 - \alpha_{t+1})x_{t+1}$ where α_{t+1} is the probability that the natural rate of interest becomes positive in period $t+1$. Here hat on the variables refers to the value of each variable contingent on that the natural rate of interest is negative. I may now use the solution for Z_{t+1} in 95 to substitute for Z_{t+1}, i.e. the value of each variable contingent on that the natural rate becomes positive again, in terms of the hatted variables. The value of x_{t+1}, for example, can be written as $x_{t+1} = \Lambda_{21}^{k_{t+1}}\tilde{\phi}_{1t} + \Lambda_{22}^{k_{t+1}}\tilde{\phi}_{2t} + \Theta_2^{k_{t+1}}$ where $\Lambda_{ij}^{k_{t+1}}$ is the ijth element of the matrix $\Lambda^{k_{t+1}}$ and the value k_{t+1} depends on the number of additional periods that the zero bound is binding (recall that I am solving the equilibrium on the assumption that I know the value of the sequence $\{k_\tau\}_{\tau=1}^{S})$. Hence I can write the system as:

$$\begin{bmatrix} \tilde{P}_t \\ \\ \tilde{Z}_t \end{bmatrix} = \begin{bmatrix} A_t & B_t \\ \\ C_t & D_t \end{bmatrix} \begin{bmatrix} \tilde{P}_{t-1} \\ \\ \tilde{Z}_{t+1} \end{bmatrix} + \begin{bmatrix} M_t \\ \\ V_t \end{bmatrix}$$

I can solve this backwards from the date K in which the natural rate returns back to normal with probability one. I can then calculate the path for each variable to date 0. Note that.

$$B_{K-1} = D_{K-1} = 0$$

By recursive substitution I can find a solution of the form:

$$\tilde{P}_t = \Omega_t \tilde{P}_{t-1} + \Phi_t \tag{96}$$

$$\tilde{Z}_t = \Lambda_t \tilde{P}_{t-1} + \Theta_t \tag{97}$$

where the coefficients are time dependent. To find the numbers $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t consider the solution of the system in period $K-1$ when $B_{K-1} = D_{K-1} = 0$. I have:

$$\Omega_{K-1} = A_{K-1}$$

$$\Phi_{K-1} = M_{K-1}$$

$$\Lambda_{K-1} = C_{K-1}$$

$$\Theta_{K-1} = V_{K-1}$$

I can find of numbers $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t for period 0 to $K-2$ by solving the system below (using the initial conditions shown above for $S-1$):

$$\Omega_t = [I - B_t\Lambda_{t+1}]^{-1}A_t$$

$$\Lambda_t = C_t + D_t\Lambda_{t+1}\Omega_t$$

$$\Phi_t = (I - B_t\Lambda_{t+1})^{-1}[B_t\Theta_{t+1} + M_t]$$

$$\Theta_t = D_t\Lambda_{t+1}\Phi_t + D_t\Theta_{t+1} + V_t$$

Using the initial condition $\tilde{P}_{-1} = 0$ I can solve for each of the endogenous variables under the contingency that the trap last to period K by (96) and (97). I then use the solution from (92)-(95)

to solve for each of the variables when the natural rate reverts back to steady state.

C.6.4 Solving for $\{k_\tau\}_{t=0}^{\infty}$

A simple way to find the value for $\{k_\tau\}_{\tau=1}^{\infty}$ is to first assume that k_τ is the same for all τ and find the k so that the zero bound is never violated. Suppose that the system has converged at $t = 25$ (i.e. the response of each of the variables is the same). Then I can move to 24 and see if $k_\tau = 4$ for $\tau = 1, 2, ...24$ is a solution that never violates the zero bound. If not move to 23 and try the same thing and so on. For preparing this paper I wrote a routine in MATLAB that applied this method to find the optimal solution and verified that the results satisfied all the necessary conditions. It turned out that in the Markov equilibrium the zero bound stopped being binding as soon as the natural rate of interest is positive again (the same is not true for the commitment equilibrium as shown in Eggertsson (2004) and Eggertsson and Woodford (2003)).

C.7 Calibration for numerical results

In the numerical examples I assume the following functional forms for preferences and technology:

$$u(C,\xi) = \frac{C^{1-\sigma^{-1}}\bar{C}^{\sigma^{-1}}}{1-\sigma^{-1}}$$

where $\bar{C}$ is a preference shock assumed to be 1 in steady state.

$$g(G,\xi) = g_1 \frac{G^{1-\sigma^{-1}}\bar{G}^{\sigma^{-1}}}{1-\sigma^{-1}}$$

where $\bar{G}$ is a preference shock assumed to be 1 in steady state

$$v(H,\xi) = \frac{\lambda_1}{1+\omega} H^{1+\omega}\bar{H}^{-\lambda_2}$$

where $\bar{H}$ is a preference shock assumed to be 1 in steady state

$$y = Ah^{\epsilon}$$

where A is a technology shock assumed to be 1 in steady state. I may substitute the production function into the disutility of working to obtain (assuming A=1):

$$\tilde{v}(Y,\xi_t) = \frac{\lambda_1}{1+\lambda_2} Y^{1+\lambda_2}\bar{H}^{-\omega}$$

When calibrating the shocks that generate the temporarily negative natural rate of interest I assume that it is the shock $\bar{\mathrm{C}}$ that is driving the natural rate of interest negative (as opposed to A) since otherwise a negative natural rate of interest would be associated with a higher natural rate of output which does not seem to be the most economically interesting case. I assume that the shock $\bar{G}$ is such that the F_t would be constant in the absence of the zero bound, in order to keep the optimal size of the government (in absence of the zero bound) constant as discussed in the text. The cost of price adjustment is assumed to take the form:

$$d(\Pi) = d_1\Pi^2$$

The cost of taxes is assumed to take to form:

$$s(T) = s_1 T^2$$

Aggregate demand implies$Y = C + F = C + G + s(F)$. I normalize $Y = 1$ in steady state and assume that the share of the government in production is $F = 0.3$. Tax collection as a share of government spending is assumed to be $\gamma = 5\%$ of government spending. This implies

$$0.1 = \frac{s(F)}{F} = s_1 F$$

so that $s_1 = \frac{\gamma}{F}$. The result for the inflation and output gap response are not very sensitive to varying γ under either commitment or discretion. The size of the public debt issued in the Markov equilibrium, however, crucially depends on this variable. In particular if γ is reduced the size of the debt issued rises substantially. For example if $\gamma = 0.5\%$ the public debt issued is about ten times bigger than reported in the figure in the paper. I assume that government spending are set at their optimal level in steady state as discussed in the text:

$$g_2 = \frac{u_c}{g_G - s' g_G} = \frac{C^{-\sigma^{-1}}}{G^{-\sigma^{-1}}(1-s')} = (\frac{G}{C})^{\sigma^{-1}} \frac{1}{1-s'} = (\frac{G}{C})^{\sigma^{-1}} \frac{1}{1-2s_1 F}$$

The IS equation and the AS equation are

$$x_t = E_t x_{t+1} - \tilde{\sigma}(i_t - E_t \pi_{t+1} - r_t^n)$$

$$\pi_t = k\pi_t + \beta E_t \pi_{t+1}$$

I assume, as Eggertsson and Woodford, that the interest rate elasticity, $\tilde{\sigma}$, is 0.5. The relationship between σ and $\tilde{\sigma}$ is

$$\sigma = \tilde{\sigma}\frac{Y}{C}$$

I assume that κ is 0.02 as in Eggertsson and Woodford (2003). The relationship between κ and the other parameters of the model is$\kappa = \theta\frac{(\tilde{\sigma}^{-1}+\lambda_2)}{d''}$. I scale hours worked so that $Y = 1$ in steady state which implies

$$v_y = \lambda_1$$

Since $u_c = \tilde{v}_y$ in steady state I have that

$$\theta = 7.87$$

Finally I assume that $\theta = 7.89$ as in Rotemberg and Woodford and that $\lambda_2 = 2$. The calibration value for the parameters are summarized in the table below:

Table 2

σ	0.71
g_1	0.33
λ_1	1.65
λ_2	2
d_1	787
s_1	0.17
θ	7.87

D Proofs

D.1 Proof of Proposition 3

Proposition 3 *If $\xi = 0$ at all times and (i)-(iii) in A4 hold there is a Markov equilibrium steady state that is given by $i = 1/\beta - 1$, $w = S^e = \phi_1 = \phi_3 = \phi_4 = \psi_1 = \psi_2 = \gamma_1 = \gamma_2 = 0$, $\Pi = 1$, $\phi_2 = g_G(\bar{F} - s(\bar{F}))s'(\bar{F})$, $f^e = u_c(\bar{Y}), F = \bar{F} = G = T + s(T)$ and $Y = \bar{Y}$ where $\bar{Y}$ and $\bar{F}$ are the unique solution to the equations: $u_c(Y - F) = v_y(Y)$ and $u_c(Y - F) + g_G(F - s(F))s'(F) =$*

$g_G(F - s(F))$

I only proof existence of this steady state here but do not discuss uniqueness (see Eggertsson (2003) for discussion about uniqueness of a Markov equilibrium in this model). In the assumption made in the proposition I assume the cashless limit and the form of the utility:

$$u(C_t, m_t\Pi_t^{-1}, \xi_t) = \tilde{u}(C_t, \xi_t) + \chi(\frac{m_t}{\bar{m}}\Pi_t^{-1}C_t^{-1}, \xi_t) \tag{98}$$

The partial derivatives with respect to each variable are given by

$$u_c = \tilde{u}_c - \chi' \frac{m}{\bar{m}} C^{-2}\Pi^{-1} \tag{99}$$

$$u_m = \frac{\chi'}{\bar{m}} C^{-1}\Pi^{-1} \tag{100}$$

$$u_{mm} = \frac{\chi''}{\bar{m}^2} C^{-2}\Pi^{-2} < 0 \tag{101}$$

$$u_{cm} = -\chi'' \frac{m}{\bar{m}^2} C^{-3}\Pi^{-2} - \frac{\chi'}{\bar{m}} C^{-2}\Pi^{-1} \tag{102}$$

As $\bar{m} -> 0$ I assume that for $\tilde{m} = \frac{m}{\bar{m}} > 0$ I have

$$lim_{\bar{m}\to 0} \frac{\chi\prime}{\bar{m}} \equiv \bar{\chi}' \geq 0 \tag{103}$$

$$lim_{\bar{m}\to 0}\frac{\chi\prime\prime}{\bar{m}^2} \equiv \bar{\chi}'' > 0 \tag{104}$$

This implies that there is a well defined money demand function, even as money held in equilibrium

approaches zero, given by

$$\frac{\bar{\chi}\prime(\tilde{m}C_t^{-1}\Pi_t^{-1}, \xi_t)C_t^{-1}\Pi_t^{-1}}{\bar{u}_c(C_t, \xi_t)} = \frac{i_t - i^m}{1 + i_t}$$

so that $\bar{\chi}' = 0$ when $i_t = i_t^m$. From the assumptions (103)-(104) it follows that:

$$lim_{\bar{m}\to 0}\chi\prime = 0$$

$$lim_{\bar{m}\to 0}\chi\prime\prime = 0$$

Then the derivatives u_c and u_{cm} in the cashless limit are:

$$\lim_{\bar{m}\to 0} u_c = \tilde{u}_c$$

and

$$\lim_{\bar{m}\to 0} u_{cm} = \lim_{\bar{m}\to 0}[-\bar{m}\frac{\chi\prime\prime}{\bar{m}^2}\frac{m}{\bar{m}}C^{-3}\Pi^{-1} - \frac{\chi\prime}{\bar{m}}C^{-2}] = -\bar{\chi}\prime C^{-2}$$

Hence in a steady state in which $\bar{m} \to 0$ and $i_t = i^m$ I have that $\bar{\chi}\prime = 0$ so that at the steady state

$$\lim_{\bar{m}\to 0} u_{cm} = 0. \tag{105}$$

Note that this does not imply that the satiation point of holding real balances is independent of consumption. To see this note that the satiation point of real money balances is is given by some finite number $S^* = \frac{m}{\bar{m}}Y$ which implies that $\chi(S \geq S^*) = \tilde{v}(S^*)$. The value of the satiation point as $\bar{m} \to 0$ is:

$$lim_{\bar{m}\to 0} S^* \equiv \bar{S} = \tilde{m}C$$

The value of this number still depends on C even as $\bar{m} \to 0$ and even if $u_{cm} = 0$ at the satiation point.

I now show that the steady state stated in Proposition 3 satisfies all the first order conditions and the constraints. The steady state candidate solution in is:

$$i = \frac{1}{\beta} - 1, w = \phi_1 = \phi_3 = \phi_4 = \psi_1 = \psi_2 = \gamma_1 = \gamma_2 = 0, \Pi = 1, \phi_2 = g_G s',\ T = F \tag{106}$$

and Y and F are the unique solution to the equations stated in the proposition. Note that (106) and the functional assumption about d (see footnote **??**) imply that:

$$d' = 0 \tag{107}$$

Let us first consider the constraints. In the steady state the AS equation is

$$\theta Y[\frac{\theta - 1}{\theta}(1+s)u_c - \tilde{v}_y] - u_c \Pi d'(\Pi) + \beta u_c \Pi d'(\Pi) = 0$$

Since by (107) d'=0, and according to assumption (ii) of the propositions $\frac{\theta-1}{\theta}(1+s) = 1$ the AS equation is only satisfied in the candidate solution if

$$u_c = v_y \tag{108}$$

Evaluated in the candidate solution the IS equation is:

$$\frac{1}{1+i} = \frac{\beta u_c}{u_c}\Pi^{-1} = \beta$$

which is always satisfied at because it simply states that $i = 1 - 1/\beta$ which is consistent with the steady state I propose in the propositions and assumption (iii). The budget constraint is:

$$w - (1+i)\Pi^{-1}w - (1+i)F + (1+i)T + (1+i)\bar{m}\tilde{m}\Pi_t^{-1} = 0$$

which is also always satisfied in our candidate solution since it states that $F = T$, $w = 0$ and $\bar{m} \to 0$. The money demand equation indicates that the candidate solutions is satisfied if

$$u_m = \Pi u_c \frac{i - i^m}{1+i} = 0 \qquad (109)$$

By (20) and (22) the expectation variables in steady state are

$$S^e = u_c \Pi d'$$

$$f^e = u_c \Pi$$

Since $\Pi = 1$ and $d' = 0$ by (107) these equations are satisfied in the candidate solution. Finally both the inequalities (9) and (19) are satisfied since $\bar{w} > w = 0$ in the candidate solution and $i = i^m$.

I now show that the first order conditions, i.e. the commitment and the Markov equilibrium first order conditions, that are given by (49)-(61), are also consistent with the steady state suggested.

I start with (49). It is

$$-u_c d' - u_m \bar{m}\tilde{m}\Pi^{-2} + \phi_1[-\frac{u_{mc}d'\Pi^{-1}}{u_c} - \frac{u_{mm}\bar{m}\tilde{m}\Pi^{-2}}{u_c} - \frac{u_m\Pi^{-2}}{u_c} + \frac{u_m u_{cc}d'\Pi^{-1}}{u_c^2} + \frac{u_m u_{cm} m\Pi^{-2}}{u_c}] \tag{110}$$

$$+[\phi_2(1+i)w\Pi^{-2} - (i-i^m)\bar{m}\tilde{m}\Pi^{-2}] + \phi_3[\frac{u_{cc}d'}{1+i} + \frac{u_{cm}\bar{m}\tilde{m}\Pi^{-2}}{(1+i)}]$$

$$+\phi_4[-Y(\theta-1)(1+s)(u_{cc}d' + \bar{m}\tilde{m}\Pi^{-2}u_{cm}) - u_{cc}\Pi d'^2 - u_{cm}\bar{m}\tilde{m}\Pi^{-1}d' + u_c\Pi d'' + u_c d']$$

$$+\beta^{-1}\psi_1[u_{cc}d'\Pi + u_{cm}\bar{m}\tilde{m}\Pi^{-1} + u_c\Pi^{-2}] + \beta^{-1}\psi_2[u_{cc}d'^2\Pi + u_{cm}d'\bar{m}\tilde{m}\Pi^{-1} - u_c d' - u_c d''\Pi] = 0$$

By (107) and (109) the first two terms are zero. The constraints that are multiplied by ϕ_1, ϕ_3, ϕ_4, ψ_1 and ψ_2 are also zero because each of these variables are zero in our candidate solution (106). Finally, the term that is multiplied by ϕ_2 (which is positive) is also zero because $w = 0$ in our candidate solution (106) and so is $i - i^m$. Thus I have shown that the candidate solution (106) satisfies (49).

Let us now turn to (51). It is

$$u_c - \tilde{v}_y + \phi_1[\frac{u_{mc}}{u_c} - \frac{u_m}{u_c^2}] - \phi_3 \frac{u_{cc}}{1+i} + \phi_4[\theta(\frac{\theta-1}{\theta}(1+s)u_c - \tilde{v}_y) - \theta Y(\frac{\theta-1}{\theta}(1+s)u_{cc} - \tilde{v}_{yy}) - u_{cc}\Pi d'] = 0 \tag{111}$$

The first two terms $u_c - v_y$ are equal to zero by (108). The next terms are also all zero because they are multiplied by the terms ϕ_1, ϕ_3, ϕ_4, ψ_1 and ψ_2 which are all zero in our candidate solution (106). Hence this equation is also satisfied in our candidate solution. Let us then consider (53). It is:

$$-\phi_1 \frac{1+i^m}{(1+i)^2} + \phi_2(\bar{m}\tilde{m} + T - w\Pi^{-1} - F) + \phi_3 \frac{u_c}{(1+i)^2} + \gamma_1 = 0$$

Again this equation is satisfied in our candidate solution because $\phi_1 = \phi_3 = w = 0$, $F = T$ and $\bar{m} \to 0$ in the candidate solution. Conditions (54) in steady state is:

$$\bar{m}\tilde{m}u_m\Pi^{-1}+\phi_1[\frac{u_{mm}}{u_c}-\frac{u_m}{u_c^2}u_{cm}\Pi^{-1}]+\phi_2(i-i^m)\bar{m}\tilde{m}-\phi_3\frac{u_{cm}}{1+i}\Pi^{-1}-\phi_4[Y(\theta-1)(1+s)u_{cm}\Pi^{-1}-u_{cm}d']=0$$

$$(112)$$

The first term is zero by (109). All the other terms are also zero because ϕ_1, ϕ_3, ϕ_4, ψ_1 and ψ_2 are all zero in our candidate solution (106). Finally $i = i^m$ in our candidate solution so that the third term is zero as well. Conditions (52) and (55) in steady state are:

$$-g_G s'(T) + \phi_2(1+i) = 0 \tag{113}$$

$$-u_c+g_G+\phi_{1t}[-\frac{u_{mc}}{u_c}+\frac{u_m}{u_c^2}]\Pi_t^{-1}+\phi_{3t}\frac{u_{cc}}{1+i_t}-\phi_{4t}[\theta(\frac{\theta-1}{\theta}(1+s)u_c+\theta Y_t(\frac{\theta-1}{\theta}(1+s)u_{cc}+u_{cc}\Pi_t d']$$

$$(114)$$

Using our candidate solution (106) I obtain:

$$u_c(Y-F) = g_G(F-s'(F)) + g_G s'(F)$$

which along with (108) is the equation that determine $\bar{Y}$ and $\bar{F}$ that was stated in the propositions

3 and 4. Using our assumption on s and standard Inada boundary conditions one may show that these equations have unique solution for $\bar{Y}$ and $\bar{F}$.

Let us now turn to (56). This equation involves three unknown functions, J_w, f_w^e and S_w^e.I can use (61) to substitute for J_w obtaining

$$-\beta\phi_2(1+i)\Pi^{-1} - \psi_1\beta f_w^e - \psi_2\beta S_w^e + \phi_2 - \gamma_2 = 0 \tag{115}$$

In general I cannot know if this equation is satisfied without making further assumption about f_w^e and S_w^e. But note that in my candidate solution $\psi_1 = \psi_2 = 0$. Thus the terms involving these two derivatives in this equation are zero. Since $\gamma_2 = 0$, this equation is satisfied if $(1+i)\Pi^{-1} = 1/\beta$. This is indeed the case in our candidate solution. Finally (57) and (58) are satisfied since $\phi_3 = \phi_4 = \psi_1 = \psi_2 = 0$ in the candidate solution. Thus I have shown that all the necessary and sufficient conditions of a Markov equilibrium are satisfied by our candidate solution (106). QED

D.2 Proof of Proposition 4

Proposition 4 *If $F_t = T_t$ at all times, then the solution under A2 and A3 are identical up to an error of order $o(||\xi||^2)$*

The proof is simple but tedious. The central bank takes $F_t = T_t$ as exogenously given and its maximization problem can be characterized by the Lagrangian:

$$L_t = (\Pi_t - 1)^2 + \lambda_x (Y_t - Y_t^n)^2$$

$$+ \phi_{1t}\left(\frac{u_m(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)\Pi_t^{-1}}{u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)} - \frac{i_t - i^m}{1 + i_t}\right)$$

$$+ \phi_{3t}\left(\beta f_t^e - \frac{u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)}{1 + i_t}\right)$$

$$+ \phi_{4t}(\theta Y_t[\frac{\theta - 1}{\theta}(1 + s)u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] + u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)\Pi_t d'(\Pi_t) - \beta$$

$$+ \psi_{1t}(f_t^e - \bar{f}^e(\xi_t)) + \psi_{2t}(S_t^e - \bar{S}^e(\xi_t)) + \gamma_{1t}(i_t - i^m)$$

taking F_t as given.

The treasury takes m_t as exogenously given and its maximization problem is:

$$L_t = u(Y_t - d(\Pi_t) - F_t, m_t \Pi_t^{-1}, \xi_t)) + g(F_t - s(T_t), \xi_t) - \tilde{v}(Y_t)$$

$$+ \phi_{1t}(\frac{u_m(Y_t - d(\Pi_t) - F_t, m_t \Pi_t^{-1}, \xi_t)\Pi_t^{-1}}{u_c(Y_t - d(\Pi_t) - F_t, m_t \Pi_t^{-1}, \xi_t)} - \frac{i_t - i^m}{1 + i_t})$$

$$+ \phi_{2t}(w_t - (1 + i_t)\Pi_t^{-1} w_{t-1} - (1 + i_t)F_t + (1 + i_t)T_t + (i_t - i^m)m_t \Pi_t^{-1})+$$

$$+ \phi_{3t}(\beta f_t^e - \frac{u_c(Y_t - d(\Pi_t) - F_t, m_t \Pi_t^{-1}, \xi_t)}{1 + i_t})$$

$$+ \phi_{4t}(\theta Y_t[\frac{\theta - 1}{\theta}(1 + s)u_c(Y_t - d(\Pi_t) - F_t, m_t \Pi_t^{-1}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] + u_c(Y_t - d(\Pi_t) - F_t, m_t \Pi_t^{-1}, \xi_t)\Pi_t d'(\Pi_t) - \beta$$

$$+ \gamma_{2t}(\bar{w} - w_t)$$

The proof is obtained by writing the first order condition of each of these maximization problems, linearizing them around the steady state in Proposition 3 and showing that the resulting equilibrium conditions are identical to the equilibrium conditions under coordination (detailed derivation is available upon request).

Notes

[24]First, as shown by Woodford (2003), for a realistic calibration parameters, this abstraction has trivial effect on the AS and the IS equation under normal circustances. Furthermore, at zero nominal interest rate, increasing money balances further *does nothing* to facilitate transactions since consumer are already satiated in liquidity. This was one of the key insights of Eggertsson and Woodford (2003), which showed that at zero nominal interest rate increasing money supply has no effect if expectations about future money supply do not change. It is thus of even *less* interest to consider this additional channel for monetary policy at zero nominal interest rates than if the short-term nominal interest rate was positive. Second, assuming m_t is a very small number is likely to change the government budget constraint very little in a realistic calibration. By assuming the cashless limit I am assuming no seignorage revenues so that the term $\frac{i_t - i^m}{1+i_t} m_t \Pi_t^{-1}$ in the budget constraint has no effect on the equilibrium. Given the low level of seignorage revenues in industrialized countries I do not think this is a bad assumption. Furthermore, in the case the bound on the interest rate is binding, this term is zero, making it of even *less* interest when the zero bound is binding than under normal circumstances.

[25]See Woodford (2003) Appendix A3 for definition and discussion of local uniqueness in stochastic general equilibrium models of this kind.

[26]The reason for this conjecture is that in this model, as opposed to Albanesi et al and Dedola work, I assume in A2 that there are no monetary frictions. The source of the multiple equilibria in those papers, however, is the payment technology they assume. The key difference between the present model and that of King and Wolman, on the other hand, is that they assume that some firms set prices at different points in time. I assume a representative firm, thus abstacting from the main channel they emphasize in generating multiple equilibria. Finally the present model is different from all the papers cited above in that I introduce nominal debt as a state variable. Even if the model I have illustrated above would be augmented to incorporate additional elements such as montary frictions and staggering prices, I conjecture that the steady state would remain unique due to the ability of the government to use nominal debt to change its future inflation incentive. That is, however, a topic for future reasearch and there is work in progress by Eggertsson and Swanson that studies this question.

[27]Even if I had written a model in which the equilibria proofed above is not the unique global equilibria the one I illustrate here would still be the one of principal interest. Furthermore a local analysis would still

be useful. The reason is twofold. First, the equilibria analyzed is identical to the commitment equilibrium (in the absence of shocks) and is thus a natural candidate for investigation. But even more importantly the work of Albanesi et al (2002) indicates that if there are non-trivial monetary frictions there are in general only two steady states.There are also two steady states in King and Wolman's model. (In Dedola's model there are three steady states, but the same point applies.) The first is a low inflation equilibria (analogues to the one in Proposition 1) and the other is a high inflation equilibria which they calibrate to be associated with double digit inflation. In the high inflation equilibria, however, the zero bound is very unlikely ever to be binding as a result of real shocks of the type I consider in this paper (since in this equilibria the nominal interest rate is very high as I will show in the next section). And it is the distortions created by the zero bound that are the central focus of this paper, and thus even if the model had a high inflation steady state, that equilibria would be of little interest in the context of the zero bound.