Real Government Spending in a Liquidity Trap

(first draft, October 2000, this version November 2001)

Gauti Eggertsson, Princeton University

PRELIMINARY AND INCOMPLETE DRAFT!

This paper explores the effects of real government spending in a New Keynesian model. A social welfare criterion is derived by a second order Taylor expansion of the representative household utility. The welfare criterion includes inflation, output gap and the deviation of government spending from a time varying target level. Using this welfare criterion, optimal monetary and fiscal policy are analyzed. This paper shows that even if Ricardian Equivalence holds, real spending can have substantial effects on output and prices. This is particularly relevant in a liquidity trap since then the effectiveness of monetary policy is reduced by the zero bound. The only way the government can use monetary policy in a liquidity trap to influence aggregate demand is by committing to future monetary actions that are dynamically inconsistent. On the other hand, increasing real government spending involves current action and is thus not subject to the same credibility problem. For a government that cannot credibly commit to future policies, varying real government spending is therefore a particularly effective policy tool in a liquidity trap.

Note: Apart from minor revisions, this draft was written in October 2000. The paper will be revised an reorganized since many of the issues in this paper also arise in "Committing to being irresponsible: Deficit Spending to Escape a Liquidity Trap" although in the context of a different model.

1 Introduction

This paper explores optimal fiscal and monetary policy in a "New Keynesian" economy in a liquidity trap. What we mean by a liquidity trap in this paper is as situation in which the short-term nominal interest rate is zero. We assume Ricardian equivalence so the choice between taxes and debt has no effect on the equilibrium outcome. Our emphasis here is on the effect of real government spending as opposed to deficit spending that is analyzed in Eggertsson (2001) in an economy with tax distortions.

We first illustrate the problem faced by the Central Bank abstracting from fiscal policy in a New Keynesian economy that has become close to a standard in the literature. In an extended version of the Barro and Gordon (1983) and Kydland and Prescott (1977) model (BG/KP) Eggertsson (2001) shows that there is a deflationary bias of a discretionary Central Bank in a liquidity trap. A common criticism the BG/KP model is that the objectives of the government are not explicitly derived from micro foundations. In this paper we assume that the Central Bank minimizes objectives that can be rationalized by a second order expansion of the representative household utility function. We illustrate that when deflationary shocks hit the economy and the zero bound is binding, the Central Bank in our model faces exactly the same problem as illustrated in Eggertsson (2001). It best achieves its objectives by committing to low nominal interest rates in the future when deflationary pressures have subsided and the zero bound is no longer binding. In doing so it raises inflation expectations and thus lower the real rate of return, stimulating aggregate demand. For a discretionary policy this is not credible. A discretionary Central Bank has incentives to promise future inflation and then to renege on the promise. The result is a liquidity trap characterized by excessive deflation and output gap. Our result indicates that the deflation bias is not a special feature of the BG/KP model. At zero nominal interest rates a deflationary bias arises in a broad class of models in which aggregate demand depends on the real rate of interest and a discretionary government seeks to minimize inflation.

Although our model economy is fully intertemporal, there is a sense in which the deflation bias puts the policy maker into an old fashion static Keynesian IS/LM model. The old fashion IS/LM model has long been criticized for treating expectations as fixed. One implication of this is that monetary policy is impotent at zero nominal interest rate. In our model a discretionary Central Bank also faces fixed expectations when its only instrument is the short-term nominal interest rate. These expectations are pinned down by what the private sector thinks is optimal future behavior for the Central Bank. The obvious question arises: Can the government raise output by increasing government spending? A necessary condition for the liquidity trap in our model is that the natural rate of interest is negative. When the natural rate of interest is negative and inflation expectations are low, the Central Bank cannot lower the nominal interest rate enough to clear the market. This stems from the zero bound. The government can, however, influence the natural rate of interest. If the government increases real spending today, holding expectations about future spending fixed, this will increases the natural rate of interest. As with monetary policy, however, the effects of government spending critically depend on expectations about future fiscal policy. Thus to explore the effects of real spending in a liquidity trap we need to make some assumption about how fiscal policy is determined in the following periods. We assume that fiscal policy is determined to maximize social welfare. We augment the standard New Keynesian model by modeling how government spending enters the utility of consumers. We suppose that government spending enters the utility function in exactly the same way as private consumption. In steady state the government determines spending so that the marginal utility of private and public consumption are equal. An objective for the government is obtained by a second order Taylor expansion of the representative household utility around steady state. The welfare criterion includes inflation, output gap and the deviation of government spending from a time varying target rate. We call this target rate of government spending the *natural rate* of government spending. It reflects the rate of government spending that would be optimal if prices were flexible. If the zero bound is not binding the government sets spending equal to the target rate at all times. If the zero bound is binding, however, optimal fiscal policy under discretion involves increasing government spending beyond the target rate. As a result, it increases the natural rate of interest, reduces deflation and increases output, thereby increasing welfare. The reason for why discretionary fiscal policy works in a liquidity trap and monetary policy does not is simple. Expansionary fiscal policy in a liquidity trap involves *actions* today that increase output and inflation without changing expectations about future policy. In contrast, monetary policy can only be expansionary by influencing expectations, which involves promising *future* actions. Since these actions are dynamically inconsistent, monetary policy is impotent for a discretionary government.

Finally we analyze optimal monetary and fiscal policy under commitment in a liquidity trap. The optimal commitment solution involves counter cyclical fiscal policy. The government increases spending beyond its target rate in the trap and reduces it below the target rate it once out. The intuition is simple. The government can increase the natural rate of interest in a liquidity trap by one of two ways: either increase spending in the trap (relative to future spending) or commit to reducing it once out (relative to current spending). A government that can commit will both increase spending beyond the target rate in the trap and commit to reducing it below the target rate once out.

The two main contributions of the paper are: First, we solve for the optimal intertemporal problem of the Central Bank under commitment and discretion in what has become close to a standard New Keynesian economy taking the zero bound explicitly into account. Second, we provide a simple way of thinking about how government spending enters the objectives of the government. This allow us to analyze the effectiveness of fiscal policy taking expectation of future policy explicitly into account.

In this paper Ricardian equivalence holds so the choice between debt and taxes has no effect on the equilibrium outcome. In Eggertsson (2001) we relax this assumption and assume that taxes are distortionary. In this case the government can increase inflation expectations, even if it is discretionary, by deficit spending (i.e. cutting taxes and issuing debt). Taken together, these two papers imply that a fully optimal policy in an economy with tax distortions involves both deficit and real government spending in a liquidity trap.

Section 2 illustrates the liquidity trap in a New Keynesian model. Section 3 shows the deflation bias of monetary policy. Section 4 illustrates how real spending can be used to increase output and the price level. Section 5 concludes. Appendixes show the model and some of the more extensive derivations.

2 The Liquidity Trap in a New Keynesian Model

Here we illustrate the New-Keynesian IS-LM model. This model has been extensively used by several authors recently to illustrate various issues regarding monetary policy (see e.g. Clarida, Gali and Gertler (2000) for a survey of several results derived in this framework and Woodford (2001) for other recent contributions). The model is derived from explicit behavioral assumptions. The basic foundation is a representative maximizing household. The two key relationships are (for details of the derivation see Appendix A):

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1} - r_t^n) \quad \text{(IS)} \tag{1}$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \quad \text{(AS)} \tag{2}$$

where x_t is the output gap, π_t is inflation, i_t is the nominal interest rate and r_t^n is the natural rate of interest. The output gap is the difference between real output (detrended) and the natural rate of output. The natural rate of output is the output that would be produced in equilibrium if prices were flexible. Similarly, the natural rate of interest is the real rate of interest that would equilibrate the market if prices were completely flexible. The (IS) equation is a linearized Euler equation that arises from the household's optimal consumption and saving decision. The (AS) curve is a Phillips curve that arises from the maximization of price setters that adjust their prices discontinuously. In this simple framework r_t^n is an *exogenous* process that *only depends on technology and preference shocks.*

The Central Bank chooses i_t and inflation and output gap are endogenously determined. By the linearized Fisher equation we have that $r_t = i_t - E_t\pi_{t+1}$. If we substitute this into the IS equation and solve forward we get

$$x_t = -\sigma E_t \sum_{j=0}^{\infty}(r_{t+j} - r_{t+j}^n) \tag{3}$$

Thus the output gap does not only depend on the current short term real interest rate. It depends on the expected long-term real rates, which in turn depend upon expected future short rates. Also note that the output gap does not only depend on the level of real rates. It depends on the difference of current and future real rates r_t and the natural real rate of interest r_t^n.

It is easy to see how a liquidity trap can arise in this simple model. Suppose the Central Bank sets $i_t = r_t^n$ at all times and the zero bound is never binding. It is easy to verify that in this case there is an equilibrium in which inflation is always zero and the output gap is zero at all times.[1] Let us now suppose that r_t^n is negative for one period and then reverts back to steady state. This can for example be rationalized by large "demand" shocks, i.e. shocks to preferences. In period 1 there will be an equilibrium with zero inflation and zero output gap. In period 0, however, the Central Bank cannot set its nominal interest rates equal to the natural rate of interest, since the nominal interest rates cannot be less than zero. The result is deflation and a negative output gap given by $x_0 = \sigma r_0^n$.[2]

3 The Deflation Bias in the New Keynesian Model

In this section we analyze the optimal solution under commitment and discretion abstracting from fiscal policy. The two main contributions of this section are: First, we analyze optimal monetary policy in the New Keynesian model taking the zero bound explictly into account. Rotemberg and Woodford (1997) take the zero bound constraint into account indirectly by imposing a constraint on the mean and variance of the nominal interest rate. This, however, does not allow for the possibility of zero interest rate policy as an optimal solution. Second, we show that there is a deflationary bias of discretionary policy in the New Keyesian economy as in the BG/KP model analyzed in Eggertsson (2001).

As showed in Woodford (2001) a second order approximation to the welfare of households results in (we discuss how this result is obtain in an extended version of the model with government spending in Appendix B):

$$E_0 \sum_{t=0}^{\infty} u(C_t, \xi_t) \approx -\Omega \sum_{t=0}^{\infty} \beta^t L_t + t.i.p. \tag{4}$$

where

$$L_t = \pi_t^2 + \lambda_x (x_t - x^*)^2 \tag{5}$$

and t.i.p stands for terms independent of policy.

We will consider the Central Bank optimization of this objective function under two different assumptions. First we consider the optimal monetary policy if the Central Bank can commit to any future policy. It can thus manipulate the expectations of the private sector by future commitments. We then show the optimal policy when the Central Bank is unable to make any commitments. This is the discretion case. Under discretion the Central Bank is unable manipulate the expectations of the private sector.

[1] This policy could for example be implemented by a policy rule of the form $i_t = r_t^n + \phi_\pi \pi_t + \phi_x x_t$ where $\phi_\pi > 1$. This would result in a determinate equilibrium of the form just described.

[2] We can for example think of this policy being implemented by modifying the policy rule in last footnote by $i_t = \max(0, r_t^n + \phi_\pi \pi_t + \phi_x x_t)$ resulting in a determinate equilibrium of the form just described.

3.1 Optimal Monetary Policy under Commitment

To illustrate the deflation bias let us suppose that x^* in the loss function is zero[3]. Consider the nonlinear minimization problem of the bank where we take the zero bound on nominal interest rates explicitly into account:

$$minE_0 \left\{ \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda_x x_t^2) \right\} \tag{6}$$

s.t.

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n) \tag{7}$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{8}$$

$$i_t \geq 0 \tag{9}$$

First combine (7) and (9):

$$E_t x_{t+1} - x_t + \sigma(E_t \pi_{t+1} + r_t^n) \geq 0 \tag{10}$$

We can write a Lagrangian for the problem with constraints (8) and (10)[4]:

$$E_0 \sum_{t=0}^{\infty} \beta^t [\frac{1}{2}(\pi_t^2 + \lambda_x x_t^2) + \phi_{1t}(x_{t+1} - x_t + \sigma(\pi_{t+1} + r_t^n)) + \phi_{2t}(\pi_t - \kappa x_t - \beta \pi_{t+1})] \tag{11}$$

We obtain the first order conditions:[5]

$$\pi_t + \phi_{2t} - \phi_{2t-1} - \beta^{-1} \sigma \phi_{1t-1} = 0 \tag{12}$$

$$\lambda_x x_t + \phi_{1t} - \beta^{-1} \phi_{1t-1} - \kappa \phi_{2t} = 0 \tag{13}$$

$$\phi_{1t} \geq 0, \qquad E_t x_{t+1} - x_t + \sigma(E_t \pi_{t+1} + r_t^n) \geq 0 \tag{14}$$

The optimal plan is characterized by the processes $\{\pi_t, x_t, \phi_{1t}, \phi_{2t}\}$ satisfying (8),(12) and (13) with equality each period. It must also satisfying the two inequalities (14) at all times and at least one of the inequalities (14) must holds with equality at each time. Note that when we consider an optimal commitment adopted at date $t = 0$, we have the initial condition:[6]

$$\phi_{1,-1} = \phi_{2,-1} = 0 \tag{15}$$

We will now do the following thought experiment: Suppose a deflationary shocks hits the economy at time $t = 0$ so that the natural real rate of return is negative. What will be the

[3] The same result would go through with a positive x^* but a larger shock would be required to create the deflation bias.

[4] See e.g. Woodford (1999a) for a discussion of the Lagrangian approach. The only difference between our approach and what is illustrated there is that we have an inequality constraint which introduces additional complications.

[5] These first order conditions are obtained by the taking the derivative of the Lagrangian with respect π_t and x_t respectively and the third condition is a complementary slackness condition.

[6] This would also be the value of these multipliers if there were no shocks to the economy. One can also interpret the commitment solution illustrated here (and the initial conditions) as a commitment made arbitarily far into the past if the probability of hitting the zero bound is very low.

optimal commitment plan at time $t = 0$? To find the solution let us suppose a simple stochastic process for r_t^n. Assume it takes on a negative value r^{nL} in period 0 and reverses to a positive "normal" r^{nH} in each period t with probability α_t. Furthermore let us suppose that it will reverse with probability 1 before some finite date T (that can be arbitrarily far in the future). Let us call the random period in which the natural rate returns to normal τ. The optimal plan of the Central Bank takes the form:[7]

$$\begin{aligned} i_t &= 0 \quad \forall \quad \text{if} \quad 0 \le t < \tau \quad \text{Thus (14) will hold with equality} \\ i_t &> 0 \quad \forall \quad \text{if} \quad t \ge \tau \quad \text{Thus } \phi_{1t} = 0 \end{aligned} \tag{16}$$

Note that under this assumption (14) becomes:

$$\begin{aligned} E_t x_{t+1} - x_t + \sigma(E_t \pi_{t+1} + r_t^n) &= 0 \qquad \text{if} \quad t < \tau \\ \phi_{1t} &= 0 \qquad \text{if} \quad t \ge \tau \end{aligned} \tag{17}$$

We first illustrate the solution of the program after the zero bound ceases to bind i.e. at the random date τ when $r_t^n = r^{nH}$. When this is the case $\phi_{1t} = 0$ and (13) becomes $x_t = (\frac{\kappa}{\lambda_x})\phi_{2t} + \frac{1}{\lambda_x}\phi_{1t-1}$. Substituting this into (8) to eliminate x_t we get the following system determining π_t, ϕ_{1t} and ϕ_{2t} for $t \ge \tau$

$$\begin{aligned} \begin{bmatrix} E_t \pi_{t+1} \\ \phi_{2t} \\ \phi_{1t} \end{bmatrix} &= \begin{bmatrix} \beta^{-1}(1 + \frac{\kappa^2}{\lambda_x}) & -\frac{\kappa^2}{\lambda_x \beta} & -\frac{\kappa}{\lambda_x \beta^2}(\kappa\sigma + 1) \\ -1 & 1 & \frac{\sigma}{\beta} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ \phi_{2t-1} \\ \phi_{1t-1} \end{bmatrix} \\ &= M \begin{bmatrix} \pi_t \\ \phi_{2t-1} \\ \phi_{1t-1} \end{bmatrix} \end{aligned} \tag{18}$$

To obtain a unique bounded solution one of the eigenvalues of the matrix M must be unstable (outside the unit circle) and the two other eigenvalues must be stable (inside the unit circle). M has the eigenvalue 0 and the two roots of the equation

$$\mu^2 - (1 + 1/\beta(1 + \kappa^2/\lambda_x)\mu + 1/\beta = 0$$

This equation has the roots $0 < \mu_1 < 1 < 1/\beta < \mu_2$, where $\mu_2 = 1/\beta/\mu_1$. Hence there is one eigenvalue that is less than one in absolute value and the 3 difference equations of (18) have a unique bounded solution of the form:

$$\pi_t = e\phi_{2t-1} + f\phi_{1t-1} \tag{19}$$

$$x_t = [\frac{1}{\beta\lambda_x} + \frac{\kappa}{\lambda_x}(\frac{\sigma}{\beta} - f)]\phi_{1t-1} + \frac{\kappa}{\lambda_x}\mu_1\phi_{2t-1} \tag{20}$$

$$\phi_{2t} = \mu_1\phi_{2t-1} + (\beta^{-1}\sigma - f)\phi_{1t-1} \tag{21}$$

[7] It is possible that it would be optimal for the Central Bank hold the nominal interest rate at zero longer than to time τ. In our numerical examples this will not be the case.

$$\phi_{1t} = 0 \tag{22}$$

Here e and f are the second and third element of the left eigenvector of the unstable eigenvalue $V = [\ -1 \quad e \quad f\ ]$. It is easy to show that $0 < e < 1$ and $f > 0$. Let us now turn to the solution for $t < \tau$. In every period when $t < \tau$ the variables $\tilde{\pi}_t$, $\tilde{x}_t$, $\tilde{\phi}_{1t}$, $\tilde{\phi}_{2t}$ satisfy 4τ linear equations given by (7) and (8) and the first order conditions. Here the hats refer to the value of the variables conditional on that the natural rate is at its low r^{nL} level:

$$\tilde{\pi}_t = \kappa \tilde{x}_t + \beta\{(1-\alpha_{t+1})\tilde{\pi}_{t+1} + \alpha_{t+1}(e\tilde{\phi}_{2t} + f\tilde{\phi}_{1t})\} \tag{23}$$

$$\begin{aligned} \tilde{x}_t = \sigma\{r_t^{nL} + (1-\alpha_{t+1})\tilde{\pi}_{t+1} + \alpha_{t+1}(e\tilde{\phi}_{2t} + f\tilde{\phi}_{1t})\} + \\ \{(1-\alpha_{t+1})\tilde{x}_{t+1} + \alpha_{t+1}(\tfrac{\kappa}{\lambda_x}\mu_1\tilde{\phi}_{2t} + (\tfrac{\kappa}{\lambda_x}(\tfrac{\sigma}{\beta} - f) + \tfrac{1}{\beta\lambda_x}))\tilde{\phi}_{1t}\} \end{aligned} \tag{24}$$

$$\tilde{\pi}_t + \tilde{\phi}_{2t} - \tilde{\phi}_{2t-1} - \beta^{-1}\sigma\tilde{\phi}_{1t-1} = 0 \tag{25}$$

$$\lambda_x \tilde{x}_t + \tilde{\phi}_{1t} - \beta^{-1}\tilde{\phi}_{1t-1} - \kappa\tilde{\phi}_{2t} = 0 \tag{26}$$

The solution for $t < \tau$ is then simply the sequence of numbers that satisfy the $\tau - 1$ difference equations given by each of the equations in (23)-(26). Along with the initial condition on $\phi_{1,-1}$, and $\phi_{2,-1}$ and the solution for $t \geq \tau$, we can solve these equations recursively and obtain the solution for the evolution of each of the variables.[8]

Using this solution method, we first consider the most simple solution. We assume that the natural rate of interest is negative at time 0 and reverses back to normal with certainty at time 1 (this is the same evolution of the natural real as is assumed by Krugman (1998)). The solid lines in figure (1) shows the path for output, inflation and nominal interest rates for optimal monetary policy in a liquidity trap. We use as calibration parameters the ones estimated by Rotemberg and Woodford (1997) (see Appendix C) and assume that the natural rate takes on the value -1% at period zero and then returns to "normal" at 3% in period 1. This simple exercise immediately establishes the first result of this paper, *optimal monetary policy under commitment in a liquidity trap results in expected inflation.*

The Central Bank creates the expectation of inflation by promising to keep nominal interest rates lower than the natural real rate once out of the trap. Thus even though the economy is in a liquidity trap in period 0 the monetary authorities can still affect output and inflation by giving promises about the future evolution of nominal interest rates. This will influence the inflation expectations of the private sector.

This result does not rely on the natural rate being negative for only one period. To illustrate, let us suppose that the natural rate becomes negative at time 0 and will return to normal in each period t with a probability α_t. Let us suppose that $\alpha_{t+1} = \frac{1}{4-t}$ so that the natural real rate of return will return to normal no later than in period 4. Figure (2) shows the evolution of the output gap and inflation assuming this simple process for the natural rate of output. The

[8] Note that we start this problem is solved "backwards" which is why it is useful to assume that the shock reverses back to normal with probability 1 at some finite date T.

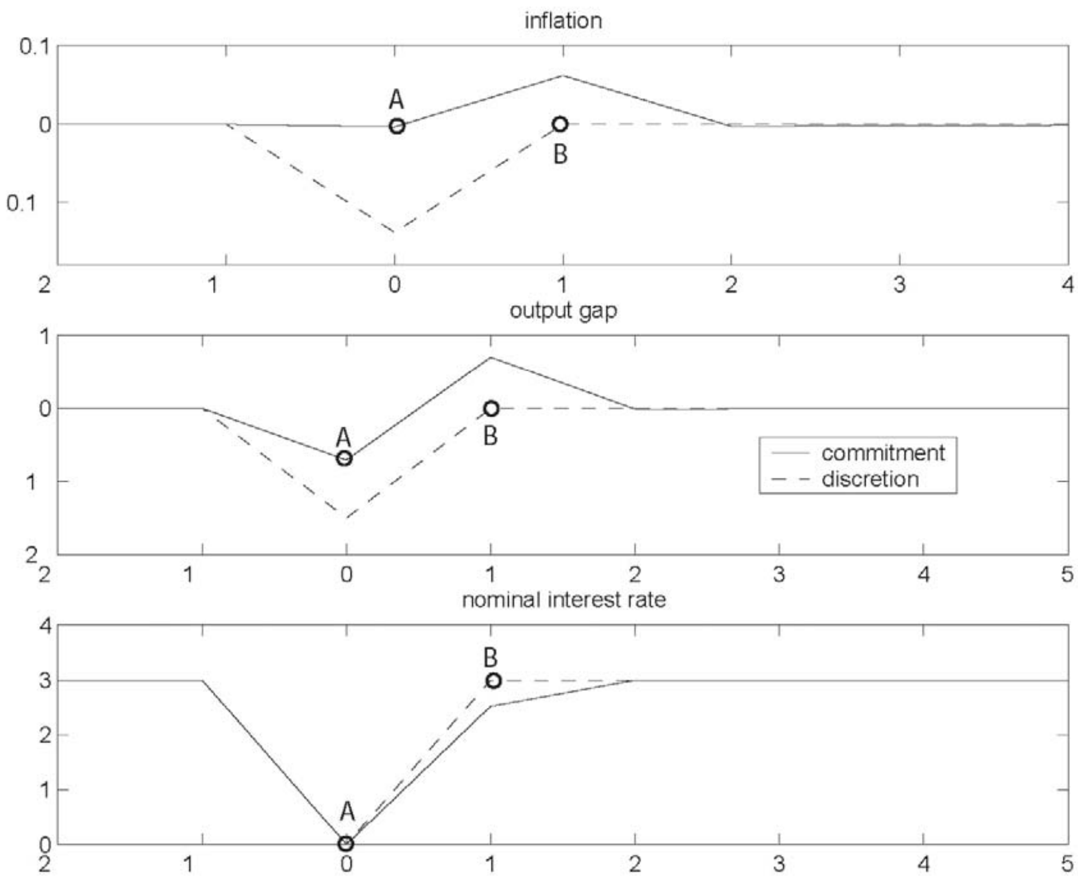

Figure 1: The deflation bias

solution method here is exactly the same as outlined above. Now the optimal policy does not only involve expected inflation once out of the trap. There is also inflation during the period in which the economy experiences negative natural rate. The intuition is exactly the same as before. To make the output gap smaller, the Central Bank keeps the difference between the real rate an the natural real rate as small as possible in all periods while not causing too much inflation.

3.2 Optimal Monetary Policy under Discretion and the Deflation Bias

Optimal policy under commitment assumes that the Central Bank can announce whatever path for the nominal interest rate it wishes and that people will take that announcement seriously. Some authors argue that this is not a realistic description of monetary policy. Clarida, Gali and Gertler (1999), for example, argue that "No major Central Bank makes any type of commitment over the future course of monetary policy." Many argue that a more realistic view of how Central Bank chooses policy is characterized by a Markovian equilibrium where the Central Bank makes no commitments about future policy. In our model the only state variable is the natural rate of interest that is only a function of the exogenous shocks. Then the expectations of the private sector about future inflation and output gap are not affected by current policy actions of the government. They are only a function of the exogenous shocks. The Central Bank then faces the same minimization problem as before but is now unable to influence the expectations of the private sector. This is a simple period by period minimization problem since the expectations of the private sector are only a function of the exogenously given state. Again we can write a Lagrangian for the problem:

$$\frac{1}{2}(\pi_t^2 + \lambda_x x_t^2) + \phi_{1t}(E_t x_{t+1} - x_t + \sigma(E_t \pi_{t+1} + r_t^n)) + \phi_{2t}(\pi_t - \kappa x_t - \beta E_t \pi_{t+1})]$$

We obtain the first order conditions:[9]

$$\pi_t + \phi_{2t} = 0 \tag{27}$$

$$\lambda_x x_t + \phi_{1t} - \kappa \phi_{2t} = 0 \tag{28}$$

$$\phi_{1t} \geq 0, \qquad E_t x_{t+1} - x_t + \sigma(E_t \pi_{t+1} + r_t^n) \geq 0 \tag{29}$$

The optimal plan is characterized by the processes $\{\pi_t, x_t, \phi_{1t}, \phi_{2t}\}$ satisfying (8),(28) and (28) with equality each period. It must also satisfy the two inequalities (29) at all times and at least one of the inequalities (29) must hold with equality each time. We illustrate the solution assuming the same stochastic process as discussed in the commitment case. The optimal plan of the Central Bank is once again of the form (16)-(17). Let us now illustrate the solution for the date $t \geq \tau$ when the natural rate of interest has reverted back to steady state. In this case $\phi_{1t} = 0$ and we can combine (8),(28) and (28) to yield:

$$\pi_t = \frac{\beta}{1 + \frac{\kappa^2}{\lambda_x}} E_t \pi_{t+1} \tag{30}$$

[9] These first order condition are obtained by the optimal choice of π_t and x_t respectively.

Since the coefficient on the right hand side is less than unity the unique bounded solution for $t \geq \tau$ is $\pi_t = x_t = 0$.

Let us now turn to the solution for $t < \tau$. In every period when $t < \tau$ the variables $\tilde{\pi}_t$, $\tilde{x}_t$, $\tilde{\phi}_{1t}$, $\tilde{\phi}_{2t}$ satisfy 4τ linear equations given by the AS equation and and the first order conditions. Again the hats refer to the value of the variables conditional on that the natural rate is at its low r^{nL} level:

$$\tilde{\pi}_t = \kappa \tilde{x}_t + \beta(1-\alpha_{t+1})\tilde{\pi}_{t+1} \tag{31}$$

$$\tilde{x}_t = \sigma\{r_t^{nL} + (1-\alpha_{t+1})\tilde{\pi}_{t+1}\} + (1-\alpha_{t+1})\tilde{x}_{t+1} \tag{32}$$

$$\tilde{\pi}_t + \tilde{\phi}_{2t} = 0 \tag{33}$$

$$\lambda_x \tilde{x}_t + \tilde{\phi}_{1t} - \kappa\tilde{\phi}_{2t} = 0 \tag{34}$$

The solution for $t < \tau$ is then simply the sequence of numbers that satisfy the $\tau - 1$ difference equations given by each of the equations in (31)-(34). Along with the solution for $t \geq \tau$ we can solve these equations recursively and obtain the solution for the evolution of each of the variables. To illustrate the solution let us again suppose that the natural rate is only negative for one period. In period 1 the Central Bank will set nominal interest rates equal to the natural rate so that $x_1 = \pi_1 = 0$. Since the nominal interest rates is set at zero in period 0 we then know that $x_0 = -\sigma r_0^n$ and $\pi_0 = -\kappa\sigma r_0^n$.

The dashed line in figure (2) shows the optimal policy under discretion in the simple case when the natural real rate of interest is negative for one period. Note that in this case once the natural real rate returns to normal the Central Bank immediately sets nominal interest rates equal to the natural rate of interest to achieve zero output gap and zero inflation. The intuition for why this must be the case is simple. Since the Central Bank wants to minimize inflation and the output gap in period 1, without any regard to what people expect it to do in period zero, it will set the output gap and inflation equal to zero. Since the private sector anticipates this behavior, it will not expect any inflation when it forms expectations at time zero. At period zero there is hence deflation and a negative output gap. This gives us the next result, *a Central Bank that minimizes $\pi_t^2 + \lambda_x x_t^2$ under discretion will generate less inflation expectations than a Central Bank that optimizes under commitment in a liquidity trap. A discretionary Central Bank will thus experience larger output gap and deflation than if it could commit.*

The Central Bank is in an interesting dilemma in a liquidity trap. The Central Bank would like the public to think it will create inflation tomorrow to lower the real rate of return but once tomorrow arrives it has incentives to deviate. It is simple to see why this must be the case by observing the two lines in figure (1). If the Central Bank announces that it will follow commitment at time 0, output and inflation will be at point A in the first and second panel in figure (1). But then when date 1 comes it can easily deviate and set interest rates at 3%. Then the economy will jump to point B in the first and second panel. The loss the Central Bank incurs in point B is much smaller than if the Central Bank would follow the commitment path. But going from point A to B cannot be a rational expectation equilibrium. The public understands the Central Bank

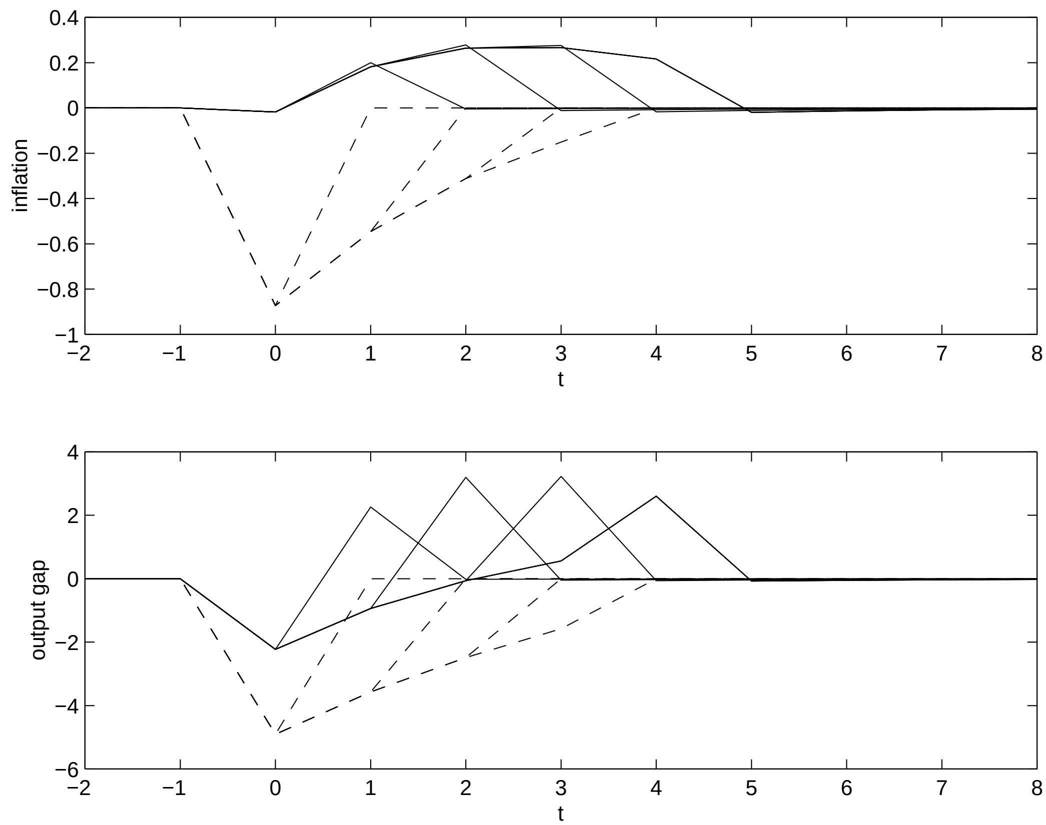

Figure 2: Figure 3: The Deflation Bias

incentives to deviate in period 1 and thus a simple announcement of the commitment path will not be taken seriously.

There is nothing special about the simple foresight path we assumed for the natural real rate of interest in figure (1). Figure (2) illustrates both the commitment and discretion path for the simple stochastic process for r_t^n discussed above giving rise to the deflation bias once again.

Our result here about the deflation bias of discretionary policy is of exactly the same nature as Eggertsson (2001). The deflation bias is therefore not a special feature of the KP/BG model. It also rises naturally in the New Keynesian model which has become the most common starting point in current discussion about monetary policy. This model has the advantage over the KP/BG model that the objectives of the government are derived from first principles.

4 Real Government Spending in a Liquidity Trap

In this section we analyze optimal monetary and fiscal policy under discretion and commitment. The two main contributions of this section are: First, we derive objectives for the government by a second order Taylor approximation of the representative household taking the welfare consequences of public spending explicitly into account. Second, assuming this objective for the government we show that fiscal policy can be particularly useful when the zero bound is binding.

The old Keynesian solution to a liquidity trap is to increase public spending. The Great Depression was explained by a collapse in private spending. In the old Keynesian models the natural way to restore full employment is to increase government spending to compensate for the private spending collapse. No role is given to monetary policy mainly because expectations are treated as exogenous. Since short term interest rates are already at zero and expectations are assumed to be fixed, monetary policy is ineffective in the old Keynesian models. In some sense our result about the deflation bias gets us back to the old Keynesian world. If the Central Bank has no credible way of influencing peoples expectations about future inflation it faces fixed expectations. Those expectations are pinned down by what the private sector thinks is optimal behavior for the Central Bank under discretion at future dates. The obvious question arises: Can the government raise output by increasing government expenditures?

Government spending can have quite powerful effect on output in our model. We can think of these effects as working through two separate channels. Through the first channel government spending increases natural level of output or the level of output that would be produced under flexible prices. This is the channel that has been extensively documented in the RBC literature see e.g. Baxter and King (1993) and the references there in. In the context of our model just as in Baxter and King the natural rate of output increases if government expenditures increases. This happens because people are willing to work more. Thus as shown in Appendix A the natural level of output can be expressed as:

$$\hat{Y}_t^n = \frac{\sigma^{-1} d_t + \omega q_t}{\omega + \sigma^{-1}} + \frac{\sigma^{-1}\gamma}{\omega + \sigma^{-1}} \hat{G}_t \tag{35}$$

where $\hat{G}_t$ is the percentage deviation of government expenditures from its steady state $G_t \equiv \log(\hat{G}_t/\bar{G})$. The shock d_t can be interpreted as a "demand" shock, i.e. shock to preferences. Alternatively it can be interpreted as variations in exogenous spending. q_t can be interpreted as productivity shocks. How the shocks relate to preferences and the production technology of the model is illustrated in Appendix A.

There is another channel through which government spending influences output in our model. We call this channel the *Keynesian channel* of government spending. The Keynesian channel of government spending is only at work if prices are sticky. To illustrate we augment the New Keynesian IS-LM model by adding government spending. The derivation is presented in the Appendix A. Both the IS and AS equations remain unchanged but we obtain additional equations that relate the natural rate of interest and the output gap to government spending:

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n) \qquad \text{(IS)} \tag{36}$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \qquad \text{(AS)} \tag{37}$$

$$r_t^n = r_t^{nG} + \sigma^{-1}(1-\delta)\gamma(\hat{G}_t - E_t \hat{G}_{t+1}) \tag{38}$$

$$x_t = \hat{Y}_t - \hat{Y}_t^{nG} - \delta\gamma \hat{G}_t \tag{39}$$

$$0 < \delta = \frac{\sigma^{-1}}{\sigma^{-1} + \omega} < 1$$

Here r_t^{nG} is the natural rate of interest holding G_t at its steady state level. Similarly $\hat{Y}_t^{nG}$ is percentage deviation of the natural level of output from steady state holding G_t at its steady state level. Both r_t^{nG} and $\hat{Y}_t^{nG}$ are exogenous terms illustrated in Appendix A. Once again r_t^n refers to the natural rate of interest. It is the real interest rate that would equilibrate the market if prices were flexible.

In our model increasing government spending does not only increase the natural rate of output, it also increases the natural real rate of interest. It is easty to see why this is the case. The natural rate of interest is simply the price of output today relative to tomorrow if prices are flexible. If the government spends more today holding spending tomorrow constant, the price of output today must rise relative to its price tomorrow in a flexible price equilibrium. To illustrate the Keynesian channel of government spending suppose that r_t^{nG} is negative at time 0 and reverts back to steady state in the next period. Similarly suppose that the expected output gap and inflation in period 1 are zero and that government spending will be at steady state in period 1. If the Central Bank reduces the nominal interest rate down to zero we can write the output gap as:

$$x_0 = \sigma r_t^{nG} + (1 - \delta)\gamma\hat{G}_0$$

A temporarily increase in government spending will increase the natural rate of interest, having positive effects on output if the Central Bank holds nominal interest rates constant. The "multiplier" of government spending in this simple example is given by $(1 - \delta)$.[10] There is work in progress by the author that shows that this "multiplier" is increased if some consumers are liquidity constrained.[11] Note that government spending is influencing output through the same channel as monetary policy does under normal circumstances. The output gap depends on the difference between the real rate, $r_t = i_t - E_t\pi_{t+1}$, and the natural rate of interest r_t^n. To keep the output gap at zero the Central Bank simply needs to track the natural rate of interest by the real rate. There is no reason to use fiscal policy under "normal circumstances" to achieve this goal. Then the Central Bank can simply lower the nominal interest rate (supposing expectations are fixed) to close the gap between r_t and r_t^n. If the zero bound is binding, however, fiscal policy can be useful. If the Central Bank keeps the nominal interest rate at zero, fiscal policy can reduce this gap by increasing the natural real rate. *In principle the government could in fact close the output gap with government spending in a liquidity trap and set inflation to zero.* To see this, suppose for example that the natural rate of interest is negative because of the temporarily collapse in exogenous spending (e.g. due to shocks to preferences or variations in autonomous spending – see Appendix A). The government can offset the effect this has on the natural rate of interest by increasing spending. If the government increases spending 1 to 1 corresponding to the negative

[10] Here we can write $\gamma\hat{G}_0 \approx \frac{G_0 - G}{Y}$ so that a 1% increase in spending as a fraction of GDP leads to $(1 - \gamma)$% increase in the output gap.

[11] In particular suppose α fraction of consumers are "liquidity contraint" so that they spend all their income. Preliminary results suggest that then this multiplier is $\frac{(1-\delta)}{1-\alpha}\gamma$.

shocks the natural rate is at steady state at all times. Then there is an equilibrium in which the output gap and inflation are zero at all times and the nominal interest rate is at steady state. This simple “solution” leaves more question than it answers. Although this may close the output gap what are the implications for welfare? Furthermore, current government expenditures are not all that matters. Expectations of the private sector about future government expenditures are just as important. Thus to understand to effects of government spending today we need a theory of how future policy is determined.

In this paper we assume that the government chooses fiscal policy in each and every period to maximize social welfare. We characterize the welfare of the economy by a second order approximation of the representative household utility. We suppose that the representative household derives utility from both private consumption and public expenditures, i.e. we will assume that utility is given by:

$$U_t = u(C_t, \xi_t) + w(G_t, \xi_t) - v(y_t(i), \xi_t) \tag{40}$$

where u is the utility of private consumption C_t, w is the utility the household derives from public spending G_t and ξ_t is a vector of random disturbances. The function $v(y_t(i))$ denotes the disutility of working (see further discussion in Appendix A about the utility of the representative household). For simplicity we assume that government purchases enters additively in the utility function. What this means is that when the government increases it's purchases it will have no substitution effect on the consumption of the representative household. What we have in mind when discussing government expenditures in this paper are not expenditures such as food stamps. Rather we mean expenditures such as roads, schools, military spending and airports whose existence has no obvious effects on the representative household consumption choices.

We show in Appendix B that a second order expansion of the representative household utility function of the form (40) can be expressed as:

$$E_0 \sum_{t=0}^{\infty} u(C_t, \xi_t) + w(G_t, \xi_t) - v(y_t(i), \xi_t) \approx -\Omega \sum_{t=0}^{\infty} \beta^t L_t + t.i.p. \tag{41}$$

where

$$L_t = \pi_t^2 + \lambda_x {x_t}^2 + \lambda_G (\hat{G}_t - \hat{G}_t^n)^2 \tag{42}$$

Here λ_x and $\lambda_{\hat{G}_t}$ are both a function of the structural parameters of the utility of the household and the production technology. $\hat{G}_t^n$ is the time varying target level of government consumption. We call this the natural rate of government consumption. It is the optimal government consumption spending that would be chosen by fiscal authorities if prices were perfectly flexible. Recall from our discussion in section 2 that if the zero bound is never binding there is an equilibrium in which the output gap and inflation are zero at all times. In this case, the government would set government spending equal to the natural rate at all times. Thus in the absence of the zero bound there is a policy in which the government can minimize its losses at zero at all times. *It is the presence of the zero bound on the short term nominal interest rate that makes it interesting to consider increasing/decreasing government spending beyond the natural rate.*

4.1 Optimal Fiscal and Monetary Policy under Discretion

We now illustrate the discretionary or Markov solution when both monetary and fiscal policy are used to maximizes social welfare. Since the only state variables are the exogenous shocks the expectations of the private sector are not affected by the governments actions. Since we are assuming Ricardian equivalence the evolution of taxes and debt have no effect on the equilibrium outcome. The problem of the governments is

$$minE_0\left\{\sum_{t=0}^{\infty}\beta^t(\pi_t^2+\lambda_x x_t^2+\lambda_G(\hat{G}_t-\hat{G}_t^n)^2)\right\} \tag{43}$$

s.t.

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1} - r_t^n) \tag{44}$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{45}$$

$$i_t \geq 0 \tag{46}$$

$$x_t = x_t^{nG} - \delta\gamma\hat{G}_t \tag{47}$$

$$r_t^n = r_t^{nG} + \sigma^{-1}(1-\delta)\gamma(\hat{G}_t - E_t\hat{G}_{t+1}) \tag{48}$$

where we have defined the variable x_t^{nG} to be $x_t^{nG} \equiv y_t - y_t^{nG}$. It is convenient to substitute for x_t and r_t^n in equation (44). Combining the resulting equation and (46) gives us the inequality constraint:

$$E_t x_{t+1} - x_t + \sigma(E_t\pi_{t+1} - r_t^n) = E_t x_{t+1}^{nG} - x_t^{nG} + \sigma(E_t\pi_{t+1} - r_t^{nG}) + \gamma(\hat{G}_t - E_t\hat{G}_{t+1}) \geq 0 \tag{49}$$

We form a Lagrangian where we minimize (43) subject to (45) (where we have substituted in for x_t by (47)) and the inequality constraint (49). This is a static minimization problem since the government actions do not influence expectations. We obtain the same first order conditions as when the government minimized under discretion using only monetary policy, i.e. (27)-(29).[12] In addition we obtain a first order condition that stems from the optimal choice of real government spending:

$$-\lambda_x\delta\gamma x_t + \lambda_G(\hat{G}_t - \hat{G}_t^n) + \delta\kappa\gamma\phi_{2t} + \gamma\phi_{1t} = 0 \tag{50}$$

where again ϕ_{1t} is the Lagrangian multiplier associated with constraint (49), and ϕ_{2t} is the multiplier associated with (45).

The optimal plan under discretion is characterized by the processes $\{\pi_t, x_t^{nG}, \hat{G}_t, \phi_{1t}, \phi_{2t}\}$ satisfying (27)-(29), (45) and (50) with equality each period. It must also satisfy the two inequalities (49) at all times and at least one of the inequalities (49) must hold with equality at each time. The optimal plan of the Central Bank is once again of the form (16)-(17). The solution for $t \geq \tau$ when the natural rate of interest has reverted back to steady state is of the same form as illustrated in

[12] In this case we obtain the first order condition by the optimal choice of x_t^{nG} which yields the same first order conditions as if x_t was the choice variable.

(30) so that $\pi_t = x_t = \phi_{2t} = 0$. Then by (50) $\hat{G}_t = \hat{G}_t^n$ for $t \geq \tau$. The solution for $t < \tau$ is again characterized by the equations (31)-(34) where one replaces the exogenous term r_t^{nL} in (24) with:

$$r_t^{nL} = r_t^{nGL} + \sigma^1(1-\delta)\gamma(\tilde{G}_t - (1-\alpha_{t+1})\tilde{G}_{t+1}).$$

In addition there is an additional equation that stems from the optimal choice of government spending:

$$-\lambda_x\delta\gamma\tilde{x}_t + \lambda_G(\tilde{G}_t - \tilde{G}_t^n) + \gamma\delta\kappa\tilde{\phi}_{2t} + \gamma\tilde{\phi}_{1t} = 0 \quad (51)$$

The solution for $t < \tau$ is then simply the sequence of numbers that satisfy the $\tau - 1$ difference equations given by each of the equations in (31)-(34) and (51). Along with the solution for $t \geq \tau$ we can solve these equations recursively and obtainv the solution for the evolution of each of the variables. To illustrate the result let us once again consider the most simple case (we will later consider a more general stochastic process). There is an unexpected shift in r_t^{nG} in period 0 so that it takes on a negative value. It returns to "normal" in period 1 with certainty. Figure (3) shows the evolution of inflation, output gap and the government expenditures in a liquidity trap assuming the simple perfect foresight path of the exogenous part of the natural real rate. Again we use as calibration parameters, the ones estimated by Rotemberg and Woodford (1997) (see Appendix C). The line with the circled marker shows the evolution for each of the variables when the government optimizes under discretion. Here the government uses not only the nominal interest rates as its instrument but also government expenditures. The dotted line shows the optimal policy of the government under discretion when it does not use fiscal spending as an instrument but only monetary policy (thus we assume that government spending are kept at the natural rate of spending at all times).

There is a considerable improvement in the equilibrium outcome when the government uses discretionary spending. The output gap and inflation are cut almost by a half in comparison with the discretionary case when there the government does not use fiscal spending. The price of this reduction in the output gap and deflation is that now government expenditures have to deviate from their optimal level. The third panel shows the how government expenditures deviate from the natural rate that is determined by G_t^n. It thus illustrates to what extent government expenditures are increased because of the zero bound. If there were no zero bound on nominal interest rates there would be no reason for the government to deviate from its target level of government expenditures. Our result thus validates the Keynesian claim that there is something special about a liquidity trap when it comes to government expenditures. This establishes our next result that *increasing government spending is not going to be subject to a credibility problem to the same extent as monetary policy. Increasing public spending in a liquidity trap will thus reduce the output gap and increase the price level as the old Keynesian literature suggests.*

4.2 Optimal Fiscal and Monetary Policy under Commitment

In the last section we only considered optimal fiscal and monetary policy if the government is unable to commit to the future path of government spending and interest rates. Let us now

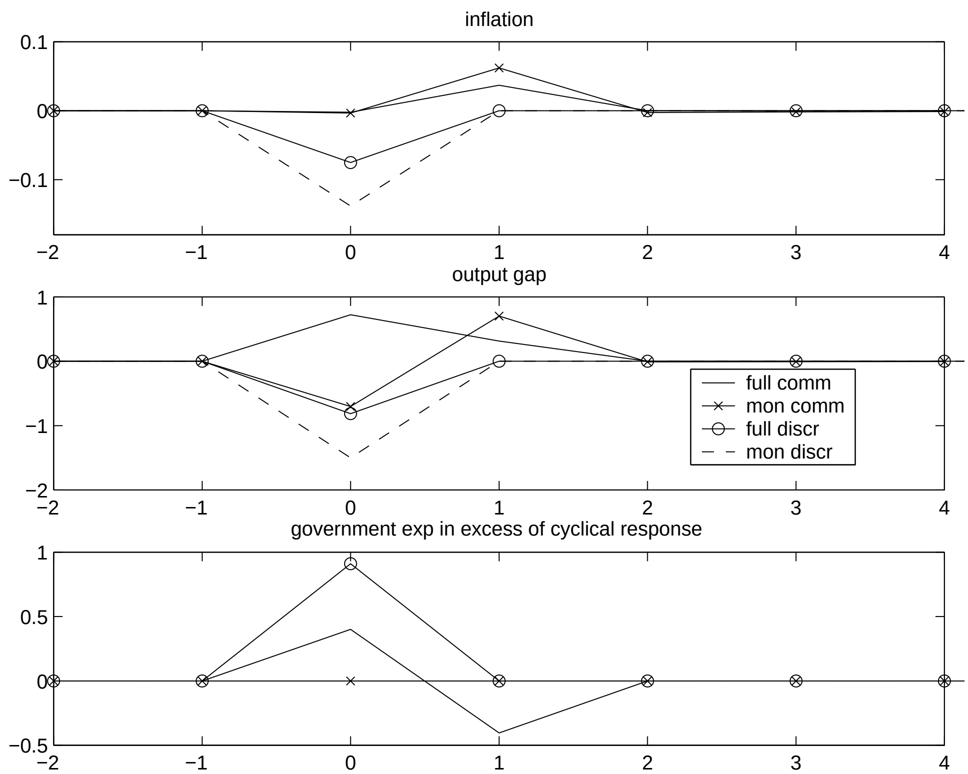

Figure 3: Optimal monetary and fiscal policy under discretion and commitment

consider what would be the optimal policy if the government could commit to the paths of both interest rates and government expenditures. The minimization problem of the government under full commitment is the same as under discretion illustrated in (43)-(48) except for that now it can influence the expectations of the private sector. The way we solve this problem is exactly the same as illustrated for optimal monetary policy under commitment section 3 except for that now we also choose $\hat{G}_t$ optimally. Once again we can combine use the inequality constraint:

$$E_t x_{t+1} - x_t + \sigma(E_t\pi_{t+1} - r_t^n) = E_t x_{t+1}^{nG} - x_t^{nG} + \sigma(E_t\pi_{t+1} - r_t^{nG}) + \gamma(\hat{G}_t - E_t\hat{G}_{t+1}) \geq 0 \quad (52)$$

We obtain exactly the same first order condition (12)-(14) as in section 5 and one additional first order condition for the optimal choice of $\hat{G}_t$[13].

$$-\lambda_x\delta x_t + \lambda_G(\hat{G}_t - \hat{G}_t^n) - \gamma\phi_{1t} + \gamma\frac{1}{\beta}\phi_{1t-1} + \gamma\delta\kappa\phi_{2t} = 0 \quad (53)$$

Once again the optimal plan is characterized by processes $\{\pi_t, x_t, \phi_{1t}, \phi_{2t}\}$ in addition to $\hat{G}_t$ satisfying (8),(12), (13) and now also (53) with equality each period, satisfying the two inequalities (14) at all times, and such that at least one of the inequalities (14)) holds with equality at each time. Our solution for period $t \geq T$ can be characterized exactly as before so that equations (19)-(22) will still hold true. $\hat{G}_t$ is then determined by (53).

The solution for $t < T$ is again characterized by the system (23) - (26) where one replaces the exogenous term r_t^{nL} in (24) with

$$\tilde{r}_t^{nL} = \tilde{r}_t^{nGL} + (1-\delta)\{\tilde{G}_t - c_1\tilde{\phi}_{1t} - c_2\tilde{\phi}_{2t} - (1-\alpha_{t+1})\tilde{G}_{t+1}\}.$$

where the coefficients c_1 and c_2 are found by substituting (19)-(22) into (53).

Again as in section 5 let us consider the most simple case when r_t^n becomes unexpectedly negative in period 0 and then goes back to normal in period 1 onwards. Figure 5 illustrates the result for a calibrated version of the model establishing the next result that *optimal fiscal and monetary policy involves counter cyclical fiscal policy and expected inflation.* Then reason for why the fiscal policy is counter cyclical is simple. Government spending is useful in a liquidity trap because it increases the natural rate of interest. This can be done in one of two ways, either increase spending in the trap relative to when out or commit to reduce it once out (beyond the natural rate). A policy maker that can commit will do both. Figure (3) illustrates the path for the output gap, inflation and government expenditures when the government optimizes under commitment. The losses that are associated with each of the policies that we have illustrated are calculated in table 1. We normalize the losses associated with the full commitment of both monetary and fiscal policy to 1. Monetary commitment refers to the policy when the government can commit to the optimal monetary policy but government expenditures are kept at the natural rate at all times. Full discretion refers to the case when the government uses both interest rates

[13] Note that here the we obtain a first order condition by making the optimal choice for x_t^{nG} but this results in exactly the same condition as if the choice varable was x_t.

and fiscal policy as instruments. Monetary discretion refers to the case when monetary policy is optimal under discretion but government expenditures are kept at the natural rate.

Full Commitment	1
Monetary Commitment	1.6
Full Discretion	2.4
Monetary discretion	4

4.3 Welfare Comparisons

<Stochastic simulations will be added here with a stochastic process for r_t^n as in previous sections and we also illustrate more general processes.>

5 Conclusions

It is often argued that government spending can only work for a short period since eventually the public debt will rise “to a limit”. Thus Krugman (1998) claims that “Japan’s debt already exceeds its gross domestic product mean that fiscal expansion has reached a limit. If the current push is no enough - and it is not - there will not be another.” In the context of our model this analysis fails for two reasons. First, increasing government spending does not require issuing government debt. We assumed that Richardian equivalence so whether or not the government issues debt is irrelevant. We could for example have assumed a balanced budget so that all spending increases are associated with higher taxes. Secondly, issuing large quantities of nominal debt is not going to have bad consequences in the context of our model even if we relax the assumption of Richardian equivalence. Ricardian equivalence could be relaxed, for example, by introducing distortionary taxes. If the government issues large volumes of nominal debt the government has strong incentives to create future inflation if taxes are distortionary. Future inflation is exactly what is needed to lower the real rate of return and stimulate private consumption in a liquidity trap. Thus the fact that the public debt typically rises when government spending are increased is rather an argument for fiscal expansion in a liquidity trap rather than against it since it will increase inflation expectations. This argument is worked out in detail in Eggertsson (2001) in the context of a Kydland/Prescott and Barro/Gordon model.

6 References:

1. Bernanke, Ben S. (1999), “Japanese Monetary Policy: A Case of Self-Induced Paralysis?” mimeo Princeton University.

2. Bernanke, Ben S., Mark Gertler and Simon Gilchrist (1998), “Financial Accelerator in a Quantitative Business Cycle Framework,” NBER Working Paper No. 6455.

3. Blinder, Alan (2000),“Monetary Policy at the Zero Lower Bound: Balancing the Risks,” mimeo, Princeton University

4. Eggertsson, Gauti (2001), "Committing to being irrsponesible: Deficit Spending to Escape a Liquidity trap," mimeo Princeton University, 2001.

5. Krugman, Paul (1998a), "It's Baaack! Japan's Slump and the return of the Liquidity Trap," Brookings Papers on Economic Activity 2:1998.

6. Krugman, Paul (1998b), "Analytical Thoughts on the Asia Crisis," http://web.mit.edu/~krugman

7. Rotemberg, Julio J., and Michael Woodford, "An Optimization-Based Econometric Framework for the calculation of Monetary Policy," NBER Macroeconomics Annual 1997, 297-346.

8. Orphanides, Athanasios, and Volker Wieland (1999), "Efficient Monetary Policy Design near Price Stability,", Working Paper, Federal Reserve Board.

9. McCallum Bennett T. (1999), "Theoretical Analysis Regarding a Zero Lower Bound on Nominal Interest Rates," mimeo Carnegie Mellon University.

10. Benhabib, Jess, Stephanie Schmitt-Grohe and Martin Uribe (2000): "The Perils of Taylor Rules," Forthcoming, Journal of Economic Theory.

11. Reifschneider, David, and John Williams (1999), "Three Lesson for Monetary Policy in a Low Inflation Era," FEDS Working Paper No. 1999-44.

12. Rogoff, Kenneth(1998), "Comments on "It's Baaack! Japan's Slump and the return of the Liquidity Trap," by Paul Krugman" Brookings Papers on Economic Activity 2:1998.

13. Sims, Christopher A. (1994), "A Simple Model for the Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," Economic Theory 4: 381-399.

14. Summers, Lawrence (1991), "Panel Discussion: Price Stability. how Should Long-Term Monetary Policy Be Determined?" Journal of Money, Credit and Banking, 23(3), 625-631.

15. Wolman, Alexander L. (1998), "Real Implication of the Zero Bound on Nominal Interest Rates," Working Paper, Federal Reserve Bank of Richmond.

16. Woodford, Michael (1996), "Control of the Public Debt: A Requirement for Price Stability?," NBER working paper No. 5684.

17. Woodford, Michael(1998), "Doing Without Money: Controlling Inflation in a Post-Monetary World," Review of Economic Dynamics 1: 173-219 .

18. Woodford, Michael (1999a), "Optimal Monetary Policy Inertia," NBER Working Paper No. 7261.

19. Woodford, Michael (1999b), "Commentary: How Should Monetary Policy Be Conducted in an Era of Price Stability?" New Challenges for Monetary Policy, Reserve Bank of Kansas City

20. Woodford, Michael (2000), "Fiscal Requirements for Price Stability," forthcoming in Journal of Money, Credit and Banking.

21. Woodford, Michael (2001), "Interest and Prices, chapter 2,3,4 and 6" unpublished, Princeton University. [Available at http://www.princeton.edu/~woodford.]

7 Appendix: The New Keynsian Model with Government spending

Here outline the new Keynesian model that has become fairly standard in the literature. See e.g. Woodford (2001) for details. There are only minor differences in our exposition stemming from the introduction of government consumption.

The household period utility is given by:

$$U_t^i = u(C_t; \xi_t) + m(\frac{M_t}{P_t}, \xi_t) + w(G_t; \xi_t) - v(y_t(i); \xi_t)$$

where β is a discount factor, ξ_t is a vector of random disturbances, and for each value ξ u and w are increasing, concave functions. The function m is increasing for each value of ξ up to a satiation level. The household directly supplies the good $y_t(i)$ and the disutility of supplying this good is given by the function v which is increasing and convex function for each value of ξ.

The argument C_t represents an index of household's purchases of all the continuum of differentiated goods produced in the economy, given by

$$C_t \equiv \left[\int c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{1-\theta}{\theta}} \tag{54}$$

with $\theta > 1$ as in Dixit Stiglitz (1977). P_t is the corresponding price index.

$$P_t \equiv \left[\int p_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{1-\theta}{\theta}}$$

The argument G_t represents government spending and is determined by the government in each period so that the representative consumer takes it as exogenously given. We assume that government purchases are made in same proportions as consumption purchases of consumers.

The household faces a budget constraint given by:

$$M_t + B_t \leq W_t + p_t(i)y_t(i) - T_t - C_t P_t$$

where M_t is money held at the end of period t, B_t is the nominal value of the bond portfolio held at the end of period t, W_t is beginning of period financial wealth, $p_t(i)$ is the price of the consumption good supplied by the household and T_t represent lump sum taxes. The households consumption plan must also satisfy the standard transversality condition.

The consumption Euler equation of the representative household implies:

$$1 + i_t = \beta^{-1} E_t[\frac{u_c(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}, \xi_{t+1})}{u_c(C_t, \frac{M_t}{P_t}, \xi_t)} \Pi_{t+1}^{-1}]^{-1} \tag{55}$$

This is what is often referred to as the "IS" equation in the literature. Optimal money holdings implies a money demand equation:

$$\frac{m_{\frac{M}{P}}(\frac{M_t}{P_t}, \xi_t)}{u_c(C_t, \xi_t)} = \frac{i_t}{1 + i_t} \tag{56}$$

This equation defines money demand or what is often referred as the "LM" equation. Utility is increasing in real money balances. As some finite level of real money balances further holding of money adds nothing to utility. The left hand side of (56) is therefore weakly positive. Thus there is a zero bound on the short term nominal interest rate.

$$\dot{i}_t \geq 0 \tag{57}$$

The real marginal cost of supplying good i is:

$$\frac{v_y(y_t(i);\xi_t)}{u_c(C_t;\xi_t)}$$

The demand for good i is given by:

$$y_t(i) = Y_t(\frac{p_t(i)}{P_t})^{-\theta}$$

In a flexible price equilibrium each supplier will equate marginal cost of supplying good i and marginal revenue:

$$\frac{p_t(i)}{P_t} = \mu\frac{v_y(y_t(i);\xi_t)}{u_c(C_t;\xi_t)}$$

where $\mu = \frac{\theta}{\theta-1}$. Following Woodford (2001) we define the natural rate of output as the output produced under flexible prices. In our computation of the equilibrium responses to shocks we make use of a log-linear approximation to the equilibrium conditions of our model, expanding in terms of percentage deviation of various state variables from their steady-state values (their constant values in the absence of all stochastic disturbances). The natural rate of output can be approximated by:

$$\hat{Y}_t^n = \frac{\sigma^{-1}d_t + \omega q_t}{\omega + \sigma^{-1}} + \frac{\sigma^{-1}\gamma}{\omega + \sigma^{-1}}\hat{G}_t \tag{58}$$

using market clearing so that $Y_t = G_t + C_t$ and defining $d_t \equiv \sigma\frac{u_{c\xi}}{u_c}\xi_t$, $q_t \equiv -\frac{v_{y\xi}\xi_t}{Y v_{yy}}$, $\sigma \equiv -\frac{u_c}{u_{cc}\bar{y}}$,$\omega \equiv -\frac{v_y}{v_{yy}y}$, $\gamma \equiv \frac{\bar{G}}{\bar{Y}}$ and $\hat{G}_t = \frac{G_t-\bar{G}}{\bar{G}}$. We define $\hat{Y}_t^{nG} = \frac{\sigma^{-1}d_t+\omega q_t}{\omega+\sigma^{-1}}$ as the percentage deviation of the natural rate of output from its steady state that cannot be explained by variation in government spending. The Euler equation (55) can be linearized as:

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1} - r_t^n)$$

where $x_t = \hat{Y}_t - \hat{Y}_t^n$ denotes the output gap and r_t^n is the natural rate of interest that can be expressed (in terms of percentage deviation from steady state) as:[14]

$$\hat{r}_t^n \equiv \frac{\sigma^{-1}\omega}{\sigma^{-1}+\omega}(d_t - d_t) - \frac{\sigma^{-1}\omega}{\sigma^{-1}+\omega}(q_t - q_{t+1}) + \frac{\sigma^{-1}\omega\gamma}{\sigma^{-1}+\omega}(\hat{G}_t - E_t\hat{G}_{t+1})$$

The natural rate of interest is the real rate of interest that would result in an equilibrium where output is equal to the natural rate of output at all times. We define $\hat{r}_t^{nG} = \frac{\sigma^{-1}\omega}{\sigma^{-1}+\omega}(d_t - d_t) -$

[14] Note that i_t and r_t^n have the same steady state value so we do not have do write them in terms of deviation from steady state in the IS equation.

$\frac{\sigma^{-1}\omega}{\sigma^{-1}+\omega}(q_t - q_{t+1})$ as the percentage deviation of the natural rate of interest from its steady state that cannot be explained by variations in government spending. We can write:

$$\hat{r}_t^n = \hat{r}_t^{nG} + \sigma^{-1}(1-\delta)\gamma(\hat{G}_t - E_t\hat{G}_{t+1})$$

where we have defined the coefficient $0 < \delta = \frac{\sigma^{-1}}{\sigma^{-1}+\omega} < 1$. We assume as Calvo (1983) that each household resets it prices with a probability α in every period that is independent of whether or not the household has reset its prices in previous periods. The aggregate supply equation resulting from the maximization problem of the representative household has been illustrated by several authors, see e.g. Woodford (2001). It can derived by the optimal pricing decision of the representative household resulting in the log-linear equation:

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1}$$

8 Appendix B: Second Order Taylor Expansion of Utility

We abstract from the effects of real money balances on the welfare criterion (see Woodford (2001) for that extension). We assume there is a continuum of households of measure 1. Thus social welfare is given by $\int_0^1 U_t(i)di$.

Following Woodford (2001) we do a second order Taylor expansion of the utility function (see detailed discussion in Woodford (2001) for the validity of this approach). The first term in the period utility function of the representative household yields:

$$\begin{aligned} u(Y_t - G_t; \xi_t) &= \bar{u} + u_c\tilde{Y}_t + u_\xi\xi_t - u_c\tilde{G}_t + \frac{1}{2}u_{cc}\tilde{Y}_t^2 - \frac{1}{2}u_{cc}\tilde{G}_t^2 \\ &\quad + u_{c\xi}\xi_t\tilde{Y}_t - u_{cc}\tilde{G}_t\tilde{Y}_t - u_{c\xi}\tilde{G}_t\xi_t + \frac{1}{2}\xi_t' u_{\xi\xi}\xi_t + \mathcal{O}\left(||\xi||^3\right) \\ &= \bar{u} + \bar{Y}u_c(\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2) - \bar{G}u_c(\hat{G}_t + \frac{1}{2}\hat{G}_t^2) + u_\xi\xi_t + \frac{1}{2}\bar{Y}^2u_{cc}\hat{Y}_t^2 \\ &\quad - \frac{1}{2}\bar{G}^2u_{cc}\hat{G}_t^2 - u_{cc}\bar{Y}\bar{G}\hat{Y}_t\hat{G}_t + u_{c\xi}\bar{Y}\xi_t\hat{Y}_t - u_{c\xi}\bar{G}\xi_t\hat{G}_t + \frac{1}{2}\xi_t' u_{\xi\xi}\xi_t + \mathcal{O}\left(||\xi||^3\right) \\ &= \bar{Y}u_c\{\hat{Y}_t + \frac{1}{2}(1-\sigma^{-1})\hat{Y}_t^2 + \sigma^{-1}d_t\hat{Y}_t\} - u_c\bar{G}\{\hat{G}_t + \frac{1}{2}(1-\sigma^{-1}\gamma)\hat{G}_t^2 \\ &\quad + \sigma^{-1}d_t\hat{G}_t\} + u_c\bar{Y}\sigma^{-1}\hat{Y}_t\hat{G}_t + t.i.p. + \mathcal{O}\left(||\xi||^3\right) \end{aligned} \tag{59}$$

Here the first line represents the usual Taylor expansion in which $u \equiv u(Y;0)$ and $\tilde{Y}_t \equiv Y_t - \bar{Y}$, and we assume that the fluctuation in $\tilde{Y}_t$ are only of order $O(\|\xi\|)$. The second line substitutes for $\tilde{Y}_t$ in terms of $\hat{Y}_t \equiv \log(Y_t/\bar{Y})$, using the Taylor series expansion $Y_t/\bar{Y} = 1 + \hat{Y}_t + \frac{1}{2}\hat{Y}_t^2 + \mathcal{O}\left(||\xi||^3\right)$. Similarly we define $\hat{G}_t = \log(G_t/\bar{G})$. The third line collects together in "t.i.p." all of the terms that are independent of policy (as they involve only constants and exogenous variables) and collects the other terms in a convenient way.

A second order expansion of the second term (by exactly the same logic as outlined above) yields:

$$w(G_t, \xi_t) = \bar{G}w_G\{\hat{G}_t + \frac{1}{2}(1-\sigma_G^{-1})\hat{G}_t^2 + \sigma_G^{-1}d_t^G\hat{G}_t\} + \mathcal{O}\left(||\xi||^3\right) \tag{60}$$

where $d_t^G \equiv \sigma_G^{-1}\frac{w_{G\xi}}{w_G}\xi_t$ and $\sigma_G \equiv -\frac{w_G}{w_{GG}\bar{G}}$. Following Woodford (2001) we can approximate the disutility of working by:

$$v(y_t(i);\xi_t) = \bar{Y}u_c\{(1-\Phi)y_t(i) + \frac{1}{2}(1+\omega)\hat{y}_t(i)^2 - \omega q_t y_t(i)\} + t.i.p. + \mathcal{O}\left(||\xi||^3\right) \quad (61)$$

where $\hat{y}_t(i) \equiv \log(y_t(i)/\bar{Y})$. Here the the parameter Φ summarizes the overall distortion in the steady-state output level as a result of both taxes and market power. The steady state output satisfies:

$$\frac{v_y(Y;\xi_t)}{u_c(Y;\xi_t)} = \frac{1-\tau}{\mu} \equiv 1-\Phi$$

where τ is the constant proportional tax rate on sales proceeds and μ is the desired markup as a result of suppliers' market power. It is assumed that Φ is small (i.e. of order $\mathcal{O}(||\xi||)$) which allows us to use our log-linear approximation to the model structural equations in welfare comparison. It also allows us to make use of the log-linear approximation $log(\bar{Y}/Y^*) = -(\omega+\sigma^{-1})^{-1}\Phi$ where Y^* is the efficient level of output (we used this expression to replace v_y by $(1-\Phi)u_c$ in (61) and the assumption that Φ is of order $\mathcal{O}(||\xi||)$). Integrating (61) over all of the differentiated goods that are of measure 1 we obtain:

$$\int_0^1 v(y_t(i);\xi_t) = \bar{Y}u_c\{(1-\Phi)\hat{Y}_t+\frac{1}{2}(1+\omega)\hat{Y}_t^2-\omega q_t\hat{Y}_t+\frac{1}{2}(\theta^{-1}+\omega)var_i\hat{y}_t(i)\}+t.i.p.+\mathcal{O}\left(||\xi||^3\right) \quad (62)$$

We linearize around a steady state level at the optimal level of government expenditures. This implies that in steady state $u_c = w_G$ i.e. the marginal utility of consumption must be equal to the marginal utility of government spending. Furthermore we assume that the intertemporal elasticity of substitution of government and private spending is equal so that $\sigma = \sigma_G$. Combining (59) and (60) yields:

$$\begin{aligned} & \bar{Y}u_c\{\hat{Y}_t + \frac{1}{2}(1-\sigma^{-1})\hat{Y}_t^2 + \sigma^{-1}d_t\hat{Y}_t\} - \bar{G}u_c\frac{1}{2}\sigma^{-1}(1-\gamma)\hat{G}_t^2 - \quad (63) \\ & u_c\bar{G}\hat{G}_t(\sigma^{-1}d_t - \sigma_G^{-1}d_t^G) + u_c\bar{Y}\sigma^{-1}\hat{Y}_t\hat{G}_t + t.i.p. + \mathcal{O}\left(||\xi||^3\right) \\ = \; & \bar{Y}u_c\{\hat{Y}_t + \frac{1}{2}(1-\sigma^{-1})\hat{Y}_t^2 + \sigma^{-1}d_t\hat{Y}_t\} + u_c\bar{Y}\sigma^{-1}\hat{Y}_t\hat{G}_t - \\ & \bar{G}u_c\frac{1}{2}\sigma^{-1}(1-\gamma)\hat{G}_t^2 - u_c\bar{G}\hat{G}_t\sigma^{-1}(d_t - d_t^G) + t.i.p. + \mathcal{O}\left(||\xi||^3\right) \\ = \; & \bar{Y}u_c\{\hat{Y}_t + \frac{1}{2}(1-\sigma^{-1})\hat{Y}_t^2 + \sigma^{-1}d_t\hat{Y}_t\} + u_c\bar{Y}\sigma^{-1}\hat{Y}_t\hat{G}_t - \\ & \frac{u_c\bar{G}}{2}\sigma^{-1}(1-\gamma)\{\hat{G}_t - \hat{G}_t^n\}^2 + t.i.p. + \mathcal{O}\left(||\xi||^3\right) \end{aligned}$$

where $\hat{G}_t^n = -\frac{d_t - d_t^G}{1-\gamma}$. Note that the natural rate of government spending varies because of variations in the marginal utility of private and public consumption. If the preference shocks in the vector ξ_t enters the utility of private and public consumption in the same fashion $\hat{G}_t^n = 0$ at all times. Combining (62) and (63) yields:

$$\begin{aligned} U_t &= \bar{Y}u_c\{\Phi\hat{Y}_t - \frac{1}{2}(\sigma^{-1}+\omega)\hat{Y}_t^2 + (\sigma^{-1}d_t + \sigma^{-1}\hat{G}_t + \omega q_t)\hat{Y}_t - \frac{1}{2}(\theta^{-1}+\omega)var_i y_t(i)\} \\ &\quad -\frac{u_c G}{2}\sigma^{-1}(1-\gamma)\{\hat{G}_t - \bar{G}_t\}^2 + t.i.p. + \mathcal{O}\left(||\xi||^3\right) \\ &= -\frac{Yu_c}{2}\{(\sigma^{-1}+\omega)(x_t - x^*)^2 + (\theta^{-1}+\omega)var_i y_t(i)\} - \frac{u_c G}{2}\sigma^{-1}(1-\gamma)\{\hat{G}_t - \hat{G}_t^n\}^2 + t.i.p. + \mathcal{O}\left(||\xi||^3\right) \end{aligned} \tag{64}$$

Our CES preferences over differentiated goods imply that each supplier faces a constant-elasticity demand curve of the form:

$$log y_t(i) = \log Y_t - \theta(\log p_t(i) - \log P_t)$$

It follows from this that

$$var_i \log y_t(i) = \theta^2 var_i \log p_t(i)$$

so that (64) can be written as:

$$U_t = -\frac{Yu_c}{2}\{(\sigma^{-1}+\omega)(x_t - x^*)^2 + \theta(1+\omega\theta)var_i \log p_t(i)\} - \frac{u_c G}{2}\sigma^{-1}(1-\gamma)\{\hat{G}_t - \bar{G}_t\}^2 + t.i.p. + \mathcal{O}\left(||\xi||^3\right) \tag{65}$$

To express the price dispersion term $Var_z \ln p_t(z)$ as a function of inflation we follow the same steps as in Woodford (2001) to write:

$$var_i \ln p_t(i) = \alpha Var_z \ln p_{t-1}(i) + \frac{\alpha}{1-\alpha}\pi_t^2 + \mathcal{O}\left(||\xi||^3\right) \tag{66}$$

Iterating backwards to time 0 this gives:

$$var_i \ln p_t(i) = \alpha^{t+1} Var_z \ln p_{-1}(i) + \frac{\alpha}{1-\alpha}\sum_{s=0}^{t}\alpha^s \pi_{t-s} + \mathcal{O}\left(||\xi||^3\right)$$

Thus if we take the discounted value of these terms over all period $t \geq 0$ we obtain:

$$\sum \beta^t var_i \ln p_t(i) = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\sum_{t=0}^{\infty}\beta^t \pi_t^2 + t.i.p. + \mathcal{O}\left(||\xi||^3\right)$$

Substituting this into (65) and expressing it as the discounted utility at time zero yields:

$$\sum_{t}^{\infty}\beta^t U_t = -\Omega\sum_{t=0}^{\infty}\beta^t L_t + t.i.p. + \mathcal{O}\left(||\xi||^3\right)$$

where in the normalized loss function is given by:

$$L_t = \pi_t^2 + \lambda_x(x_t - x^*)^2 + \lambda_G(\hat{G}_t - \hat{G}_t^n)^2$$

where $\lambda_x = \kappa/\theta$ and $\lambda_G = \frac{\kappa\sigma^{-1}\gamma(1-\gamma)}{\theta(\sigma^{-1}+\omega)}$ and $x^* \equiv \log(Y^*/\bar{Y})$

9 Appendix C: The Rotemberg Woodford Calibration

For detailed discussion of the estimation and interpretation of the estimation of the structural parameters in Rotemberg and Woodford (1997) see Woodford (2001). We use the following parameter values: $\kappa = 0.024$, $\beta = 0.99$, $\sigma = 6.36$, $\theta = 7.88$, $\omega = .4729$, $\gamma = 0.25$. In the numerical exercise we assume that $d_t^G = 0$ at all times so that the shocks only affect the marginal utility of consumption. This would be the case if one interprets the shocks as exogenous variations in autonomous spending. The results do not hinge on this simplification and revised drafts will include other specification of the shock process.