The Deflation Bias and Committing to being Irresponsible

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Abstract

I model deflation, at zero nominal interest rate, in a microfounded general equilibrium model. I show that deflation can be analyzed as a credibility problem if the government has only one policy instrument, i.e. increasing money supply by open market operations in short-term bonds, is faced with temporary negative demand shocks and cannot commit to future policies. This is the deflation bias of discretionary policy. I propose several policies to solve the deflation bias. They involve printing money or issuing nominal debt and either 1) cutting taxes, 2) buying real assets such as stocks, or 3) purchasing foreign exchange. The government credibly "commits to being irresponsible" by using these policy instruments. It commits to higher money supply in the future so that the private sector expects inflation instead of deflation. This is optimal since it curbs deflation and increases output by lowering the real rate of return.

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Can the government lose control over the price level so that no matter how much money it prints, it has no effect on inflation or output? Ever since Keynes' General Theory this question has been hotly debated. Keynes answered yes, Friedman and the monetarists said no. Keynes argued that increasing the money supply has no effect at low nominal interest rates. This has been coined as the liquidity trap. The zero short-term nominal interest rate in Japan today, together with the lowest short-term interest rate in the US in 45 years, make this old question interesting again. The Bank of Japan (BOJ) has nearly doubled the monetary base over the past 5 years, yet the economy still suffers deflation, and growth is stagnant.[1] Was Keynes right? Is increasing money supply ineffective when the interest rate is zero? This paper revisits this classic question using a microfounded intertemporal general equilibrium model and assuming rational expectations. The results suggest that both the Keynesian and the monetarist view can be supported under different assumptions about policy expectations.

The paper has three key results. The first is that monetary and fiscal policy have no effect in a liquidity trap *if* expectations about future money supply are independent of past policy decisions, and certain restrictions on fiscal policy apply. This is shown in a standard New Keynesian general equilibrium model widely used in the literature. The message is not that monetary and fiscal policy are irrelevant. Rather, the point is that monetary and fiscal policy have their largest impact in a liquidity trap through expectations. This indicates that the old fashion IS-LM model is a blind alley since expectations are assumed to be exogenous in that model. In contrast, expectations are at the heart of this study.

I assume that expectations are rational. The government maximizes social welfare and I analyze two different equilibria in a liquidity trap. First I assume that the government is able to commit to future policy. This is the commitment equilibrium. Then I assume that the government is unable to commit to any future policy apart from paying back the nominal value of its debt. This is the Markov equilibrium (formally defined by Maskin and Tirole (2001)). I explore optimal policy when the natural rate of interest – assumed to be exogenous in the model – is temporarily negative, causing the zero bound to be binding. The optimal commitment is to commit to low future interest rates, modest inflation and an output boom once the natural rate of interest returns back to normal as in Jung et al (2001) and Eggertsson and Woodford (2003a). This reduces the real rate of return in a liquidity trap and increases demand. In a Markov equilibrium, however, this commitment may not be feasible.

The second key result of the paper is that in a Markov equilibrium, deflation can be modelled as a credibility problem. This problem arises if the government has only one policy instrument, i.e. open market operations in government bonds, and the natural rate of interest is temporarily negative. Under these conditions there is excessive deflation if the government cannot commit to future policy. This is the deflation bias of discretionary policy. This theory of deflation, derived from the analysis of a Markov equilibrium, is in sharp contrast to conventional wisdom about deflation in Japan today (or, for that matter, US during the Great Depression). The conventional wisdom blames deflation on policy mistakes by the central bank or bad policy rules (see e.g. Friedman and Schwartz (1963), Krugman (1998), Bernanke (2000), Benabib et al (2002) and Buiter (2003)).[2] Deflation in this paper, however, is not attributed to an inept central bank or bad policy rules. It is a consequence of the central bank's *policy constraints* and inability to commit to the optimal policy when faced with negative demand shocks.[3] This result, however, does not absolve the government of responsibility for deflation. Rather, it identifies the possible policy constraints that result in inefficient deflation in equilibrium (without resorting to an irrational policy maker). I identify two sources of inefficient deflation of equal importance. The first is the inability of the government to commit. The second is that open market operations in short-term government bonds is the only policy instrument. The central question of the paper, therefore, is how the government can use additional policy instruments to fight deflation even if it cannot commit to future policy.

The third key result of the paper is that in a Markov equilibrium the government can eliminate deflation by deficit spending. Deficit spending eliminates deflation for the following reason: If the government cuts taxes and increases nominal debt, and taxation is costly, inflation expectations increase (i.e. the private sector expects higher money supply in the future). Inflation expectation increase because higher nominal debt gives the government an incentive to inflate to reduce the real value of the debt. To eliminate deflation the government simply cuts taxes until the private sector expects inflation instead of deflation. At zero nominal interest rates higher inflation expectations reduce the real rate of return, and thereby raise aggregate demand and the price level. The two main assumption underlying this result is that (i) there is some cost of taxation which makes this policy credible and that (ii) monetary and fiscal policy are coordinated.[4]

Deficit spending has exactly the same effect as the government following Friedman's famous suggestion to "drop money from helicopters" to increase inflation. At zero nominal interest rates money and bonds are perfect substitutes. They are one and the same: A government issued piece of paper that carries no interest but has nominal value. It does not matter, therefore, if the government drops money from helicopters or issues government bonds. Friedman's proposal thus increases the price level through the same mechanism as deficit spending. Dropping money from helicopters, however, does *not* increase prices in a Markov equilibrium because it increases the *current* money supply. It creates inflation by increasing government debt which is defined as the sum of money and bonds. In a Markov equilibrium *it is government debt that determines the price level in a liquidity trap* because it determines expectations about *future* money supply.

The key mechanism that increases inflation expectation in this paper, and thus eliminating deflation, is government debt. The government, however, can increase its debt in several ways. Cutting taxes or dropping money from helicopters are only two examples. The government can also increase debt by printing money (or issuing nominal bonds) and buy private assets, such as stocks, or foreign exchange. In a Markov equilibrium these operations increase prices and output *because they change the inflation incentive of the government by increasing government debt* (money+bonds). Hence, when the short-term nominal interest rate is zero, open market operations in real assets and/or foreign exchange increase prices through the same mechanism as deficit spending in a Markov equilibrium. This channel of monetary policy does not rely on the portfolio effect of buying real assets or foreign exchange. This paper thus compliments Meltzer's (1999) and McCallum (1999) arguments for foreign exchange interventions that rely on the portfolio channel. The argument in this paper is also complimentary to Svensson's (2001) "foolproof"' way of escaping the liquidity trap, although in that paper foreign exchange intervention are only useful to maintain or establish a currency peg rather than creating inflation incentives.

Deflationary pressures in this paper are due to temporary exogenous real shocks that shift aggregate demand.[5] The paper, therefore, does not address the origin of the deflationary shocks during the Great Depression in the US or in Japan today. These deflationary shocks are most likely due to a host of factors, including the stock market crash and banking problems. I take these deflationary pressures as given and ask: How can the government eliminate deflation by monetary and fiscal policy even if the zero bound is binding and it cannot commit to future policy? There is no doubt that there are several other policy challenges for a government that faces large negative shocks, and various structural problems, as in Japan.[6] Stabilizing the price level (and reducing real rates) by choosing the optimal mix of monetary and fiscal policy, however, is an obvious starting point and does not preclude other policy measures and/or structural reforms.

I study this model, and some extensions, in a companion paper Eggertsson (2004) with explicit reference to the current situation in Japan and some historical episodes (the Great Depression in particular). That paper also demonstrates that deficit spending may have little or no effect if the central bank is "goal independent". It follows that monetary and fiscal policy need to be coordinated for deficit spending to be effective, an assumption that is maintained in this paper (see also Eggertsson and Woodford (2003b) for further discussions about Japan). Eggertsson (2003) and Jeanne and Svensson (2004) suggest that a "goal independent" central bank may be able to commit to future inflation by purchasing foreign exchange reserves or real assets if it cares about balance sheet losses, but Eggertsson (2003) points out that this commitment device is not used by the central bank if it is too risk averse. Thus coordination between the central bank and the treasury may be required even if the central bank is concerned about balance sheet losses, and can use its balance sheet as a commitment device.

Benhabib, Schmitte-Grohe and Uribe (2002) (BSU hereafter) and Woodford (2003) also emphasize the importance of fiscal policy to eliminate deflation in a liquidity trap. They stress that appropriate fiscal policy implies tax-cuts in response to deflation and suggest tax rules based on this principle to eliminate "bad" deflationary equilibrium. The analysis by BSU (2002) and Woodford (2003) (and the emphasis on fiscal policy in particular) is closely related to the present paper but with some important differences. First, in BSU (2002) and Woodford (2003) deflation is due to self-fulfilling expectations and is therefore an example of a "bad" equilibrium in a model with multiple ones, but in this paper deflation is due to a series of bad real shocks that make the zero bound binding. The suggested policy rules in BSU and Woodford are therefore only effective to exclude the self-fulfilling equilibrium but do nothing to respond to the real shocks that make the zero bound binding in this paper (in fact it can be shown that the policy rules suggested by BSU (2002) and Woodford (2003) lead to exactly the same inefficient deflation bias as shown in section 4). A second difference is that BSU (2002) and Woodford (2003) assume that the government can commit to future fiscal and monetary policy and the commitment to "bad" policy rules is the reason for deflation in the first place. In this paper I assume that the government cannot commit to future policy and the inability of the government to commit – coupled with a series of bad shocks and policy constraints – is the culprit for deflation. The role of fiscal policy here is that it is a commitment mechanism to solve the credibility problem posed by deflationary shocks. Inappropriate fiscal policy is not the source of a deflationary equilibrium in itself as in the work cited above.

1 The Model

Here I outline a simple sticky prices general equilibrium model and define the set of feasible equilibrium allocations. This prepares the grounds for the next section, which considers whether "quantitative easing" – a policy currently in effect at the Bank of Japan – and/or deficit spending have any effect on the feasible set of equilibrium allocations.

1.1 The private sector

1.1.1 Households

The representative household that maximizes expected utility over the infinite horizon:

$$E_t \sum_{T=t}^{\infty} \beta^T U_T = E_t \left\{ \sum_{T=t}^{\infty} \beta^T [u(C_T, \frac{M_T}{P_T}, \xi_T) + g(G_T, \xi_T) - \int_0^1 v(h_T(i), \xi_T) di] \right\} \quad (1)$$

where C_t is a Dixit-Stiglitz aggregate of consumption of each of a continuum of differentiated goods,

$$C_t \equiv [\int_0^1 c_t(i)^{\frac{\theta}{\theta-1}}]^{\frac{\theta-1}{\theta}}$$

with elasticity of substituting equal to $\theta > 1$, G_t is is a Dixit-Stiglitz aggregate of government consumption, ξ_t is a vector of exogenous shocks, M_t is end-of-period money balances, P_t is the Dixit-Stiglitz price index,

$$P_t \equiv [\int_0^1 p_t(i)^{1-\theta}]^{\frac{1}{1-\theta}}$$

and $h_t(i)$ is quantity supplied of labor of type i. $u(.)$ is concave and strictly increasing in C_t for any possible value of ξ. The utility of holding real money balances is increasing in $\frac{M_t}{P_t}$ for any possible value of ξ up to a satiation point at some finite level of real money balances as in Friedman (1969).[7] $g(.)$ is the utility of government consumption and is concave and strictly increasing in G_t for any possible value of ξ. $v(.)$ is the disutility of supplying labor of type i and is increasing and convex in $h_t(i)$ for any possible value of ξ. E_t denotes mathematical expectation conditional on information available in period t. ξ_t is a vector of r exogenous shocks. The vector of shocks ξ_t follows a stochastic process as described below.[8]

A1 (i) $pr(\xi_{t+j}|\xi_t) = pr(\xi_{t+j}|\xi_t, \xi_{t-1}, ...)$ for $j \geq 1$ where $pr(.)$ is the conditional probability density function of ξ_{t+j}. (ii) All uncertainly is resolved before a finite date K that can be arbitrarily high.

For simplicity I assume complete financial markets and no limit on borrowing against future income. As a consequence, a household faces an intertemporal budget constraint of the form:

$$E_t \sum_{T=t}^{\infty} Q_{t,T}[P_T C_T + \frac{i_T - i^m}{1+i_T} M_T] \leq W_t + E_t \sum_{T=t}^{\infty} Q_{t,T}[\int_0^1 Z_T(i)di + \int_0^1 n_T(j)h_T(j)dj - P_T T_T] \quad (2)$$

looking forward from any period t. Here $Q_{t,T}$ is the stochastic discount factor that financial markets use to value random nominal income at date T in monetary units at date t; i_t is the riskless nominal interest rate on one-period obligations purchased in period t, i^m is the nominal interest rate paid on money balances held at the end of period t, W_t is the beginning of period nominal wealth at time t (note that its composition is determined at time $t-1$ so that it is equal to the sum of monetary holdings from period $t-1$ and the (possibly stochastic) return on non-monetary assets), $Z_t(i)$ is the time t nominal profit of firm i, $n_t(i)$ is the nominal wage rate for labor of type i, T_t is net real tax collections by the government. Households maximize utility subject to the budget constraint.[9]

1.1.2 Firms

The production function of the representative firm that produces good i is:

$$y_t(i) = f(h_t(i), \xi_t) \quad (3)$$

where f is an increasing concave function for any ξ. I abstract from capital dynamics. As in Rotemberg (1983), firms face a cost of price changes given by the function $d(\frac{p_t(i)}{p_{t-1}(i)})$[10] but I can derive exactly the same result assuming that firms adjust their prices at stochastic intervals as assumed by Calvo (1983).[11] Price variations have a welfare cost that is separate from the cost of expected inflation due to real money balances in utility. I show that the key results of the paper do not depend on this cost being particularly large, indeed they hold even if the cost of price changes is arbitrarily small. The Dixit-Stiglitz preferences of the household imply a demand function for the product of firm i given by

$$y_t(i) = Y_t(\frac{p_t(i)}{P_t})^{-\theta}.$$

The firm maximizes

$$E_t \sum_{T=t}^{\infty} Q_{t,T} Z_T(i) \tag{4}$$

where

$$Q_{t,T} = \beta^{T-t} \frac{u_c(C_T, \frac{M_T}{P_T}, \xi_T)}{u_c(C_t, \frac{M_t}{P_t}, \xi_t)} \frac{P_t}{P_T}. \tag{5}$$

I can write firms period profits as

$$Z_t(i) = (1+s) Y_t P_t^{\theta} p_t(i)^{1-\theta} - n_t(i) f^{-1}(Y_t P_t^{\theta} p_t^{-\theta}) - P_t d(\frac{p_t(i)}{p_{t-1}(i)}) \tag{6}$$

where s is an exogenously given production subsidy that I introduce for computational convenience.[12] Firm is to maximize profits.[13]

1.1.3 Private Sector Equilibrium Conditions: AS, IS and LM Equations

In this subsection I show the necessary conditions for equilibrium that stem from the maximization problems of the private sector. These conditions must hold for *any* government policy. The first order conditions of the household maximization imply an Euler equation

$$\frac{1}{1+i_t} = E_t \{ \frac{\beta u_c(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}, \xi_{t+1})}{u_c(C_t, \frac{M_t}{P_t}, \xi_t)} \frac{P_t}{P_{t+1}} \} \tag{7}$$

where i_t is the nominal interest rate on a one period riskless bond. This equation is often referred to as the IS equation. Optimal money holding implies that

$$\frac{u_{\frac{M}{P}}(C_t, \frac{M_t}{P_t}, \xi_t)}{u_c(C_t, \xi_t)} = \frac{i_t - i^m}{1+i_t}. \tag{8}$$

This equation defines money demand or what is often referred to as the "LM" equation. Utility is weakly increasing in real money balances. Utility does not increase further at some finite level of real money balances. The left hand side of (8) is therefore weakly positive. Thus there is bound on the short-term nominal interest rate given by

$$i_t \geq i^m. \tag{9}$$

In most economic discussions it is assumed that the interest paid on the monetary base is zero so that (9) becomes $i_t \geq 0$.[14]

The optimal consumption plan of the representative household must also satisfy the transversality condition

$$\lim_{T\to\infty} E_t(Q_{t,T}\frac{W_T}{P_t}) = 0 \quad (10)$$

to ensure that the household exhausts its intertemporal budget constraint. I assume that workers are wage takers so that households optimal choice of labor supplied of type j satisfies

$$n_t(j) = \frac{P_t v_h(h_t(j);\xi_t)}{u_c(C_t, \frac{M_t}{P_t}, \xi_t)}. \quad (11)$$

I restrict my attention to a symmetric equilibria where all firms charge the same price and produce the same level of output so that

$$p_t(i) = p_t(j) = P_t; \quad y_t(i) = y_t(j) = Y_t; \quad n_t(i) = n_t(j) = n_t; \quad h_t(i) = h_t(j) = h_t \quad \text{for} \quad \forall\ j, i \quad (12)$$

Given the wage demanded by households I can derive the aggregate supply function from the first order conditions of the representative firm, assuming competitive labor market so that each firm takes its wage as given. I obtain the equilibrium condition often referred to as the AS or the "New Keynesian" Phillips curve:

$$\theta Y_t[\frac{\theta-1}{\theta}(1+s)u_c(C_t, \frac{M_t}{P_t}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] + u_c(C_t, \frac{M_t}{P_t}, \xi_t)\frac{P_t}{P_{t-1}}d'(\frac{P_t}{P_{t-1}}) \quad (13)$$
$$- E_t\beta u_c(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}, \xi_{t+1})\frac{P_{t+1}}{P_t}d'(\frac{P_{t+1}}{P_t}) = 0$$

where for notational simplicity I have defined the function $\tilde{v}_y(y_t(i), \xi_t) \equiv v(f^{-1}(y_t(i)), \xi_t)$

1.2 The Government

There is an output cost of taxation (e.g. due to tax collection costs as in Barro (1979)) captured by the function $s(T_t)$.[15] For every dollar collected in taxes $s(T_t)$ units of output are waisted without contributing anything to utility. Government real spending is then given by:

$$F_t = G_t + s(T_t) \quad (14)$$

I could also define cost of taxation as one that would result from distortionary taxes on income or consumption and obtain similar results.[16] I assume a representative household so that in a symmetric equilibrium, all nominal claims held are issued by the government. It follows that the government flow budget constraint is

$$B_t + M_t = W_t + P_t(F_t - T_t) \tag{15}$$

where B_t is the end-of-period nominal value of bonds issued by the government. Finally, market clearing implies that aggregate demand satisfies:

$$Y_t = C_t + d(\frac{P_t}{P_{t-1}}) + F_t \tag{16}$$

I now define the set of possible equilibria that are consistent with the private sector equilibrium conditions and the technological constraints on government policy.

Definition 1 A *Private Sector Equilibrium (PSE) is a collection of stochastic processes*

$\{P_t, Y_t, W_{t+1}, B_t, M_t, i_t, F_t, T_t, Q_t, Z_t, G_t, C_t, n_t, h_t, \xi_t\}$ *for* $t \geq t_0$ *that satisfy equations (3)-(16) for each* $t \geq t_0$, *given* W_{t_0}, P_{t_0-1} *and the exogenous stochastic process* $\{\xi_t\}$ *that satisfies A1 for* $t \geq t_0$.

Having defined the set of feasible equilibrium allocations I now consider how government policy affects it.

2 Equilibrium with exogenous policy expectations

According to Keynes (1936) famous analysis, monetary policy loses its power when the short term nominal interest rate is zero. Others argue, most notably Friedman and Schwartz (1963) and the monetarist, that a monetary expansion increases aggregate demand even under such circumstances, and this is what lies behind the "quantitative easing" policy of the BOJ since 2001.

One of Keynes better known suggestions is to increase demand in a liquidity trap by government deficit spending. Many have raised doubts recently about the importance of this channel, pointing to Japan's mountains of nominal debt, citing the Ricardian equivalence, i.e. the principle that any decrease in government savings should be offset by an increase in private savings (to pay for higher future taxes). Yet another group of economists argue that the Ricardian equivalence fails if deficit spending is financed by money creation (see e.g. Buiter (2003) and Bernanke (2000,2003)).

Here I consider whether or not "quantitative easing" and deficit spending are separate policy tools in the explicit intertemporal general equilibrium model laid out in the last section. The key result is that neither "quantitative easing" nor deficit spending have any effect on the feasible set of equilibrium allocations *if* expectations about future money supply remain unchanged – or alternatively – expectations about future interest rate policy remain unchanged. Furthermore, this result is unchanged if these two operations are used together, hence our analysis does not support the proposition that "money financed deficit spending" increases demand independently of the expectation channel. This result is an extension of the irrelevance results by Krugman (1998) and Eggertsson and Woodford (2003a), extended to include fiscal policy.

I do not contend that deficit spending and/or quantitative easing are irrelevant in a liquidity trap. Rather, my the point is that the main effect of these policies is best illustrated by analyzing how they change expectations about future policy, in particular expectations about future money supply.

2.1 The irrelevance of monetary and fiscal policy when policy expectations are exogenous

Here I characterize a policy regime that allows for the possibility that the government increases money supply by "quantitative easing" when the zero bound is binding and/or engages in deficit spending.

The money supply is determined by a policy function:

$$M_t = M(q_t, \xi_t) I_t \tag{17}$$

where q_t is a vector that may include any of the endogenous variables *that are determined at time* t (note that as a consequence q_t cannot include W_t that is predetermined at time t). The multiplicative factor I_t satisfies the conditions

$$I_t = 1 \text{ if } i_t > 0 \quad \text{otherwise} \tag{18}$$

$$I_t = \psi(q_t, \xi_t) \geq 1. \tag{19}$$

The rule (17) is a fairly general specification of policy (since I assume that M_t is a function of all the endogenous variables). It could for example include simple Taylor type rules, monetary targeting, and any policy that does not depend on the past values of the endogenous variables. Following Eggertsson and Woodford (2003a) I define the multiplicative factor $I_t = \psi(q_t, \xi_t)$ when the zero bound is binding. A policy of "quantitative easing" is represented by a value of the function ψ that is greater than 1. Note that I assume that the functions M and ψ are only functions of the endogenous variables and the shocks at time t. This separates the direct effect of a quantitative easing from the effect of a policy that influences expectation about future money supply. I impose the restriction on the policy rule (17) that:

$$M_t \geq M^*. \tag{20}$$

This restriction says the nominal value of the monetary base can never be smaller than some finite number M^*. This number can be arbitrarily small, so I do not view this as a very restrictive (or unrealistic) assumption since I am not modelling any technological innovation in the payment technology (think of M^* as being equal to one cent!). I assume, for simplicity, that the central bank does quantitative easing by buying government bonds, but the model can be extended to allow for the possibility of buying a range of other long or short term financial assets (see Eggertsson and Woodford (2003a)). I also assume that the government only issues one period riskless nominal bonds so that B_t in equation (15) refers to a one period riskless nominal debt.

Fiscal policy is defined by a function for real government spending:

$$F_t = F \tag{21}$$

and a policy function for deficit spending

$$T_t = T(q_t, \xi_t) \tag{22}$$

I assume that real government spending F_t is constant at all times in order to focus on deficit spending which is defined by the function $T(.)$. Debt issued at the end of period t is then defined by the consolidated government budget constraint (15) and the policy specifications (17)-(22).

Finally I assume that the government is neither a debtor or a creditor asymtotically so that

$$\lim_{T \to \infty} E_t Q_{t,T} B_T = 0 \tag{23}$$

This is a fairly weak condition stating that the government cannot accumulate real debt asymtotically at a higher rate than the real rate of interest.[17] Note that (23) is a restriction on fiscal policy so that it has an effect on the set of functions $T(.)$ that are consistent with the policy regime.

The idea behind the policy rules (17)-(23) is to separate the "direct" effect of a quantitative easing and deficit spending in a liquidity trap from any effect these policies may have on expectations about future policy, i.e. I hold expectation about policy at positive interest rates constant. One simple special case of the policy rules above is that money supply is some constant $\bar{M}$ at positive interest rates and taxes are a constant value of debt that is rolled over to the next period. In this example I can consider the question of whether quantitative easing or deficit spending have any effect holding expectations of future money supply and fiscal policy constant. This is the sense in which policy expectation are constant, that is, I assume that policy actions when the zero bound is binding have no effect on the policy rules at positive interest rates. The policy rules (17)-(23) are much more general than the simple special case just given since they allow me to consider a broad range of monetary and fiscal policies that have only one thing in common, i.e. policy cannot depend on the past values of the endogenous variables. A simple Taylor rule is another special case.[18] The sense in which monetary policy expectation are constant in that case is that quantitative easing at zero interest rate will have no effect on the central banks commitment to the Taylor rule at positive interest rates.[19]

Using the policy rules above I can now obtain the following irrelevance result for monetary and fiscal policy:

Proposition 1 *The set of paths* $\{P_t, Y_t, i_t, Q_t, Z_t, C_t, n_t, h_t, \xi_t\}$ *consistent with a PSE and the monetary and fiscal policy regimes (17)-(23) is independent of the specification of the functions* $\psi(.)$ *and* $T(.)$.

The proof of this proposition is fairly simple, and the formal details are provided in the Technical Appendix. The proof is that I show that I can write all the equilibrium conditions in a way that does not involve the functions T or ψ. First, I use market clearing to show that the intertemporal budget constraint of the household can be written without reference to either function. This relies on the Ricardian properties of the model. Second, I show that (10) is satisfied regardless of the specification of these functions using the two restrictions we imposed on policy given by (20) and (23). Finally I can write the remaining conditions without any reference to the function $\psi(.)$, following the proof by Eggertsson and Woodford (2003a).

2.2 Discussion

Proposition 1 says that a policy of quantitative easing and/or deficit spending in a liquidity trap has no effect on the set of feasible equilibrium allocations that are consistent with the policy regimes I specified. It may seem that this result contradicts Keynes' view that deficit spending is an effective tool to escape the liquidity trap. It may also seem to contradict the monetarist view (see e.g. Friedman and Schwartz) that increasing the money supply is effective at low interest rates. But this would only be true if one took a narrow view of these schools of thought like Hicks (1937) does in his ground breaking paper "Mr. Keynes and the Classics". Hicks develops a static version of the General Theory and contrasts it to the monetarist view assuming that expectation are exogenous constants. This is the IS-LM model. My analysis, however, indicates is that it is the intertemporal elements of the liquidity trap that are crucial to understand the effects of different policy actions, namely their effect on expectations (to be fair to Hick he was very explicit that he was abstracting from expectation and recognized this was a major issues). Both Keynes (1936) and many monetarists (e.g. Friedman and Schwartz (1963)) discussed the importance of expectations in their work and a static model is therefore not going to do full justice to their claims.

My result is that deficit spending has no effect on whether a given deflationary path represents an equilibrium if it does not change expectations about future policy. But as we shall see in later sections (when analyzing a Markov equilibrium) deficit spending can be very effective to *change expectations*. Thus the irrelevance result still leaves an important role for deficit spending, namely, it can be useful to change expectations. My result that quantitative easing is ineffective also relies on constant policy expectations. But as we shall also see (when analyzing a Markov equilibrium) quantitative easing changes expectation if the money printed is used to buy some private asset. Thus the irrelevance result also leaves an important role for quantitative easing through the expectation channel. Thus by modelling expectations explicitly, I believe my result neither contradicts Friedman and Schwartz' interpretation of the "Classics" , i.e. the Quantity Theory of Money, nor Keynes' General Theory, at least if one takes a generous view of the main policy implications of these theories. On the contrary, it may serve to integrate the two through modelling the expectation channel.

Proposition 1 may also seem to contradict the claims of Bernanke (2003) and Buiter (2003). Both authors indicate that money financed tax cuts increase demand. Buiter, for example, writes that "base money-financed tax cuts or transfer payments – the mundane version of Friedman's helicopter drop of money – will always boost aggregate demand." But what Buiter implicitly has in mind, is that tax cuts permanently increase the money supply. Thus a tax cut today, in his model, increases expectations about future money supply. Thus my proposition does not disprove Buiter's or Bernanke's claims since I assume that money supply in the future is set without any reference to past policy actions. The propositions, therefore, clarifies that tax cuts will only increase demand to the extent that they change beliefs about future money supply. The higher demand equilibrium that Buiter analyses, therefore, does not depend on the tax cut itself, only on expectations about future money supply. A similar comment applies to Auerbach and Obstfeld's (2003) result. They argue that open-market operations will increase aggregate demand. But their assumption is that open-market operations increase expectations about future money supply. It is that belief that matters and not the open market operation itself, even is cost of taxation.[20]

3 Equilibrium with Endogenous Policy Expectations

The main lesson from the last section is that expectations about future monetary and fiscal policy are crucial. Deficit spending and quantitative easing have no effect if they do not change expectations about future policy. But does deficit spending have no effect on expectations under reasonable assumptions about how these expectation are formed? Suppose, for example, that the government prints unlimited amounts of money and drops it from helicopters, distributes it by tax cuts, or prints money and buys unlimited amounts of some private asset. Would this not alter expectations about future money supply? To answer this question I need an explicit model of how the government sets policy in the future. To do this I assume that the government sets monetary and fiscal policy optimally at all future dates. By optimal, I mean that the government maximizes social welfare that is given by the utility of the representative agent. I analyze equilibrium under two assumptions about policy formulation. Under the first assumption, which I call the commitment equilibrium, the government can commit to future policy in order to influence the equilibrium outcome by choosing future policy actions (at all different states of the world).

Rational expectations require that these commitments are fulfilled in equilibrium. Under the second assumption, the government cannot commit to future policy. In this case the government maximizes social welfare under discretion in every period, disregarding any past policy actions, except insofar as they have affected the endogenous state of the economy at that date (defined more precisely below). Thus the government can only choose its current policy instruments, it cannot directly influence future governments actions. This is what I call the Markov equilibrium. In the Markov equilibrium, following Lucas and Stockey (1983) and a large literature that followed, I assume that the government is capable of issuing one period riskless nominal debt and committing to paying it back with certainty. In this sense, even under discretion, the government is capable of limited commitment.

3.1 Recursive representation

To analyze the commitment and Markov equilibrium it is useful to rewrite the model in a recursive form so that I can identify the endogenous state variables at each date. When the government can only issue one period nominal debt I can write the total nominal claims of the government (which in equilibrium are equal to the total nominal wealth of the representative household) as $W_{t+1} = (1 + i_t)B_t + (1 + i^m)M_t$. Substituting this into (15) and defining the variables $w_t \equiv \frac{W_{t+1}}{P_t}$, $m_t \equiv \frac{M_t}{P_{t-1}}$ and $\Pi_t = \frac{P_t}{P_{t-1}}$ I can write the government budget constraint as:

$$w_t = (1 + i_t)(w_{t-1}\Pi_t^{-1} + (F - T_t) - \frac{i_t - i^m}{1 + i_t} m_t \Pi_t^{-1}) \tag{24}$$

Note that I use the time subscript t on w_t (even if it denotes the real claims on the government at the beginning of time $t+1$) to emphasize that this variable is determined at time t. I assume that $F_t = F$ so that real government spending is an exogenous constant at all times. In Eggertsson (2004) I treat F_t as a choice variable. Instead of the restrictions (20) and (23) I imposed in the last section on government policies, I impose a borrowing limit on the government that rules out Ponzi schemes:

$$u_c w_t \leq \bar{w} < \infty \tag{25}$$

where $\bar{w}$ is an arbitrarily high finite number. This condition can be justified by the fact the government can never borrow more than the equivalence of the expected discounted value of its maximum tax base.[21] It is easy to show that this limit ensures that the transversality condition of the representative household is satisfied at all times.

The treasury's policy instruments is taxation, T_t, that determines the end-of-period government debt which is equal to $B_t + M_t$. The central bank determines how the end-of-period debt is split between bonds and money by open market operations. Thus the central banks policy instrument is $M_{t.}$ Note that since P_{t-1} is determined in the previous period, I may think of $m_t \equiv \frac{M_t}{P_{t-1}}$ as the instrument of monetary policy.

It is useful to note that I can reduce the number of equations that are necessary and sufficient for a private sector equilibrium substantially from those listed in Definition 1. First, note that the equations that determine $\{Q_t,\ Z_t, G_t, C_t, n_t, h_t\}$ are redundant, i.e. each of them is only useful to determine one particular variable but has no effect on the any of the other variables. Thus I can define necessary and sufficient condition for a private sector equilibrium without specifying the stochastic process for $\{Q_t,\ Z_t, G_t, C_t, n_t, h_t\}$ and do not need to consider equations (3), (5), (6), (11), (14) and (16). Furthermore, condition (25) ensures that the transversality condition of the representative household is satisfied at all times so I do not need to include (10) in the list of necessary and sufficient conditions. For the remaining conditions I use (16) to substitute out for C_t.

It is useful to define the expectation variable

$$f_t^e \equiv E_t u_c(Y_{t+1} - d(\Pi_{t+1}) - F, m_{t+1}\Pi_{t+1}^{-1}, \xi_{t+1})\Pi_{t+1}^{-1} \tag{26}$$

as the part of the nominal interest rates that is determined by the expectations of the private sector formed at time t. The IS equation can then be written as

$$1 + i_t = \frac{u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)}{\beta f_t^e} \tag{27}$$

Similarly it is useful to define the expectation variable

$$S_t^e \equiv E_t u_c(Y_{t+1} - d(\Pi_{t+1}) - F, m_{t+1}\Pi_{t+1}^{-1}, \xi_{t+1})\Pi_{t+1} d'(\Pi_{t+1}) \tag{28}$$

The AS equation can be written as

$$\theta Y_t[\frac{\theta - 1}{\theta}(1+s)u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] + u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)\Pi_t d'(\Pi_t) - \beta S_t^e = 0. \tag{29}$$

Finally the money demand equation (8) can be written in terms of m_t and Π_t as

$$\frac{u_m(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)\Pi_t^{-1}}{u_c(Y_t - d(\Pi_t) - F, \xi_t)} = \frac{i_t - i^m}{1 + i_t} \tag{30}$$

The next two propositions are useful to characterize equilibrium outcomes. Proposition 2 follows directly from our discussion above:

Proposition 2 *A necessary and sufficient condition for a PSE at each time $t \geq t_0$ is that the variables $(\Pi_t, Y_t, w_t, m_t, i_t, T_t)$ satisfy: (i) conditions (9), (24),(25), (27), (29), (30) given w_{t-1} and the expectations f_t^e and S_t^e. (ii) in each period $t \geq t_0$, expectations are rational so that f_t^e is given by (26) and S_t^e by (28).*

Proposition 3 *The possible PSE equilibrium defined by the necessary and sufficient conditions for any date $t \geq t_0$ onwards depend only on w_{t-1} and ξ_t.*

The second proposition follows from observing that w_{t-1} is the only endogenous variable that enters with a lag in the necessary and sufficient conditions in (i) of Proposition 2 and using the assumption that ξ_t is Markovian (i.e. using A1) so that the conditional probability distribution of ξ_t for $t > t_0$ only depends on ξ_{t_0}. It follows from this proposition that (w_{t-1}, ξ_t) are the only state variables at time t that directly affect the PSE. I may economize on notation by introducing vector notation. I define vectors

$$\Lambda_t \equiv \begin{bmatrix} \Pi_t & Y_t & m_t & i_t & T_t \end{bmatrix}^T, \text{ and } e_t \equiv \begin{bmatrix} f_t^e \\ S_t^e \end{bmatrix}.$$

Since Proposition 3 indicates that w_t is the only relevant endogenous state variable, I prefer not to include it in either vector but keep track of it separately. It simplifies notation a bit to write the utility function as a function of Λ_t i.e. I define the function $U : \mathbb{R}^{5+r} \rightarrow \mathbb{R}$

$$U_t = U(\Lambda_t, \xi_t)$$

using (14) and (16) to solve for G_t and C_t as a function of Λ_t, along with (3) and (12) to solve for $h_t(i)$ as a function of Y_t.

3.2 The Commitment Equilibrium

Using Proposition (3) I can now define the commitment solution.

Definition 2 *The optimal commitment solution at date $t \geq t_0$ is the PSE that maximizes the utility of the representative household given w_{t_0-1}* and ξ_{t_0}.

Necessary conditions for the commitment equilibrium can be found by using a Lagrangian method fairly standard in the literature (apart from the inequality constraints present here). The Technical Appendix shows the Lagrangian and the first order conditions of the governments maximization problem.

3.3 The Markov equilibrium

Here I consider an equilibrium that occurs when policy is conducted under discretion so that the government is unable to commit to any future actions. To do this I solve for a Markov equilibrium (it is formally defined by Maskin and Tirole (2001)) that has been extensively applied in the monetary literature. The basic idea behind this equilibrium concept is to define a minimum set of state variables that directly affect market conditions and assume that the strategies of the government and the private sector expectations depend only on this minimum state. Proposition 3 indicates that a Markov equilibrium requires that the variables (Λ_t, w_t) only depend on (w_{t-1}, ξ_t), since this is the minimum set of state variables that affect the PSE.

The timing of events in the game is as follows: At the beginning of each period t, w_{t-1} is a predetermined state variable. At the beginning of the period, the vector of exogenous disturbances ξ_t is realized and observed by the private sector and the government. The monetary and fiscal authorities choose policy for period t given the state and the private sector forms expectations e_t. Note that I assume that the private sector may condition its expectation at time t on w_t, i.e. it observes the policy actions of the government in that period so that Λ_t and e_t are jointly determined. This is important because w_t is the relevant endogenous state variable at date $t+1$. Since the state in this game is captured by (w_{t-1}, ξ_t) a Markov equilibrium requires that there exist policy functions $\bar{\Pi}_t(.), \bar{Y}_t(.), \bar{m}_t(.), \bar{\imath}_t(.), \bar{T}_t(.)$, that I denote by the vector valued function $\bar{\Lambda}_t(.)$ and a function $\bar{w}_t(.)$, such that each period:[22]

$$\begin{bmatrix} \Lambda_t \\ w_t \end{bmatrix} \equiv \begin{bmatrix} \bar{\Lambda}_t(w_{t-1}, \xi_t) \\ \bar{w}_t(w_{t-1}, \xi_t) \end{bmatrix} \tag{31}$$

Note that the functions $\bar{\Lambda}_t(.)$ and $\bar{w}_t(.)$ will also define a set of functions of (w_{t-1}, ξ_t) for $(Q_t, Z_t, G_t, C_t, n_t, h_t)$ by the redundant equations from Definition 1. Using $\bar{\Lambda}_t(.)$ I may also use (26) and (28) to define a function $\bar{e}_t(.)$ so so that

$$e_t = \begin{bmatrix} f_t^e \\ S_t^e \end{bmatrix} = \begin{bmatrix} \bar{f}_t^e(w_t, \xi_t) \\ \bar{S}_t^e(w_t, \xi_t) \end{bmatrix} = \bar{e}_t(w_{t,}\xi_t) \tag{32}$$

Rational expectations imply that the function $\bar{e}_t$ satisfies

$$\bar{e}_t(w_{t,}\xi_t) = \begin{bmatrix} E_t u_c(\bar{C}_t(w_t, \xi_{t+1}), \bar{m}_t(w_t, \xi_{t+1})\bar{\Pi}_t(w_t, \xi_{t+1})^{-1}; \xi_{t+1})\bar{\Pi}_t(w_t, \xi_{t+1})^{-1} \\ E_t u_c(\bar{C}_t(w_t, \xi_{t+1}), \bar{m}_t(w_t, \xi_{t+1})\bar{\Pi}_t(w_t, \xi_{t+1})^{-1}; \xi_{t+1})\bar{\Pi}_t(w_t, \xi_{t+1})d'(\bar{\Pi}_t(w_t, \xi_{t+1})) \end{bmatrix} \tag{33}$$

I define a value function $J_t(w_{t-1}, \xi_t)$ as the expected discounted value of the utility of the representative household, looking forward from period t, given the evolution of the endogenous variable from period t onwards that is determined by $\bar{\Lambda}_t(.)$, $\bar{w}_t(.)$ and $\{\xi_t\}$. Thus I define:

$$J_t(w_{t-1}, \xi_t) \equiv E_t \left\{ \sum_{T=t}^{\infty} \beta^T [U(\bar{\Lambda}_T(.), \xi_T] \right\} \tag{34}$$

The optimizing problem of the government is as follows. Given w_{t-1} and ξ_t, the government chooses the values for (Λ_t, w_t) (by its choice of the policy instruments m_t and T_t) to maximize the utility of the representative household subject to the conditions in Proposition 2 and (32). Thus its problem can be written as:

$$\max_{m_t, w_t} [U(\Lambda_t, \xi_t) + \beta E_t J(w_t, \xi_{t+1})] \tag{35}$$

s.t. (9), (24),(25), (27), (29), (30) and (32)

I can now define a Markov equilibrium.

Definition 3 *A Markov equilibrium is a collection of functions $\bar{\Lambda}_t(.)$,$\bar{w}_t()$,$J_t(.)$,$\bar{e}_t(.)$, such that (i) given the function $J_t(w_{t-1}, \xi_t)$ and the vector function $\bar{e}_t(w_t, \xi_t)$ the solution to the policy maker's optimization problem (35) is given by $\Lambda_t = \bar{\Lambda}_t(w_{t-1}, \xi_t)$ and $w_t = \bar{w}_t(w_{t-1}, \xi_t)$ for each possible state $(w_{t-1,}\xi_t)$ (ii) given the vector function $\bar{\Lambda}_t(w_{t-1}, \xi_t)$* and *$\bar{w}_t(w_{t-1}, \xi_t)$ then $e_t = \bar{e}_t(w_t, \xi_t)$ is formed under rational expectations (see equation (33)). (iii) given the vector function $\bar{\Lambda}_t(w_{t-1}, \xi_t)$ and $\bar{w}_t(w_{t-1}, \xi_t)$ the function $J_t(w_{t-1}, \xi_t)$ satisfies (34).*

I will only look for a Markov equilibrium in which the functions $\bar{\Lambda}_t(.)$, $J_t(.)$, $\bar{e}_t(.)$ are continuous and have well defined derivatives. Then the value function satisfies the Bellman equation:

$$J_t(w_{t-1}, \xi_t) = \max_{m_t, w_t} [U(\Lambda_t, \xi_t) + E_t \beta J_t(w_t, \xi_{t+1})] \tag{36}$$

s.t. (9), (24),(25), (27), (29), (30) and (32).

Necessary conditions for the Markov Equilibrium can now be characterized by using a Lagrangian method for the maximization problem on the right hand side of (36). In addition, the solution satisfies an envelope conditions. The Lagrangian, associated with the appropriate first order condition, and the envelope condition, are shown in the Technical Appendix.

3.4 Approximation method

The necessary condition for the Markov and commitment solution can be linearized by a first order Taylor expansion around a steady state. The solution can then be obtained using the linearized equations. I define a steady state as a solution in the absence of shocks in which each of the variables (Π_t,Y_t,m_t,i_t,T_t,w_t,f_t^e,S_t^e)= (Π,Y,m,i,T,w,f^e,S^e) are constants. Following Woodford (2003), I define a steady state where monetary frictions are trivial. To do this I parameterize the utility function by the technology parameter $\bar{m}$ so that as $\bar{m}$ is reduced the household will demand ever lower real money balances. I denote the policy instrument as $\tilde{m}_t \equiv \frac{m_t}{\bar{m}}$ and it is still meaningful to discuss the evolution of the nominal stock of money even as $\bar{m} \to 0$ (see Technical Appendix for details). Furthermore I assume, following Woodford (2003), that the steady state is fully efficient so that $1 + s = \frac{\theta - 1}{\theta}$. Finally I suppose that in steady state $i^m = 1/\beta - 1$. To summarize:

A2 Steady state assumptions. (i) $\bar{m} \to 0$, (ii) $1 + s = \frac{\theta-1}{\theta}$ (iii) $i^m = 1/\beta - 1$.

Using A2 I prove in the Technical Appendix the existence of a steady state for both the commitment and the Markov solution given by $(\Pi, Y, \frac{m}{\bar{m}}, i, T, w, f^e, S^e) = (1, \bar{Y}, \tilde{m}, \frac{1}{\beta}{-}1, \bar{F}, 0, u_c(\bar{Y}{-}\bar{F}), 0)$ and show the equations the values $\bar{Y}$, $\bar{F}$ and $\tilde{m}$ satisfy. Furthermore I discuss how the state state of the Markov equilibrium relates to the results in Dedola (2002), King and Wolman (2003), Albanesi et al (2003) and Klein et al (2003). I then show that the solution can be approximated around this steady state and that the resulting solution, which is locally unique, is accurate to the order $O(||\xi, \bar{\delta}||)$ where $\bar{\delta} \equiv \frac{i - i^m}{1+i}$ (this latter approximation error arises because I analyze an equilibrium where $i^m = 0$ in the following sections). A complication is introduced by the presence of the interest rate bound inequality and I discuss how I treat this problem in the Technical Appendix. A further complication arises because in the Markov equilibrium the expectation functions $\bar{e}_t(.)$ are in general unknown. I illustrate a simple way of approximate these functions in Proposition 7.

4 The Deflation Bias

In the last section I showed how an equilibrium with endogenous policy expectations can be defined and approximated. I now analyze the approximate equilibrium and show that deflation can be modeled as a credibility problem. The point of this section is not to absolve the government of responsibility for deflation. Rather, the point is to identify the policy constraints that result in inefficient deflation. The policy constraint in this section, apart from the governments inability to commit to future policy, is the assumption that government spending and taxes are constant. Money supply, by open market operations in short-term government bonds, is the governments only policy instrument. This is equivalent to assuming that the interest rate is the only policy instrument. In the next section I relax this assumption. An appealing interpretation of the results is that they apply if the central bank does not coordinate its action with the treasury, i.e. if the central bank is "goal independent". This interpretation is discussed further in a companion paper Eggertsson (2004).

The assumption about the policy instruments of the government in this section is as follows:

A3 *Limited instruments: Open market operations in government bonds, i.e.* $\tilde{m}_t$, *is the only policy instrument. Fiscal policy is constant so that* $w_t = 0$ *and* $T_t = F$ *at all times*

To gain insights it is useful to consider the linear approximation of the private sector equilibrium constraints. The AS equation (29) can be written to the first order as the "New Keynesian Phillips curve"

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{37}$$

and equation (27) can be written to the first order as the forward looking "IS relation"

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n). \tag{38}$$

Here $\pi_t \equiv \Pi_t - 1$ is the inflation rate, $x_t \equiv \frac{Y_t - Y_t^n}{Y_t^n}$ is the output gap, i.e. it is the percentage deviation of output from the natural rate of output.[23] The term r_t^n is a composite exogenous disturbance (its exact form is given in the Technical Appendix) that shifts the IS equation. It represents exogenous variations in the Wicksellian natural rate of interest, that is, the equilibrium real rate of interest in the case output is equal to its natural rate at all times. In this model $r_t^n = \frac{1-\beta}{\beta} + \frac{\sigma^{-1}\omega}{\sigma^{-1}+\omega}[g_t - E_t g_{t+1} - (q_t - E_t q_{t+1})]$ summarizes to the first order all the relevant shocks (when the model is written in terms of the output gap). The coefficients κ and σ are both positive and given by $\kappa \equiv \theta \frac{(\sigma^{-1}+\omega)}{d''}$ and $\sigma \equiv -\frac{\bar{u}_{cc}\bar{Y}}{\bar{u}_c}$ where $\omega \equiv \frac{\bar{v}_y}{\bar{v}_{yy}\bar{Y}}$ and $g_t \equiv -\frac{\bar{u}_{c\xi}}{\bar{Y}\bar{u}_{cc}}\xi_t$ summarizes the shocks to consumption preferences and $q_t \equiv \frac{v_{y\xi}}{\bar{Y}v_{yy}}\xi_t$ summarize the shocks to the disutitility of working.

I first show that if the natural rate of interest is positive at all times, and A2 and A3 hold, the commitment and the Markov solution are identical and the zero bound is never binding. To be precise, the assumption on the natural rate of interest is:

A4 $r_t^n \in [0, S]$ *at all times where* S *is a finite positive number.*

Assuming this restriction on the natural rate of interest I can proof the following proposition.

Proposition 4 ***Markov and the commitment equivalence.*** *If A2,A3(i),A3(ii) and A4 then the following must hold at least locally to the steady state: There is a unique bounded Markov and commitment solution given by* $i_t = r_t^n \geq 0$ *and* $\pi_t = x_t = 0$. *The equilibrium is accurate up to an error of the order* $o(||\xi, \bar{\delta}||^2)$

Proof: See Technical Appendix

The intuition for this result is straight forward and can be understood by inspecting the linear approximation of the IS and AS equations in addition to a second order expansion of the representative household utility (but the household utility is the objective of the government). When fiscal policy is held constant, the utility of the representative household, to the second order, is equal to:[24]

$$U_t = -[\pi_t^2 + \frac{\kappa}{\theta}(x_t - x^*)^2] + O(||\xi, \bar{\delta}, 1 + s - \frac{\theta}{\theta - 1}||^3) + t.i.p. \tag{39}$$

where $x^* = (\omega + \sigma^{-1})^{-1}(1 - \frac{\theta-1}{\theta}(1+s))$ and $t.i.p$ is terms independent of policy. In A2(ii) I assume that $(1+s) = \frac{\theta}{\theta-1}$ an therefore $x^* = 0$. One can then observe by the IS and the AS equation that the government can completely stabilize the loss function at zero inflation and zero output gap in an equilibrium where $i_t = r_t^n$ at all times. Since this policy maximizes the government's objective at all times, there is no incentive for the government to deviate. Therefore the government's ability to commit has no effect on the equilibrium outcome, which is the intuition behind the formal proof of Proposition 4 in the Technical Appendix.

Proposition 4 only applies when $x^* = 0$ as in A2. When $x^* > 0$, the commitment and Markov solutions differ because of the classic inflation bias (stemming from monopoly powers of the firms) as first demonstrated by Kydland and Prescott (1977). I will now show that even when $x^* = 0$, the commitment and Markov solutions may also differ because of shocks that render the zero bound binding and which in turn trigger temporary excessive deflation in the Markov equilibrium. This new dynamic inconsistency problem is the deflation bias. I assume that $x^* > 0$ in the next subsection and show the connection between the inflation and the deflation bias.

The deflation bias can be derived by a simple assumption about the natural rate of interest r_t^n (recall that all the shocks that change the private sector equilibrium constraints can be captured by the natural rate of interest). Here I assume that the natural rate of interest becomes unexpectedly negative in period 0 and then reverts back to a positive steady state in every subsequent period with some probability. At the time r_t^n reverts back to steady state, a stochastic date denoted τ, it stays there forever. Assuming that all uncertainty is resolved before a finite date K simplifies the proofs. This is not a very restrictive assumptions since K may be arbitrarily high. To be more precise I assume:

A5 $r_t^n = r_L^n < 0$ *at* $t = 0$ *and* $r_t^n = r_{ss}^n = \frac{1}{\beta} - 1$ *at all* $0 < t < K$ *with probability* α *if* $r_{t-1}^n = r_L^n$ *and probability 1 if* $r_{t-1}^n = r_{ss}^n$ *at all* $t > 0$. *The stochastic date when* r_t^n reverts to r_{ss}^n is denoted τ. *There is an arbitrarily large number* K *so that* $r_t^n = r_{ss}^n$ *with probability 1 for all* $t \geq K$ and thus $\tau \leq K$.

The natural rate of interest can be negative due to a series of negative demand shocks (i.e. shifts in the utility of consumption) or expectations of lower future productivity (i.e. shift in the dis-utility of working). A temporary collapse in some autonomous component of aggregate spending (that is separate from private consumption) can also be interpreted as preference shocks. More generally, the most plausible candidate for a collapse in aggregate spending is a decline in investment. A host of candidates could lead to an investment collapse, such as problems in financial intermediation, adverse shocks to the balance sheets of firms, or a productivity slowdown. These shocks are not modelled in detail at this level of abstraction (but arguably correspond most closely to an autonomous decline in aggregate spending in the current setup) but could be studied more thoroughly in a model with endogenous capital.[25]

The commitment and the Markov solutions derived in Proposition 4 are not feasible if A5 holds because the solution in Proposition 4 requires that $i_t = r_t^n$ at all times. If the natural rate of interest is temporarily negative, as in A5, this would violate the zero bound. How does the solution change when the natural rate of interest is negative?

Consider first the commitment solution. A simple numerical example is useful. Suppose that in period 0 the natural rate of interest is unexpectedly negative so that $r_L^n = -2\%$ and then reverts back to steady state of $r_{ss}^n = 2\%$ with 10 percent probability in each period (taken to be a quarter here). The calibration parameters I use are the same as in Eggertsson and Woodford (2003a) (see details in the Technical Appendix). Figure 1 shows the solution for inflation, the output gap, and the interest rate using the approximation method described in the Technical Appendix. The first line in the first panel shows inflation when the natural rate of interest reverts to the steady state in period 1, the second if it returns back in period 2 and so on.[26] The central bank offsets a low natural rate of interest by lowering the interest rate correspondingly. But when the natural rate of interest is negative this is not feasible. To offset the shock the government commits to inflation and a temporary boom *in the future*, i.e. once the natural rate of interest returns to normal, and keeping the nominal interest rate low for a substantial period. Furthermore the optimal commitment implies a higher price level in the future and a higher money supply (see figure 7 and 8 and section 5 for further discussion). The expectations of future inflation and output boom are beneficial when $r_L^n < 0$ because they offset the negative demand effect of the shock. To see this consider the IS equation (38). Even if the nominal interest rate cannot fall below 0 in period t, the real rate of return (i.e. $i_t - E_t\pi_{t+1}$) is what is relevant for aggregate demand and it can still be lowered by increasing inflation expectations. This is captured by the second element of the right hand side of equation (38). Furthermore, a commitment to a temporary boom, i.e. higher $E_t x_{t+1}$, also stimulates demand by the permanent income hypothesis. This is represented by the first term on the right hand side of equation (38).

Bank of Japan officials have objected to an inflation target on the grounds that it is not be "credible" since they cannot lower the nominal interest rate to manifest their intentions. The optimal commitment depends on manipulating expectations and one should consider the extent to which this policy commitment is credible, i.e. if the government has an incentive to deviate from the optimal plan. Consider now the Markov equilibrium. For the case $K \to \infty$ it can be shown to yield the simple closed form solution:[27]

$$x_t = \frac{1-\beta(1-\alpha)}{\alpha(1-\beta(1-\alpha))-\sigma\kappa(1-\alpha)}\sigma r_L^n \text{ if } r_t^n = r_L^n \text{ and } x_t = 0 \text{ otherwise}$$

$$\pi_t = \frac{1}{\alpha(1-\beta(1-\alpha))-\sigma\kappa(1-\alpha)}\kappa\sigma r_L^n \text{ if } r_t^n = r_L^n \text{ and } \pi_t = 0 \text{ otherwise}$$

This solution is shown in figure 2 for the calibrated example. It shows excessive deflation in the periods in which the natural rate of interest is negative. A key reason for the excessive deflation is the expectation channel. The 90 percent chance of the natural rate of interest remaining negative for the next quarter creates the expectation of future deflation and a continued negative output gap, which creates even further deflation. Even if the central bank lowers the short-term nominal interest rate to zero, the real rate of return is positive, because the private sector expects deflation.

The reason for the sharp difference between the commitment and the Markov solution is that the Markov solution mandates zero inflation and zero output gap as soon as the natural rate of interest is positive. Thus the government cannot commit to a higher future price level as the optimal commitment implies and this lack of commitment is the main culprit for deflation. This is the deflation bias of discretionary policy.

Proposition 5 ***The deflation bias****. If A2(i), A2(ii), A3 and A4 then the following must hold at least locally to the steady state. The Markov equilibrium for $t \geq \tau$ is given by $\pi_t = x_t = 0$ and the result is excessive deflation and output gap for $t < \tau$ relative to a policy that implies $\pi_\tau > 0$ and $x_\tau > 0$ and $i_t = 0$ when $t \leq \tau$. The equilibrium is accurate up to an error of the order $O(||\xi, \bar{\delta}||^2)$*

Proof: See Technical Appendix

What is the logic behind the deflation bias? Consider one realization of the shock from the numerical example. Figure 3 shows the commitment and the Markov solution for $\tau = 15$. The optimal commitment is to keep the nominal interest rate low for a substantial period of time after the natural rate becomes positive resulting in $x^C_{\tau=15} > 0$ and $\pi^C_{\tau=15} > 0$. If the government is discretionary, however, this type of commitment is not credible. In period 15, once the natural rate becomes positive again, the government raises the nominal interest rate to steady state, thus achieving zero inflation and zero output gap from period 15 onward. The result of this policy, however, is excessive deflation in period 0 to 14. Why does the government choose this suboptimal policy if it cannot commit? Consider the objectives of the government (recall that I assume that $x^* = 0$). Once the natural rate of interest has become positive again, at time $t = 15$, the optimal policy is to set the nominal interest rate at the steady state from then on since this policy will result in zero output gap and zero inflation *at that time onwards* — thus the Markov policy is maximizing the objectives (39) from period 15 onwards. The government, therefore, has an incentive to renege on the optimal commitment because the optimal commitment results in a temporary boom and inflation in period 15 and thus implies higher utility losses in period 15 onwards relative to the Markov solution. In rational expectation, however, the private sector understands the government's incentives. If the government is unable to commit the result is excessive deflation and an output gap in period 0 to 14 when the zero bound is binding. The deflation bias is not an artifact of the numerical values assumed in the example. Proposition 5 is proofed analytically in the Technical Appendix without the cost of changing prices being above any critical value. Thus it remains true even if the cost of changing prices is made arbitrarily small, as long as it is not exactly zero.[28]

In the Markov solution any increase in the monetary base at zero interest rate will *always* be expected to be reversed. This can help explain why BOJ aggressive increase in the monetary base has had little effect. It cannot credibly promise higher *future* money supply – the private sector expects the BOJ to contract as soon as there is any sign of inflation. It is a credibility problem of a *rational* central bank that *cannot commit to future policy.* Krugman (1998) recognizes a commitment problem at zero interest rate. He assumes that the government follows a monetary policy targeting rule so that $M_t = M^*$. He then shows that if expectation about future money supply are fixed at M^*, increasing money supply at time t has no effect at zero interest rate. Krugman calls this "the inverse of the usual credibility problem." The key to effective policy, according to Krugman, is to commit to higher money supply in the future (as is verified by our numerical example), i.e. to "commit to being irresponsible". My result illustrates that this problem is not isolated to a government that is expected to follow a monetary targeting rule. The problem arises for a government that maximizes social welfare and has only one policy instrument but is unable to commit to *not re-optimize in the future disregarding past actions.* This is of practical importance. According to my solution, inefficient deflation is consistent with a rational government, as long as it is unable to commit to future policy. It may, therefore, be hard for it to change expectations for a government that has little credibility. In contrast, Krugman's government is committed to some monetary targeting policy rule that is suboptimal. It may, therefore, seem that it is easy to change policy expectations and that the only problem is to find the optimal policy. This result, however, indicates that more may be required.

4.1 Extension: The inflation bias vs the deflation bias

In this section I explore the connection between the deflation bias derived in last section and the inflation bias shown by Kydland and Prescott (1977) and Barro and Gordon (1983). The government's inability to commit in this model results in chronic inflation if $x^* > 0$. It is easy to show that if the zero bound is never binding (e.g under A3) inflation is given by

$$\pi_t = \bar{\pi} = \frac{1-\beta}{1-\beta+\theta\kappa}x^* > 0 \tag{40}$$

which is inefficient. This implies that the equilibrium nominal interest rate is given by

$$i_t = r_t^n + \bar{\pi}$$

Thus when there is an inflation bias in the economy, denoted by $\bar{\pi}$, a necessary condition for avoiding the zero bound is $r_t^n + \bar{\pi} \geq 0$. If the natural rate of interest is low enough, however, there is a deflation bias. Thus exactly the same commitment problem as shown in last section arises in an economy with an inflation bias if the shock is large enough, i.e. if $r_t^n < -\bar{\pi}$. To summarize:

Proposition 6 ***The inflation bias vs the deflation bias.*** *If A2(i), A3, A5 and* $0 \leq s < \frac{1}{\theta-1}$ *then* $\pi_t = \frac{\kappa}{1-\beta}\bar{x} = \bar{\pi}$ *for* $t \geq \tau$ *and there is excessive deflation and an output gap in period* $t < \tau$ *if* $r_L^n < -\bar{\pi}$ *relative to a policy that implies* $\pi_\tau > \bar{\pi}$ *and* $x_\tau > \bar{x}$ *and* $i_t = 0$ *when* $t < \tau$*. Here* $\bar{\pi}$ *is a solution to the equation* $\bar{\pi} = \frac{1-\beta}{1-\beta+\theta\kappa}x^* \geq 0$*. The equilibrium is accurate up to an error of the order* $O(||\xi, \bar{\delta}, 1+s-\frac{\theta}{\theta-1}||^2)$.

Proof: See Technical Appendix

Figure 4 shows the solution for inflation and the output gap for different values of x^*. Note that according to equation (40) a different value of x^* translates into different inflation targets for the government in a Markov equilibrium . The figure shows values of x^* that corresponds to 1%, 2% and 4% inflation targets respectively (I may vary this number by assuming different values for s in the expression for x^*). I assume A5 but the natural rate of interest is -4% in the low state and reverts back to steady state with 10 percent probability in each period. Note that only when the inflation bias corresponds to $\bar{\pi} = 4\%$ there is no deflation bias. If $\bar{\pi} < -r_L^n = 4\%$, the result is excessive deflation. The picture also illustrates, and this is the lesson of Proposition 6, that the deflation bias is a problem even in an economy with an average inflation bias, as long as the negative shock is large enough. The higher the average inflation bias, however, the larger the shock required for the deflation bias to be problematic.

What is a realistic inflation bias in an industrial economy? If I use the same values as in the numerical example above (see Computational Appendix) the implied inflation bias is below one percent per year. If the model is applied to Japan, this is indeed quite consistent with average inflation rates during the 80's and early 90's (before deflationary pressures emerged). The inflation bias, therefore, is relatively low and a deflationary bias is a considerable concern. I think it is fairly realistic to assume a low inflation bias for Japan. Throughout the 80's an early 90's, for example, there was virtually no unemployment, and the government had a small incentive to inflate, consistent with that x^* close to zero. The assumption that $x^* = 0$, therefore, does not seem grossly at odds with the evidence for Japan, and as argued by Rogoff (2003) the great disinflation in the world indicates that the inflation bias may be small (and shrinking) throughout the rest of the world.

Two aspects of a liquidity trap render the deflation bias a particularly acute problem, and possibly a more serious than the inflation bias. First, announcing a higher inflation target in a liquidity trap involves no direct policy action - since the short-term nominal interest rate is at zero it cannot be lowered any further. The central bank has, therefore, no obvious means to demonstrate its desire for inflation. Thus announcing an inflation target in a liquidity trap may be less credible then under normal circumstances when the central bank can take direct actions to show its commitment. Second, unfavorable shocks create the deflation bias. It may be hard for the central bank to acquire any reputation for dealing shocks if they are infrequent – which is presumably the case with shocks that make the zero bound binding given the few historical examples of the liquidity trap. To make matters worse, optimal policy in a liquidity trap involves committing to inflation. In an era of price stability the optimal policy under commitment is fundamentally different from what has been observed in the past.

5 Committing the Being Irresponsible

Last section demonstrated that deflation can be modelled as a credibility problem if the government is unable to commit to future policy and it's only instrument is open market operations. This section illustrates how the result changes if the government can use fiscal policy as an additional policy instrument. I first explore if deficit spending increases demand. When the government coordinates fiscal and monetary policy it can commit to future inflation and low nominal interest rate by cutting taxes and issuing nominal debt. I then use the result to interpret the effect of open market operations in a large spectrum of private assets, such as foreign exchange or stocks.

The assumption about monetary and fiscal policy is:

A6 *Coordinated fiscal and monetary policy instruments: Open market operations in government bonds, i.e. $\tilde{m}_t$, and deficit spending, $B_t - T_t$, are the instruments of policy.*

Using this assumption I can proof the following proposition.

Proposition 7 ***Committing to being irresponsible.*** *If A2, A5 and A6 then there is a solution at date $t \geq \tau$ for each of the endogenous variables given by $\Lambda_t = \Lambda^1 w_{t-1}$, and $w_t = w^1 w_{t-1}$ where Λ^1 and w^1 are constants. For a given value of w^1 there is a unique solution for Λ^1. The coefficient w^1 is a number that solves equation (118) in the the Technical Appendix. The solution for inflation is $\pi_t = \pi^1 w_{t-1}$ and the government can use deficit spending to increase inflation expectations when $\pi^1 \neq 0$, curbing deflation and the output gap in period $t < \tau$. The equilibrium is accurate up to an error of the order $O(||\xi, \bar{\delta}||^2)$*

I prove this proposition in the Technical Appendix. The solution shows that nominal debt effectively commits the government to inflation even if it is discretionary. It is instructive to write out the algebraic expression for the inflation coefficient in the solution. I show in the Appendix that at $t \geq \tau$ the solution for inflation is

$$\pi_t = \pi^1 w_{t-1} \text{ where } \pi^1 = \frac{s' g_G}{d'' u_c} \beta^{-1} + \phi_4^1 \tag{41}$$

The government can reduce the real value of its debt (and future interest payments) by either increasing taxes or inflation. Since both inflation and taxes are costly, it chooses a combination of the two. The presence of debt creates inflation through two channels in our model: (1) If the government has outstanding *nominal* debt it has incentives to create inflation to reduce the real value of the debt. This incentive is captured by the term $\frac{s' g_G}{d'' u_c} \beta^{-1}$ in equation (41). The marginal cost of taxation is $s' g_G$ and the marginal cost of inflation is $d'' u_c$. (2) If the government issues debt at time t, it has incentives to lower the real rate of return its pays on the debt it rolls over to time $t+1$. This incentive also translates into higher inflation.[29] This incentive is reflected in the value of the coefficient ϕ_4^1 which is the coefficient in the solution for the Lagrangian multiplier on the AS equation i.e. $\phi_{4t} = \phi_4^1 w_{t-1}$. This coefficient reflects the value of relaxing the aggregate supply constraint, which can be beneficial because of the reduction in the real interest rate paid on debt associated with higher output; i.e. the government has an incentive to create a boom (by lowering the real rate of interest) to lower the service on the debt it rolls over to the next period.

As I showed in the previous section, committing to future inflation and an output boom is exactly what is mandated by the optimal commitment. Using the same numerical example as in previous section, figures 5 and 6 show that it is optimal for a discretionary government to issue debt when the zero bound is binding. This effectively commits it to future inflation and an output boom once the natural rate of interest is positive again.[30] By cutting taxes and issuing debt in a liquidity trap the government curbs deflation and increases output to nearly the optimal commitment level. Figure 5 also shows that the nominal interest rate stays below the steady state after the natural rate of interest returns to normal and rises only slowly.

The Markov solution is still not fully optimal since it does not replicate the commitment solution perfectly. Table 1 shows welfare under three policy regimes. Welfare is evaluated by utility of the representative household. The first regime, R1, is a government that can fully commit to future policy and uses both monetary and fiscal policy to achieve its objective. The second, R2, is a government that cannot commit to future policy but uses both monetary and fiscal policy to maximize utility. The third regime, R3, is a government that is unable to commit to future policy and has only one policy instrument, i.e. open market operations in short-term government bonds. This table shows that the government's ability to use debt as a commitment device nearly eliminates all the costs of discretion. The interpretation of this utility index is that under R1 the representative household would pay 0.02 percent of its steady state quarterly consumption (forever) to avoid moving to regime R2. Thus the number 0.02 reflects that value of commitment if the government can coordinate monetary and fiscal policy. In contrast the loss in utility to move from R1 to R3 is very large or 13.48 percent.[31]

Table 1

Policy regime	Utility in cons. eq. units
R1	100
R2	99.98
R3	86.52

Proposition 7, figures 5 and 6, and Table 1 summarize the central results of this paper. Even if the government cannot commit it can stabilize the price level in a liquidity trap. A simple way of increasing inflation expectations is coordinating fiscal and monetary policy and running budget deficits, which in turn increases output and prices. The channel is simple. Budget deficits generate nominal debt. Nominal debt, in turn, makes a higher inflation target credible because the real value of the debt increases if the government reneges on the target. Higher inflation expectations lower the real rate of interest and thus stimulate aggregate demand. This policy involves direct actions by the government which can be useful to communicate the policy (a criticism that is sometimes raised about the commitment policy is that it does not require any actions, only announcements about future intentions, see e.g. Friedman (2003)). The government can announce an inflation target and proceed to increase budget deficits until the target is reached.

Discussion To contrast the commitment and the discretion solutions, it is useful to consider the evolution of the price level. Figure 7 shows the evolution of the price level under the three policy regimes reported in Table 1. The optimal solution (i.e. R1) is to commit to a higher future price level as can be seen in panel a of figure 7, although the extent to which the price level increases is small. If the government is unable to commit, however, this policy is not credible. A dramatic decline in the price level occurs under monetary discretion (i.e. R3) as shown in panel b. The price level declines by 35 percent, for example, if the natural rate of interest becomes positive in period 15 (this is the case I showed in figure 3). Panel c of figure 2 shows the large price decline can be avoided if the government uses fiscal policy to "commit to being irresponsible" (i.e. R2). This commitment involves increasing the price level once the natural rate becomes positive. When the natural rate of interest reverts to steady state in period 15, for example, the long run price level falls by less than 1 percent, compared to 35 percent decline under monetary discretion (R3).

It is worth considering the evolution of money supply in these different equilibria.[32] Figure 8 shows the long run nominal stock of money under each of the three policy regimes discussed above. In the figure I show the future level of the nominal stock of money in the case when the natural rate of interest reverts back to steady state in periods 3, 6, 9,12 and 15. The figure shows the level of money supply under each policy once the price level has converged back to its new steady state (so I do not need to make any assumptions here about the interest rate elasticity or output elasticity of money demand.)[33] I assume that the value of the money supply is 1 before the shocks hit the economy. The figure illustrates that the optimal commitment (R1) involves committing to a nominal money supply in the future that is only marginally larger than before the shock. In contrast the monetary discretion (R3) involves a considerable contraction of the monetary base. The government will accommodate any deflation at $t < \tau$ by contracting the monetary base as soon as the natural rate of interest becomes positive again in order to prevent inflation at $t \geq \tau$. Under a monetary and fiscal discretion regime (R2) aggressive deficit spending allows the government to credibly commit to a higher money supply, thus suppressing deflationary expectations. As a result the government achieves an equilibrium outcome that is close to the commitment solution, as illustrated in the welfare evaluation above and shown in figures 5 and 6.

An obvious question that arises if this model is applied to Japan. The gross national debt is currently over 130 percent of GDP. Why has the high level of outstanding debt in Japan failed to increase inflation expectations? There are at least two possible explanations of this. First, a large part of Japans debt is held by public institution and therefore not creating any inflation incentive. A better measure of the actual inflation incentive is net government debt. Net debt government debt as a fraction of GDP is not as high in Japan, about 70 percent, and only slightly above the G7 average. The other explanation (see Eggertsson (2004)) is that the Bank of Japan (BOJ) does not internalize the inflation incentive of outstanding government debt, i.e. that it has an objective that is more narrow than social welfare (that paper proofs that if the objective of BOJ is given by $\pi_t^2 + \lambda x_t^2$ deficit spending has no effect because it does not change the future incentive of the bank to inflate). Eggertsson (2004) argues that this indicates that there may be benefits of monetary and fiscal coordination, as suggested by Bernanke (2003), and verified by our welfare evaluation, and maintains that such cooperation may only need to be temporary to be effective.

5.1 Extension: Dropping money from helicopters and open market operations in foreign exchange as a commitment device

The model can be extended to analyze non-standard open market operations such as the purchase of foreign exchange and other private assets, or even more exotically, dropping money from helicopters. Here I discuss how these extensions enrich the results (an earlier version of this paper works out the details analytically – see Eggertsson (2003)).

Friedman suggests that the government can always control the price level by increasing money the supply, even in a liquidity trap. According to Friedman's famous *reductio ad absurdum* argument, if the government wants to increase the price level it can simply "drop money from helicopters." Eventually this should increase the price level – liquidity trap or not. Bernanke

(2000) revisits this proposal and suggests that Japanese government should make "money-financed transfers to domestic households–the real-life equivalent of that hoary thought experiment, the "helicopter drop" of newly printed money." This analysis supports Friedman and Bernanke's suggestions. The analysis suggests, however, that it is the increase in government liabilities (money+bonds), rather than the increase in the money supply that has this effect. Since money and bonds are equivalent in a liquidity trap dropping money from helicopters is exactly equivalent to issuing nominal bonds. If the treasury and the central bank coordinate policy the effect of dropping money from helicopters will have exactly the same effect as deficit spending. Thus this paper's model can be interpreted as establishing a "fiscal theory" of dropping money from helicopters.

The model can also be extended to consider the effects of the government buying foreign exchange (or any other private assets). It is often suggested that the central bank can depreciate the exchange rate and stimulate spending by buying foreign exchange (and similar arguments are sometimes raised about some other private assets and their corresponding price). Due to the interest rate parity (and similar asset pricing equations for other private assets), however, buying foreign exchange should have no effect on the exchange rate unless it changes expectations about future policy (since the interest rate parity says that the exchange rate should depend on current and expected interest rate differentials). Will such operations have any effect on expectations about future policy? Open market operations in foreign exchange (or any other private asset) would lead to a corresponding increase in public debt defined as money plus government bonds. This gives the government an incentives to create inflation *through exactly the same channel as I have explored in this paper and, therefore, leads to a corresponding depreciation in the nominal exchange rate hand-in-hand with the rise in inflation expectations.* An advantage of buying private assets, as opposed to cutting taxes, is that it does not worsen the net fiscal position of the government. It only changes the inflation incentive of the government.[34]

6 Conclusion

The great inflation of the 1970's was a key motivation for the rational expectation revolution and the analysis of the celebrated inflation bias first illustrated by Kydland and Prescott (1977). The main motivation behind this paper is the large decline in inflation in recent years (towards deflation – or very close to it – in some countries such as Japan, China, Hong Kong, Singapore, Taiwan, Israel and Swiss) together with extraordinary low interest rates throughout the world (interest rates have not been lower since the Great Depression in the countries listed above as well as in the US and the Euro area to name a few). I have shown that a similar dynamic inconsistency problem as Kydland and Prescott (1977) identify as the source of inefficient inflation (i.e. the inflation bias) can also cause inefficient deflation if the zero bound is binding. I coined this new dynamic inconsistency problem the deflation bias and contrasted it to the classic inflation bias. The source of the deflation bias, however, is inefficient response to temporary shocks, due to the governments inability to commit, whereas the inflation bias arises even in the absence of shocks. This implies that it may be even harder for a central bank to accrue reputation for fighting deflation than inflation (since the main culprit for deflation is infrequent shocks). Accordingly, the main focus of the paper has been policy measures to fight deflation that do not depend on reputation mechanism.

The paper establishes that deficit spending, i.e. cutting taxes and issuing nominal debt, is a simple way of fighting deflation. This may seem to resurrect an old Keynesian dictum. To draw that conclusion, however, is somewhat tenuous. Deficit spending in this paper works entirely through expectations. It increases output and prices only because it increases expectations about future money supply. If money supply in the future (when the zero bound is not binding anymore) is set without any regard to past policy decisions, there is no effect of deficit spending, as the irrelevance result in section 2.1 illustrated. In Eggertsson (2004) I show that a similar irrelevance result applies if the central bank is "goal independent," i.e. if it does not internalize the fiscal benefits of monetary expansion.

Another result of this paper is that open market operations in private assets can also be analyzed in a similar framework. Two interesting examples of private assets that can be bought by open market operations are stocks or foreign exchange. Open market operations in these assets are useful to fight deflation because they change the inflation incentives of the government in the future and thus change expectation from being deflationary to being inflationary.

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Notes

[1]Although recent signs indicates that the Japaneese economy may finally be recovering.

[2]There is a large literture that discusses optimal monetray policy rules when the zero bound is binding. Contributions include Summers (1991), Fuhrer and Madigan (1997), Woodford and Rotemberg (1997), Wolman (1999), Reifschneider and Williams (2000) and references there in. Since monetary policy rules arguably become credible over time these contributions can be viewed as illustration of how to *avoid* a liquidity trap rather than a prescription of how to escape them which is the focus here.

[3]The deflation bias is closely related, and in some sense a formalization of, a common objection to Krugman's policy proposal for the BoJ. To battle deflation he suggested that the BoJ should announce and inflation target of 5% for 15 years. Responding to this proposal, Kunio Okina, director of the Institute for Monetary Studies at the BoJ, said in DJN (1999): "Because short-term interest rates are already at zero setting an inflatio target of say 2 percent would not carry much credibility." Similar objections were raised by economists such as e.g. Dominigues (1998), Woodford (1999) and Svensson (2001).

[4]The Fiscal Theory of the Price Level (FTPL) popularized by Leeper (1991), Sims (1994) and Woodford (1994,1996) also stresses that fiscal policy can influence the price level. What separates this analysis from the FTPL (and the seminal contribution of Sargent and Wallace (1981)) is that in my setting fiscal policy only affects the price level because it changes the *inflation incentive* of the government. In contrast, according to the FTPL, fiscal policy affects the price level because it is *assumed* that the monetary authority commits to a (possibly suboptimal) interest rate rule and fiscal policy is modelled as a (possibly suboptimal) exogenous path of real government surpluses. Under these assumptions innovations in real government surpluses can influence the price level, since the prices may have to move for the government budget constraint to be satisfied. In my setting, however, the government budget constraint is a *constraint* on the policy choices of the government.

The approach taken here is more closely related to Calvo (1978) classic paper on the inflationary impact of government nominal liabilities when the government cannot commit to future policy (see Person et al (1987) for further references on this literature). The inflationary impact of debt analysed is essentially of the same source as analyzed by Calvo. The analysis here is different from Calvo's in that I explicitly analyse the inflationary impact of debt in a sticky price model (so that an increase in inflation expectation can increase output as well as prices) and show that increasing inflation expectation through this channel can be beneficial when the zero bound is binding.

[5]In contrast to Benabib et al (2002) where deflation is due to selffulfilling deflationary spirals.

[6]See for example Caballero et al (2003) that argue that banking problems are at the heart of the Japaneese recession.

[7]The idea is that real money balances enter the utility because they facilitate transactions. At some finite level of real money balances, e.g. when the representative household holds enough cash to pay for all consumption purchases in that period, holding more real money balances will not facilitate transaction any further and thereby add nothing to utility. This is at the "satiation" point of real money balances. I assume that there is no storage cost of holding money so increasing money holding can never reduce utility directly through $u(.)$. A satiation level in real money balances is also implied by several cash-in-advance models such as Lucas and Stokey (1987).

[8]Assumption A1 (i) is the Markov property. This assumption is not very restrictive since the vector ξ_t can be augmented by lagged values of a particular shock. Assumption A1 (ii) is added for tractability. Since K can be arbitrarily high it is not very restrictive.

[9]The problem of the household is: at every time t the household takes W_t and $\{Q_{t,T}, n_T(i), P_T, T_T, Z_T(i), \xi_T; T \geq t\}$ as exogenously given and maximizes (1) subject to (2) by choice of $\{M_T, h_T(i), C_T; T \geq t\}$.

[10]I assume that $d'(\Pi) > 0$ if $\Pi > 1$ and $d'(\Pi) < 0$ if $\Pi < 1$. Thus both inflation and deflation are costly. $d(1) = 0$ so that the optimal inflation rate is zero (consistent with the interpretation that this represent a cost of changing prices). Finally, $d'(1) = 0$ so that in the neighborhood of zero inflation the cost of price changes is of second order.

[11]The reason I do not assume Calvo prices is that it complicates to solution by introducing an additional state variable, i.e. price dispersion. This state variable, however, has only second order effects local to the steady state I approximate around and the resulting equilibrium is exactly the same as derived here (to the first order). This is shown formally in Eggertsson and Swanson (2004).

[12]I introduce it so that I can calibrate an inflationary bias that is independent of the other structural parameters, and this allows me to define a steady state at the fully efficient equilibrium allocation. I abstract from any tax costs that the financing of this subsidy may create.

[13]At every time t the firm takes $\{n_T(i), Q_{t,T}, P_T, Y_T, C_T, \frac{M_T}{P_T}, \xi_T; T \geq t\}$ as exogenously given and maximizes (4) by choice of $\{p_T(i); T \geq t\}$.

[14]The intuition for this bound is simple. There is no storage cost of holding money in the model and money can be held as an asset. It follows that i_t cannot be a negative number. No one would lend 100 dollars if he or she would get less than 100 dollars in return. I do not address here the possibility of imposing tax on currency as in Goodfriend (2000).

[15]The function s(T) is assumed to be differentiable with derivatives $s'(T) > 0$ and $s''(T) > 0$ for $T > 0$.

[16]The specification used here, however, focuses the analysis on the channel of fiscal policy that I am interested in. This is because for a constant F_t the level of taxes has no effect on the private sector equilibrium conditions (see equations above) but only affect the equilibrium by reducing the utility of the households (because a higher tax costs mean lower government consumption G_t). This allows me to isolate the effect current tax cuts will have on expectation about future monetary and fiscal policy, abstracting away from any effect on relative prices that those tax cuts may have. This is the key reason that I can obtain Propostion 1 in the next section even if taxation is costly. There is no doubt that tax policy can change relative prices and that these effects may be important. Those effects, however, are quite separate from the main focus of this paper. Eggertsson and Woodford (2004) consider how taxes that change relative prices can be used to affect the equilibrium allocations. That work considers both labor and consumption taxes assuming that the government can commit to future policy.

[17]One plausible sufficient condition that would guarantee that (23) must always hold is to assume that the private sector would never hold more government debt that correpondes to expected future discounted level of some maximum tax level – that would be a sum of the maximum seignorage revenues and some technology constraint on taxation.

[18]The Taylor rule is a member of this family in the following sense. The Taylor rule is

$$i_t = \max(\phi_\pi(\Pi_t - 1) + \phi_y Y_t, 0)$$

The money demand equation (8) defines the the interest rate as a function of the monetary base, inflation and output. This relation may then be used to infer the money supply rule that would result in an indentical equilibrium outcome as a Taylor rule described above and would be a member of the rules we consider.

[19]The reason why the variable q_t in the policy specification (17)-(23) can only include variables datet at time t is that if it included lagged variables this may give the central bank to influence policy expectation by effecting the variables when the zero bound is binding that will enter the policy rules when the interest rate are positive. In that case policy expectation would not be "exogenous" in the way defined above.

[20]An obvious criticism of the irrelevance result for fiscal policy in Proposition 1 is that it relies on Ricardian equivalence. This aspect of the model is unlikely to hold exactly in actual economies. If taxes effect relative prices, for example if I consider income or consumption taxes, changes in taxation changes demand in a way that

is independent of expectations about future policy. Similarly, if some households have finite-life horizons and no bequest motive, current taxing decisions affect their wealth and thus aggregate demand in a way that is also independent of expectation about future policy. The latter point developed by Ireland (2001) who show that in an overlapping generation model wealth transfers increase demand at zero nominal interest rate (this of course would also be true at positive interest rate). The assumption of Ricardian equivalence is not applied here, however, to downplay the importance of these additional policy channels. Rather, it is made to focus the attention on how fiscal policy may change policy expectations. That exercise is most clearly defined by specifying taxes so that they can *only* affect the equilibrium through expectations about future policy. Furthermore, since our model indicates that expectations about future monetary policy have large effects in equilibrium, my conjecture is that this channel is of first order in a liquidity trap and thus a good place to start.

[21] Since this constraint is never binding in equilibrium and $\bar{w}$ can be any arbitrarily high number for the results to be obtained, I do not model in detail the endogenous value of the debt limit.

[22] Note that if the conditional expectation of ξ_{t+1} at time t does not depend on calender time, these functions will be time invariant and one may drop the subscript t.

[23] The natural rate of output is the output that would be produced if prices where completely flexible. It is the output that solves the equation

$$v_y(Y_t^n, \xi_t) = \frac{\theta - 1}{\theta}(1 + s)u_c(Y_t^n, \xi_t). \tag{42}$$

Note that this definition of the natural rate of output is different from the efficient level of output which is obtained if $(1 + s) = \frac{\theta}{\theta - 1}$ and prices are flexible.

[24] Please note that the proposition 4 does not rely on this expansions since I derive the first order conditions of the government problem in the fully nonlinear model. The expansion is only reported to clarify the intution behind the propositions. In (39) I have expanded utility around the steady state discussed in section (3.4) and allowed for stochastic variations in ξ and also assumed that s and i^m may be deviate from the steady state I expand around. Derivation is available upon request.

[25] There is work in progress by Eggertsson and Vigfusson (2004) that shows that in a model with time-to-build and capital adjustment costs a permanent decline in the growth rate of productivity can lead to temporary negative natural rate of interest. The reason for this is that a model with time-to-build and a permanent productivity slowdown leads to a capital overhang and thus there is a transition period in which investment collapses and the natural rate of interest is negative.

[26] The numerical solution reported here is exactly the same as the one shown by Eggertsson and Woodford (2003) in a model that is similar but has Calvo prices (instead of the quadratic adjustment costs I assume here). Their solution also differs in that they compute the optimal policy in a linear quadratic framework. As our numerical solution illustrates, however, the results for the commitment equilibrium are identical. Jung et al (2001) also derive the commitment equilibrium in a linear quadratic framework but assume a deterministic process for the natural rate.

[27] Note that to ensure that the solution is bounded I need to assume that α satisfies the inequalities $\beta\alpha^2 + (1 + \sigma\kappa - \beta)\alpha - \sigma\kappa > 0$ and $0 < \alpha < 1$. If this condition is not satisfied the solution explodes and a linear approximation of the IS and the AS equation is not valid for shocks of any order of magnitude. Thus I would need to use other nonlinear solution methods to solve for the equilibrium if the value of α does not satisfy these bounds. Here I simply assume parameters so that these two inequalities are satisfied and a linear approximation of the IS and AS is feasible and the solution is accurate of order $o(||\xi, \bar{\delta}||^2)$ (see Technical Appendix).

[28] It is easiest to see this for a special case of A5. If $\alpha = 1$ the natural rate of interest is positive with probability 1 in period 1. Then Proposition 6 indicates that the solution in period 1 onwards is given by $\pi_t = x_t = 0$ for $t \geq 1$. The IS indicates that in period 0 the output gap is $x_0 = \sigma r_t^n$. Note that the output gap in period 0 is independent of the cost of changing prices since neither r_t^n nor σ are a function of the cost of price changes. This is because the output gap only depends on the difference between the current interest rate and the natural rate of interest and expectations about future inflation and output gap, and the latter are zero in period 1 onwards. The AS equation, however, indicates that the deflation in period 0 is going to depend on the cost of changing prices, i.e. $\pi_0 = \kappa x_0$. The lower the cost of changing prices the higher is $\kappa = \frac{\theta}{d''}(\sigma^{-1} + \omega)$ which indicates that there will be more deflation, the lower the cost of price changes (since x_0 is given by the IS equation which does not depend on d"). The intuition for this is that the lower the cost of price changes, the more prices need to adjust for the equation $x_0 = \sigma r_t^n$ to be satisfied. Thus the deflation bias is worse – in terms of actual fall in the price level – the lower the cost of changing prices. This basic intuition will also carry through to the stochastic case.

[29] Obstfeld (1991,1997) analyses a flexible price model with real debt (as opposed to nominal as in our model) but seignorage revenues due to money creation. He obtains a solution similar to mine (i.e. debt in his model creates inflation but is paid down over time). Calvo and Guidotti (1992) similarly illustrate a flexible price model that has

a similar solution. The influence of debt on inflation these authors illustrate is closely related to the first channel we discuss above. The second channel we show, however, is not present in these papers since they assume flexible prices.

[30] In general there are more than one solution for w^1 in equation 118. In the numerical examples I have done, however, all but one of the values that satisfy this equation are explosive and imply that by equation (118) that the value of γ_{2t} is negative once the debt limit of the government is reached. This in turn, violates the inequality constraint of this mulitplier, imlying that an explosive solution does not solve the first order conditions of the governments maximization problem. It can be proofed in a simplified version of the model that there is always a unique solution w^1 that solves the model and that it implies that debt converges back to steady state. For this version of the model, however, an analytic proof is not available, but in all the calibrated examples that I have explored this is indeed the case.

[31] Here I normalize the utility flow by transforming the utility stream (which is the future discounted stream of utility from private and public consumption – in all states of the world – minus the flow from the disutility of working) into a stream of a constant private consumption endowment.

[32] I have assumed that monetary frictions are very small, but as I discuss in the Technical Appendix money demand is still well defined so that it remains meaningful to discuss the growth rate of money supply (even if the real monetary base relative to output is very small). The money demand equation defines the evolution for real money balances in the equilibrium, i.e the variables $\tilde{m}_t$ which is normalized by the transaction technology parameter, and the growth rate of money supply can then be inferred from equation (66) in the Technical Appendix. I can then calculate the money supply for each of the different equilibria.

[33] It is not very instructive to consider the evolution of the nominal stock in the transition periods because the large movement in the nominal interest rate cause large swings in the nominal stock of money).

[34] Note that in a model with private asset the value of the assets becomes an additional state variable as shown in Eggertsson (2003).

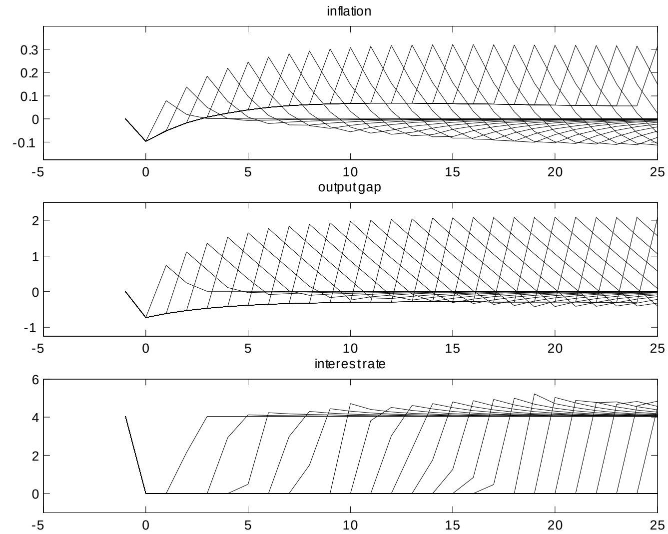

Figure 1: Inflation, the output gap, and the short-term nominal interest rate under optimal policy committment when the goverment can only use open market operations as its policy instrument. Each line represent the response of inflation, the output gap or the nominal interest rate when the natural rate of interest returns to its steady-state value in that period.

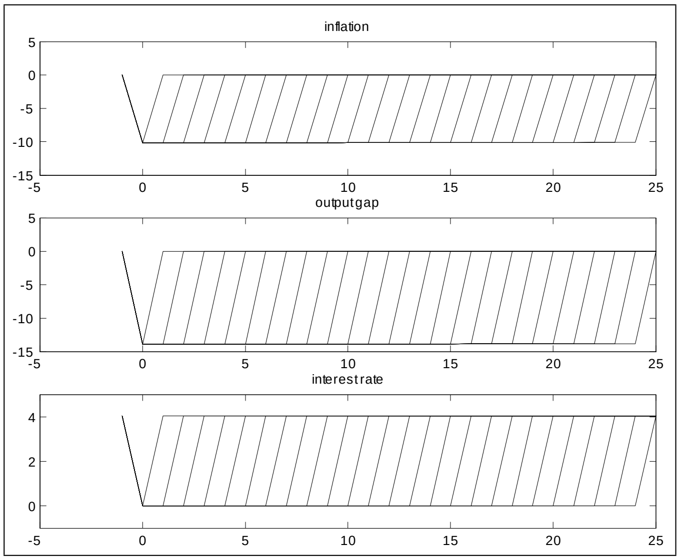

Figure 2: Inflation, the output gap, and the short-term nominal interest rate in a Markov equilibrium under discretion when the goverment can only use open market operations as its policy instrument. Each line represent the response of inflation, the output gap or the nominal interest rate when the natural rate of interest returns to its steady-state value in that period.

A Technical Appendix (not for publication)

This Technical Appendix contains the numerical solution methods used and some further details for the proofs, for readers interested in the technical details. The appendix is not intended for publication so that it contains quite extensive details to facilitate the verification of the results. Some of this material is also contained in the Technical Appendix of a companion paper Eggertsson (2004) and the computation method shown in section (A.2.5) is also applied (with appropriate modifications) in Eggertsson and Woodford (2003).

A.1 Explicit first order conditions for the commitment and Markov solution

This section shows the non-linear first order conditions of the governments maximization problem in the Markov and the commitment equilibrium. These give the necessary conditions for each equilibrium which can then be approximated to yield a solution.

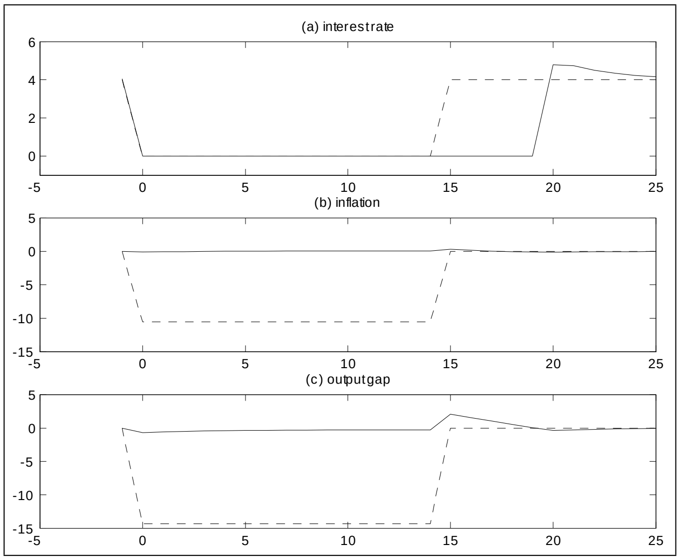

Figure 3: Response of the nominal interest rate, inflation and the output gap to a shocks that lasts for 15 quarters.

A.1.1 Commitment FOC

The commitment equilibrium Lagrangian is

$$
\begin{aligned}
L_t = E_{t_0} \sum_{t=t_0}^{\infty} \beta^t [&u(Y_t - d(\Pi_t) - F, m_t \Pi_t^{-1}, \xi_t)) + g(F - s(T_t), \xi_t) - \tilde{v}(Y_t) \\
&+ \phi_{1t}(\frac{u_m(Y_t - d(\Pi_t) - F, m_t \Pi_t^{-1}, \xi_t)\Pi_t^{-1}}{u_c(Y_t - d(\Pi_t) - F, m_t \Pi_t^{-1}, \xi_t)} - \frac{i_t - i^m}{1 + i_t}) \\
&+ \phi_{2t}(w_t - (1 + i_t)\Pi_t^{-1} w_{t-1} - (1 + i_t)F + (1 + i_t)T_t + (i_t - i^m) m_t \Pi_t^{-1}) \\
&+ \phi_{3t}(\beta f_t^e - \frac{u_c(Y_t - d(\Pi_t) - F, m_t \Pi_t^{-1}, \xi_t)}{1 + i_t}) \\
&+ \phi_{4t}(\theta Y_t [\frac{\theta - 1}{\theta}(1 + s) u_c(Y_t - d(\Pi_t) - F, m_t \Pi_t^{-1}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] \\
&+ u_c(Y_t - d(\Pi_t) - F, m_t \Pi_t^{-1}, \xi_t)\Pi_t d'(\Pi_t) - \beta S_t^e) \\
&+ \psi_{1t}(f_t^e - u_c(Y_{t+1} - d(\Pi_{t+1}) - F, m_{t+1}\Pi_{t+1}^{-1}, \xi_{t+1})\Pi_{t+1}^{-1}) \\
&+ \psi_{2t}(S_t^e - u_c(Y_{t+1} - d(\Pi_{t+1}) - F, m_{t+1}\Pi_{t+1}^{-1}, \xi_{t+1})\Pi_{t+1} d'(\Pi_{t+1})) + \gamma_{1t}(i_t - i^m) + \gamma_{2t}(\bar{w} - w_t)]
\end{aligned}
$$

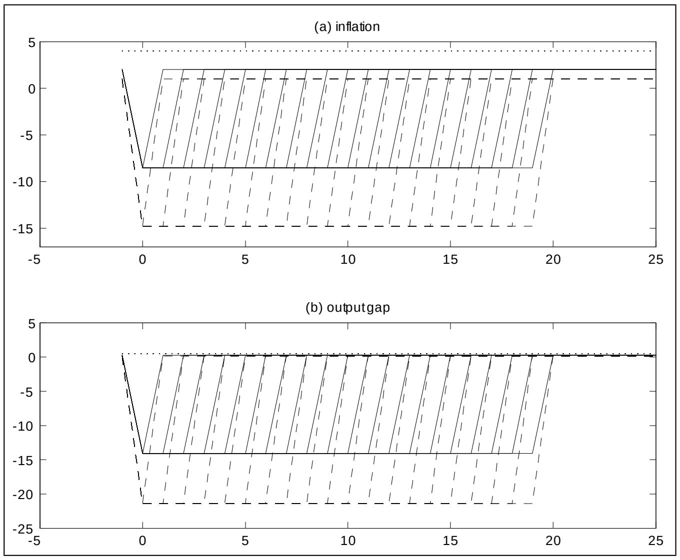

Figure 4: Inflation and the output gap under different assumption about steady state inflation bias when the natural rate of interest is temporarily -4 percent. The dotted lines correpond to a 4 percent steady state inflation bias, the solid line 2 percent and the dashed line 1 percent.

FOC (all the derivatives should be equated to zero) at all dates $t \geq 1$.

$$\begin{aligned}\frac{\delta L_s}{\delta \Pi_t} &= -u_c d'(\Pi_t) - u_m m_t \Pi_t^{-2} \\ &+\phi_{1t}[-\frac{u_{mc} d' \Pi_t^{-1}}{u_c} - \frac{u_{mm} m_t \Pi_t^{-3}}{u_c} - \frac{u_m \Pi_t^{-2}}{u_c} + \frac{u_m u_{cc} d' \Pi_t^{-1}}{u_c^2} + \frac{u_m u_{cm} m_t \Pi_t^{-2}}{u_c^2}] \\ &+\phi_{2t}[(1+i_t) w_{t-1} \Pi_t^{-2} - (i_t - i^m) m_t \Pi_t^{-2}] + \phi_{3t}[\frac{u_{cc} d'}{1+i_t} + \frac{u_{cm} m_t \Pi_t^{-2}}{1+i_t}] \\ &+\phi_{4t}[-Y_t(\theta-1)(1+s)(u_{cc} d' + m_t \Pi_t^{-2} u_{cm}) - u_{cc} \Pi_t d'^2 - u_{cm} m_t \Pi_t^{-2} d' + u_c \Pi_t d'' + u_c d'] \\ &+\beta^{-1} \psi_{1t-1}[u_{cc} d' \Pi_t^{-1} + u_{cm} m_t \Pi_t^{-3} + u_c \Pi_t^{-2}] \\ &+\beta^{-1} \psi_{2t-1}[u_{cc} d'^2 \Pi_t + u_{cm} d' m_t \Pi_t^{-1} - u_c d' - u_c d'' \Pi_t]\end{aligned} \tag{43}$$

$$\begin{aligned}\frac{\delta L_s}{\delta Y_t} &= u_c - \tilde{v}_y + \phi_{1t}[\frac{u_{mc} \Pi_t^{-1}}{u_c} - \frac{u_m u_{cc} \Pi_t^{-1}}{u_c^2}] - \phi_{3t} \frac{u_{cc}}{1+i_t} \\ &+\phi_{4t}[\theta(\frac{\theta-1}{\theta}(1+s) u_c - \tilde{v}_y) + \theta Y_t(\frac{\theta-1}{\theta}(1+s) u_{cc} - \tilde{v}_{yy}) + u_{cc} \Pi_t d'] \\ &-\beta^{-1} \psi_{1t-1} u_{cc} \Pi_t^{-1} - \beta^{-1} \psi_{2t-1} u_{cc} d' \Pi_t\end{aligned} \tag{44}$$

$$\frac{\delta L_s}{\delta i_t} = -\phi_{1t} \frac{1+i^m}{(1+i_t)^2} + \phi_{2t}(m_t \Pi_t^{-1} + T_t - w_{t-1} \Pi_t^{-1} - F) + \phi_{3t} \frac{u_c}{(1+i_t)^2} + \gamma_{1t} \tag{45}$$

$$\begin{aligned}\frac{\delta L_s}{\delta m_t} &= u_m \Pi_t^{-1} + \phi_{1t}[\frac{u_{mm} \Pi_t^{-2}}{u_c} - \frac{u_m u_{cm} \Pi_t^{-2}}{u_c^2}] + \phi_{2t}(i_t - i^m) \Pi_t^{-1} - \phi_{3t} \frac{u_{cm}}{1+i_t} \Pi_t^{-1} \\ &+\phi_{4t}[Y_t(\theta-1)(1+s) u_{cm} \Pi_t^{-1} + u_{cm} d'] - \beta^{-1} \psi_{1t-1} u_{cm} \Pi_t^{-2} - \beta^{-1} \psi_{2t-1} u_{cm} d'\end{aligned} \tag{46}$$

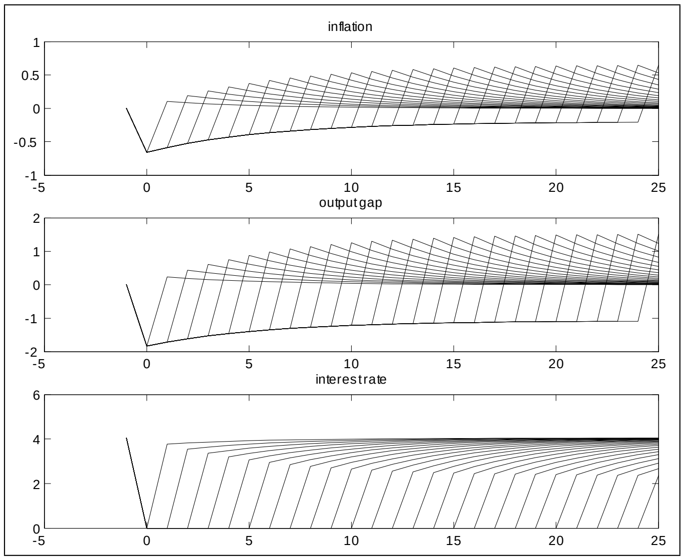

Figure 5: Inflation and output gap in a Markov equilibrium under discretion, when the government can use both monetary and fiscal policy to respond to a negative natural rate of interest.

$$\frac{\delta L_s}{\delta T_t} = -g_G s'(T_t) + \phi_{2t}(1 + i_t) \tag{47}$$

$$\frac{\delta L_s}{\delta w_t} = \phi_{2t} - \beta E_t \phi_{2t+1}(1 + i_{t+1})\Pi_{t+1}^{-1} - \gamma_{2t} \tag{48}$$

$$\frac{\delta L_s}{\delta f_t^e} = \beta \phi_{3t} + \psi_{1t} \tag{49}$$

$$\frac{\delta L_s}{\delta S_t^e} = -\beta \phi_{4t} + \psi_{2t} \tag{50}$$

The complementary slackness conditions are:

$$\gamma_{1t} \geq 0, \ i_t \geq i^m, \ \ \gamma_{1t}(i_t - i^m) = 0 \tag{51}$$

$$\gamma_{2t} \geq 0, \ \ \bar{w} - w_t \geq 0, \ \ \gamma_{2t}(\ \bar{w} - w_t) = 0 \tag{52}$$

For date $t = 0$ the same condition apply if I set $\psi_{1t-1} = \psi_{2t-1} = 0$. The first order conditions above, together with the constraints of the Lagrangian, give necessary conditions for the commitment equilibrium.

A.1.2 Markov equilibrium FOC

Markov equilibrium Lagrangian is:

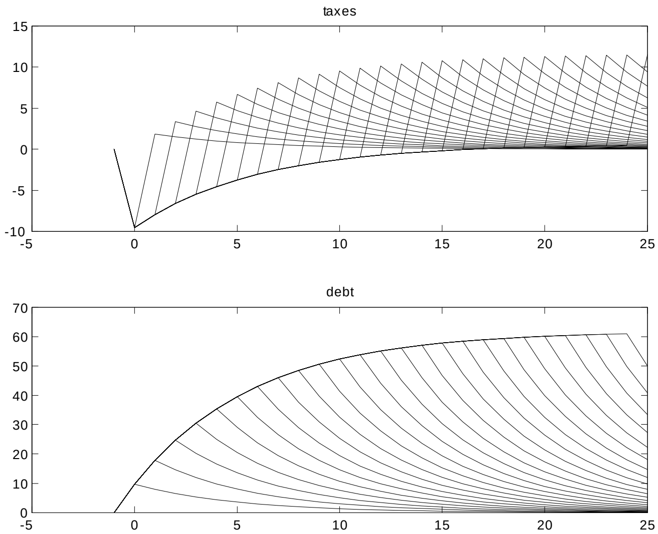

Figure 6: Taxes and debt in a Markov equilibrium under discretion, when the government can use both monetary and fiscal policy to respond to a negative natural rate of interest.

$$
\begin{aligned}
L_t = {} & u(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)) + g(F - s(T_t), \xi_t) - \tilde{v}(Y_t) + E_t\beta J(w_t, \xi_{t+1}) \\
& + \phi_{1t}(\frac{u_m(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)\Pi_t^{-1}}{u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)} - \frac{i_t - i^m}{1 + i_t}) \\
& + \phi_{2t}(w_t - (1 + i_t)\Pi_t^{-1}w_{t-1} - (1 + i_t)F + (1 + i_t)T_t + (i_t - i^m)m_t\Pi_t^{-1}) \\
& + \phi_{3t}(\beta f_t^e - \frac{u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)}{1 + i_t}) \\
& + \phi_{4t}(\theta Y_t[\frac{\theta - 1}{\theta}(1 + s)u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] \\
& + u_c(Y_t - d(\Pi_t) - F, m_t\Pi_t^{-1}, \xi_t)\Pi_t d'(\Pi_t) - \beta S_t^e) \\
& + \psi_{1t}(f_t^e - \bar{f}^e(w_t, \xi_t)) + \psi_{2t}(S_t^e - \bar{S}^e(w_t, \xi_t)) + \gamma_{1t}(i_t - i^m) + \gamma_{2t}(\bar{w} - w_t)
\end{aligned}
$$

FOC (all the derivatives should be equated to zero)

$$
\begin{aligned}
\frac{\delta L_s}{\delta \Pi_t} = {} & -u_c d'(\Pi_t) - u_m m_t \Pi_t^{-2} && (53) \\
& +\phi_{1t}[-\frac{u_{mc}d'\Pi_t^{-1}}{u_c} - \frac{u_{mm}m_t\Pi_t^{-3}}{u_c} - \frac{u_m\Pi_t^{-2}}{u_c} + \frac{u_m u_{cc} d'\Pi_t^{-1}}{u_c^2} + \frac{u_m u_{cm} m_t \Pi_t^{-2}}{u_c^2}] \\
& +\phi_{2t}[(1 + i_t)w_{t-1}\Pi_t^{-2} - (i_t - i^m)m_t\Pi_t^{-2}] + \phi_{3t}[\frac{u_{cc}d'}{1 + i_t} + \frac{u_{cm}m_t\Pi_t^{-2}}{1 + i_t}] && (54) \\
& +\phi_{4t}[-Y_t(\theta - 1)(1 + s)(u_{cc}d' + m_t\Pi_t^{-2}u_{cm}) - u_{cc}\Pi_t d'^2 - u_{cm}m_t\Pi_t^{-2}d' + u_c\Pi_t d'' + u_c d']
\end{aligned}
$$

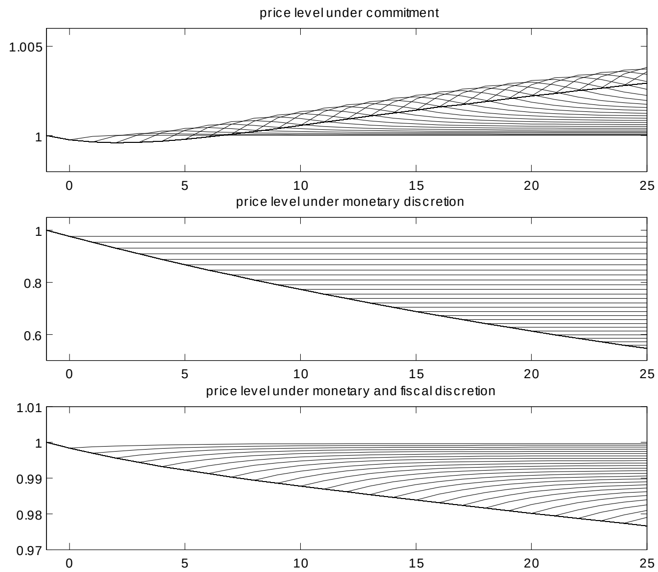

Figure 7: The evolution of the price level under different assumptions about policy.

$$\frac{\delta L_s}{\delta Y_t} = u_c - \tilde{v}_y + \phi_{1t}[\frac{u_{mc}\Pi_t^{-1}}{u_c} - \frac{u_m u_{cc}\Pi_t^{-1}}{u_c^2}] - \phi_{3t}\frac{u_{cc}}{1+i_t} + \phi_{4t}[\theta(\frac{\theta-1}{\theta}(1+s)u_c - \tilde{v}_y) + \theta Y_t(\frac{\theta-1}{\theta}(1+s)u_{cc} - \tilde{v}_{yy}) + u_{cc}\Pi_t d'] \tag{55}$$

$$\frac{\delta L_s}{\delta i_t} = -\phi_{1t}\frac{1+i^m}{(1+i_t)^2} + \phi_{2t}(m_t\Pi_t^{-1} + T_t - w_{t-1}\Pi_t^{-1} - F) + \phi_{3t}\frac{u_c}{(1+i_t)^2} + \gamma_{1t} \tag{56}$$

$$\frac{\delta L_s}{\delta m_t} = u_m\Pi_t^{-1} + \phi_{1t}[\frac{u_{mm}\Pi_t^{-2}}{u_c} - \frac{u_m u_{cm}\Pi_t^{-2}}{u_c^2}] + \phi_{2t}(i_t - i^m)\Pi_t^{-1} - \phi_{3t}\frac{u_{cm}}{1+i_t}\Pi_t^{-1} + \phi_{4t}[Y_t(\theta-1)(1+s)u_{cm}\Pi_t^{-1} + u_{cm}d'] \tag{57}$$

$$\frac{\delta L_s}{\delta T_t} = -g_G s'(T_t) + \phi_{2t}(1+i_t) \tag{58}$$

$$\frac{\delta L_s}{\delta w_t} = \beta E_t J_w(w_t, \xi_{t+1}) - \psi_{1t}f_w^e - \psi_{2t}S_w^e + \phi_{2t} - \gamma_{2t} \tag{59}$$

$$\frac{\delta L_s}{\delta f_t^e} = \beta\phi_{3t} + \psi_{1t} \tag{60}$$

$$\frac{\delta L_s}{\delta S_t^e} = -\beta\phi_{4t} + \psi_{2t} \tag{61}$$

The complementary slackness conditions are:

$$\gamma_{1t} \geq 0,\ i_t \geq i^m,\ \ \gamma_{1t}(i_t - i^m) = 0 \tag{62}$$

$$\gamma_{2t} \geq 0,\ \ \bar{w} - w_t \geq 0,\ \ \gamma_{2t}(\ \bar{w} - w_t) = 0 \tag{63}$$

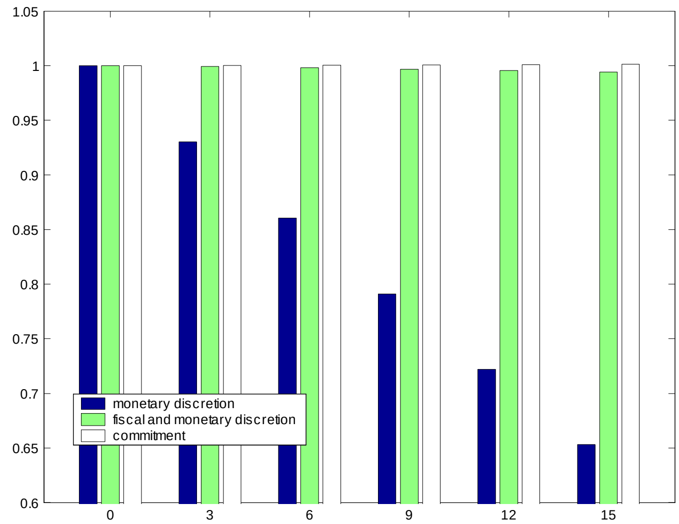

Figure 8: Long run nominal stock of money under different contingencies for the natural rate of interest.

The optimal plan under discretion also satisfies an envelope condition:

$$J_w(w_{t-1}, \xi_t) = -\phi_{2t}(1 + i_t)\Pi_t^{-1} \tag{64}$$

The first order conditions above, together with the constraints of the Lagrangian, give necessary conditions for the Markov equilibrium.

A.2 Approximation Method

This section show the approximation method used to approximate the commitment and Markov equilibrium. In section A.2.1 I discuss some simplifying assumption that I use to derive the steady state, in section A.2.2 I show the steady state, and in section A.2.3 I linearize the necessary condition and discuss the order of accuracy of the approximation. In section A.2.5 I show a solution method that uses the linearized condition to a find solution while taking into account the nonlinearity imposed by the zero bound.

A.2.1 Equilibrium in the absence of seigniorage revenues

As discussed in the text it simplifies the discussion to assume that the equilibrium base money is small, i.e. that m_t is a small number (see Woodford (2003), chapter 2, for a detailed treatment). I discuss in the footnote some reasons for why I conjecture that this abstraction has no significant effect.[35]

To analyze an equilibrium with a small monetary base I parameterize the utility function by the parameter $\bar{m}$ and assume that the preferences are of the form:

$$u(C_t, m_t\Pi_t^{-1}, \xi_t) = \tilde{u}(C_t, \xi_t) + \chi(\frac{m_t}{\bar{m}}\Pi_t^{-1}C_t^{-1}, \xi_t) \tag{65}$$

As the parameter $\bar{m}$ approaches zero the equilibrium value of m_t approaches zero as well. At the same time it is possible for the value of u_m to be a nontrivial positive number, so that money demand is well defined and the government's control over the short-term nominal interest rate is still well defined (see discussion in the proofs of Propositions 8 and 9 in the Appendix). I can define $\tilde{m}_t = \frac{m_t}{\bar{m}}$ as the policy instrument of the government, and this quantity can be positive even as $\bar{m}$ and m_t approach zero. Note that even as the real monetary base approaches the cashless limit the growth rate of the nominal stock of money associated with different equilibria is still well defined. I can then still discuss the implied path of money supply for different policy options. To see this note that

$$\frac{\tilde{m}_t}{\tilde{m}_{t-1}} = \frac{\frac{M_t}{P_{t-1}\bar{m}}}{\frac{M_{t-1}}{P_{t-2}\bar{m}}} = \frac{M_t}{M_{t-1}}\Pi_{t-1}^{-1} \tag{66}$$

which is independent of the size of $\bar{m}$. For a given equilibrium path of inflation and $\tilde{m}_t$ I can infer the growth rate of the nominal stock of money that is required to implement this equilibrium by the money demand equation. Since much of the discussion of the zero bound is phrased in terms of the implied path of money supply, I devote some space in the text to discuss how money supply adjusts in different equilibria. By assuming $\bar{m} \to 0$ I only abstract from the effect this adjustment has on the marginal utility of consumption and seigniorage revenues, both of which would be trivial in a realistic calibration (see footnote 35).

A.2.2 Steady state discussion and relation to literature on Markov Equilibrium

I define a steady state as a solution in the absence of shocks were each of the variables $(\Pi_t, Y_t, m_t, i_t, T_t, w_t, f_t^e, S_t^e) = (\Pi, Y, m, i, T, w, f^e, S^e)$ are constants. The steady state for the commitment equilibrium is straight forward. In general a steady-state of a Markov equilibrium is non-trivial to compute, as emphasized by Klein et al (2003). This is because each of the steady state variables depend on the mapping between the endogenous state (i.e. debt) and the unknown functions $J(.)$ and $\bar{e}(.)$, so that one needs to know the derivative of these functions with respect to the endogenous policy state variable to calculate the steady state. Klein et al suggest an approximation method by which one may approximate this steady state numerically by using perturbation methods. Here I take a different approach. Below I show that a steady state may be calculated under assumptions that are fairly common in the monetary literature (i.e. A2), without any further assumptions about the unknown functions $J(.)$ and $e(.)$.

Proposition 8 *If $\xi = 0$ at all times and A2(i)-(iii) hold there is a commitment equilibrium steady state that is given by $i = 1/\beta - 1$, $w = S^e = \phi_1 = \phi_3 = \phi_4 = \psi_1 = \psi_2 = \gamma_1 = \gamma_2 = 0$, $\Pi = 1$, $\phi_2 = g_G(\bar{F} - s(\bar{F}))s'(\bar{F})$, $f^e = u_c(\bar{Y}), F = \bar{F} = G = T + s(T)$ and $Y = \bar{Y}$ where $\bar{Y}$ is the unique solution to the equation $u_c(Y - F) = v_y(Y)$*

Proposition 9 *If $\xi = 0$ at all times and A2(i)-(iii) hold there is a Markov equilibrium steady state that is given by $i = 1/\beta - 1$, $w = S^e = \phi_1 = \phi_3 = \phi_4 = \psi_1 = \psi_2 = \gamma_1 = \gamma_2 = 0$, $\Pi = 1$, $\phi_2 = g_G(\bar{F} - s(\bar{F}))s'(\bar{F})$, $f^e = u_c(\bar{Y}), F = \bar{F} = G = T + s(T)$ and $Y = \bar{Y}$ where $\bar{Y}$ is the unique solution to the equation $u_c(Y - F) = v_y(Y)$.*

To proof these propositions I look at the algebraic expressions of the first order conditions of the government maximization problem. The proof is in section (A.4) of this Appendix. A noteworthy feature of the proof is that the mapping between the endogenous state and the functions $J(.)$ and $e(.)$ does not matter (i.e. the derivatives of these functions cancel out). The reason is that the Lagrangian multipliers associated with the expectation functions are zero in steady state and I may use the envelope condition to substitute for the derivative of the value function. The intuition for why these Lagrangian multipliers are zero in equilibrium is simple. At the steady state the distortions associated with monopolistic competition are zero (because of A2 (ii) in the text).

This implies that there is no gain of increasing output from steady state. In the steady the real debt is zero and according to assumption (i) seigniorage revenues are zero as well. This implies that even if there is cost of taxation in the steady state, increasing inflation does not reduce taxes. It follows that all the Lagrangian multipliers are zero in the steady state apart from the one on the government budget constraint. That multiplier, i.e. ϕ_2, is positive because there are steady state tax costs. Hence it would be beneficial (in terms of utility) to relax this constraint.

Proposition 8 and 9 give a convenient point to approximate around because the commitment and Markov solution are identical in this steady state. In the text, I relax both assumption A2(ii) and A3(iii) and investigate the behavior of the model local to this steady state. A major convenience of using A2 is that I can proof all of the key propositions in the paper analytically but do not need to rely on numerical simulation except to graph up the solutions.

There is by now a rich literature studying the question whether there can be multiple Markov equilibria in monetary models that are similar in many respects to the one I have described here (see e.g. Albanesi et al (2003), Dedola (2002) and King and Wolman (2003)). I do not proof the global uniqueness of the steady state in Proposition 9 but show that it is locally unique.[36] I conjecture, however, that the steady state is globally unique under A2.[37] But even if I would have written the model so that it had more than one steady state, the one studied here would still be the one of principal interest as discussed in the footnote.[38]

A.2.3 Approximate system and order of accuracy

The conditions that characterize equilibrium, in both the Markov and the commitment solution, are given by the constraints of the model and the first order conditions of the governments problem. A linearization of this system is complicated by the Kuhn-Tucker inequalities (51) and (52). I look for a solution in which the bound on government debt is never binding, and then verify that this bound is never binding in the equilibrium I calculate. Under this conjectured the solution to the inequalities (51) and (52) can be simplified into two cases:

$$\text{Case 1} : \gamma_t^1 = 0 \;\; \text{if } i_t > i^m \tag{67}$$

$$\text{Case 2} : i_t = i^m \;\; \text{otherwise} \tag{68}$$

Thus in both Case 1 and 2 I have equalities characterizing equilibrium. In the case of commitment, for example, these equations are (9), (24),(25), (27), (26), (29), (28), (30) and (43)-(50) and either (67) when $i_t > i^m$ or (68) otherwise. Under the condition A1(i) and A1(ii) but $i^m < \frac{1}{\beta} - 1$ then $i_t > i^m$ and Case 1 applies in the absence of shocks. In the knife edge case when $i^m = \frac{1}{\beta} - 1$, however, the equations that solve the two cases (in the absence of shocks) are identical since then both $\gamma_{1t} = 0$ and $i_t = i^m$. Thus both Case 1 and Case 2 have the same steady state in the knife edge case $i_t = i^m$. If I linearize around this steady state (which I show exists in Proposition 8 and 9) I obtain a solution that is accurate up to a residual $(||\xi||^2)$ for both Case 1 and Case 2. As a result I have one set of linear equations when the bound is binding, and another set of equations when it is not. The challenge, then, is to find a solution method that, for a given stochastic process for $\{\xi_t\}$, finds in which states of the world the interest rate bound is binding and the equilibrium has to satisfy the linear equations of Case 1, and in which states of the world it is not binding and the equilibrium has to satisfy the linear equations in Case 2. Since each of these solution are accurate to a residual $(||\xi||^2)$ the solutions can be made arbitrarily accurate by reducing the amplitude of the shocks. The next subsection shows a solution method, assuming the simple process for the natural rate of interest in the text, that numerically calculates when Case 1 applies and when Case 2 applies.

Note that I may also consider solutions when i^m is below the steady state nominal interest rate. A linear approximation of the equations around the steady state in Proposition 8 and 9 is still valid if the opportunity cost of holding money, i.e. $\bar{\delta} \equiv (i - i^m)/(1+i)$, is small enough. Specifically, the result will be exact up to a residual of order $(||\xi, \bar{\delta}||^2)$. In the text I assume that $i^m = 0$ (see Eggertsson and Woodford (2003) for further discussion about the accuracy of this approach when the zero bound is binding). A non-trival complication of approximating the Markov equilibrium is that I do not know the unknown expectation functions $\bar{e}(.)$. I illustrate a simple way of matching coefficients to approximate this function in the proof of Propositions 7.

A.2.4 Linearized solution

Here I linearize the first order conditions and the constraints around the steady state in Propositions 8 and 9. I assume the form of the utility discussed in section A.2.1. I allow for deviations in the vector of shocks ξ_t, the production subsidy s (the latter deviation is used in Proposition 6) and in i^m (which I assume is zero) so that the equations are accurate of order $o(||\xi, \bar{\delta}, 1+s-\frac{\theta}{\theta-1}||^2)$. I abstract from the effect of the shocks on the disutility of labor. Here I use the notation $dz_t = z_t - z_{ss}$ The economic constraints under both commitment and discretion are:

$$\bar{u}_c \bar{Y} d'' d\Pi_t + \theta \bar{Y}(\bar{u}_{cc} - \bar{v}_{yy}) dY_t + \bar{Y}(\theta - 1)\bar{u}_c ds + \theta \bar{u}_{c\xi} d\xi_t - \beta dS_t^e = 0 \tag{69}$$

$$\bar{u}_{cc} dY_t + \bar{u}_{c\xi} d\xi_t - \beta \bar{u}_c di_t - \beta df_t^e = 0 \tag{70}$$

$$dw_t - \frac{1}{\beta} dw_{t-1} + \frac{1}{\beta} dT_t = 0 \tag{71}$$

$$dS_t^e - \bar{u}_c d'' E_t d\Pi_{t+1} = 0 \tag{72}$$

$$df_t^e + \bar{u}_c E_t d\Pi_{t+1} - \bar{u}_{cc} E_t dY_{t+1} - \bar{u}_{c\xi} E_t \xi_{t+1} = 0 \tag{73}$$

The equation determining the natural rate of output is (see footnote (23) for the nonlinear equation):

$$(\bar{v}_{yy} - \bar{u}_{cc}) dY_t^n - \bar{u}_{c\xi} d\xi_t - \frac{(\theta - 1)}{\theta} \bar{u}_c ds = 0 \tag{74}$$

The equation determining the natural rate of interest is:

$$\beta E_t(\bar{u}_{cc} dY_{t+1}^n - \bar{u}_{c\xi} d\xi_{t+1}) - (\bar{u}_{cc} dY_t^n - \bar{u}_{c\xi} d\xi_t) + \beta \bar{u}_{cc} dr_t^n = 0 \tag{75}$$

Equations (69),(70) and (72)-(75) can be reduced to the IS and the AS equations in the text. Note that the real money balances deflated by $\bar{m}$, i.e. $\tilde{m}_t$, are well defined in the cashless limit so that equation 66 is

$$d\tilde{m}_t - d\tilde{m}_{t-1} - d\frac{M_t}{M_{t-1}} + d\pi_{t-1} = 0$$

and money demand is approximated by

$$\frac{\bar{\chi}_{mm}}{u_c} d\tilde{m}_t - \frac{\bar{\chi}_{mm}}{u_c} \tilde{m} d\Pi_t - \frac{\bar{\chi}_{mm}}{u_c} \tilde{m} dY_t - \beta di_t + \beta di^m = 0$$

The Kuhn Tucker conditions imply that

Case 1 when $i_t > i^m$

$$d\gamma_{1t} = 0 \tag{76}$$

Case 2 when $i_t = i^m$

$$di_t = 0 \tag{77}$$

I look for a solution in which case the debt limit is never binding so that $d\gamma_{2t} = 0$ at all times and verify that this is satisfied in equilibrium. The linearized FOC in a commitment equilibrium are (where I have substituted out for ψ_1 and ψ_2):

$$-d''\bar{u}_c d\Pi_t + \bar{\phi}_2 \beta^{-1} dw_{t-1} + d''\bar{u}_c d\phi_{4t} - \bar{u}_c d\phi_{3t-1} - d''\bar{u}_c d\phi_{4t-1} = 0 \tag{78}$$

$$(\bar{u}_{cc} - \bar{v}_{yy})dY_t + \bar{u}_{c\xi} d\xi_t - \bar{v}_{y\xi} d\xi_t - \bar{u}_{cc}\beta d\phi_{3t} + \theta(\bar{u}_{cc} - \bar{v}_{yy})d\phi_{4t} + \bar{u}_{cc} d\phi_{3t-1} = 0 \tag{79}$$

$$\bar{\phi}_2 dT_t - \bar{\phi}_2 dw_{t-1} + \bar{u}_c \beta^2 d\phi_{3t} + d\gamma_{1t} = 0 \tag{80}$$

$$\bar{g}_{GG}(s')^2 dT_t - \bar{g}_G s'' dT_t - \bar{g}_{G\xi} d\xi_t + \beta^{-1} d\phi_{2t} + \bar{\phi}_2 di_t = 0 \tag{81}$$

$$d\phi_{2t} - E_t d\phi_{2t+1} - \beta\bar{\phi}_2 E_t di_{t+1} + \bar{\phi}_2 E_t d\Pi_{t+1} - d\gamma_{2t} = 0 \tag{82}$$

Linearized FOC in a Markov Equilibrium

$$-d''\bar{u}_c d\Pi_t + \bar{\phi}_2 \beta^{-1} dw_{t-1} + d''\bar{u}_c d\phi_{4t} = 0 \tag{83}$$

$$(\bar{u}_{cc} - \bar{v}_{yy})dY_t + \bar{u}_{c\xi} d\xi_t - \bar{v}_{y\xi} d\xi_t - \bar{u}_{cc}\beta d\phi_{3t} + \theta(\bar{u}_{cc} - \bar{v}_{yy})d\phi_{4t} = 0 \tag{84}$$

$$\bar{\phi}_2 dT_t - \bar{\phi}_2 dw_{t-1} + \bar{u}_c \beta^2 d\phi_{3t} + d\gamma_{1t} = 0 \tag{85}$$

$$\bar{g}_{GG}(s')^2 dT_t - \bar{g}_G s'' dT_t - \bar{g}_{G\xi} d\xi_t + \beta^{-1} d\phi_{2t} + \bar{\phi}_2 di_t = 0 \tag{86}$$

$$d\phi_{2t} - E_t d\phi_{2t+1} - \beta\bar{\phi}_2 E_t di_{t+1} + \bar{\phi}_2 E_t d\Pi_{t+1} + \beta f_w d\phi_{3t} - \beta S_w d\phi_{4t} - d\gamma_{2t} = 0 \tag{87}$$

Note that the first order condition with respect to m_t does not play any role in the cashless limit so that it is omitted above.

A.2.5 Computational method

Here I illustrate a solution method for the optimal commitment solution. This method can also be applied, with appropriate modification of each of the steps, to find the Markov solution. I assume shocks so that the natural rate of interest becomes unexpectedly negative in period 0 and the reverts back to normal with probability α_t in every period t as in A5 (one may use (74) and (75) to find what a given negative number for the natural rate of interest implies for the underlying exogenous shocks). I assume that there is a final date K in which the natural rate becomes positive with probability one (this date can be arbitrarily far into the future).

The solution takes the form:

$$\begin{array}{lcccl} \text{Case 2 } i_t & = & 0 & \forall \quad t & 0 \le t < \tau + k \\ \text{Case 1 } i_t & > & 0 & \forall \quad t & t \ge \tau + k \end{array}$$

Here τ is he stochastic date at which the natural rate of interest returns to steady state. I assume that τ can take any value between 1 and the terminal date K that can be arbitrarily far into the future. The number $\tau + k_\tau$ is the period in which the zero bound stops being binding in the contingency when the natural rate of interest becomes positive in period τ. Note that the value of k_τ can depend on the value of τ. I first show the solution for the problem as if I knew the sequence $\{k_\tau\}_{\tau=1}^{K}$. I then describe a numerical method to find the sequence $\{k_\tau\}_{\tau=1}^{K}$.

The solution for $t \geq \tau + k_\tau$ The system of linearized equations (78)-(82) (but (83)-(87) in the case of the Markov solution), (69)-(73), and (76) can be written in the form:

$$\begin{bmatrix} E_t Z_{t+1} \\ P_t \end{bmatrix} = M \begin{bmatrix} Z_t \\ P_{t-1} \end{bmatrix}$$

where $Z_t \equiv \begin{bmatrix} \Lambda_t & e_t & \phi_t \end{bmatrix}^T$ and $P_t \equiv \begin{bmatrix} w_t & e_t & \psi_t & \gamma_t^1 \end{bmatrix}^T$. If there are eleven eigenvalues of the matrix M outside the unit circle this system has a unique bounded solution of the form:

$$P_t = \Omega^0 P_{t-1} \tag{88}$$

$$Z_t = \Lambda^0 P_{t-1} \tag{89}$$

The solution for $\tau \leq t < \tau + k$ Again this is a perfect foresight solution but with the zero bound binding. The solution now satisfies the equations (78)-(82) (but (83)-(87) in the case of the Markov solution), (69)-(73) but (77) instead of (76). The system can be written on the form:

$$\begin{bmatrix} P_t \\ Z_t \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} P_{t-1} \\ Z_{t+1} \end{bmatrix} + \begin{bmatrix} M \\ V \end{bmatrix}$$

This system has a solution of the form:

$$P_{\tau+j} = \Omega^{k_\tau - j} P_{\tau+j-1} + \Phi^{k_\tau - j} \tag{90}$$

$$Z_{\tau+j} = \Lambda^{k_\tau - j} P_{\tau+j-1} + \Theta^{k_\tau - j} \tag{91}$$

where $j = 0, 1, 2, ..., k$. Here $\Omega^{k_\tau - j}$ is the coefficient in the solution when there are $k_\tau - j$ periods until the zero bound stops being binding (i.e. when $j - k_\tau = 0$ the zero bound is not binding anymore and the solution is equivalent to (88)-(89)). We can find the numbers $\Lambda^j, \Omega^j, \Theta^j$ and Φ^j for $j = 1, 2, 3, ..., k$ by solving the equations below using the initial conditions $\Phi^0 = \Theta^0 = 0$ for $j = 0$ and the initial conditions for Λ^j and Ω^j given in (88)-(89):

$$\Omega^j = [I - B\Lambda^{j-1}]^{-1} A$$

$$\Lambda^j = C + D\Lambda^{j-1}\Omega^j$$

$$\Phi^j = (I - B\Lambda^{j-1})^{-1}[B\Theta^{j-1} + M]$$

$$\Theta^j = D\Lambda^{j-1}\Phi^j + D\Theta^{j-1} + V$$

The solution for $t < \tau$ The solution satisfies (78)-(82) (but (83)-(87) in the case of the Markov solution), (69)-(73), and (77). Note that each of the expectation variables can be written as $\tilde{x}_t = E_t x_{t+1} = \alpha_{t+1}\tilde{x}_{t+1} + (1 - \alpha_{t+1})x_{t+1}$ where α_{t+1} is the probability that the natural rate of interest becomes positive in period $t + 1$. Here hat on the variables refers to the value of each variable contingent on that the natural rate of interest is negative. I may now use the solution for Z_{t+1} in 91 to substitute for Z_{t+1}, i.e. the value of each variable contingent on that the natural rate becomes positive again in terms of the hatted variables. The value of x_{t+1}, for example, can be written as $x_{t+1} = \Lambda_{21}^{k_{t+1}}\tilde{\phi}_{1t} + \Lambda_{22}^{k_{t+1}}\tilde{\phi}_{2t} + \Theta_2^{k_{t+1}}$ where $\Lambda_{ij}^{k_{t+1}}$ is the ijth element of the matrix $\Lambda^{k_{t+1}}$ and the value k_{t+1} depends on the number of additional periods that the zero bound is binding (recall that I am solving the equilibrium under the assumption that I know the value of the sequence $\{k_\tau\}_{\tau=1}^{S}$) . Hence I can write the system as:

$$\begin{bmatrix} \tilde{P}_t \\ \tilde{Z}_t \end{bmatrix} = \begin{bmatrix} A_t & B_t \\ C_t & D_t \end{bmatrix} \begin{bmatrix} \tilde{P}_{t-1} \\ \tilde{Z}_{t+1} \end{bmatrix} + \begin{bmatrix} M_t \\ V_t \end{bmatrix}$$

I can solve this backwards from the date K in which the natural rate returns back to normal with probability one. I can then calculate the path for each variable to date 0. Note that.

$$B_{K-1} = D_{K-1} = 0$$

By recursive substitution I can find a solution of the form:

$$\tilde{P}_t = \Omega_t \tilde{P}_{t-1} + \Phi_t \tag{92}$$

$$\tilde{Z}_t = \Lambda_t \tilde{P}_{t-1} + \Theta_t \tag{93}$$

where the coefficients are time dependent. To find the numbers $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t consider the solution of the system in period $K-1$ when $B_{K-1} = D_{K-1} = 0$. I have:

$$\Omega_{K-1} = A_{K-1}$$

$$\Phi_{K-1} = M_{K-1}$$

$$\Lambda_{K-1} = C_{K-1}$$

$$\Theta_{K-1} = V_{K-1}$$

I can find of numbers $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t for period 0 to $K-2$ by solving the system below (using the initial conditions shown above for $S-1$):

$$\Omega_t = [I - B_t \Lambda_{t+1}]^{-1} A_t$$

$$\Lambda_t = C_t + D_t \Lambda_{t+1} \Omega_t$$

$$\Phi_t = (I - B_t \Lambda_{t+1})^{-1} [B_t \Theta_{t+1} + M_t]$$

$$\Theta_t = D_t \Lambda_{t+1} \Phi_t + D_t \Theta_{t+1} + V_t$$

Using the initial condition $\tilde{P}_{-1} = 0$ I can solve for each of the endogenous variables under the contingency that the trap last to period K by (92) and (93). This initial condition corresponds to the assumption that the system is at its steady state at time $t-1$ and the initial shock is unexpected. I then use the solution from (88)-(91) to solve for each of the variables when the natural rate reverts back to steady state.

Solving for $\{k_\tau\}_{t=0}^{\infty}$ A simple way to find the value for $\{k_\tau\}_{\tau=1}^{\infty}$ is to first assume that k_τ is the same for all τ and find the k so that the zero bound is never violated. Suppose that the system has converged at $t = 25$ (i.e. the response of each of the variables is the same). Then I can move to 24 and see if $k_\tau = 4$ for $\tau = 1, 2, ...24$ is a solution that never violates the zero bound. If not move to 23 and try the same thing and so on. For preparing this paper I wrote a routine in MATLAB that applied this method to find the optimal solution and verified that the results satisfied all the necessary conditions.

A.3 Calibration parameters

In the numerical examples I assume the following functional forms for preferences and technology:

$$u(C,\xi) = \frac{C^{1-\tilde{\sigma}^{-1}}\bar{C}^{\tilde{\sigma}^{-1}}}{1-\tilde{\sigma}^{-1}}$$

where $\bar{C}$ is a preference shock assumed to be 1 in steady state.

$$g(G,\xi) = g_1 \frac{G^{1-\tilde{\sigma}^{-1}}\bar{G}^{\tilde{\sigma}^{-1}}}{1-\tilde{\sigma}^{-1}}$$

where $\bar{G}$ is a preference shock assumed to be 1 in steady state

$$v(H,\xi) = \frac{\lambda_1}{1+\lambda_2} H^{1+\lambda_2}\bar{H}^{-\lambda_2}$$

where $\bar{H}$ is a preference shock assumed to be 1 in steady state

$$y = Ah^{\epsilon}$$

where A is a technology shock assumed to be 1 in steady state. I may substitute the production function into the disutility of working to obtain (assuming A=1):

$$\tilde{v}(Y,\xi_t) = \frac{\lambda_1}{1+\omega} Y^{1+\omega}\bar{H}^{-\lambda_2}$$

having defined $1+\omega \equiv \frac{1+\lambda_2}{\epsilon}$. When calibrating the shocks that generate the temporarily negative natural rate of interest I assume that it is the shock $\bar{C}$ that is driving the natural rate of interest negative (as opposed to A) since otherwise a negative natural rate of interest would be associated with a higher natural rate of output which does not seem to be the most economically interesting case. I assume that the shock $\bar{G}$ is such that the F_t would be constant in the absence of the zero bound, in order to keep the optimal size of the government (in absence of the zero bound) constant (see Eggertsson (2004) for details)). The cost of price adjustment is assumed to take the form:

$$d(\Pi) = d_1\Pi^2$$

The cost of taxes is assumed to take to form:

$$s(T) = s_1 T^2$$

Aggregate demand implies $Y = C + F = C + G + s(F)$. I normalize $Y = 1$ in steady state and assume that the share of the government in production is $F = 0.3$. Tax collection as a share of government spending is assumed to be $\gamma = 5\%$ of government spending. This implies

$$\gamma = \frac{s(F)}{F} = s_1 F$$

so that $s_1 = \frac{\gamma}{F}$. The result for the inflation and output gap response are not very sensitive to varying γ under either commitment or discretion. The size of the public debt issued in the Markov equilibrium, however, crucially depends on this variable. In particular if γ is reduced the size of the debt issued rises substantially. For example if $\gamma = 0.5\%$ the public debt issued is about ten times bigger than reported in the figure in the paper. I assume that government spending are set at their optimal level in steady state giving me the relationship (see Eggertsson 2004 for details on how this is derived)

$$g_1 = \frac{u_c}{g_G - s' g_G} = \frac{C^{-\tilde{\sigma}^{-1}}}{G^{-\tilde{\sigma}^{-1}}(1-s')} = (\frac{G}{C})^{\tilde{\sigma}^{-1}} \frac{1}{1-s'} = (\frac{G}{C})^{\tilde{\sigma}^{-1}} \frac{1}{1-2s_1 F}$$

The IS equation and the AS equation are

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n)$$

$$\pi_t = k\pi_t + \beta E_t \pi_{t+1}$$

I assume, as Eggertsson and Woodford (2003a), that the interest rate elasticity, σ, is 0.5. The relationship between $\tilde{\sigma}$ and σ is

$$\tilde{\sigma} = \sigma \frac{Y}{C}$$

I assume that κ is 0.02 as in Eggertsson and Woodford (2003a). The relationship between κ and the other parameters of the model is $\kappa = \theta \frac{(\tilde{\sigma}^{-1} + \lambda_2)}{d''}$. I scale hours worked so that $Y = 1$ in steady state which implies

$$v_y = \lambda_1$$

Finally I assume that $\theta = 7.89$ as in Rotemberg and Woodford (1997) and that $\lambda_2 = 2$. The calibration value for the parameters are summarized in the table below:

Table 2

σ	0.5
g_1	0.33
λ_1	1.65
λ_2	2
d_1	787
s_1	0.17
θ	7.87

A.4 Proofs

A.4.1 Proof of Proposition 1:

I proof this proposition by showing that all the necessary and sufficient conditions for a PSE listed in Definition 1 (i.e. equation (3)-(16)) can be written without any reference to either $T(.)$ or $\psi_t(.)$. Note that the irrelevance result does his not mean that one may specify any function for $T(.)$ and $\psi(.)$ – since these functions need to be consistent with the policy regime (for example one may not set taxes equal to zero forever!).

1. I first show that the equilibrium conditions can be written without any references to the function $T(.)$. This function only appears in equation (14) and (15). Since F_t is a constant by (21) and G_t appears in no equation but (14) this equation is redundant and only defines G_t as a function of taxes. Since only one period bonds are issued I can write $W_{t+1} = (1+i_t)B_t + (1+i^m)M_t$ and equation (15) becomes

$$\frac{1}{1+i_t} W_{t+1} = W_t + P_t F - P_t T_t - \frac{i_t - i^m}{1+i_t} M_t \tag{94}$$

which defines W_{t+1} as a function of T_t and M_t and W_t. For this equation to be redundant I must show that I can write all the other equilibrium conditions without any reference to W_t. The only condition that involves W_t is (10). Using $W_{t+1} = (1+i_t)B_t + (1+i^m)M_t$ and (23) this condition can be simplified to yield:

$$\lim_{T\to\infty} \beta^T E_t[u_c(Y_T - d(\Pi_T) - F_T, M_T/P_T; \xi_T) M_T/P_T] = 0 \tag{95}$$

which neither depends on $T(.)$ or W_t. Thus I have shown that the equilibrium conditions can be expressed without any reference to $T(.)$.

2. I now show that all the constraint of required for a private sector equilibrium can be expressed independently of the specification of $\psi(.)$. I first consider equation (95). If the nominal interest rate is never binding asymtotically M_t will not depend on $\psi(.)$ according (17). The specification of $\psi(.)$ could be important if the zero bound is asymtotically binding. Assuming A1 the equilibrium is deterministic at all dates $t \geq K$. It is easy to show that for the zero bound to be asymtotically binding I must have $\Pi_t = \frac{P_t}{P_{t-1}} = \beta$ and $Y_t = \bar{Y}$. Then I can write, for all $t \geq K$ (i.e. all dates after the uncertainty is resolved) $P_t = \beta^{t-K} P_K$. Then (95) becomes

$$\lim_{T\to\infty} \beta^K [u_c(Y_T - d(\Pi_T) - F_T, M_T/P_T; \xi_T) \frac{M_T}{P_K} = 0$$

This condition is only satisfied if $M_T \to 0$. But this would violate (20) and thus an asymtotic deflationary equilibrium is not consistent with my specification of fiscal and monetary policy. It follows that the specification of $\psi(.)$ has no effect on whether or not (95) is satisfied since I have just shown that the zero bound cannot be asymtotically binding under the monetary and fiscal regime specified. What remains to be shown is that all the other equilibrium conditions can be written without any reference to the function $\psi(.)$. That part of the proof follows exactly the same steps as the proof of Proposition 1 in Eggertsson and Woodford (2003a).

A.4.2 Proof of Proposition 4

In this equilibrium there is only one policy instrument so that $dT_t = dw_t = 0$ and I may ignore the linearized first order conditions (81), (82) for commitment and (86) and (87) in the Markov equilibrium. The remaining FOC along with the constraint (69), (70) and (76) determine the equilibrium.

1. I first consider the commitment case. Equation (81) indicates that $\phi_{3t} = 0$. Then I can write (78) and (79) in terms of inflation and output gap as (using (74) to solve it in terms of the output gap):

$$\pi_t - \phi_{4t} + \phi_{4t-1} = 0$$

$$x_t + \theta\phi_{4t} = 0$$

Substituting these two equations into the AS equation (37) combined with (74) I can write the solution in terms of a second order difference equation:

$$\beta E_t x_{t+1} - (1+\beta+\kappa\theta)x_t + x_{t-1} \tag{96}$$

The characteristic polynomial

$$\beta\mu^2 - (1+\beta+\kappa\theta)\mu + 1 = 0$$

has two real roots

$$0 < \mu_1 < 1 < \beta^{-1} < \mu_2 = (\beta\mu_1)^{-1}$$

and it follows that (96) has an unique bounded solution $x_t = 0$ consistent with the the initial condition that $x_{-1} = 0$. Substituting this solution into (37) I can verify that the unique bounded solution for inflation is $\pi_t = 0$.

2. In the case of the Markov solution equation (86) indicates that $\phi_{3t} = 0$. Then I can write (83) and (84) so that (using (74) to solve it in terms of the output gap):

$$-\pi_t + \phi_{4t} = 0 \tag{97}$$

$$x_t + \theta\phi_{4t} = 0 \tag{98}$$

I can substitute these equations into the AS (37) together with (74) and write the solution in terms of the difference equation:

$$(1 + \theta\kappa)x_t - \beta E_t x_{t+1} = 0 \tag{99}$$

This equation has a unique bounded solution $x_t = 0$ and it follows that the unique bounded solution for inflation is $\pi_t = 0$.

A.4.3 Proof of Proposition 5

1. The first part of the proposition is that $\pi_t = x_t = 0$ for $t \geq \tau$. The proof for this follows directly from the second part of the proof for Proposition 4 since for $t \geq \tau$ there are no shocks and the Markov equilibrium is the one given in that Proposition. To see this note that the first order condition for $t \geq \tau$ are again given by (97) and (98) and I can again write the difference equation (99). Since this equation involves no history dependence (i.e. initial conditions do not matter) it follows once again that the unique bounded solution when $t \geq \tau$ is $x_t^M = \pi_t^M = 0$.

2. The second part of the proposition is that the Markov solution results in excessive deflation and output gap in period $0 < t < \tau$ relative to a policy that implies $\pi_\tau^C > 0$ and $x_\tau^C > 0$. I proof this by first showing that this must hold true for $\tau = K$ and then show that this implies it must hold for any $\tau < K$. Note first that our solution for the Markov equilibrium at any date $t \geq \tau$ implies that

$$\pi_\tau^C - \pi_\tau^M > 0 \tag{100}$$

$$x_\tau^C - x_\tau^M > 0 \tag{101}$$

The IS and AS equation implies that in the Markov equilibrium at date $K - 1$ is

$$\tilde{x}_{K-1}^M = \sigma\tilde{r}_{K-1}^n$$

$$\tilde{\pi}_{K-1}^M = \kappa\tilde{x}_{K-1}^M$$

where I denote each of the variables by a hat to state that it is their value conditional on the natural rate of interest being negative at that time. Compared to a solution that implies that $x_K^C > 0$ and $\pi_K^C > 0$ I can use the AS and the IS equations to write the inequalities:

$$\tilde{x}_{K-1}^C - \tilde{x}_{K-1}^M = x_K^C + \sigma\pi_K^C > 0$$

and

$$\tilde{\pi}_{K-1}^C - \tilde{\pi}_{K-1}^M = \kappa(\tilde{x}_{K-1}^M - \tilde{x}_{K-1}^C) + \beta\pi_K^C > 0$$

Using these two equation I can use the IS and AS equations at time $K-2$, (100)-(101), and the assumption about the natural rate of interest to write:

$$\tilde{x}^C_{K-2}-\tilde{x}^M_{K-2} = \alpha[(x^C_{K-1}-x^M_{K-1})+\sigma(\pi^C_{K-1}-\pi_{K-1})]+(1-\alpha)[(\tilde{x}^C_{K-1}-\tilde{x}^M_{K-1})+\sigma(\tilde{\pi}^C_{K-1}-\tilde{\pi}_{K-1})] > 0 \tag{102}$$

$$\tilde{\pi}^M_{K-2} - \tilde{\pi}^C_{K-2} = \kappa(\tilde{x}^M_{K-2} - \tilde{x}^M_{K-2}) + \alpha\beta\alpha(\pi^C_{K-1} - \pi_{K-1}) + (1-\alpha)\beta(\tilde{\pi}^C_{K-1} - \tilde{\pi}_{K-1}) > 0 \tag{103}$$

I can similarly solve the system backwards and write equation (102) and (103) for $K-2, K-3, ..., 0$ using at each time t the solution for $t+1$ and thereby the proposition is proofed.

A.4.4 Proof of Proposition 6

1. I first proof the that the solution for $t \geq \tau$ in the Markov solution is given by the one solution stated in the proposition. In the case of the Markov solution equation (86) indicates that $\phi_{3t} = 0$. When s is away from $\frac{\theta}{\theta-1}$ I can write (83) and (84) so that:

$$-\pi_t + \phi_{4t} = 0 \tag{104}$$

$$(x_t - x^*) + \theta\phi_{4t} = 0 \tag{105}$$

where $x^* = (\omega + \sigma^{-1})^{-1}(1 - \frac{\theta-1}{\theta}(1+s))$. These two equation imply that $\pi_t = -\theta^{-1}(x_t - x^*)$.I can substitute this into the AS (37) equation and write the solution in terms of the difference equation:

$$(1+\theta\kappa)x_t - \beta E_t x_{t+1} = (1-\beta)x^* \tag{106}$$

This equation has a unique bounded solution given by $x_t = \frac{1-\beta}{1-\beta+\theta\kappa}x^*$ and it follows that the unique bounded solution for inflation is $\pi_t = \frac{\kappa}{1-\beta+\theta\kappa}x^*$.

2. The second part of the proposition follows exactly the same steps as the second part of Proposition 7.

A.4.5 Proof of Proposition 7

At time $t \geq \tau$ the system is deterministic. In this case the functions $\Lambda_t = \bar{\Lambda}_t(w_{t-1}, \xi)$ and $w_t = \bar{w}(w_{t-1}, \xi)$ are independent of the calender time. Then I can approximate these functions to yield $w_t = w^1 w_{t-1}$ and $d\Lambda_t = \Lambda^1 w_{t-1}$, where the first element of the vector $d\Lambda_t$ is $d\pi_t = \pi^1 w_{t-1}$, the second $dY_t = Y^1 w_{t-1}$ and so on and $w_t = w^1 w_{t-1}$ where the vector Λ^1 and the number w^1 are some unknown constants. To find the value of each of these coefficients I substitute this solution into the system (69)-(73) and (83)-(87) and match coefficients. For example equation (69) implies that

$$\bar{u}_c d'' \pi^1 w_{t-1} + \theta(\bar{u}_{cc} - \bar{v}_{yy})Y^1 w_{t-1} - \bar{u}_c d'' \beta\pi^1 w^1 w_{t-1} = 0 \tag{107}$$

where I have substituted for $d\pi_t = \pi^1 w_{t-1}$ and for $d\pi_{t+1} = \pi^1 w_t = \pi^1 w^1 w_{t-1}$. Note that I assume that $t \geq \tau$ so that there is perfect foresight and I may ignore the expectation symbol. This equation implies that the coefficients π^1, y^1 and w^1 must satisfy the equation:

$$\bar{u}_c d'' \pi^1 + \theta(\bar{u}_{cc} - \bar{v}_{yy})Y^1 - \bar{u}_c d'' \beta\pi^1 w^1 = 0 \tag{108}$$

I may similarly substitute the solution into each of the equation (69)-(73) and (83)-(87) to obtain a system of equation that the coefficients must satisfy:

$$\bar{u}_c d'' \pi^1 + \theta(\bar{u}_{cc} - \bar{v}_{yy})Y^1 - \bar{u}_c d'' \beta\pi^1 w^1 = 0 \tag{109}$$

$$\bar{u}_{cc}Y^1 - \beta\bar{u}_{cc}Y^1w^1 - \beta\bar{u}_c i^1 + \beta\bar{u}_c\pi^1 w^1 = 0 \tag{110}$$

$$w^1 - \frac{1}{\beta} + \frac{1}{\beta}T^1 = 0 \tag{111}$$

$$S^1 - \bar{u}_c d''\pi^1 w^1 = 0 \tag{112}$$

$$f^1 + \bar{u}_c\pi^1 w^1 - \bar{u}_{cc}Y^1 w^1 = 0 \tag{113}$$

$$-d_{\mathrm{J}}\bar{u}_c\pi^1 + \frac{s'\bar{g}_G}{\beta} + d''\bar{u}_c\phi_4^1 = 0 \tag{114}$$

$$(\bar{u}_{cc} - \bar{v}_{yy})Y^1 - \bar{u}_{cc}\beta\phi_3^1 + \theta(\bar{u}_{cc} - \bar{v}_{yy})\phi_4^1 = 0 \tag{115}$$

$$s'\bar{g}_G T^1 - s'\bar{g}_G + \bar{u}_c\beta^2\phi_3^1 = 0 \tag{116}$$

$$\bar{g}_{GG}(s')^2T^1 - \bar{g}_G s'' T^1 + \beta^{-1}\phi_2^1 + \bar{g}_G s' i^1 = 0 \tag{117}$$

$$\phi_2^1 - \phi_2^1 w^1 - \beta\bar{g}_G s' i^1 w^1 + \bar{g}_G s'\pi^1 w^1 + \beta f^1\phi_3^1 - \beta S^1\phi_4^1 = 0 \tag{118}$$

There are 10 unknown coefficients in this system i.e. $\pi^1, Y^1, i^1, S^1, f^1, T^1, \phi_2^1, \phi_3^1, \phi_4^1, w^1$. For a given value of w^1, (109)-(117) is a linear system of 9 equations with 9 unknowns, and thus there is a unique value given for each of the coefficients as long as the system is non-singular (which can be verified to be the case for standard functional forms for the utility and technology functions).

A.4.6 Proof of Propositions 8 and 9

In the assumption made in the proposition I assume the cashless limit and the form of the utility given by (65) so that

$$u(C_t, m_t\Pi_t^{-1}, \xi_t) = \tilde{u}(C_t, \xi_t) + \chi(\frac{m_t}{\bar{m}}\Pi_t^{-1}C_t^{-1}, \xi_t) \tag{119}$$

The partial derivatives with respect to each variable are given by

$$u_c = \tilde{u}_c - \chi'\frac{m}{\bar{m}}C^{-2}\Pi^{-1} \tag{120}$$

$$u_m = \frac{\chi'}{\bar{m}}C^{-1}\Pi^{-1} \tag{121}$$

$$u_{mm} = \frac{\chi''}{\bar{m}^2}C^{-2}\Pi^{-2} < 0 \tag{122}$$

$$u_{cm} = -\chi''\frac{m}{\bar{m}^2}C^{-3}\Pi^{-2} - \frac{\chi'}{\bar{m}}C^{-2}\Pi^{-1} \tag{123}$$

As $\bar{m}- > 0$ I assume that for $\tilde{m} = \frac{m}{\bar{m}} > 0$ I have

$$lim_{\bar{m}\to 0}\frac{\chi\prime}{\bar{m}} \equiv \bar{\chi}' \geq 0 \tag{124}$$

$$lim_{\bar{m}\to 0}\frac{\chi\prime\prime}{\bar{m}^2} \equiv \bar{\chi}'' > 0 \tag{125}$$

This implies that there is a well defined money demand function, even as money held in equilibrium approaches zero, given by

$$\frac{\bar{\chi}\prime(\tilde{m}C_t^{-1}\Pi_t^{-1}, \xi_t)C_t^{-1}\Pi_t^{-1}}{\bar{u}_c(C_t, \xi_t)} = \frac{i_t - i^m}{1 + i_t}$$

so that $\bar{\chi}' = 0$ when $i_t = i_t^m$. From the assumptions (124)-(125) it follows that:

$$lim_{\bar{m}\to 0}\chi\prime = 0$$

$$lim_{\bar{m}\to 0}\chi\prime\prime = 0$$

Then the derivatives u_c and u_{cm} in the cashless limit are:

$$\lim_{\bar{m}\to 0} u_c = \tilde{u}_c$$

and

$$\lim_{\bar{m}\to 0} u_{cm} = \lim_{\bar{m}\to 0}[-\bar{m}\frac{\chi\prime\prime}{\bar{m}^2}\frac{m}{\bar{m}}C^{-3}\Pi^{-1} - \frac{\chi\prime}{\bar{m}}C^{-2}] = -\bar{\chi}\prime C^{-2}$$

Hence in a steady state in which $\bar{m} \to 0$ and $i_t = i^m$ I have that $\bar{\chi}\prime = 0$ so that at the steady state

$$\lim_{\bar{m}\to 0} u_{cm} = 0. \tag{126}$$

Note that this does not imply that the satiation point of holding real balances is independent of consumption. To see this note that the satiation point of real money balances is is given by some finite number $S^* = \frac{m}{\bar{m}}Y$ which implies that $\chi(S \geq S^*) = \tilde{v}(S^*)$. The value of the satiation point as $\bar{m} \to 0$ is:

$$lim_{\bar{m}\to 0}S^* \equiv \bar{S} = \tilde{m}C$$

The value of this number still depends on C even as $\bar{m} \to 0$ and even if $u_{cm} = 0$ at the satiation point.

I now show that the steady state stated in Proposition 3 and 4 satisfies all the first order conditions and the constraints. The steady state candidate solution in both proposition is:

$$i = \frac{1}{\beta} - 1, w = \phi_1 = \phi_3 = \phi_4 = \psi_1 = \psi_2 = \gamma_1 = \gamma_2 = 0, \Pi = 1, \phi_2 = g_G s',\ T = F \tag{127}$$

Note that (127) and the functional assumption about d (see footnote 5) imply that:

$$d' = 0 \tag{128}$$

Let us first consider the constraints. In the steady state the AS equation is

$$\theta Y[\frac{\theta - 1}{\theta}(1+s)u_c - \tilde{v}_y] - u_c\Pi d'(\Pi) + \beta u_c \Pi d'(\Pi) = 0$$

Since by (128) d'=0, and according to assumption (ii) of the propositions $\frac{\theta-1}{\theta}(1+s) = 1$ the AS equation is only satisfied in the candidate solution if

$$u_c = v_y \tag{129}$$

Evaluated in the candidate solution the IS equation is:

$$\frac{1}{1+i} = \frac{\beta u_c}{u_c}\Pi^{-1} = \beta$$

which is always satisfied at because it simply states that $i = 1 - 1/\beta$ which is consistent with the steady state I propose in the propositions and assumption (iii). The budget constraint is:

$$w-(1+i)\Pi^{-1}w-(1+i)F+(1+i)T+(1+i)\bar{m}\tilde{m}\Pi_t^{-1}=0$$

which is also always satisfied in our candidate solution since it states that $F=T$, $w=0$ and $\bar{m}\to 0$. The money demand equation indicates that the candidate solutions is satisfied if

$$u_m=\Pi u_c\frac{i-i^m}{1+i}=0 \tag{130}$$

By (26) and (28) the expectation variables in steady state are

$$S^e=u_c\Pi d'$$

$$f^e=u_c\Pi$$

Since $\Pi=1$ and $d'=0$ by (128) these equations are satisfied in the candidate solution. Finally both the inequalities (9) and (25) are satisfied since $\bar{w}>w=0$ in the candidate solution and $i=i^m$.

I now show that the first order conditions, i.e. the commitment and the Markov equilibrium first order conditions, that are given by (43)-(52) and (53)-(64) respectively, are also consistent with the steady state suggested. I first show the commitment equilibrium. The proof for the Markov equilibrium will follow along the same lines.

Commitment equilibrium steady state Let us start with (43). It is

$$\begin{aligned}
&-u_cd'-u_m\bar{m}\tilde{m}\Pi^{-2}+\phi_1[-\frac{u_{mc}d'\Pi^{-1}}{u_c}-\frac{u_{mm}\bar{m}\tilde{m}\Pi^{-3}}{u_c}-\frac{u_m\Pi^{-2}}{u_c}+\frac{u_mu_{cc}d'\Pi^{-1}}{u_c^2}+\frac{u_mu_{cmmm}\bar{m}\tilde{m}\Pi^{-2}}{u_c}]\\
&+\phi_2[(1+i)w\Pi^{-2}-(i-i^m)\bar{m}\tilde{m}\Pi^{-2}]+\phi_3[\frac{u_{cc}d'}{1+i}+\frac{u_{cm}\bar{m}\tilde{m}\Pi^{-2}}{(1+i)}]\\
&+\phi_4[-Y(\theta-1)(1+s)(u_{cc}d'+\bar{m}\tilde{m}\Pi^{-2}u_{cm})-u_{cc}\Pi d'^2-u_{cm}\bar{m}\tilde{m}\Pi^{-2}d'+u_c\Pi d''+u_cd']\\
&+\beta^{-1}\psi_1[u_{cc}d'\Pi+u_{cm}\bar{m}\tilde{m}\Pi^{-1}+u_c\Pi^{-2}]+\beta^{-1}\psi_2[u_{cc}d'^2\Pi+u_{cm}d'\bar{m}\tilde{m}\Pi^{-1}-u_cd'-u_cd''\Pi]=0
\end{aligned}$$

By (128) and (130) the first two terms are zero. The constraints that are multiplied by ϕ_1,ϕ_3, ϕ_4, ψ_1 and ψ_2 are also zero because each of these variables are zero in our candidate solution (127). Finally, the term that is multiplied by ϕ_2 (which is positive) is also zero because $w=0$ in our candidate solution (127) and so is $i-i^m$. Thus I have shown that the candidate solution (127) satisfies (43).

Let us now turn to (44). It is

$$\begin{aligned}
&u_c-\tilde{v}_y+\phi_1[\frac{u_{mc}\Pi^{-1}}{u_c}-\frac{u_mu_{cc}\Pi^{-1}}{u_c^2}]-\phi_3\frac{u_{cc}}{1+i}\\
&+\phi_4[\theta(\frac{\theta-1}{\theta}(1+s)u_c-\tilde{v}_y)+\theta Y(\frac{\theta-1}{\theta}(1+s)u_{cc}-\tilde{v}_{yy})+u_{cc}\Pi d']\\
&-\psi_1\beta^{-1}u_{cc}\Pi^{-1}-\psi_2\beta^{-1}u_{cc}d'\Pi\\
=\ &0
\end{aligned}$$

The first two terms u_c-v_y are equal to zero by (129). The next terms are also all zero because they are multiplied by the terms $\phi_1, \phi_3, \phi_4, \psi_1$ and ψ_2 which are all zero in our candidate solution (127). Hence this equation is also satisfied in our candidate solution. Let us then consider (45). It is:

$$-\phi_1 \frac{1+i^m}{(1+i_t)^2} + \phi_2(\bar{m}\tilde{m} + T - w\Pi^{-1} - F) + \phi_3 \frac{u_c}{(1+i)^2} + \gamma_1 = 0$$

Again this equation is satisfied in our candidate solution because $\phi_1 = \phi_3 = w = 0$, $F = T$ and $\bar{m} \to 0$ in the candidate solution. Conditions (46) in steady state is:

$$\begin{aligned}&\bar{m}\tilde{m}u_m\Pi^{-1} + \phi_1[\frac{u_{mm}\Pi^{-2}}{u_c} - \frac{u_m u_{cm}\Pi^{-2}}{u_c^2}] + \phi_2(i - i^m)\Pi^{-1} - \phi_3 \frac{u_{cm}}{1+i}\Pi^{-1} \\ &+\phi_4[Y(\theta-1)(1+s)u_{cm}\Pi^{-1} + u_{cm}d'] - \beta^{-1}\psi_1 u_{cm}\Pi^{-2} - \beta^{-1}\psi_2 u_{cm}d' = 0\end{aligned} \tag{131}$$

The first term is zero by (130). All the other terms are also zero because $\phi_1, \phi_3, \phi_4, \psi_1$ and ψ_2 are all zero in our candidate solution (127). Finally $i = i^m$ in our candidate solution so that the third term is zero as well. Condition (47) in steady state is:

$$-g_G s'(T) + \phi_2(1+i) = 0 \tag{132}$$

which is satisfied in the candidate solution. Condition (48) is

$$\phi_2 - \beta\phi_2(1+i)\Pi^{-1} - \gamma_2 = 0$$

This condition is also satisfied in our candidate solution because $\gamma_2 = 0$ and $(1+i)\Pi^{-1} = \frac{1}{\beta}$. Conditions (49) and (50) are:

$$\beta\phi_3 + \psi_1 = 0 \tag{133}$$

$$-\beta\phi_4 + \psi_2 = 0 \tag{134}$$

Since $\phi_3 = \phi_4 = \psi_1 = \psi_2 = 0$ in our candidate solution, these conditions are also satisfied. Finally our candidate solution (127) indicates that (51) and (52) are also satisfied in steady state. I have now showed that our candidate solution satisfies all necessary and sufficient conditions for an equilibrium and Proposition 8 is thus proofed.

Markov equilibrium steady state Let us now turn to the Markov equilibrium. The first order conditions in steady state are

$$\begin{aligned}&-u_c d' - u_m\bar{m}\tilde{m}\Pi^{-2} \\ &+\phi_1[-\frac{u_{mc}d'\Pi^{-1}}{u_c} - \frac{u_{mm}\bar{m}\tilde{m}\Pi^{-3}}{u_c} - \frac{u_m\Pi^{-2}}{u_c} + \frac{u_m u_{cc}d'\Pi^{-1}}{u_c^2} + \frac{u_m u_{cm}\bar{m}\tilde{m}\Pi^{-2}}{u_c^2}] \\ &+\phi_2[(1+i)w\Pi^{-2} - (i-i^m)\bar{m}\tilde{m}\Pi^{-2}] + \phi_3[\frac{u_{cc}d'}{1+i} + \frac{u_{cm}\bar{m}\tilde{m}\Pi^{-2}}{(1+i)}] \\ &+\phi_4[-Y(\theta-1)(1+s)(u_{cc}d' + \bar{m}\tilde{m}\Pi^{-2}u_{cm}) - u_{cc}\Pi d'^2 - u_{cm}\bar{m}\tilde{m}\Pi^{-2}d' + u_c\Pi d'' + u_c d']\end{aligned} \tag{135}$$

$$u_c - \tilde{v}_y + \phi_1[\frac{u_{mc}\Pi^{-1}}{u_c} - \frac{u_m u_c\Pi^{-1}}{u_c^2}] - \phi_3\frac{u_{cc}}{1+i} + \phi_4[\theta(\frac{\theta-1}{\theta}(1+s)u_c - \tilde{v}_y) - \theta Y(\frac{\theta-1}{\theta}(1+s)u_{cc} - \tilde{v}_{yy}) + u_{cc}\Pi d'] = 0 \tag{136}$$

$$-\phi_1 \frac{1+i^m}{(1+i)^2} + \phi_2(\bar{m}\tilde{m} + T - w\Pi^{-1} - F) + \phi_3 \frac{u_c}{(1+i)^2} + \gamma_1 = 0 \tag{137}$$

$$u_m\Pi^{-1}+\phi_1[\frac{u_{mm}\Pi^{-2}}{u_c}-\frac{u_m u_{cm}\Pi^{-2}}{u_c^2}]+\phi_2(i-i^m)\Pi^{-1}\bar{m}\tilde{m}-\phi_3\frac{u_{cm}}{1+i}\Pi^{-1}+\phi_4[Y(\theta-1)(1+s)u_{cm}\Pi^{-1}+u_{cm}d'] = 0 \tag{138}$$

$$-g_G s'(T) + \phi_2(1+i) = 0 \tag{139}$$

$$\beta J_w - \psi_1\beta f_w^e - \psi_2\beta S_w^e + \phi_2 - \gamma_2 = 0 \tag{140}$$

$$\beta\phi_3 + \psi_1 = 0 \tag{141}$$

$$-\beta\phi_4 + \psi_2 = 0 \tag{142}$$

$$J_w = -\phi_2(1+i)\Pi^{-1} \tag{143}$$

Condition (135)-(139) and (141)-(142) are the same as in the commitment equilibrium, apart from the presence of ψ_1 and ψ_2 in the equations corresponding to (135) and (136). Since $\psi_1 = \psi_2 = 0$ in the candidate solution this does not change our previous proof. Thus, exactly the same arguments as I made to show that the candidate solution (127) satisfied the first order conditions in the commitment equilibrium can be used in the Markov equilibrium for equations (135)-(139) and (141)-(142). The crucial difference between the first order conditions in the Markov and the commitment equilibrium is in equation (140). This equation involves three unknown functions, J_w, f_w^e and S_w^e.I can use (143) to substitute for J_w in (140) obtaining

$$-\beta\phi_2(1+i)\Pi^{-1} - \psi_1\beta f_w^e - \psi_2\beta S_w^e + \phi_2 - \gamma_2 = 0 \tag{144}$$

In general I cannot know if this equation is satisfied without making further assumption about f_w^e and S_w^e. But note that in our candidate solution $\psi_1 = \psi_2 = 0$. Thus the terms involving these two derivatives in this equation are zero. Since $\gamma_2 = 0$, this equation is satisfied if $(1+i)\Pi^{-1} = 1/\beta$. This is indeed the case in our candidate solution. Thus I have shown that all the necessary and sufficient conditions of a Markov equilibrium are satisfied by our candidate solution (127). QED

Notes

[35] First, as shown by Woodford (2003), for a realistic calibration parameters, this abstraction has trivial effect on the AS and the IS equation under normal circustances. Furthermore, at zero nominal interest rate, increasing money balances further *does nothing* to facilitate transactions since consumer are already satiated in liquidity. This was one of the key insights of Eggertsson and Woodford (2003), which showed that at zero nominal interest rate increasing money supply has no effect if expectations about future money supply do not change. It is thus of even *less* interest to consider this additional channel for monetary policy at zero nominal interest rates than if the short-term nominal interest rate was positive. Second, assuming m_t is a very small number is likely to change the government budget constraint very little in a realistic calibration. By assuming the cashless limit I am assuming no seignorage revenues so that the term $\frac{i_t-i^m}{1+i_t}m_t\Pi_t^{-1}$ in the budget constraint has no effect on the equilibrium. Given the low level of seignorage revenues in industrialized countries (see King and Plosser (1985) I do not think this is a bad assumption. Furthermore, in the case the bound on the interest rate is binding, this term is zero, making it of even *less* interest when the zero bound is binding than under normal circumstances.

[36] See Woodford (2003) Appendix A3 for definition and discussion of local uniqueness in stochastic general equilibrium models of this kind.

[37] The reason for this conjecture is that in this model, as opposed to Albanesi et al and Dedola work, I assume in A2 that there are no monetary frictions. The source of the multiple equilibria in those papers, however, is the payment technology they assume. The key difference between the present model and that of King and Wolman, on the other hand, is that they assume that some firms set prices at different points in time. I assume a representative firm, thus abstacting from the main channel they emphasize in generating multiple equilibria. Finally the present model is different from all the papers cited above in that I introduce nominal debt as a state variable. Even if the model I have illustrated above would be augmented to incorporate additional elements such as montary frictions and staggering prices, I conjecture that the steady state would remain unique due to the ability of the government to use nominal debt to change its future inflation incentive. That is, however, a topic for future reasearch and there

is work in progress by Eggertsson and Swanson that studies this question.

[38]Even if I had written a model in which the equilibria proofed above is not the unique global equilibria the one I illustrate here would still be the one of principal interest. Furthermore a local analysis would still be useful. The reason is twofold. First, the equilibria analyzed is identical to the commitment equilibrium (in the absence of shocks) and is thus a natural candidate for investigation. But even more importantly the work of Albanesi et al (2002) indicates that if there are non-trivial monetary frictions there are in general only two steady states.There are also two steady states in King and Wolman's model. (In Dedola's model there are three steady states, but the same point applies.) The first is a low inflation equilibria (analogues to the one in Proposition 1) and the other is a high inflation equilibria which they calibrate to be associated with double digit inflation. In the high inflation equilibria, however, the zero bound is very unlikely ever to be binding as a result of real shocks of the type I consider in this paper (since in this equilibria the nominal interest rate is very high as I will show in the next section). And it is the distortions created by the zero bound that are the central focus of this paper, and thus even if the model had a high inflation steady state, that equilibria would be of little interest in the context of the zero bound.