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The Deflation Bias and Committing to Being Irresponsible

I model deflation, at zero nominal interest rate, in a microfounded general equilibrium model. I show that one can analyze deflation as a credibility problem if three conditions are satisfied. First: The government's only policy instrument is increasing the money supply by open market operations in short-term bonds. Second: The economy is subject to large negative demand shocks. Third: The government cannot commit to future policy. I call the credibility problem that arises under these conditions the deflation bias. I propose several policies to solve it. They all involve printing money or issuing nominal debt. In addition they require cutting taxes, buying real assets such as stocks, or purchasing foreign exchange. The government "credibly commits to being irresponsible" by pursuing these policies. It commits to higher money supply in the future so that the private sector expects inflation instead of deflation. This is optimal since it curbs deflation and increases output by lowering the real rate of return.

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CAN THE GOVERNMENT lose control over the general price level so that no matter how much money it prints, it's actions have no effect on inflation or output? Economists have debated this question ever since Keynes' General Theory. Keynes answered yes, Friedman and the monetarists said no. Keynes argued that increasing the money supply has no effect at low nominal interest rates. This has been coined as the liquidity trap. The zero short-term nominal

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interest rate in Japan today, together with the lowest short-term interest rate in the U.S. in 45 years in 2003, makes this old question interesting again, since the Bank of Japan (BOJ) cannot lower interest rates below zero. The BOJ has nearly doubled the monetary base over the past 5 years, yet the economy still suffers deflation, and growth is stagnant.[1] Was Keynes right? Is increasing money supply ineffective when the interest rate is zero? This paper revisits this classic question using a microfounded intertemporal general equilibrium model and assuming rational expectations. The results suggest that both the Keynesian and the monetarist view can be supported under different assumptions about policy expectations.

The paper has three key results. The first is that monetary and fiscal policies have no effect in a liquidity trap *if* expectations about future money supply are independent of past policy decisions, and certain restrictions on fiscal policy apply. This is shown in a standard New Keynesian general equilibrium model widely used in the literature. The message is not that monetary and fiscal policies are irrelevant. Rather, the point is that monetary and fiscal policies have their largest impact in a liquidity trap through expectations. This indicates that the old fashion IS–LM model is a blind alley since expectations are assumed to be exogenous in that model. In contrast, expectations are at the heart of this study.

I assume that expectations are rational. The government maximizes social welfare and I analyze two different equilibria in a liquidity trap. First, I assume that the government is able to commit to future policy. This is the commitment equilibrium. Then I assume that the government is unable to commit to any future policy apart from paying back the nominal value of its debt. This is the Markov equilibrium (formally defined by Maskin and Tirole, 2001). I explore optimal policy when the natural rate of interest—assumed to be exogenous in the model—is temporarily negative, causing the zero bound on the short-term nominal interest rate to be binding. The optimal commitment is to commit to low future interest rates, modest inflation, and an output boom once the natural rate of interest returns back to normal as in Jung, Teranishi, and Watanabe (2001) and Eggertsson and Woodford (2003a). This reduces the real rate of return in a liquidity trap and increases demand. In a Markov equilibrium, however, this commitment may not be feasible.

The second key result of the paper is that in a Markov equilibrium, deflation can be modelled as a credibility problem. This problem arises if the government's only policy instrument is open market operations in government bonds and the natural rate of interest is temporarily negative. Under these conditions we see excessive deflation if the government cannot commit to future policy. This is the deflation bias of discretionary policy. This theory of deflation, derived from the analysis of a Markov equilibrium, is in sharp contrast to conventional wisdom about deflation in Japan today (or, for that matter, U.S. during the Great Depression). The conventional wisdom blames deflation on policy mistakes by the central bank or bad policy rules (see e.g., Friedman and Schwartz, 1963, Krugman, 1998, Bernanke, 2000, Benhabib,

1. Although recent signs indicate that the Japanese economy may finally be recovering.

Schmitte-Grohe, and Uribe, 2002, Buiter, 2003).[2] Deflation in this paper, however, is not attributed to an inept central bank or bad policy rules. It is a consequence of the central bank's *policy constraints* and inability to commit to the optimal policy when faced with negative demand shocks.[3] This result, however, does not absolve the government of responsibility for deflation. Rather, it identifies the possible policy constraints that result in inefficient deflation in equilibrium (without resorting to an irrational policy maker). I identify two sources of inefficient deflation of equal importance. The first source is the inability of the government to commit. The second source is that open market operations in short-term government bonds is the government's only policy instrument. The central question of the paper, therefore, is how the government can use additional policy instruments to fight deflation even if it cannot commit to future policy.

The third key result of the paper is that in a Markov equilibrium the government can eliminate deflation by deficit spending. Deficit spending eliminates deflation for the following reason: If the government cuts taxes and increases nominal debt, and taxation is costly, inflation expectations increase (i.e., the private sector expects higher money supply in the future). Inflation expectations increase because higher nominal debt gives the government an incentive to inflate to reduce the real value of the debt. To eliminate deflation the government simply cuts taxes until the private sector expects inflation instead of deflation. At zero nominal interest rates higher inflation expectations reduce the real rate of return, and thereby raise aggregate demand and the price level. The two main assumptions underlying this result is that (1) there is some cost of taxation which makes this policy credible and that (2) monetary and fiscal policies are coordinated.[4]

2. There is a large literature that discusses optimal monetary policy rules when the zero bound is binding. Contributions include Summers (1991), Fuhrer and Madigan (1997), Woodford and Rotemberg (1997), Wolman (1999), Reifschneider and Williams (2000), and references therein. Since monetary policy rules arguably become credible over time these contributions can be viewed as illustration of how to *avoid* a liquidity trap rather than a prescription of how to escape them which is the focus here.

3. The deflation bias is closely related, and in some sense, a formalization of, a common objection to Krugman's policy proposal for the BOJ. To battle deflation he suggested that the BOJ should announce an inflation target of 5% for 15 years. Responding to this proposal, Kunio Okina, director of the Institute for Monetary Studies at the BOJ, said in DJN (1999): "Because short-term interest rates are already at zero setting an inflation target of say 2% would not carry much credibility." Similar objections were raised by economists such as, e.g., Dominiguez (1998), Woodford (1999), and Svensson (2001).

4. The Fiscal Theory of the Price Level (FTPL) popularized by Leeper (1991), Sims (1994), and Woodford (1994, 1996) also stresses that fiscal policy can influence the price level. What separates this analysis from the FTPL (and the seminal contribution of Sargent and Wallace, 1981) is that in my setting fiscal policy only affects the price level because it changes the *inflation incentive* of the government. In contrast, according to the FTPL, fiscal policy affects the price level because it is *assumed* that the monetary authority commits to a (possibly suboptimal) interest rate rule and fiscal policy is modelled as a (possibly suboptimal) exogenous path of real government surpluses. Under these assumptions, innovations in real government surpluses can influence the price level, since the prices may have to move for the government budget constraint to be satisfied. In my setting, however, the government budget constraint is a *constraint* on the policy choices of the government.

The approach taken here is more closely related to Calvo's (1978) classic paper on the inflationary impact of government nominal liabilities when the government cannot commit to future policy (see Persson, Persson, and Svensson, 1987 for further references on this literature). The inflationary impact of debt analysed is essentially of the same source as analyzed by Calvo. The analysis here is different from Calvo's in that I explicitly analyse the inflationary impact of debt in a sticky price model (so that an increase in inflation expectation can increase output as well as prices) and show that increasing inflation expectation through this channel can be beneficial when the zero bound is binding.

Deficit spending has exactly the same effect as the government following Friedman's famous suggestion to "drop money from helicopters" to increase inflation. At zero nominal interest rates money and bonds are perfect substitutes. They are one and the same: A government issued piece of paper that carries no interest but has nominal value. It does not matter, therefore, if the government drops money from helicopters or issues government bonds. Friedman's proposal thus increases the price level through the same mechanism as deficit spending. Dropping money from helicopters, however, does *not* increase prices in a Markov equilibrium because it increases the *current* money supply. It creates inflation by increasing government debt which is defined as the sum of money and bonds. In a Markov equilibrium, *it is government debt that determines the price level in a liquidity trap* because it determines expectations about *future* money supply.

Government debt is the key mechanism that increases inflation expectation in this paper, thus eliminating deflation. The government, however, can increase its debt in several ways. Cutting taxes and dropping money from helicopters are only two examples. The government can also increase debt by printing money (or issuing nominal bonds) and buying private assets, such as stocks, or foreign exchange. In a Markov equilibrium, these operations increase prices and output *because they change the inflation incentive of the government by increasing government debt* (money + bonds). Hence, when the short-term nominal interest rate is zero, open market operations in real assets and/or foreign exchange increase prices through the same mechanism as deficit spending in a Markov equilibrium. This channel of monetary policy does not rely on the portfolio effect of buying real assets or foreign exchange. This paper thus compliments Meltzer's (1999) and McCallum's (1999) arguments for foreign exchange interventions that rely on the portfolio channel. The argument in this paper is also complimentary to Svensson's (2001) "foolproof" way of escaping the liquidity trap, although in that paper foreign exchange intervention is only useful in maintaining or establishing a currency peg rather than creating inflation incentives.

Deflationary pressures in this paper are due to temporary exogenous real shocks that shift aggregate demand.[5] The paper, therefore, does not address the origin of the deflationary shocks during the Great Depression in the U.S. or in Japan today. These deflationary shocks are most likely due to a host of factors, including the stock market crash and banking problems. I take these deflationary pressures as given and ask: How can the government eliminate deflation by monetary and fiscal policies even if the zero bound is binding and it cannot commit to future policy? There is no doubt that there are several other policy challenges for a government that faces large negative shocks, and various structural problems, as in Japan.[6] Stabilizing the price level (and reducing real rates) by choosing the optimal mix of monetary and fiscal policies, however, is an obvious starting point and does not preclude other policy measures and/or structural reforms.

5. In contrast to Benhabib, Schmitt-Grohe, and Uribe (2002) where deflation is due to self-fulfilling deflationary spirals.

6. See for example Caballero, Kashyap, and Hoshi (2003) who argue that banking problems are at the heart of the Japanese recession.

I study this model, and some extensions, in a companion paper, Eggertsson (2006), with explicit reference to the current situation in Japan and some historical episodes (the Great Depression in particular). That paper also demonstrates that deficit spending may have little or no effect if the central bank is "goal independent." It follows that monetary and fiscal policies need to be coordinated for deficit spending to be effective, an assumption that is maintained in this paper (see also Eggertsson and Woodford, 2003b for further discussions about Japan). Eggertsson (2003) and Jeanne and Svensson (2004) suggest that a "goal independent" central bank may be able to commit to future inflation by purchasing foreign exchange reserves or real assets if it cares about balance sheet losses, but Eggertsson (2003) argues that this commitment device is not used by the central bank if it is too risk averse. Thus coordination between the central bank and the treasury, according to Eggertsson (2003), may be required even if the central bank is concerned about balance sheet losses, and can use its balance sheet as a commitment device.

Benhabib, Schmitte-Grohe, and Uribe (2002) (BSU hereafter) and Woodford (2003) also emphasize the importance of fiscal policy to eliminate deflation in a liquidity trap. They stress that appropriate fiscal policy implies tax cuts in response to deflation and suggest tax rules based on this principle to eliminate "bad" deflationary equilibrium. The analysis by BSU (2002) and Woodford (2003) (and the emphasis on fiscal policy in particular) is closely related to the present paper but with some important differences. First, in BSU (2002) and Woodford (2003) deflation is due to self-fulfilling expectations and is therefore an example of a "bad" equilibrium in a model with multiple ones, but in this paper deflation is due to a series of bad real shocks that make the zero bound binding. The suggested policy rules in BSU and Woodford are therefore only effective to exclude the self-fulfilling equilibrium but do nothing to respond to the real shocks that make the zero bound binding in this paper (in fact it can be shown that the policy rules suggested by BSU (2002) and Woodford (2003) lead to exactly the same inefficient deflation bias as shown in Section 4). A second difference is that BSU (2002) and Woodford (2003) assume that the government can commit to future fiscal and monetary policies and the commitment to "bad" policy rules is the reason for deflation in the first place. In this paper, I assume that the government cannot commit to future policy and the inability of the government to commit—coupled with a series of bad shocks and policy constraints—is the culprit for deflation. The role of fiscal policy here is that it is a commitment mechanism to solve the credibility problem posed by deflationary shocks. Inappropriate fiscal policy is not the source of a deflationary equilibrium in itself as in the work cited above.

1. THE MODEL

Here I outline a simple sticky price general equilibrium model and define the set of feasible equilibrium allocations. This prepares the grounds for the next section, which considers whether "quantitative easing"—a policy currently in effect at the

Bank of Japan—and/or deficit spending have any effect on the feasible set of equilibrium allocations.

1.1 The Private Sector

Households. The representative household that maximizes expected utility over the infinite horizon:

$$E_t\sum_{T=t}^{\infty}\beta^T U_T = E_t\left\{\sum_{T=t}^{\infty}\beta^T\left[u\left(C_T, \frac{M_T}{P_T}, \xi_T\right)\right.\right.$$
$$\left.\left. + g\,(G_T, \xi_T) - \int_0^1 v(h_T(i), \xi_T)\, di\right]\right\}, \tag{1}$$

where C_t is a Dixit–Stiglitz aggregate of consumption of each of a continuum of differentiated goods,

$$C_t \equiv \left[\int_0^1 c_t\,(i)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

with elasticity of substituting equal to $\theta > 1$, G_t is a Dixit–Stiglitz aggregate of government consumption, ξ_t is a vector of exogenous shocks, M_t is end-of-period money balances, P_t is the Dixit–Stiglitz price index,

$$P_t \equiv \left[\int_0^1 p_t\,(i)^{1-\theta}\right]^{\frac{1}{1-\theta}},$$

and $h_t(i)$ is quantity supplied of labor of type i. $u(\cdot)$ is concave and strictly increasing in C_t for any possible value of ξ. The utility of holding real money balances is increasing in M_t/P_t for any possible value of ξ up to a satiation point at some finite level of real money balances as in Friedman (1969).[7] $g(\cdot)$ is the utility of government consumption and is concave and strictly increasing in G_t for any possible value of ξ. $v(\cdot)$ is the disutility of supplying labor of type i and is increasing and convex in $h_t(i)$ for any possible value of ξ. E_t denotes mathematical expectation conditional on information available in period t. ξ_t is a vector of r exogenous shocks. The vector of shocks ξ_t follows a stochastic process as described below.[8]

7. The idea is that real money balances enter the utility because they facilitate transactions. At some finite level of real money balances, e.g., when the representative household holds enough cash to pay for all consumption purchases in that period, holding more real money balances will not facilitate transaction any further and thereby add nothing to utility. This is at the "satiation" point of real money balances. I assume that there is no storage cost of holding money, so increasing money holding can never reduce utility directly through $u(\cdot)$. A satiation level in real money balances is also implied by several cash-in-advance models such as Lucas and Stokey (1987).

8. Assumption 1(i) is the Markov property. This assumption is not very restrictive since the vector ξ_t can be augmented by lagged values of a particular shock. Assumption 1(ii) is added for tractability. Since K can be arbitrarily high it is not very restrictive.

Assumption 1: (i) $pr(\xi_{t+j}|\xi_t) = pr(\xi_{t+j}|\xi_t, \xi_{t-1}, ...)$ for $j \geq 1$ where $pr(\cdot)$ is the conditional probability density function of ξ_{t+j}. (ii) All uncertainty is resolved before a finite date K that can be arbitrarily high.

For simplicity I assume complete financial markets and no limit on borrowing against future income. As a consequence, a household faces an intertemporal budget constraint of the form:

$$E_t \sum_{T=t}^{\infty} Q_{t,T} \left[P_T C_T + \frac{i_T - i^m}{1+i_T} M_T \right] \leq W_t + E_t \sum_{T=t}^{\infty} Q_{t,T} \left[\int_0^1 Z_T(i)\, di + \int_0^1 n_T(j)\, h_T(j)\, dj - P_T T_T \right] \quad (2)$$

looking forward from any period t. Here $Q_{t,T}$ is the stochastic discount factor that financial markets use to value random nominal income at date T in monetary units at date t; i_t is the riskless nominal interest rate on one period obligations purchased in period t, i^m is the nominal interest rate paid on money balances held at the end-of-period t, W_t is the beginning of period nominal wealth at time t (note that its composition is determined at time $t - 1$ so that it is equal to the sum of monetary holdings from period $t - 1$ and the (possibly stochastic) return on non-monetary assets), $Z_t(i)$ is the time t nominal profit of firm i, $n_t(i)$ is the nominal wage rate for labor of type i, and T_t is net real tax collections by the government. Households maximize utility subject to the budget constraint.[9]

Firms. The production function of the representative firm that produces goods i is:

$$y_t(i) = f(h_t(i), \xi_t)\,, \quad (3)$$

where f is an increasing concave function for any ξ. I abstract from capital dynamics. As in Rotemberg (1982), firms face a cost of price changes given by the function $d(p_t(i)/p_{t-1}(i))$[10] but I can derive exactly the same result assuming that firms adjust their prices at stochastic intervals as assumed by Calvo (1983).[11] Price variations have a welfare cost that is separate from the cost of expected inflation due to real money balances in utility. I show that the key results of the paper do not depend on this cost being particularly large, indeed they hold even if the cost of price

9. The problem of the household is: at every time t the household takes W_t and $\{Q_{t,T}, n_T(i), P_T, T_T, Z_T(i), \xi_T; T \geq t\}$ as exogenously given and maximizes Equation (1) subject to Equation (2) by choice of $\{M_T, h_T(i), C_T; T \geq t\}$.

10. I assume that $d'(\Pi) > 0$ if $\Pi > 1$ and $d'(\Pi) < 0$ if $\Pi < 1$. Thus both inflation and deflation are costly. $d(1) = 0$ so that the optimal inflation rate is zero (consistent with the interpretation that this represents a cost of changing prices). Finally, $d'(1) = 0$ so that in the neighborhood of zero inflation the cost of price changes is of second order.

11. The reason I do not assume Calvo's prices is that it complicates the solution by introducing an additional state variable, i.e., price dispersion. This state variable, however, has only second order effects local to the steady state I approximate around and the resulting equilibrium is exactly the same as derived here (to the first order). This is shown formally in Eggertsson and Swanson (2006).

changes is arbitrarily small. The Dixit–Stiglitz preferences of the household imply a demand function for the product of firm i given by

$$y_t(i) = Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\theta}.$$

The firm maximizes

$$E_t \sum_{T=t}^{\infty} Q_{t,T} Z_T(i), \tag{4}$$

where

$$Q_{t,T} = \beta^{T-t} \frac{u_c\left(C_T, \frac{M_T}{P_T}, \xi_T\right)}{u_c\left(C_t, \frac{M_t}{P_t}, \xi_t\right)} \frac{P_t}{P_T}. \tag{5}$$

I can write firms' period profits as

$$Z_t(i) = (1+s) Y_t P_t^{\theta} p_t(i)^{1-\theta} - n_t(i) f^{-1}(Y_t P_t^{\theta} p_t^{-\theta}) - P_t d\left(\frac{p_t(i)}{p_{t-1}(i)}\right), \tag{6}$$

where s is an exogenously given production subsidy that I introduce for computational convenience.[12] Firms maximize profits.[13]

Private sector equilibrium conditions: AS, IS, and LM equations. In this subsection, I show the necessary conditions for equilibrium that stem from the maximization problems of the private sector. These conditions must hold for *any* government policy. The first order conditions of the household maximization imply an Euler equation

$$\frac{1}{1+i_t} = E_t \left\{ \frac{\beta u_c\left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}, \xi_{t+1}\right)}{u_c\left(C_t, \frac{M_t}{P_t}, \xi_t\right)} \frac{P_t}{P_{t+1}} \right\}. \tag{7}$$

This equation is often referred to as the IS equation. Optimal money holding implies that

$$\frac{u_{M/P}\left(C_t, \frac{M_t}{P_t}, \xi_t\right)}{u_c(C_t, \xi_t)} = \frac{i_t - i^m}{1+i_t}. \tag{8}$$

This equation defines money demand or what is often referred to as the "LM"

12. I introduce it so that I can calibrate an inflationary bias that is independent of the other structural parameters, and this allows me to define a steady state at the fully efficient equilibrium allocation. I abstract from any tax costs that the financing of this subsidy may create.

13. At every time t the firm takes $\{n_T(i), Q_{t,T}, P_T, Y_T, C_T, M_T/P_T, \xi_T; T \geq t\}$ as exogenously given and maximizes Equation (4) by choice of $\{p_T(i); T \geq t\}$.

equation. Utility is weakly increasing in real money balances. Utility does not increase further at some finite level of real money balances. The left hand side of Equation (8) is therefore weakly positive. Thus there is bound on the short-term nominal interest rate given by

$$i_t \geq i^m \, . \tag{9}$$

In most economic discussions it is assumed that the interest paid on the monetary base is zero so that Equation (9) becomes $i_t \geq 0$.[14]

The optimal consumption plan of the representative household must also satisfy the transversality condition,

$$\lim_{T\to\infty} E_t \left(Q_{t,T} \frac{W_T}{P_t} \right) = 0 \, , \tag{10}$$

to ensure that the household exhausts its intertemporal budget constraint. I assume that workers are wage takers so that households' optimal choice of labor supplied of type j satisfies

$$n_t\,(j) = \frac{P_t\, v_h\,(h_t\,(j);\, \xi_t)}{u_c\left(C_t, \dfrac{M_t}{P_t}, \xi_t\right)} \, . \tag{11}$$

I restrict my attention to a symmetric equilibria where all firms charge the same price and produce the same level of output so that

$$\begin{aligned} &p_t(i) = p_t(j) = P_t; \quad y_t(i) = y_t(j) = Y_t; \\ &\qquad n_t(i) = n_t(j) = n_t;\, h_t(i) = h_t(j) = h_t \quad \text{for } \forall\, j, i \, . \end{aligned} \tag{12}$$

Given the wage demanded by households I can derive the aggregate supply function from the first order conditions of the representative firm, assuming competitive labor market so that each firm takes its wage as given. I obtain the equilibrium condition often referred to as the AS or the "New Keynesian" Phillips curve:

$$\begin{aligned} &\theta Y_t \left[\frac{\theta - 1}{\theta} (1 + s) u_c \left(C_t, \frac{M_t}{P_t}, \xi_t \right) - \tilde{v}_y(Y_t, \xi_t) \right] + u_c\left(C_t, \frac{M_t}{P_t}, \xi_t \right) \frac{P_t}{P_{t-1}} d' \left(\frac{P_t}{P_{t-1}} \right) \\ &\qquad - E_t \beta u_c \left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}, \xi_{t+1} \right) \frac{P_{t+1}}{P_t} d' \left(\frac{P_{t+1}}{P_t} \right) = 0 \, , \end{aligned} \tag{13}$$

where for notational simplicity I have defined the function $\tilde{v}_y\,(y_t(i), \xi_t) \equiv v(f^{-1}\,(y_t\,(i)), \xi_t)$.

14. The intuition for this bound is simple. There is no storage cost of holding money in the model and money can be held as an asset. It follows that i_t cannot be a negative number. No one would lend 100 dollars if he or she would get less than 100 dollars in return. I do not address here the possibility of imposing tax on currency as in Goodfriend (2000).

1.2 The Government

There is an output cost of taxation (e.g., due to tax collection costs as in Barro, 1979) captured by the function $s(T_t)$.[15] For every dollar collected in taxes $s(T_t)$ units of output are wasted without contributing anything to utility. Government real spending is then given by:

$$F_t = G_t + s(T_t)\ . \tag{14}$$

I could also define cost of taxation as one that would result from distortionary taxes on income or consumption and obtain similar results.[16] I assume a representative household so that in a symmetric equilibrium, all nominal claims held are issued by the government. It follows that the government flow budget constraint is

$$B_t + M_t = W_t + P_t\,(F_t - T_t)\ , \tag{15}$$

where B_t is the end-of-period nominal value of bonds issued by the government. Finally, market clearing implies that aggregate demand satisfies:

$$Y_t = C_t + d\left(\frac{P_t}{P_{t-1}}\right) + F_t\ . \tag{16}$$

I now define the set of possible equilibria that are consistent with the private sector equilibrium conditions and the technological constraints on government policy.

DEFINITION 1: *A Private Sector Equilibrium (PSE) is a collection of stochastic processes* $\{P_t, Y_t, W_{t+1}, B_t, M_t, i_t, F_t, T_t, Q_t, Z_t, G_t, C_t, n_t, h_t, \xi_t\}$ *for* $t \geq t_0$ *that satisfy Equations (3)–(16) for each* $t \geq t_0$*, given* W_{t_0}, P_{t_0-1} *and the exogenous stochastic process* $\{\xi_t\}$ *that satisfies Assumption 1 for* $t \geq t_0$.

Having defined the set of feasible equilibrium allocations I now consider how government policy affects it.

2. EQUILIBRIUM WITH EXOGENOUS POLICY EXPECTATIONS

According to Keynes' (1936) famous analysis, monetary policy loses its power when the short-term nominal interest rate is zero. Others argue, most notably Friedman and Schwartz (1963) and the monetarist, that a monetary expansion increases

15. The function $s(T)$ is assumed to be differentiable with derivatives $s'(T) > 0$ and $s''(T) > 0$ for $T > 0$.

16. The specification used here, however, focuses the analysis on the channel of fiscal policy that I am interested in. This is because for a constant F_t the level of taxes has no effect on the private sector equilibrium conditions (see equations above) but only affect the equilibrium by reducing the utility of the households (because higher tax costs mean lower government consumption G_t). This allows me to isolate the effect current tax cuts will have on expectation about future monetary and fiscal policies, abstracting away from any effect on relative prices that those tax cuts may have. This is the key reason why I can obtain Proposition 1 in the next section even if taxation is costly. There is no doubt that tax policy can change relative prices and that these effects may be important. These effects, however, are quite separate from the main focus of this paper. Eggertsson and Woodford (2004) consider how taxes that change relative prices can be used to affect the equilibrium allocations. That work considers both labor and consumption taxes assuming that the government can commit to future policy.

aggregate demand even under such circumstances, and this is what lies behind the "quantitative easing" policy of the BOJ since 2001.

One of Keynes' better known suggestions is to increase demand in a liquidity trap by government deficit spending. Many have raised doubts recently about the importance of this channel, pointing to Japan's mountains of nominal debt, citing the Ricardian equivalence, i.e., the principle that any decrease in government savings should be offset by an increase in private savings (to pay for higher future taxes). Yet another group of economists argue that the Ricardian equivalence fails if deficit spending is financed by money creation (see e.g., Buiter, 2003, Bernanke, 2000, 2003).

Here I consider whether or not "quantitative easing" and deficit spending are separate policy tools in the explicit intertemporal general equilibrium model laid out in the last section. The key result is that neither "quantitative easing" nor deficit spending has any effect on the feasible set of equilibrium allocations *if* expectations about future money supply remain unchanged—or alternatively—expectations about future interest rate policy remain unchanged. Furthermore, this result is unchanged if these two operations are used together, hence our analysis does not support the proposition that "money-financed deficit spending" increases demand independently of the expectation channel. This result is an extension of the irrelevance results by Krugman (1998) and Eggertsson and Woodford (2003a), extended to include fiscal policy.

I do not contend that deficit spending and/or quantitative easing are irrelevant in a liquidity trap. Rather, my point is that the main effect of these policies is best illustrated by analyzing how they change expectations about future policy, in particular expectations about future money supply.

2.1 The Irrelevance of Monetary and Fiscal Policies When Policy Expectations are Exogenous

Here I characterize a policy regime that allows for the possibility that the government increases money supply by "quantitative easing" when the zero bound is binding and/or engages in deficit spending.

The money supply is determined by a policy function:

$$M_t = M(q_t, \xi_t)I_t \, , \tag{17}$$

where q_t is a vector that may include any of the endogenous variables *that are determined at time t* (note that as a consequence q_t cannot include W_t that is predetermined at time t). The multiplicative factor I_t satisfies the conditions

$$I_t = 1 \text{ if } i_t > 0 \text{ otherwise} \tag{18}$$

$$I_t = \psi(q_t, \xi_t) \geq 1 \, . \tag{19}$$

The rule (Equation 17) is a fairly general specification of policy (since I assume that M_t is a function of all the endogenous variables). It could for example include simple Taylor type rules, monetary targeting, and any policy that does not depend on the past values of the endogenous variables. Following Eggertsson and Woodford (2003a), I define the multiplicative factor $I_t = \psi(q_t, \xi_t)$ when the zero bound is binding. A policy of "quantitative easing" is represented by a value of the function ψ that is greater than 1. Note that I assume that the functions M and ψ are only functions of the endogenous variables and the shocks at time t. This separates the direct effect of a quantitative easing from the effect of a policy that influences expectation about future money supply. I impose the restriction on the policy rule (Equation 17) that:

$$M_t \geq M^* . \tag{20}$$

This restriction says the nominal value of the monetary base can never be smaller than some finite number M^*. This number can be arbitrarily small, so I do not view this as a very restrictive (or unrealistic) assumption since I am not modelling any technological innovation in the payment technology (think of M^* as being one cent!). I assume, for simplicity, that the central bank does quantitative easing by buying government bonds, but the model can be extended to allow for the possibility of buying a range of other long or short-term financial assets (see Eggertsson and Woodford 2003a). I also assume that the government only issues one period riskless nominal bonds so that B_t in Equation (15) refers to a one period riskless nominal debt.

Fiscal policy is defined by a function for real government spending:

$$F_t = F \tag{21}$$

and a policy function for deficit spending

$$T_t = T(q_t, \xi_t) . \tag{22}$$

I assume that real government spending F_t is constant at all times in order to focus on deficit spending which is defined by the function $T(\cdot)$. Debt issued at the end-of-period t is then defined by the consolidated government budget constraint (Equation 15) and the policy specification Equations (17)–(22). Finally, I assume that the government is neither a debtor or a creditor asymptotically so that

$$\lim_{T \to \infty} E_t Q_{t,T} B_T = 0 . \tag{23}$$

This is a fairly weak condition stating that the government cannot accumulate real debt asymptotically at a higher rate than the real rate of interest.[17] Note that Equation (23) is a restriction on fiscal policy so that it has an effect on the set of functions $T(\cdot)$ that are consistent with the policy regime.

The idea behind the policy rule Equations (17)–(23) is to separate the "direct" effect of a quantitative easing and deficit spending in a liquidity trap from any effect these policies may have on expectations about future policy, i.e., I hold expectation

17. One plausible sufficient condition that would guarantee that Equation (23) must always hold is to assume that the private sector would never hold more government debt that corresponds to expected future discounted level of some maximum tax level—that would be a sum of the maximum seignorage revenues and some technology constraint on taxation.

about policy at positive interest rates constant. One simple special case of the policy rules above is that money supply is some constant $\bar{M}$ at positive interest rates and taxes are a constant value of debt that is rolled over to the next period. In this example, I can consider the question of whether quantitative easing or deficit spending have any effect holding expectations of future money supply and fiscal policy constant. This is the sense in which policy expectations are constant, that is, I assume that policy actions when the zero bound is binding have no effect on the policy rules at positive interest rates. The policy rule in Equations (17)–(23) are much more general than the simple special case just discussed since they allow me to consider a broad range of monetary and fiscal policies that have only one thing in common, i.e., policy cannot depend on the past values of the endogenous variables. A simple Taylor rule is another special case.[18] The sense in which monetary policy expectations are constant in that case is that quantitative easing at zero interest rate will have no effect on the central banks' commitment to the Taylor rule at positive interest rates.[19]

Using the policy rules above I can now obtain the following irrelevance result for monetary and fiscal policies:

PROPOSITION 1: *The set of paths* $\{P_t, Y_t, i_t, Q_{t,T}, Z_t, C_t, n_t, h_t, \xi_t\}$ *consistent with a PSE and the monetary and fiscal policy regime Equations (17)–(23) is independent of the specification of the functions* $\psi(\cdot)$ *and* $T(\cdot)$.

The proof of this proposition is fairly simple, and the formal details are provided in the Technical Appendix.[20] The proof is that I show that I can write all the equilibrium conditions in a way that does not involve the functions T or ψ. First, I use market clearing to show that the intertemporal budget constraint of the household can be written without reference to either function. This relies on the Ricardian properties of the model. Second, I show that Equation (10) is satisfied regardless of the specification of these functions using the two restrictions we imposed on policy given by Equations (20) and (23). Finally, I can write the remaining conditions without any reference to the function $\psi(\cdot)$, following the proof by Eggertsson and Woodford (2003a).

2.2 Discussion

Proposition 1 says that a policy of quantitative easing and/or deficit spending in a liquidity trap has no effect on the set of feasible equilibrium allocations that are

18. The Taylor rule is a member of this family in the following sense. The Taylor rule is $i_t = \max(\phi_\pi(\Pi_{t-1}) + \phi_y Y_t, 0)$.
The money demand Equation (8) defines the interest rate as a function of the monetary base, inflation, and output. This relation may then be used to infer the money supply rule that would result in an identical equilibrium outcome as a Taylor rule described above and would be a member of the rules we consider.

19. The reason why the variable q_t in the policy specification (Equations 17–23) can only include variables data at time t is that if it included lagged variables this may give the central bank to influence policy expectation by effecting the variables when the zero bound is binding that will enter the policy rules when the interest rates are positive. In that case policy expectation would not be "exogenous" in the way defined above.

20. The Technical Appendix is available on request.

consistent with the policy regimes I specified. It may seem that this result contradicts Keynes' view that deficit spending is an effective tool to escape the liquidity trap. It may also seem to contradict the monetarist view (see e.g., Friedman and Schwartz) that increasing the money supply is effective at low interest rates. But this would only be true if one took a narrow view of these schools of thought like Hicks (1937) does in his ground breaking paper "Mr. Keynes and the Classics." Hicks develops a static version of the General Theory and contrasts it to the monetarist view assuming that expectations are exogenous constants. This is the IS–LM model. My analysis, however, indicates that it is the intertemporal elements of the liquidity trap that are crucial in understanding the effects of different policy actions, namely their effect on expectations (to be fair to Hicks, he was very explicit that he was abstracting from expectation and recognized this was a major issue). Both Keynes (1936) and many monetarists (e.g., Friedman and Schwartz 1963) discussed the importance of expectations in their work and a static model is therefore not going to do full justice to their claims.

My result is that deficit spending has no effect on whether a given deflationary path represents an equilibrium if it does not change expectations about future policy. But as we shall see in later sections (when analyzing a Markov equilibrium) deficit spending can be very effective to *change expectations*. Thus the irrelevance result still leaves an important role for deficit spending, namely, it can be useful to change expectations. My result that quantitative easing is ineffective also relies on constant policy expectations. But as we shall also see (when analyzing a Markov equilibrium) quantitative easing changes expectation if the money printed is used to buy some private assets. Thus the irrelevance result also leaves an important role for quantitative easing through the expectation channel. Thus by modelling expectations explicitly, I believe my result neither contradicts Friedman and Schwartz' interpretation of the "Classics," i.e., the Quantity Theory of Money, nor Keynes' General Theory, at least if one takes a generous view of the main policy implications of these theories. On the contrary, it may serve to integrate the two by modelling the expectation channel.

Proposition 1 may also seem to contradict the claims of Bernanke (2003) and Buiter (2003). Both authors indicate that money-financed tax cuts increase demand. Buiter, for example, writes that "base money-financed tax cuts or transfer payments—the mundane version of Friedman's helicopter drop of money—will always boost aggregate demand." But what Buiter implicitly has in mind is that tax cuts permanently increase the money supply. Thus a tax cut today, in his model, increases expectations about future money supply. Thus my proposition does not disprove Buiter's or Bernanke's claims since I assume that money supply in the future is set without any reference to past policy actions. The propositions, therefore, clarify that tax cuts will only increase demand to the extent that they change beliefs about future money supply. The higher demand equilibrium that Buiter analyses, therefore, does not depend on the tax cut itself, but only on expectations about future money supply. A similar comment applies to Auerbach and Obstfeld's (2003) result. They argue that open market operations will increase aggregate demand. But their assumption is that open market operations increase expectations about future money supply. It is that belief that matters and not the open market operation itself, even if there is no cost of taxation.[21]

3. EQUILIBRIUM WITH ENDOGENOUS POLICY EXPECTATIONS

The main lesson from the last section is that expectations about future monetary and fiscal policies are crucial. Deficit spending and quantitative easing have no effect if they do not change expectations about future policy. But does deficit spending have no effect on expectations under reasonable assumptions about how these expectations are formed? Suppose, for example, the government prints unlimited amounts of money and drops it from helicopters, distributes it by tax cuts, or prints money and buys unlimited amounts of some private asset. Would this not alter expectations about future money supply? To answer this question I need an explicit model of how the government sets policy in the future. To do this I assume that the government sets monetary and fiscal policies optimally at all future dates. By optimal, I mean that the government maximizes social welfare that is given by the utility of the representative agent. I analyze equilibrium under two assumptions about policy formulation. Under the first assumption, which I call the commitment equilibrium, the government can commit to future policy in order to influence the equilibrium outcome by choosing future policy actions (at all different states of the world). Rational expectations require that these commitments are fulfilled in equilibrium. Under the second assumption, the government cannot commit to future policy. In this case the government maximizes social welfare under discretion in every period, disregarding any past policy actions, except insofar as they have affected the endogenous state of the economy at that date (defined more precisely below). Thus the government can only choose its current policy instruments, it cannot directly influence future government actions. This is what I call the Markov equilibrium. In the Markov equilibrium, following Lucas and Stokey (1983) and a large literature that followed, I assume that the government is capable of issuing one period riskless nominal debt and committing to paying it back with certainty. In this sense, even under discretion, the government is capable of limited commitment.

21. An obvious criticism of the irrelevance result for fiscal policy in Proposition 1 is that it relies on Ricardian equivalence. This aspect of the model is unlikely to hold exactly in actual economies. If taxes effect relative prices, for example if I consider income or consumption taxes, changes in taxation changes demand in a way that is independent of expectations about future policy. Similarly, if some households have finite-life horizons and no bequest motive, current taxing decisions affect their wealth and thus aggregate demand in a way that is also independent of expectation about future policy. The latter point is developed by Ireland (2001) who show that in an overlapping generation model wealth transfers increase demand at zero nominal interest rate (this of course would also be true at positive interest rate). The assumption of Ricardian equivalence is not applied here, however, to downplay the importance of these additional policy channels. Rather, it is made to focus the attention on how fiscal policy may change policy expectations. That exercise is most clearly defined by specifying taxes so that they can *only* affect the equilibrium through expectations about future policy. Furthermore, since our model indicates that expectations about future monetary policy have large effects in equilibrium, my conjecture is that this channel is of first order in a liquidity trap and thus a good place to start.

3.1 Recursive Representation

To analyze the commitment and Markov equilibrium it is useful to rewrite the model in a recursive form so that I can identify the endogenous state variables at each date. When the government can only issue one period nominal debt I can write the total nominal claims of the government (which in equilibrium are equal to the total nominal wealth of the representative household) as $W_{t+1} = (1 + i_t) B_t + (1 + i^m) M_t$. Substituting this into Equation (15) and defining the variables $w_t \equiv W_{t+1}/P_t$, $m_t \equiv M_t/P_{t-1}$, and $\Pi_t = P_t/P_{t-1}$ I can write the government budget constraint as:

$$w_t = (1 + i_t)\left(w_{t-1}\,\Pi_t^{-1} + (F - T_t) - \frac{i_t - i^m}{1 + i_t}\, m_t\,\Pi_t^{-1}\right). \tag{24}$$

Note that I use the time subscript t on w_t (even if it denotes the real claims on the government at the beginning of time $t + 1$) to emphasize that this variable is determined at time t. I assume that $F_t = F$ so that real government spending is an exogenous constant at all times. In Eggertsson (2006), I treat F_t as a choice variable. Instead of the restrictions (Equations 20 and 23) I imposed in the last section on government policies, I impose a borrowing limit on the government that rules out Ponzi schemes:

$$u_c\, w_t \leq \bar{w} < \infty\,, \tag{25}$$

where $\bar{w}$ is an arbitrarily high finite number. This condition can be justified by the fact the government can never borrow more than the equivalence of the expected discounted value of its maximum tax base.[22] It is easy to show that this limit ensures that the transversality condition of the representative household is satisfied at all times.

The treasury's policy instrument is taxation, T_t, that determines the end-of-period government debt which is equal to $B_t + M_t$. The central bank determines how the end-of-period debt is split between bonds and money by open market operations. Thus the central banks' policy instrument is M_t. Note that since P_{t-1} is determined in the previous period, I may think of $m_t \equiv M_t/P_{t-1}$ as the instrument of monetary policy.

It is useful to note that I can reduce the number of equations that are necessary and sufficient for a private sector equilibrium substantially from those listed in Definition 1. First, note that the equations that determine $\{Q_t, Z_t, G_t, C_t, n_t, h_t\}$ are redundant, i.e., each of them is only useful to determine one particular variable but has no effect on any of the other variables. Thus I can define necessary and sufficient condition for a private sector equilibrium without specifying the stochastic process for $\{Q_t, Z_t, G_t, C_t, n_t, h_t\}$ and do not need to consider Equations (3), (5), (6), (11), (14), and (16). Furthermore, Condition (25) ensures that the transversality

22. Since this constraint is never binding in equilibrium and $\bar{w}$ can be any arbitrarily high number for the results to be obtained, I do not model in detail the endogenous value of the debt limit.

condition of the representative household is satisfied at all times, so I do not need to include Equation (10) in the list of necessary and sufficient conditions. For the remaining conditions I use Equation (16) to substitute out for C_t.

It is useful to define the expectation variable

$$f_t^e \equiv E_t\, u_c\, (Y_{t+1} - d(\Pi_{t+1}) - F,\, m_{t+1}\, \Pi_{t+1}^{-1},\, \xi_{t+1})\, \Pi_{t+1}^{-1} \tag{26}$$

as the part of the nominal interest rate that is determined by the expectations of the private sector formed at time t. The IS equation can then be written as

$$1 + i_t = \frac{u_c(Y_t - d(\Pi_t) - F,\, m_t\Pi_t^{-1},\, \xi_t)}{\beta f_t^e}\,. \tag{27}$$

Similarly, it is useful to define the expectation variable

$$S_t^e \equiv E_t\, u_c\, (Y_{t+1} - d(\Pi_{t+1}) - F,\, m_{t+1}\, \Pi_{t+1}^{-1},\, \xi_{t+1})\, \Pi_{t+1}\, d'(\,\Pi_{t+1})\,. \tag{28}$$

The AS equation can be written as

$$\theta Y_t \left[\frac{\theta - 1}{\theta}\,(1 + s)\, u_c\, (Y_t - d(\Pi_t) - F,\, m_t\Pi_t^{-1},\, \xi_t) - \tilde{v}_y\, (Y_t,\, \xi_t)\right] \\ + u_c\, (Y_t - d(\Pi_t) - F,\, m_t\, \Pi_t^{-1},\, \xi_t)\, \Pi_t d'\, (\Pi_t) - \beta S_t^e = 0\,. \tag{29}$$

Finally, the money demand Equation (8) can be written in terms of m_t and Π_t as

$$\frac{u_m(Y_t - d(\Pi_t) - F,\, m_t\, \Pi_t^{-1},\, \xi_t)\, \Pi_t^{-1}}{u_c\, (Y_t - d(\Pi_t) - F,\, \xi_t)} = \frac{i_t - i^m}{1 + i_t}\,. \tag{30}$$

The next two propositions are useful to characterize equilibrium outcomes. Proposition 2 follows directly from our discussion above:

PROPOSITION 2: *A necessary and sufficient condition for a PSE at each time $t \geq t_0$ is that the variables $(\Pi_t, Y_t, w_t, m_t, i_t, T_t)$ satisfy: (i) Conditions (9), (24), (25), (27), (29), and (30) given w_{t-1} and the expectations f_t^e and S_t^e, (ii) in each period $t \geq t_0$, expectations are rational so that f_t^e is given by Equation (26) and S_t^e by Equation (28).*

PROPOSITION 3: *The possible PSE equilibrium defined by the necessary and sufficient conditions for any date $t \geq t_0$ onwards depend only on w_{t_0-1} and ξ_{t_0}.*

The second proposition follows from observing that w_{t-1} is the only endogenous variable that enters with a lag in the necessary and sufficient conditions in (i) of Proposition 2 and using the assumption that ξ_t is Markovian (i.e., using Assumption 1) so that the conditional probability distribution of ξ_t for $t > t_0$ only depends on ξ_{t_0}. It follows from this proposition that (w_{t-1}, ξ_t) are the only state variables at time t that directly affect the PSE. I may economize on notation by introducing vector notation. I define vectors

$$\Lambda_t \equiv [\Pi_t\; Y_t\; m_t\; i_t\; T_t]^T \text{ and } e_t \equiv \begin{bmatrix} f_t^e \\ S_t^e \end{bmatrix}.$$

Since Proposition 3 indicates that w_t is the only relevant endogenous state variable, I prefer not to include it in either vector but keep track of it separately. It simplifies notation a bit to write the utility function as a function of Λ_t i.e., I define the function $U : \mathrm{R}^{5+r} \rightarrow \mathrm{R}$

$$U_t = U(\Lambda_t, \xi_t)$$

using Equations (14) and (16) to solve for G_t and C_t as a function of Λ_t, along with Equations (3) and (12) to solve for $h_t(i)$ as a function of Y_t.

3.2 The Commitment Equilibrium

Using Proposition (3), I can now define the commitment solution.

DEFINITION 2: *The optimal commitment solution at date $t \geq t_0$ is the PSE that maximizes the utility of the representative household given w_{t_0-1}* and ξ_{t_0}.

Necessary conditions for the commitment equilibrium can be found by using a Lagrangian method fairly standard in the literature (apart from the inequality constraints present here). The Technical Appendix shows the Lagrangian and the first order conditions of the government's maximization problem.

3.3 The Markov Equilibrium

Here I consider an equilibrium that occurs when policy is conducted under discretion so that the government is unable to commit to any future actions. To do this I solve for a Markov equilibrium (it is formally defined by Maskin and Tirole, 2001) that has been extensively applied in the monetary literature. The basic idea behind this equilibrium concept is to define a minimum set of state variables that directly affect market conditions and assume that the strategies of the government and the private sector expectations depend only on this minimum state. Proposition 3 indicates that a Markov equilibrium requires that the variables (Λ_t, w_t) only depend on (w_{t-1}, ξ_t), since this is the minimum set of state variables that affect the PSE.

The timing of events in the game is as follows: At the beginning of each period t, w_{t-1} is a predetermined state variable. At the beginning of the period, the vector of exogenous disturbances ξ_t is realized and observed by the private sector and the government. The monetary and fiscal authorities choose policy for period t given the state and the private sector forms expectations e_t. Note that I assume that the private sector may condition its expectation at time t on w_t, i.e., it observes the policy actions of the government in that period so that Λ_t and e_t are jointly determined. This is important because w_t is the relevant endogenous state variable at date $t+1$. Since the state in this game is captured by (w_{t-1}, ξ_t), a Markov equilibrium requires that there exist policy functions $\bar{\Pi}_t(\cdot)$, $\bar{Y}_t(\cdot)$, $\bar{m}_t(\cdot)$, $\bar{i}_t(\cdot)$, $\bar{T}_t(\cdot)$ that I denote by the vector valued function $\bar{\Lambda}_t(\cdot)$ and a function $\bar{w}_t(\cdot)$, such that each period:[23]

23. Note that if the conditional expectation of ξ_{t+1} at time t does not depend on calendar time, these functions will be time invariant and one may drop the subscript t.

$$\begin{bmatrix}\Lambda_t \\ w_t\end{bmatrix} \equiv \begin{bmatrix}\bar{\Lambda}_t(w_{t-1}, \xi_t) \\ \bar{w}_t(w_{t-1}, \xi_t)\end{bmatrix}. \tag{31}$$

Note that the functions $\bar{\Lambda}_t(\cdot)$ and $\bar{w}_t(\cdot)$ will also define a set of functions of (w_{t-1}, ξ_t) for $(Q_t, Z_t, G_t, C_t, n_t, h_t)$ by the redundant equations from Definition 1. Using $\bar{\Lambda}_t(\cdot)$ I may also use Equations (26) and (28) to define a function $\bar{e}_t(\cdot)$ so that

$$e_t = \begin{bmatrix}f_t^e \\ S_t^e\end{bmatrix} = \begin{bmatrix}\bar{f}_t^e\ (w_t, \xi_t) \\ \bar{S}_t^e(w_t, \xi_t)\end{bmatrix} = \bar{e}_t(w_t, \xi_t)\ . \tag{32}$$

Rational expectations imply that the function $\bar{e}_t$ satisfies

$$\bar{e}_t(w_t, \xi_t) = \tag{33}$$
$$\begin{bmatrix} E_t u_c(\bar{C}_t(w_t, \xi_{t+1}), \bar{m}_t(w_t, \xi_{t+1})\bar{\Pi}_t(w_t, \xi_{t+1})^{-1}; \xi_{t+1})\ \bar{\Pi}_t(w_t, \xi_{t+1})^{-1} \\ E_t u_c(\bar{C}_t(w_t, \xi_{t+1}), \bar{m}_t(w_t, \xi_{t+1})\ \bar{\Pi}_t(w_t, \xi_{t+1})^{-1}; \xi_{t+1})\ \bar{\Pi}_t(w_t, \xi_{t+1})\ d'(\bar{\Pi}_t(w_t, \xi_{t+1}))\end{bmatrix}.$$

I define a value function $J_t(w_{t-1}, \xi_t)$ as the expected discounted value of the utility of the representative household, looking forward from period t, given the evolution of the endogenous variable from period t onwards that is determined by $\bar{\Lambda}_t(\cdot)$, $\bar{w}_t(\cdot)$, and $\{\xi_t\}$. Thus I define:

$$J_t\ (w_{t-1}, \xi_t) \equiv E_t \left\{ \sum_{T=t}^{\infty} \beta^T\ [U(\bar{\Lambda}_T\ (\cdot), \xi_T] \right\}. \tag{34}$$

The optimizing problem of the government is as follows. Given w_{t-1} and ξ_t, the government chooses the values for (Λ_t, w_t) (by its choice of the policy instruments m_t and T_t) to maximize the utility of the representative household subject to the conditions in Proposition 2 and Equation (32). Thus its problem can be written as:

$$\max_{m_t, w_t} [U(\Lambda_t, \xi_t) + \beta E_t\, J_{t+1}(w_t, \xi_{t+1})]\ , \tag{35}$$

such that Equations (9), (24),(25), (27), (29), (30), and (32) are satisfied.

I can now define a Markov equilibrium.

Definition 3: *A Markov equilibrium is a collection of functions* $\bar{\Lambda}_t(\cdot)$, $\bar{w}_t(\ )$, $J_t(\cdot)$, *and* $\bar{e}_t(\cdot)$, *such that (i) given the function* $J_t(w_{t-1}, \xi_t)$ *and the vector function* $\bar{e}_t(w_t, \xi_t)$ *the solution to the policy maker's optimization problem (Equation 35) is given by* $\Lambda_t = \bar{\Lambda}_t\ (w_{t-1}, \xi_t)$ *and* $w_t = \bar{w}_t(w_{t-1}, \xi_t)$ *for each possible state* (w_{t-1}, ξ_t) *and (ii) given the vector function* $\bar{\Lambda}_t(w_{t-1}, \xi_t)$ and $\bar{w}_t(w_{t-1}, \xi_t)$ *then* $e_t = \bar{e}_t(w_t, \xi_t)$ *is formed under rational expectations (see Equation (33)). (iii) Given the vector function* $\bar{\Lambda}_t(w_{t-1}, \xi_t)$ *and* $\bar{w}_t(w_{t-1}, \xi_t)$ *the function* $J_t(w_{t-1}, \xi_t)$ *satisfies Equation (34).*

I will only look for a Markov equilibrium in which the functions $\bar{\Lambda}_t(\cdot)$, $J_t(\cdot)$, and $e_t(\cdot)$ are continuous and have well defined derivatives. Then the value function satisfies the Bellman equation:

$$J_t(w_{t-1},\xi_t) = \max_{m_t, w_t}\ [U(\Lambda_t, \xi_t) + E_t \beta J_{t+1}(w_t, \xi_{t+1})]\ , \tag{36}$$

such that Equations (9), (24),(25), (27), (29), (30), and (32) are satisfied.

Necessary conditions for the Markov equilibrium can now be characterized by using a Lagrangian method for the maximization problem on the right hand side of Equation (36). In addition, the solution satisfies envelope conditions. The Lagrangian, associated with the appropriate first order condition, and the envelope conditions are shown in the Technical Appendix.

3.4 Approximation Method

The necessary condition for the Markov and commitment solutions can be linearized by a first order Taylor expansion around a steady state. The solution can then be obtained using the linearized equations. I define a steady state as a solution in the absence of shocks in which each of the variables $(\Pi_t, Y_t, m_t, i_t, T_t, w_t, f_t^e, S_t^e) = (\Pi, Y, m, i, T, w, f^e, S^e)$ are constants. Following Woodford (2003), I define a steady state where monetary frictions are trivial. To do this I parameterize the utility function by the technology parameter $\bar{m}$ so that as $\bar{m}$ is reduced the household will demand ever lower real money balances. I denote the policy instrument as $\tilde{m}_t \equiv m_t/\bar{m}$ and it is still meaningful to discuss the evolution of the nominal stock of money even as $\bar{m} \to 0$ (see Technical Appendix for details). Furthermore I assume, following Woodford (2003), that the steady state is fully efficient so that $1 + s = \theta/(\theta - 1)$. Finally, I suppose that in steady state $i^m = 1/\beta - 1$. To summarize:

Assumption 2: Steady state assumptions. (i) $\bar{m} \to 0$, (ii) $1 + s = \theta/(\theta - 1)$, and (iii) $i^m = 1/\beta - 1$.

Using Assumption 2, I prove in the Technical Appendix the existence of a steady state for both the commitment and the Markov solutions given by $(\Pi, Y, m/\bar{m}, i, T, w, f^e, S^e) = (1, \bar{Y}, \tilde{m}, (1/\beta) - 1, \bar{F}, 0, u_c(\bar{Y} - \bar{F}), 0)$ and show the equations the values $\bar{Y}$, $\bar{F}$, and $\tilde{m}$ satisfy. Furthermore, I discuss how the state of the Markov equilibrium relates to the results in Dedola (2002), King and Wolman (2003), Albanesi, Lawrence, and Chari (2003), and Klein, Krussel, and Rios-Rull (2003). I then show that the solution can be approximated around this steady state and that the resulting solution, which is locally unique, is accurate to the order $O(\|\xi, \bar{\delta}\|)$ where $\bar{\delta} \equiv (i - i^m)/(1 + i)$ (this latter approximation error arises because I analyze an equilibrium where $i^m = 0$ in the following sections). A complication is introduced by the presence of the interest rate bound inequality and I discuss how I treat this problem in the Technical Appendix. A further complication arises because in the Markov equilibrium the expectation functions, $\bar{e}_t(\cdot)$, are in general unknown. I illustrate a simple way of approximating these functions in Proposition 7.

4. THE DEFLATION BIAS

In the last section, I showed how an equilibrium with endogenous policy expectations can be defined and approximated. I now analyze the approximate equilibrium and show that deflation can be modeled as a credibility problem. The point of this section is not to absolve the government of responsibility for deflation. Rather, the point is to identify the policy constraints that result in inefficient deflation. The policy constraint in this section, apart from the government's inability to commit to future policy, is the assumption that government spending and taxes are constant. Money supply, by open market operations in short-term government bonds, is the government's only policy instrument. This is equivalent to assuming that the interest rate is the only policy instrument. In the next section, I relax this assumption. An appealing interpretation of the results is that they apply if the central bank does not coordinate its action with the treasury, i.e., if the central bank is "goal independent." This interpretation is discussed further in a companion paper (Eggertsson 2006).

The assumption about the policy instruments of the government in this section is as follows:

Assumption 3: Limited instruments: Open market operation in government bonds, i.e., $\tilde{m}_t$, is the only policy instrument. Fiscal policy is constant so that $w_t = 0$ and $T_t = F$ at all times.

To gain insights, it is useful to consider the linear approximation of the private sector equilibrium constraints. The AS Equation (29) can be written to the first order as the "New Keynesian Phillips curve"

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \qquad (37)$$

and Equation (27) can be written to the first order as the forward looking "IS relation"

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n) . \qquad (38)$$

Here $\pi_t \equiv \Pi_t - 1$ is the inflation rate, $x_t \equiv (Y_t - Y_t^n)/Y_t^n$ is the output gap, i.e., it is the percentage deviation of output from the natural rate of output.[24] The term r_t^n is a composite exogenous disturbance that shifts the IS equation. It represents exogenous variations in the Wicksellian natural rate of interest, that is, the equilibrium real rate of interest in the case output is equal to its natural rate at all times. In this model $r_t^n = [(1 - \beta)/\beta + (\sigma^{-1}\omega)/(\sigma^{-1} + \omega)][g_t - E_t g_{t+1} - (q_t - E_t q_{t+1})]$ summarizes to the first order all the relevant shocks (when the model is written in terms of the output gap). The coefficients κ and σ are both positive and given by $\kappa \equiv \theta(\sigma^{-1} + \omega)/d''$ and $\sigma \equiv -\bar{u}_{cc}\bar{Y}/\bar{u}_c$ where $\omega \equiv \bar{v}_y/\bar{v}_{yy}\bar{Y}$. The term $g_t \equiv -\bar{u}_{c\xi}/\bar{Y}\bar{u}_{cc}\xi_t$ summarizes the shocks to consumption preferences and $q_t \equiv \bar{v}_{y\xi}/\bar{Y}\bar{v}_{yy}\xi_t$ summarizes the shocks to the disutility of working.

I first show that if the natural rate of interest is positive at all times, and Assumptions 2 and 3 hold, the commitment and the Markov solutions are identical and the

24. The natural rate of output is the output that would be produced if prices were completely flexible. It is the output that solves the equation

$$v_y(Y_t^n, \xi_t) = \frac{\theta - 1}{\theta}(1 + s)u_c(Y_t^n, \xi_t) . \qquad (42)$$

Note that this definition of the natural rate of output is different from the efficient level of output which is obtained if $(1 + s) = \theta/(\theta - 1)$ and prices are flexible. The variable i_t in Equation (38) actually refers to i_t times β^{-1} in our previous notation so that it does not refer to deviation from steady state. I do this so I can write the zero bound as the simple requirement that i_t be non-negative.

zero bound is never binding. To be precise, the assumption on the natural rate of interest is:

Assumption 4: $r_t^n \in [0, S]$ at all times where S is a finite positive number.

Assuming this restriction on the natural rate of interest I can prove the following proposition.

PROPOSITION 4: *Markov and the commitment equivalence. If Assumptions 2, 3(i), 3(ii), and 4 are satisfied then the following must hold at least locally to the steady state: There is a unique bounded Markov and commitment solution given by* $i_t = r_t^n \geq 0$ *and* $\pi_t = x_t = 0$. *The equilibrium is accurate up to an error of the order* $O(\|\xi, \bar{\delta}\|^2)$.

PROOF: See Technical Appendix.

The intuition for this result is straight forward and can be understood by inspecting the linear approximation of the IS and AS equations in addition to a second order expansion of the representative household utility (but the household utility is the objective of the government). When fiscal policy is held constant, the utility of the representative household, to the second order, is equal to:[25]

$$U_t = -\left[\pi_t^2 + \frac{\kappa}{\theta}(x_t - x^*)^2\right] + O\left(\left\|\xi, \bar{\delta}, 1 + s - \frac{\theta}{\theta - 1}\right\|^3\right) + \text{t.i.p}\,. \tag{39}$$

where $x^* = (\omega + \sigma^{-1})^{-1}(1 - (1 - 1/\theta)(1 + s))$ and t.i.p. is terms independent of policy. In Assumption 2(ii), I assume that $(1 + s) = \theta/(\theta - 1)$ and therefore $x^* = 0$. One can then observe by the IS and AS equations that the government can completely stabilize the loss function at zero inflation and zero output gap in an equilibrium where $i_t = r_t^n$ at all times. Since this policy maximizes the government's objective at all times, there is no incentive for the government to deviate. Therefore the government's ability to commit has no effect on the equilibrium outcome, which is the intuition behind the formal proof of Proposition 4 in the Technical Appendix.

Proposition 4 only applies when $x^* = 0$ as in Assumption 2. When $x^* > 0$, the commitment and Markov solutions differ because of the classic inflation bias (stemming from monopoly powers of the firms) as first demonstrated by Kydland and Prescott (1977). I will now show that even when $x^* = 0$, the commitment and Markov solutions may also differ because of shocks that render the zero bound binding and which in turn trigger temporary excessive deflation in the Markov equilibrium. This new dynamic inconsistency problem is the deflation bias. I assume that $x^* > 0$ in the next subsection and show the connection between the inflation and the deflation bias.

The deflation bias can be derived by a simple assumption about the natural rate of interest r_t^n (recall that all the shocks that change the private sector equilibrium constraints can be captured by the natural rate of interest). Here I assume that the natural rate of interest becomes unexpectedly negative in period 0 and then reverts

25. Please note that Proposition 4 does not rely on these expansions since I derive the first order conditions of the government problem in the fully nonlinear model. The expansion is only reported to clarify the intuition behind the propositions. In Equation (40), I have expanded utility around the steady state discussed in Section 3.4 and allowed for stochastic variations in ξ and also assumed that s and i^m may deviate from the steady state I expand around. Derivation is available upon request.

back to a positive steady state in every subsequent period with some probability. At the time r_t^n reverts back to steady state, a stochastic date denoted τ, it stays there forever. Assuming that all uncertainty is resolved before a finite date K simplifies the proofs. This is not a very restrictive assumption since K may be arbitrarily high. To be more precise I assume:

Assumption 5: $r_t^n = r_L^n < 0$ at $t = 0$ and $r_t^n = r_{ss}^n = (1/\beta) - 1$ at all $0 < t < K$ with probability α if $r_{t-1}^n = r_L^n$ and probability 1 if $r_{t-1}^n = r_{ss}^n$ at all $t > 0$. The stochastic date when r_t^n reverts to r_{ss}^n is denoted τ. There is an arbitrarily large number K so that $r_t^n = r_{ss}^n$ with probability 1 for all $t \geq K$ and thus $\tau \leq K$.

The natural rate of interest can be negative due to a series of negative demand shocks (i.e., shifts in the utility of consumption) or expectations of lower future productivity (i.e., shift in the disutility of working). A temporary collapse in some autonomous component of aggregate spending (that is separate from private consumption) can also be interpreted as preference shocks. More generally, the most plausible candidate for a collapse in aggregate spending is a decline in investment. A host of candidates could lead to an investment collapse, such as problems in financial intermediation, adverse shocks to the balance sheets of firms, or a productivity slowdown. These shocks are not modelled in detail at this level of abstraction (but arguably correspond most closely to an autonomous decline in aggregate spending in the current setup) but could be studied more thoroughly in a model with endogenous capital.

The commitment and the Markov solutions derived in Proposition 4 are not feasible if Assumption 5 holds because the solution in Proposition 4 requires that $i_t = r_t^n$ at all times. If the natural rate of interest is temporarily negative, as in Assumption 5, this would violate the zero bound. How does the solution change when the natural rate of interest is negative?

Consider first the commitment solution. A simple numerical example is useful. Suppose that in period 0 the natural rate of interest is unexpectedly negative so that $r_L^n = -2\%$ and then reverts back to steady state of $r_{ss}^n = 2\%$ with 10% probability in each period (taken to be a quarter here). The calibration parameters I use are the same as in Eggertsson and Woodford (2003a) (see details in the Technical Appendix). Figure 1 shows the solution for inflation, the output gap, and the interest rate using the approximation method described in the Technical Appendix. The first line in the top panel shows inflation when the natural rate of interest reverts to the steady state in period 1, the second if it returns back in period 2 and so on.[26] The central bank offsets a low natural rate of interest by lowering the interest rate correspondingly. But when the natural rate of interest is negative this is not feasible. To offset the shock the government commits to inflation and a temporary boom *in the future*, i.e., once the

26. The numerical solution reported here is exactly the same as the one shown by Eggertsson and Woodford (2003) in a model that is similar but has Calvo prices (instead of the quadratic adjustment costs I assume here). Their solution also differs in that they compute the optimal policy in a linear quadratic framework. As our numerical solution illustrates, however, the results for the commitment equilibrium are identical. Jung, Teranishi, and Watanabe (2001) also derive the commitment equilibrium in a linear quadratic framework but assume a deterministic process for the natural rate.

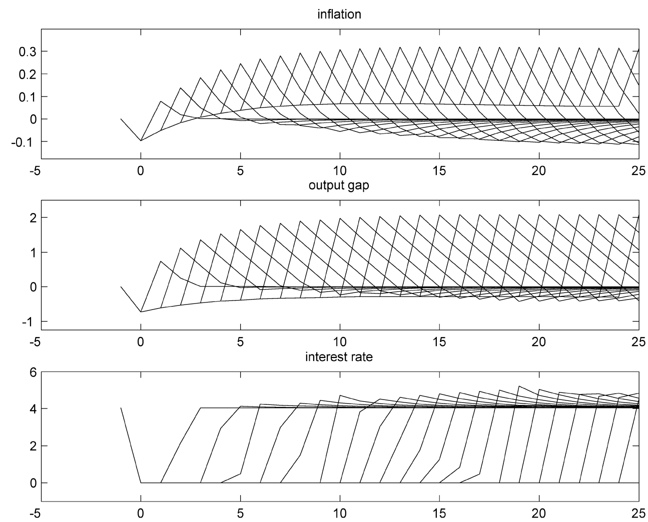

FIG. 1. Inflation, the Output Gap, and the Short-Term Nominal Interest Rate under Optimal Policy Commitment When The Government Can only Use Open Market Operations as Its Policy Instrument. Each line represents the response of inflation, the output gap, or the nominal interest rate when the natural rate of interest returns to its steady-state value in that period.

natural rate of interest returns to normal, and keeping the nominal interest rate low for a substantial period. Furthermore, the optimal commitment implies a higher price level in the future and a higher money supply (see Figures 7 and 8 and Section 5 for further discussion). The expectations of future inflation and output boom are beneficial when $r_L^n < 0$ because they offset the negative demand effect of the shock. To see this consider the IS Equation (38). Even if the nominal interest rate cannot fall below 0 in period t, the real rate of return (i.e., $i_t - E_t \pi_{t+1}$) is what is relevant for aggregate demand and it can still be lowered by increasing inflation expectations. This is captured by the second element of the right hand side of Equation (38). Furthermore, a commitment to a temporary boom, i.e., higher $E_t x_{t+1}$, also stimulates demand by the permanent income hypothesis. This is represented by the first term on the right hand side of Equation (38).

Bank of Japan officials have objected to an inflation target on the grounds that it is not "credible" since they cannot lower the nominal interest rate to manifest their intentions. The optimal commitment depends on manipulating expectations and one should consider the extent to which this policy commitment is credible, i.e., if the government has an incentive to deviate from the optimal plan. Consider now the Markov equilibrium. For the case $K \to \infty$ it can be shown to yield the simple closed form solution:[27]

27. Note that to ensure that the solution is bounded I need to assume that α satisfies the inequalities $\beta\alpha^2 + (1 + \sigma\kappa - \beta)\alpha - \sigma\kappa > 0$ and $0 < \alpha < 1$. If this condition is not satisfied the solution explodes and a linear approximation of the IS and the AS equations is not valid for shocks of any order of magnitude. Thus I would need to use other nonlinear solution methods to solve for the equilibrium if the value of α does not satisfy these bounds. Here I simply assume parameters so that these two inequalities are satisfied and a linear approximation of the IS and AS is feasible and the solution is accurate of the order $o(\|\xi, \bar{\delta}\|^2)$ (see Technical Appendix).

$$x_t = \frac{1 - \beta(1 - \alpha)}{\alpha(1 - \beta(1 - \alpha)) - \sigma\kappa(1 - \alpha)} \sigma r_L^n \quad \text{if } r_t^n = r_L^n \text{ and } x_t = 0 \text{ otherwise },$$

$$\pi_t = \frac{1}{\alpha(1 - \beta(1 - \alpha)) - \sigma\kappa(1 - \alpha)} \kappa\sigma r_L^n \quad \text{if } r_t^n = r_L^n \text{ and } \pi_t = 0 \text{ otherwise }.$$

This solution is shown in Figure 2 for the calibrated example. It shows excessive deflation in the periods in which the natural rate of interest is negative. A key reason for the excessive deflation is the expectation channel. The 90% chance of the natural rate of interest remaining negative for the next quarter creates the expectation of future deflation and a continued negative output gap, which creates even further deflation. Even if the central bank lowers the short-term nominal interest rate to zero, the real rate of return is positive, because the private sector expects deflation.

The reason for the sharp difference between the commitment and the Markov solution is that the Markov solution mandates zero inflation and zero output gap as soon as the natural rate of interest is positive. Thus the government cannot commit to a higher future price level as the optimal commitment implies and this lack of commitment is the main culprit for deflation. This is the deflation bias of discretionary policy.

PROPOSITION 5: *The deflation bias. If Assumptions 2(i), 2(ii), 3, and 4 are satisfied then the following must hold at least locally to the steady state. The Markov equilibrium for* $t \geq \tau$ *is given by* $\pi_t = x_t = 0$ *and the result is excessive deflation and output gap for* $t < \tau$ *relative to a policy that implies* $\pi_\tau > 0$ *and* $x_\tau > 0$ *and* $i_t = 0$ *when* $t \leq \tau$. *The equilibrium is accurate up to an error of the order* $O(||\xi, \bar{\delta}||^2)$.

PROOF: See Technical Appendix.

What is the logic behind the deflation bias? Consider one realization of the shock from the numerical example. Figure 3 shows the commitment and the Markov solutions for $\tau = 15$. The optimal commitment is to keep the nominal interest rate low for a substantial period of time after the natural rate becomes positive resulting in $x_{\tau=15}^C > 0$ and $\pi_{\tau=15}^C > 0$. If the government is discretionary, however, this type of commitment is not credible. In period 15, once the natural rate becomes positive again, the government raises the nominal interest rate to steady state, thus achieving zero inflation and zero output gap from period 15 onwards. The result of this policy, however, is excessive deflation in period 0 to 14. Why does the government choose this suboptimal policy if it cannot commit? Consider the objectives of the government (recall I assumed that $x^* = 0$). Once the natural rate of interest has become positive again, at time $t = 15$, the optimal policy is set to the nominal interest rate at the steady state from then on since this policy will result in zero output gap and zero inflation *at that time onwards*—thus the Markov policy is maximizing the objectives (Equation 39) from period 15 onwards. The government, therefore, has an incentive to renege on the optimal commitment because the optimal commitment results in a temporary boom and inflation in period 15 and thus implies higher utility losses from period 15 onwards relative to the Markov solution. In rational expectation, however, the private sector understands

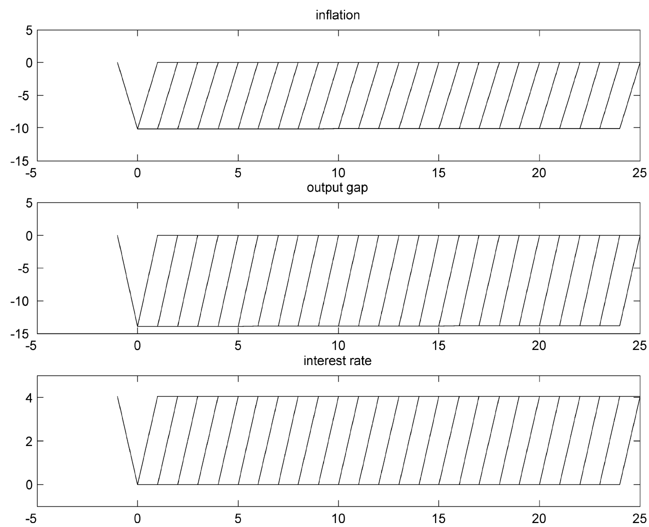

FIG. 2. Inflation, the Output Gap, And the Short-Term Nominal Interest Rate in a Markov Equilibrium under Discretion When the Government Can only Use Open Market Operations as Its Policy Instrument. Each line represents the response of inflation, the output gap, or the nominal interest rate when the natural rate of interest returns to its steady-state value in that period.

the government's incentives. If the government is unable to commit, the result is excessive deflation and an output gap in period 0 to 14 when the zero bound is binding. The deflation bias is not an artifact of the numerical values assumed in the example. Proposition 5 is proved analytically in the Technical Appendix without the cost of changing prices being above any critical value. Thus it remains true even if the cost of changing prices is made arbitrarily small, as long as it is not exactly zero.[28]

In the Markov solution any increase in the monetary base at zero interest rate will *always* be expected to be reversed. This can help explain why BOJ aggressive increase in the monetary base has had little effect. It cannot credibly promise higher *future* money supply—the private sector expects the BOJ to contract as soon as there is any sign of inflation. It is a credibility problem of a *rational* central bank that

28. It is easier to see this as a special case of Assumption 5. If $\alpha = 1$ the natural rate of interest is positive with probability 1 in period 1. Then Proposition 6 indicates that the solution from period 1 onwards is given by $\pi_t = x_t = 0$ for $t \geq 1$. The IS indicates that in period 0 the output gap is $x_0 = \sigma r_t^n$. Note that the output gap in period 0 is independent of the cost of changing prices since neither r_t^n nor σ is a function of the cost of price changes. This is because the output gap only depends on the difference between the current interest rate and the natural rate of interest and expectations about future inflation and output gap, and the latter are zero from period 1 onwards. The AS equation, however, indicates that the deflation in period 0 is going to depend on the cost of changing prices, i.e., $\pi_0 = \kappa x_0$. The lower the cost of changing prices the higher $\kappa = \theta/d''(\sigma^{-1} + \omega)$ which indicates that there will be more deflation, the lower the cost of price changes (since x_0 is given by the IS equation which does not depend on d''). The intuition for this is that the lower the cost of price changes, the more the prices need to adjust for the equation $x_0 = \sigma r_t^n$ to be satisfied. Thus the deflation bias is worse—in terms of actual fall in the price level—the lower the cost of changing prices. This basic intuition will also carry through to the stochastic case.

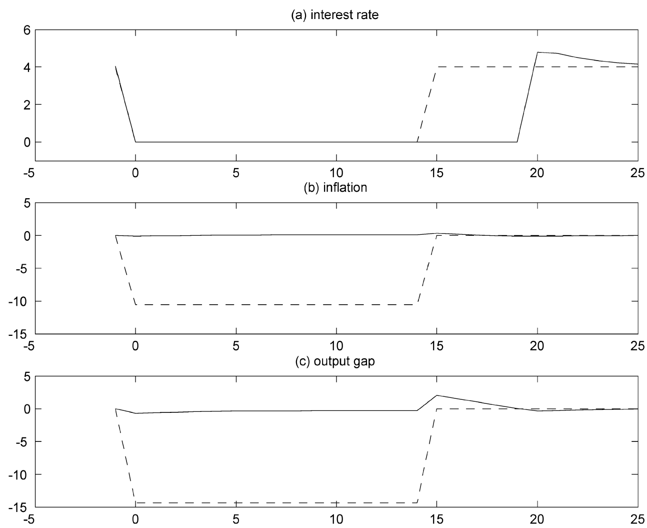

FIG. 3. Response of the Nominal Interest Rate, Inflation, And the Output Gap to Shocks That Lasts for 15 Quarters

cannot commit to future policy. Krugman (1998) recognizes a commitment problem at zero interest rate. He assumes that the government follows a monetary policy targeting rule so that $M_t = M^*$. He then shows that if expectations about future money supply are fixed at M^*, increasing money supply at time t has no effect at zero interest rate. Krugman calls this "the inverse of the usual credibility problem." The key to effective policy, according to Krugman, is to commit to higher money supply in the future (as is verified by our numerical example), i.e., to "commit to being irresponsible." My result illustrates that this problem is not isolated to a government that is expected to follow a monetary targeting rule. The problem arises for a government that maximizes social welfare and has only one policy instrument but is unable to commit to *not re-optimize in the future disregarding past actions*. This is of practical importance. According to my solution, inefficient deflation is consistent with a rational government, as long as it is unable to commit to future policy. It may, therefore, be hard for it to change expectations for a government that has little credibility. In contrast, Krugman's government is committed to some monetary targeting policy rule that is suboptimal. It may, therefore, seem that it is easy to change policy expectations and that the only problem is to find the optimal policy. This result, however, indicates that more may be required.

4.1 Extension: The Inflation Bias vs the Deflation Bias

In this section, I explore the connection between the deflation bias derived in last section and the inflation bias shown by Kydland and Prescott (1977) and Barro and Gordon (1983). The government's inability to commit in this model results in chronic inflation if $x^* > 0$. It is easy to show that if the zero bound is never binding (e.g., under Assumption 3) inflation is given by

$$\pi_t = \bar{\pi} = \frac{1 - \beta}{1 - \beta + \theta\kappa} x^* > 0 \,, \tag{40}$$

which is inefficient. This implies that the equilibrium nominal interest rate is given by

$$i_t = r_t^n + \bar{\pi} \,.$$

Thus when there is an inflation bias in the economy, denoted by $\bar{\pi}$, a necessary condition for avoiding the zero bound is $r_t^n + \bar{\pi} \geq 0$. If the natural rate of interest is low enough, however, there is a deflation bias. Thus exactly the same commitment problem as shown in last section arises in an economy with an inflation bias if the shock is large enough, i.e., if $r_t^n < -\bar{\pi}$. To summarize:

PROPOSITION 6: *The inflation bias vs the deflation bias. If Assumptions 2(i), 3, 5, and* $0 \leq s < 1/(\theta - 1)$ *then* $\pi_t = (\kappa/(1 - \beta))\bar{x} = \bar{\pi}$ *for* $t \geq \tau$ *and there is excessive deflation and an output gap in period* $t < \tau$ *if* $r_L^n < -\bar{\pi}$ *relative to a policy that implies* $\pi_\tau > \bar{\pi}$ *and* $x_\tau > \bar{x}$ *and* $i_t = 0$ *when* $t \geq \tau$. *Here* $\bar{\pi}$ *is a solution to the equation* $\bar{\pi} = ((1 - \beta)/(1 - \beta + \theta\kappa))\, x^* \geq 0$. *The equilibrium is accurate up to an error of the order* $O(\|\xi, \bar{\delta}, 1 + s - \theta/(\theta - 1)\|^2)$.

PROOF: See Technical Appendix.

Figure 4 shows the solution for inflation and the output gap for different values of x^*. Note that according to Equation (40) a different value of x^* translates into different inflation targets for the government in a Markov equilibrium. The figure shows values of x^* that corresponds to 1%, 2%, and 4% inflation targets, respectively (I may vary this number by assuming different values for s in the expression for x^*). I assume Assumption 5 but the natural rate of interest is -4% in the low state and reverts back to steady state with 10% probability in each period. Note that only when the inflation bias corresponds to $\bar{\pi} = 4\%$, there is no deflation bias. If $\bar{\pi} < -r_L^n = 4\%$, the result is excessive deflation. The picture also illustrates, and this is the lesson of Proposition 6, that the deflation bias is a problem even in an economy with an average inflation bias, as long as the negative shock is large enough. The higher the average inflation bias, however, the larger the shock required for the deflation bias to be problematic.

What is a realistic inflation bias in an industrial economy? If I use the same values as in the numerical example above (see Technical Appendix) the implied inflation bias is below 1% per year. If the model is applied to Japan, this is indeed quite consistent with average inflation rates during the 1980s and early 1990s (before deflationary pressures emerged). The inflation bias, therefore, is relatively low and a deflationary bias is a considerable concern. I think it is fairly realistic to assume a low inflation bias for Japan. Throughout the 1980s and early 1990s, for example, there was virtually no unemployment, and the government had a small incentive to inflate, consistent with that x^* is close to zero. The assumption that $x^* = 0$, therefore, does not seem grossly at odds with the evidence for Japan, and as argued by Rogoff

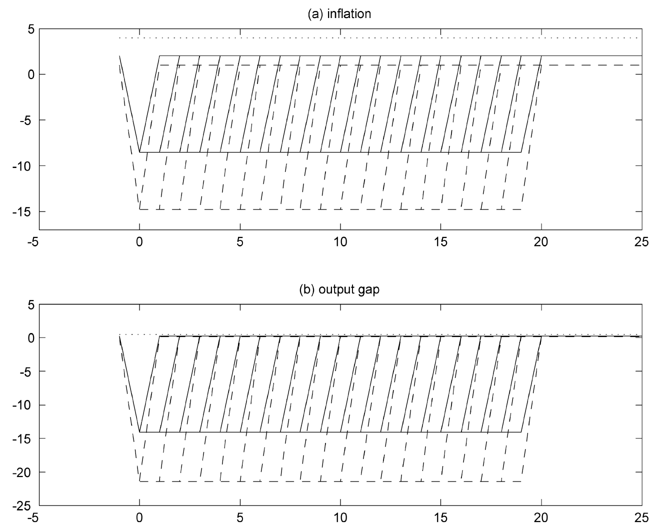

FIG. 4. Inflation and the Output Gap under Different Assumptions about Steady State Inflation Bias When the Natural Rate of Interest is Temporarily −4%. The dotted lines correspond to a 4% steady state inflation bias, the solid line 2%, and the dashed line 1%.

(2003) the great disinflation in the world indicates that the inflation bias may be small (and shrinking) throughout the rest of the world.

Two aspects of a liquidity trap render the deflation bias a particularly acute problem, and possibly a more serious than the inflation bias. First, announcing a higher inflation target in a liquidity trap involves no direct policy action—since the short-term nominal interest rate is at zero, it cannot be lowered any further. The central bank has, therefore, no obvious means to demonstrate its desire for inflation. Thus announcing an inflation target in a liquidity trap may be less credible than under normal circumstances when the central bank can take direct actions to show its commitment. Second, unfavorable shocks create the deflation bias. It may be hard for the central bank to acquire any reputation for dealing shocks if they are infrequent—which is presumably the case with shocks that make the zero bound binding given the few historical examples of the liquidity trap. To make matters worse, optimal policy in a liquidity trap involves committing to inflation. In an era of price stability the optimal policy under commitment is fundamentally different from what has been observed in the past.

5. COMMITTING TO BEING IRRESPONSIBLE

The last section demonstrated that deflation can be modelled as a credibility problem if the government is unable to commit to future policy and it's only instrument is open market operations. This section illustrates how the result changes if the government can use fiscal policy as an additional policy instrument. I first explore if deficit spending increases demand. When the government coordinates fiscal and monetary policies it can commit to future inflation and low nominal interest rate by cutting taxes and issuing nominal debt. I then use the result to interpret the effect of open market operations in a large spectrum of private assets, such as foreign exchange or stocks.

The assumption about monetary and fiscal policies is:

Assumption 6: Coordinated fiscal and monetary policy instruments: Open market operations in government bonds, i.e., $\tilde{m}_t$, and deficit spending, $B_t - T_t$, are the instruments of policy.

Using this assumption, I can prove the following proposition.

Proposition 7: *Committing to being irresponsible. If Assumptions 2, 5, and 6 hold then there is a solution at date* $t \geq \tau$ *for each of the endogenous variables given by* $\Lambda_t = \Lambda^1 w_{t-1}$, *and* $w_t = w^1 w_{t-1}$ *where* Λ^1 *and* w^1 *are constants. For a given value of* w^1 *there is a unique solution for* Λ^1. *The coefficient* w^1 *is a number that solves Equation (118) in the Technical Appendix. The solution for inflation is* $\pi_t = \pi^1 w_{t-1}$ *and the government can use deficit spending to increase inflation expectations when* $\pi^1 \neq 0$, *curbing deflation and the output gap in period* $t < \tau$. *The equilibrium is accurate up to an error of the order* $O(\|\xi, \delta\|^2)$.

I prove this proposition in the Technical Appendix. The solution shows that nominal debt effectively commits the government to inflation even if it is discretionary. It is instructive to write out the algebraic expression for the inflation coefficient in the solution. I show in the Appendix that at $t \geq \tau$ the solution for inflation is

$$\pi_t = \pi^1 w_{t-1} \quad \text{where } \pi^1 = \frac{s' \bar{g}_G}{d'' \bar{u}_c} \beta^{-1} + \phi_4^1 . \tag{41}$$

The government can reduce the real value of its debt (and future interest payments) by increasing either taxes or inflation. Since both inflation and taxes are costly, it chooses a combination of the two. The presence of debt creates inflation through two channels in our model: (1) If the government has outstanding *nominal* debt it has incentives to create inflation to reduce the real value of the debt. This incentive is captured by the term $(s' g_G / d'' u_c)\beta^{-1}$ in Equation (41). The marginal cost of taxation is $s' g_G$ and the marginal cost of inflation is $d'' u_c$. (2) If the government issues debt at time t, it has incentives to lower the real rate of return it pays on the debt it rolls over to time $t + 1$. This incentive also translates into higher inflation.[29] This incentive is reflected in the value of the coefficient ϕ_4^1 which is the coefficient in the solution for the Lagrangian multiplier on the AS equation i.e., $\phi_{4t} = \phi_4^1 w_{t-1}$. This coefficient reflects the value of relaxing the aggregate supply constraint, which can be beneficial

29. Obstfeld (1997) analyses a flexible price model with real debt (as opposed to nominal as in our model) but seignorage revenues due to money creation. He obtains a solution similar to mine (i.e., debt in his model creates inflation but is paid down over time). Calvo and Guindotti (1992) similarly illustrate a flexible price model that has a similar solution. The influence of debt on inflation these authors illustrate is closely related to the first channel we discuss above. The second channel we show, however, is not present in these papers since they assume flexible prices.

because of the reduction in the real interest rate paid on debt associated with higher output; i.e., the government has an incentive to create a boom (by lowering the real rate of interest) to lower the service on the debt it rolls over to the next period.

As I showed in the previous section, committing to future inflation and an output boom is exactly what is mandated by the optimal commitment. Using the same numerical example as in previous section, Figures 5 and 6 show that it is optimal for a discretionary government to issue debt when the zero bound is binding. This effectively commits it to future inflation and an output boom once the natural rate of interest is positive again.[30] By cutting taxes and issuing debt in a liquidity trap the government curbs deflation and increases output to nearly the optimal commitment level. Figure 5 also shows that the nominal interest rate stays below the steady state after the natural rate of interest returns to normal and rises only slowly.

The Markov solution is still not fully optimal since it does not replicate the commitment solution perfectly. Table 1 shows welfare under three policy regimes. Welfare is evaluated by utility of the representative household. The first regime, R1, is a government that can fully commit to future policy and uses both monetary and fiscal policies to achieve its objective. The second, R2, is a government that cannot commit to future policy but uses both monetary and fiscal policies to maximize utility. The third regime, R3, is a government that is unable to commit to future policy and has only one policy instrument, i.e., open market operations in short-term government bonds. This table shows that the government's ability to use debt as a commitment device nearly eliminates all the costs of discretion. The interpretation of this utility index is that under R1 the representative household would pay 0.02% of its steady state quarterly consumption (forever) to avoid moving to regime R2. Thus the number 0.02 reflects that value of commitment if the government can coordinate monetary and fiscal policies. In contrast the loss in utility to move from R1 to R3 is very large or 13.48%.[31]

Proposition 7, Figures 5 and 6, and Table 1 summarize the central results of this paper. Even if the government cannot commit it can stabilize the price level in a liquidity trap. A simple way of increasing inflation expectations is coordinating fiscal and monetary policies and running budget deficits, which in turn increases output and prices. The channel is simple. Budget deficits generate nominal debt. Nominal debt, in turn, makes a higher inflation target credible because the real value of the debt increases if the government reneges on the target. Higher inflation expectations lower the real rate of interest and thus stimulate aggregate demand.

30. In general, there is more than one solution for w^1 in Equation (118). In the numerical examples I have done, however, all but one of the values that satisfy this equation are explosive and imply that by Equation (118) the value of γ_{2t} is negative once the debt limit of the government is reached. This in turn, violates the inequality constraint of this multiplier, implying that an explosive solution does not solve the first order conditions of the government's maximization problem. It can be proved in a simplified version of the model that there is always a unique solution w^1 that solves the model and that it implies that debt converges back to steady state. For this version of the model, however, an analytic proof is not available, but in all the calibrated examples that I have explored this is indeed the case.

31. Here I normalize the utility flow by transforming the utility stream (which is the future discounted stream of utility from private and public consumption—in all states of the world—minus the flow from the disutility of working) into a stream of a constant private consumption endowment.

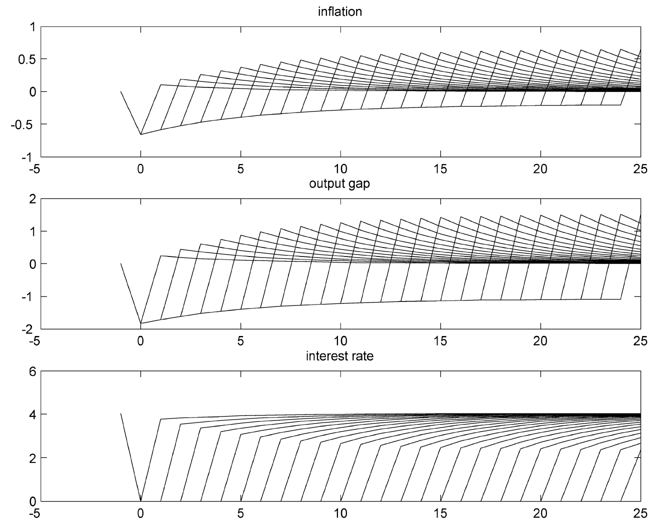

FIG. 5. Inflation and Output Gap in a Markov Equilibrium under Discretion, When the Government Can Use Both Monetary And Fiscal Policies to Respond to a Negative Natural Rate of Interest

This policy involves direct actions by the government which can be useful to communicate the policy (a criticism that is sometimes raised about the commitment policy is that it does not require any actions, only announcements about future

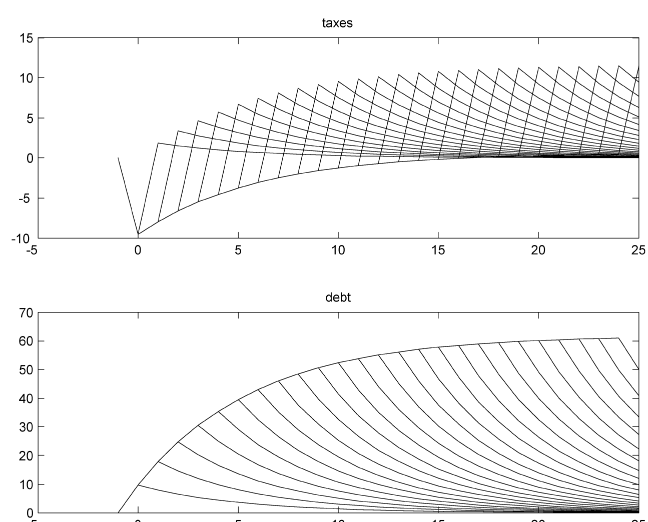

FIG. 6. Taxes and Debt in a Markov Equilibrium under Discretion, When the Government Can Use Both Monetary And Fiscal Policies to Respond to a Negative Natural Rate of Interest

TABLE 1
WELFARE UNDER POLICY REGIMES

Policy regime	Utility in cons. eq. units
R1	100
R2	99.98
R3	95.73

intentions, see e.g., Friedman, 2003). The government can announce an inflation target and proceed to increase budget deficits until the target is reached.

Discussion. To contrast the commitment and the discretion solutions, it is useful to consider the evolution of the price level. Figure 7 shows the evolution of the price level under the three policy regimes reported in Table 1. The optimal solution (i.e., R1) is to commit to a higher future price level as can be seen in panel A of Figure 7, although the extent to which the price level increases is small. If the government is unable to commit, however, this policy is not credible. A dramatic decline in the price level occurs under monetary discretion (i.e., R3) as shown in panel B. The price level declines by 35%, for example, if the natural rate of interest becomes positive in period 15 (this is the case I showed in Figure 3). Panel C of Figure 2 shows the large price decline can be avoided if the government uses fiscal policy to "commit to being irresponsible" (i.e., R2). This commitment involves increasing the price level once the natural rate becomes positive. When the natural

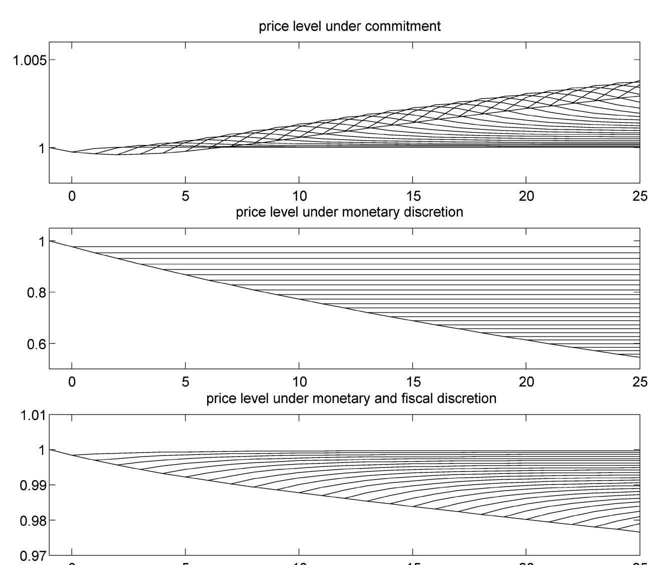

FIG. 7. The Evolution of the Price Level under Different Assumptions about Policy

rate of interest reverts to steady state in period 15, for example, the long run price level falls by less than 1%, compared to 35% decline under monetary discretion (R3).

It is worth considering the evolution of money supply in these different equilibria.[32] Figure 8 shows the long run nominal stock of money under each of the three policy regimes discussed above. In the figure, I show the future level of the nominal stock of money in the case when the natural rate of interest reverts back to steady state in periods 3, 6, 9, 12, and 15. The figure shows the level of money supply under each policy once the price level has converged back to its new steady state (so I do not need to make any assumptions here about the interest rate elasticity or output elasticity of money demand).[33] I assume that the value of the money supply is 1 before the shocks hit the economy. The figure illustrates that the optimal commitment (R1) involves committing to a nominal money supply in the future that is only marginally larger than before the shock. In contrast, the monetary discretion (R3) involves a considerable contraction of the monetary base. The government will accommodate any deflation at $t < \tau$ by contracting the monetary base as soon as the natural rate of interest becomes positive again in order to prevent inflation at $t \geq \tau$. Under a monetary and fiscal discretion regime (R2) aggressive deficit spending allows the government to credibly commit to a higher money supply, thus suppressing deflationary expectations. As a result the government achieves an equilibrium outcome that is close to the commitment solution, as illustrated in the welfare evaluation above and shown in Figures 5 and 6.

An obvious question arises if this model is applied to Japan. The gross national debt is currently over 130% of GDP. Why has the high level of outstanding debt in Japan failed to increase inflation expectations? There are at least two possible explanations of this. First, a large part of Japan's debt is held by public institution and therefore does not create any inflation incentive. A better measure of the actual inflation incentive is net government debt. Net debt government debt as a fraction of GDP is not as high in Japan, about 70%, and only slightly above the G7 average. The other explanation (see Eggertsson 2006) is that the Bank of Japan (BOJ) does not internalize the inflation incentive of outstanding government debt, i.e., it has an objective that is more narrow than social welfare (that paper proves that if the objective of BOJ is given by $\pi_t^2 + \lambda x_t^2$ deficit spending has no effect because it does not change the future incentive of the bank to inflate). Eggertsson (2006) argues that this indicates that there may be benefits of monetary and fiscal coordination, as suggested by Bernanke (2003), and verified by our welfare evaluation, and maintains that such cooperation may only need to be temporary to be effective.

32. I have assumed that monetary frictions are very small, but as I discuss in the Technical Appendix money demand is still well defined so that it remains meaningful to discuss the growth rate of money supply (even if the real monetary base relative to output is very small). The money demand equation defines the evolution for real money balances in the equilibrium, i.e., the variables $\tilde{m}_t$ which is normalized by the transaction technology parameter, and the growth rate of money supply can then be inferred from Equation (66) in the Technical Appendix. I can then calculate the money supply for each of the different equilibria.

33. It is not very instructive to consider the evolution of the nominal stock in the transition periods because the large movement in the nominal interest rate causes large swings in the nominal stock of money.

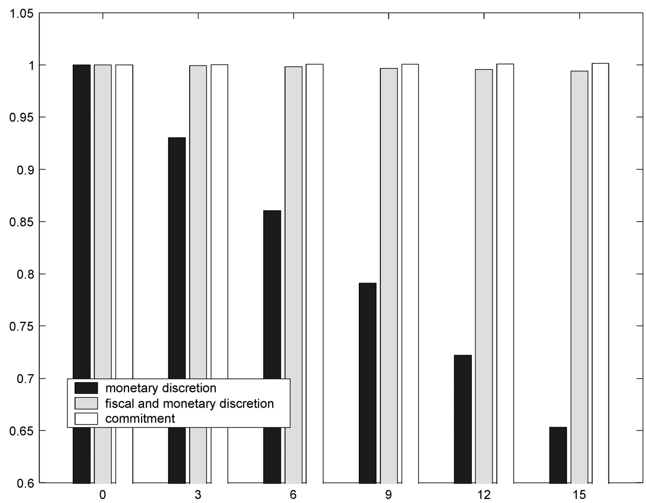

FIG. 8. Long Run Nominal Stock of Money under Different Contingencies for the Natural Rate of Interest

5.1 Extension: Dropping Money from Helicopters and Open Market Operations in Foreign Exchange as a Commitment Device

The model can be extended to analyze non-standard open market operations such as the purchase of foreign exchange and other private assets, or even more exotically, dropping money from helicopters. Here I discuss how these extensions enrich the results (an earlier version of this paper works out the details analytically—see Eggertsson, 2003).

Friedman suggests that the government can always control the price level by increasing the money supply, even in a liquidity trap. According to Friedman's famous *reductio ad absurdum* argument, if the government wants to increase the price level it can simply "drop money from helicopters." Eventually this should increase the price level—liquidity trap or not. Bernanke (2000) revisits this proposal and suggests that Japanese government should make "money-financed transfers to domestic households—the real-life equivalent of that hoary thought experiment, the "helicopter drop" of newly printed money." This analysis supports Friedman and Bernanke's suggestions. The analysis suggests, however, that it is the increase in government liabilities (money + bonds), rather than the increase in the money supply that has this effect. Since money and bonds are equivalent in a liquidity trap dropping money from helicopters is exactly equivalent to issuing nominal bonds. If the treasury and the central bank coordinate policy the effect of dropping money from helicopters will have exactly the same effect as deficit spending. Thus this paper's model can be interpreted as establishing a "fiscal theory" of dropping money from helicopters.

The model can also be extended to consider the effects of the government buying foreign exchange (or any other private assets). It is often suggested that the central bank can depreciate the exchange rate and stimulate spending by buying foreign exchange (and similar arguments are sometimes raised about some other private assets and their corresponding price). Due to the interest rate parity (and similar asset pricing equations for other private assets), however, buying foreign exchange should have no effect on the exchange rate unless it changes expectations about future policy (since the interest rate parity says that the exchange rate should depend on current and expected interest rate differentials). Will such operations have any effect on expectations about future policy? Open market operations in foreign exchange (or any other private asset) would lead to a corresponding increase in public debt defined as money plus government bonds. This gives the government an incentive to create inflation *through exactly the same channel as I have explored in this paper and, therefore, leads to a corresponding depreciation in the nominal exchange rate hand-in-hand with the rise in inflation expectations.* An advantage of buying private assets, as opposed to cutting taxes, is that it does not worsen the net fiscal position of the government. It only changes the inflation incentive of the government.[34]

6. CONCLUSION

The great inflation of the 1970s was a key motivation for the rational expectation revolution and the analysis of the celebrated inflation bias first illustrated by Kydland and Prescott (1977). The main motivation behind this paper is the large decline in inflation in recent years (towards deflation—or very close to it—in some countries such as Japan, China, Hong Kong, Singapore, Taiwan, Israel, and Swiss) together with extraordinary low interest rates throughout the world (interest rates have not been lower since the Great Depression in the countries listed above as well as in the U.S. and the Euro area to name a few). I have shown that a similar dynamic inconsistency problem as Kydland and Prescott (1977) identify as the source of inefficient inflation (i.e., the inflation bias) can also cause inefficient deflation if the zero bound is binding. I coined this new dynamic inconsistency problem as the deflation bias and contrasted it to the classic inflation bias. The source of the deflation bias, however, is inefficient response to temporary shocks, due to the government's inability to commit, whereas the inflation bias arises even in the absence of shocks. This implies that it may be even harder for a central bank to accrue reputation for fighting deflation than inflation (since the main culprit for deflation is infrequent shocks). Accordingly, the main focus of the paper has been policy measures to fight deflation that do not depend on reputation mechanism.

The paper establishes that deficit spending, i.e., cutting taxes and issuing nominal debt, is a simple way of fighting deflation. This may seem to resurrect an old Keynesian dictum. To draw that conclusion, however, is somewhat tenuous. Deficit spending in this paper works entirely through expectations. It increases output and

34. Note that in a model with private asset the value of the assets becomes an additional state variable as shown in Eggertsson (2003).

prices only because it increases expectations about future money supply. If money supply in the future (when the zero bound is not binding anymore) is set without any regard to past policy decisions, there is no effect of deficit spending, as the irrelevance result illustrated in Section 2.1. In Eggertsson (2006), I show that a similar irrelevance result applies if the central bank is "goal independent," i.e., if it does not internalize the fiscal benefits of monetary expansion.

Another result of this paper is that open market operations in private assets can also be analyzed in a similar framework. Two interesting examples of private assets that can be bought by open market operations are stocks or foreign exchange. Open market operations in these assets are useful to fight deflation because they change the inflation incentives of the government in the future and thus change expectation from being deflationary to being inflationary.

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