Time Consistency and Duration of Government Debt: A Model of Quantitative Easing*

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Abstract

This paper presents a model of quantitative easing (QE) at the zero lower bound (ZLB) on the short-term nominal interest rate. QE, which reduces the maturity of government debt, is effective at the ZLB because it generates expectations of future monetary expansion in a time-consistent equilibrium. Numerical experiments show that this effect can be substantial.

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The problem with Quantitative Easing is it works in practice, but it doesn't work in theory.
Ben Bernanke, chairman of the Federal Reserve

1 Introduction

Starting at the onset of the economic crisis in 2008, the Federal Reserve expanded its balance sheet by large amounts, on the order of $3 trillion, mostly under the heading of quantitative easing (QE). At its peak in 2012, the cumulative amount of QE was equivalent to about 20% of annual GDP. The enormous scale of this policy has largely been explained by the fact that the Federal Reserve was unable to cut the federal funds rate further than it already had because of the zero lower bound (ZLB) on the short-term nominal interest rate. Meanwhile, high unemployment, slow growth, and low inflation called for further stimulus measures.

Many commentators argued that QE in the US prevented a much stronger contraction during the Great Recession and that QE was a key reason why the US recovered more rapidly than other countries. As pointed out by Federal Reserve chairman Ben Bernanke, however, while this might be true in practice, a coherent theoretical rationale has been hard to formulate. This paper formulates such a rationale for QE: it works because it allows the central bank to credibly commit, in a time-consistent equilibrium, to expansionary future policy at the ZLB. The paper accounts for QE in theory and shows numerical examples in which the effect is considerable.

What is QE? Our paper defines it as a policy in which a central bank buys long-term government debt with money.[1] Since the nominal interest rate was zero when QE was implemented in the US, it makes no difference whether QE is implemented by printing money (or more precisely creating bank reserves) or by issuing short-term government debt: both types of assets are government-issued papers that yield a zero interest rate. From the perspective of the government as a whole, QE at the ZLB can then be thought of as reducing the maturity of outstanding government debt held by the public, as the government is replacing long-term debt with short-term debt. The main finding of this paper is that shortening the maturity of government debt is an effective way to commit to a future monetary expansion, an especially valuable tool when the central bank's policy rate is constrained by the ZLB.

The main goal of QE in the US was to reduce long-term interest rates at the ZLB and thereby stimulate the economy. Indeed, several empirical studies find evidence of a reduction in long-term interest rates following these policy interventions by the Federal Reserve (see, for

[1]As we emphasize later, our paper is thus best interpreted as providing a model for what has been called QE2 and QE3, as those policies were largely focused on purchases of long-term government bonds.

example, Gagnon et al. (2011), Krishnamurthy and Vissing-Jorgensen (2011), Hamilton and Wu (2012), Swanson and Williams (2013), and Bauer and Rudebusch (2013)).[2]

From a theoretical perspective, however, the effect of such a policy is not obvious since open market operations of this kind are neutral (or irrelevant) in standard macroeconomic models if we hold constant the future interest rate reaction function of the central bank.[3] An "irrelevance result" of this form was first stated by Wallace (1981). The Wallace irrelevance result was extended by Eggertsson and Woodford (2003) to a model with sticky prices and an explicit ZLB. Those results illustrate that, absent certain restrictions on asset trading that prevent arbitrage, a change in the relative supplies of various assets in the hands of the private sector has no effect on equilibrium quantities.

As pointed out by Eggertsson and Woodford (2003), and further illustrated in Woodford (2012) in the context of the financial crisis, QE may be effective not only because it reduces risk premia due to limits on arbitrage and/or differences in liquidity between private and public bonds. QE can also reduce long-term interest rates if it *communicates* to the private sector that the central bank will keep the short-term interest rates low once the ZLB is no longer a constraint; that is, it signals a change in the policy rule taken as given in Eggertsson and Woodford (2003). Krishnamurthy and Vissing-Jorgensen (2011) and Bauer and Rudebusch (2013) find strong evidence in support of such a "signaling channel" of various QE programs; that is, the programs generated expectations of low future federal funds rates.

The first stage of QE, which has been dubbed QE1, was implemented in response to the collapse of the shadow banking system after a large disruption in credit markets in the fall of 2008. The Federal Reserve extended large amounts of credit in exchange for private assets—for example, via the commercial paper facility. Models of private credit disruption, such as those by Gertler and Karadi (2011, 2012) and Del Negro et al. (2017), have found that QE considerably mitigated the effect of the financial crisis, largely by decreasing the credit spreads that arose in that period of acute financial distress. This class of models finds a much smaller effect of QE2 and QE3, but those interventions were undertaken in a period of more normal credit markets and largely by exchanging short-dated government bonds (or reserves) for long-dated government bonds. The focus of this paper is on this latter form of intervention, and QE2 is the benchmark for a numerical calibration. In contrast to the existing literature, we find that the effect may have been considerable because of the effect it had on expectations about future interest rates, rather than via any direct effect on credit premia.

The main contribution of this paper is to provide a formal theoretical model showing how

[2]For example, Gagnon et al. (2011) estimate that the 2009 program, worth $1.75 trillion, reduced long-term interest rates by 58 basis points, while Krishnamurthy and Vissing-Jorgensen (2011) estimate that the 2010 program, worth $600 billion, reduced long-term interest rates by 33 basis points.

[3]This may have motivated Ben Bernanke's comment cited in the epigraph.

QE generates a credible commitment to future monetary expansion in a standard general equilibrium model. We formulate a Markov perfect equilibrium (MPE) as a game played by the government and the private sector. In the MPE, agents use the natural state variables of the game to predict the behavior of future governments. QE changes the dynamics of the endogenous state variables of the game, thereby affecting expectations about the future path of the policy instrument in a time-consistent equilibrium.

This paper proposes the *rollover incentive* as the main force that makes the government more keen to set lower rates the more short-term debt it issues. This is the principal effect of QE. The rollover incentive is best explained via a simple example that glosses over several subtleties discussed in detail in the body of the paper. Consider yourself as evaluating two mortgage contracts: a thirty-year loan with fixed interest rates and a loan contract with floating interest rates that are determined monthly. Now consider your incentives if you get the opportunity to set the federal funds rate. If you have a thirty-year loan, then your own interest rate is unaffected by an increase in the federal funds rate, leaving your interest costs unchanged. But if you have a flexible-rate loan, you have much to lose by increasing interest rates, for it would directly increase your interest payments. You are now rolling over your debt from one period to the next with higher interest rates.[4] Accordingly, the more short-term debt you hold, the less willing you are to raise the federal funds rate.

A better-known effect is termed the *balance sheet incentive*: interest rate policy can change inflation and thus can change the real value of outstanding nominal debt. The balance sheet incentive, however, as we show below, is independent of the maturity structure of debt (in sharp contrast to the rollover incentive).

One objection to the main thesis of the paper is that many argue that fiscal considerations do not play a fundamental role in the decision making of the Federal Reserve. While the main analysis assumes a consolidated government budget constraint, we show that under certain conditions, the model can equivalently be interpreted as referring to an independent central bank that cares about it's own balance sheet losses. We furthermore provide anecdotal evidence in support of this interpretation: Based upon recently declassified documents from the deliberations of the Federal Reserve Open Market Committee, we show that the Federal Reserve closely tracked possible balance sheet losses associated with its QE policy.

Our paper is organized as follows. Section 2 considers a three-period model that can be solved without resorting to any approximations. First, we show the deflation bias of discretionary monetary policy at the ZLB: in the presence of a deflationary shock, a central

[4] As we discuss later, the rollover incentive operates under fully flexible prices, fully fixed prices, or anywhere in between. In this example, a key subtlety that is glossed over is how to properly define the state variables of the game played by successive governments—namely, the maturity value of debt and its composition—and which asset-pricing conditions are constraints faced by the government in a given period in an MPE.

bank would like to commit to lower future policy rates but, because of a fundamental credibility problem, cannot do so. The paper then shows that the government can issue or buy a mix of long- and short-term debt to make the optimal monetary policy incentive compatible. This is the role QE plays in the paper. Section 3 considers an infinite-horizon model in a quantitative analysis and shows a substantial role for QE. Section 4 considers extensions, presents empirical evidence, and discusses the relationship of the results to the literature. Section 5 concludes.

2 A Simple Model

This section illustrates the role of the maturity of nominal government debt in affecting expectations about future policy in a time-consistent equilibrium, using minimal modeling ingredients.

2.1 Environment

2.1.1 Private Sector

There are three periods: 0, 1, and 2 (labeled short, medium, and long run respectively). A representative household solves

$$\max_{C_t,H_t,L_t,B_t} \mathbb{E}_0 \sum_{t=0}^{2} \beta^t \left[\log C_t - \chi H_t\right] \xi_t \tag{1}$$

subject to the flow budget constraints

$$\begin{aligned} P_0C_0 + B_0 + L_0 &= B_{-1} + \int_0^1 Z_0(i)di + N_0H_0 - P_0T_0, \\ P_1C_1 + B_1 &= (1+i_0)B_0 + \int_0^1 Z_1(i)di + N_1H_1 - P_1T_1, \text{ and} \\ P_2C_2 &= (1+i_1)B_1 + (1+R_0)L_0 + \int_0^1 Z_2(i)di + N_2H_2 - P_2T_2, \end{aligned}$$

where C_t is consumption, H_t hours, and ξ_t a preference shock. B_{-1} is the initial wealth the household holds in period 0; B_0 and B_1 are one-period nominal risk-free bonds; and L_0 represents long-term nominal bonds issued in period 0 and repaid in period 2. The short-term interest rates in periods 0 and 1 are i_0 and i_1, while R_0 is the long-term interest rate in period 0. N_t is the wage rate, P_t the price index, $Z_t(i)$ firms' profits (where i indexes firms), and T_t taxes. The model abstracts from money but imposes the ZLB directly[5].

[5]Eggertsson and Woodford (2003) motivate this abstraction in more detail.

Consumption C_t is a Dixit-Stiglitz aggregator, $C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ is the elasticity of substitution across goods with i indexing varieties of goods. For each variety, there is a single firm, so that $y_t(i) = c_t(i)$. The firm has a linear production function, $y_t(i) = h_t(i)$, where $h_t(i)$ is labor. The firm maximizes profits

$$\max_{p_t(i), y_t(i)} \{(1+\tau) p_t(i) y_t(i) - N_t y_t(i)\}$$

subject to $y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} Y_t$, where $Y_t = C_t$ is aggregate output and τ denotes a production subsidy. In periods 0 and 1, a fraction γ of firms set prices flexibly while the remaining fraction index their prices to the past aggregate price index. In the terminal period, all prices are flexible. Further details are provided in Appendix A.1.

We abstract from government spending but assume that the government needs to repay the debt inherited in period 0, B_{-1}. As in Barro (1979), taxes are not one-to-one transfers of purchasing power from individuals to the government. Instead, they entail some collection costs or indirect misallocation costs. More concretely, the "production" of government revenues, T_t, requires labor input $f(T_t)$, where $f(.)$ is increasing and convex in T_t.[6] This, together with the fact that production is linear in labor, implies that total hours can be expressed as

$$H_t = Y_t \Delta_t + f(T_t),$$

where $\Delta_t \equiv \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{-\theta} di \geq 1$ represents price dispersion. A major convenience of this set of assumptions about taxes (and the fact that hours enter linearly in utility) is that taxes do not appear directly in the equilibrium conditions that stem from the private sector maximization problem (see (3)-(6) below). Since taxes imply wasted resources, however, they contribute negatively to social welfare. A benevolent government will therefore try to minimize taxation.[7]

2.1.2 The Government's Problem: A Maturity Value of Debt Characterization

The model can be summarized in three steps (details are provided in Appendix A). We obtain the government's objective via a second-order approximation of household utility (1):

$$-\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{2} \beta^t \left[(\Pi_t - 1)^2 + \lambda_y (Y_t - \bar{Y})^2 + \lambda_T (T_t)^2 + t.i.p.\right], \tag{2}$$

[6] Here, T_t denotes tax revenues net of government workers' salaries.

[7] In the quantitative model, utility is not assumed to be linear in labor, but the implications for taxes are the same. The reason is that taxes only affect allocations up to a second order in the quantitative model.

where we have normalized the weight on inflation to 1 and the weights λ_y and λ_T are functions of the structural parameters, whose derivation is relegated to Appendix A.2.[8] The term including taxes appears because of the taxation costs discussed in Section 2.1.1. The cost of inflation is due to inefficient price dispersion.[9]

The optimality conditions for the household's and firm's maximization problems are

$$\frac{1}{1+i_t} = \beta \mathbb{E}_t \left[\frac{Y_t}{Y_{t+1}} \Pi_{t+1}^{-1} \frac{\xi_{t+1}}{\xi_t} \right] \text{ for } t = 0, 1, \tag{3}$$

$$\frac{1}{1+R_0} = \beta^2 \mathbb{E}_0 \left[\frac{Y_0}{\bar{Y}} \Pi_1^{-1} \Pi_2^{-1} \frac{\xi_2}{\xi_0} \right], \tag{4}$$

$$i_t \geq 0 \text{ for } t = 0, 1, \tag{5}$$

$$(\Pi_t - 1) = \kappa (Y_t - \bar{Y}) \text{ for } t = 0, 1, \text{ and } Y_2 = \bar{Y}. \tag{6}$$

The household budget constraints, together with market clearing, imply the following flow government budget constraints:

$$x_0 \frac{w_0}{1+R_0} + (1-x_0) \frac{w_0}{1+i_0} = \frac{w_{-1}}{\Pi_0} - T_0, \tag{7}$$

$$W_1 = \frac{w_0}{\Pi_1} + i_1 (1-x_0) \frac{w_0}{\Pi_1} - (1+i_1) T_1, \text{ and} \tag{8}$$

$$0 = \frac{W_1}{\Pi_2} - T_2. \tag{9}$$

Here, $w_0 \equiv b_0(1+i_0) + l_0(1+R_0)$, $x_0 \equiv \frac{l_0(1+R_0)}{b_0(1+i_0)+l_0(1+R_0)}$, and $W_1 \equiv w_1 + x_0 \frac{w_0}{\Pi_1}$, where $w_1 \equiv (1+i_1)b_1$. The lower-case letters refer to the real value of the corresponding nominal debt; that is, $b_0 \equiv \frac{B_0}{P_0}$, $b_1 \equiv \frac{B_1}{P_1}$, $l_0 \equiv \frac{L_0}{P_0}$, and $b_{-1} \equiv w_{-1} \equiv \frac{B_{-1}}{P_{-1}}$.

The government's problem is to maximize (2) by choosing $\{\Pi_t, T_t\}$ for $t = 0, 1, 2$, $\{i_t, Y_t\}$ for $t = 0, 1$, and $\{w_0, x_0, R_0, W_1\}$, subject to (3)-(9), taking as given the initial value of debt w_{-1}. We next discuss the government's problem and interpret the policy variables.

[8] We use the approximated welfare function, rather than the exact nonlinear utility, as it makes the results more transparent and connects with existing literature that uses ad hoc welfare functions that take the same form. In Appendix A.7.5, we show that the results from using this welfare function, relative to the exact nonlinear household utility function, are almost exactly the same in our benchmark numerical experiment.

[9] Because all firms are the same in the model, with the exception that some firms index their prices, any inflation implies firms are charging prices different from one another—that is, generating price dispersion—which is inefficient, as it would be socially optimal for them all to charge the same price. Because there is no price dispersion in period 2, the penalty on inflation is in reduced form for that period.

2.1.3 The Government's Problem: A Discussion

The most important feature of the characterization of the government's problem in the last subsection is that its budget constraint in period 1, (8), is written in terms of the *maturity value of debt issued in period 0*, w_0, and a variable that denotes the fraction of that debt that is long term, x_0, while its budget constraint in period 2, (9), is written in terms of the *maturity value of total debt* issued in periods 0 and 1 that is due in period 2, W_1. We call this characterization the *maturity-value notation.* The reason we use this notation is that our paper focuses on an MPE (see Maskin and Tirole (2001) for a formal definition of this concept). The key restriction of an MPE is that the strategies of each player depend on the *minimum set* of state variables.[10]

What are the state variables of the policy game in perid 2? This depends on how the constraints are written.[11] To see this, consider the government budget constraint in the terminal period. It might seem natural to write the constraint in terms of the real value of the short-term debt issued in period 0, $b_0 = \frac{B_0}{P_0}$, and the long-term debt issued in period 0, $l_0 = \frac{L_0}{P_0}$. Let us call this the *debt-issuance notation.* Written in this way, the period-2 budget constraint is

$$0 = (1 + i_1)\frac{b_1}{\Pi_2} + (1 + R_0)\frac{l_0}{\Pi_1 \Pi_2} - T_2. \tag{10}$$

Comparing (10) with (9), we see that using the debt-issuance notation implies there are five state variables in period 2—that is, $i_1, b_1, R_0, l_0,$ and Π_1—while according to the maturity-value notation, there is only a single state variable, W_1.

Deriving (7)-(9) and using those constraints to define a policy game can be summarized in four steps. First, all government debt issued is written in terms of its *maturity value*—that is, the total number of (real) future dollars that the debt issued in a given period promises to pay, inclusive of future interest rates. This yields w_0 and w_1. Second, define the fraction of the debt in period 0 issued in terms of long-term debt, yielding x_0. Third, identify the state variables of the policy game in each period. Fourth, write the budget constraints today in terms of both the state variable(s) for the next period and the state variables today.

What are the state variables in the three periods? The other constraints, (3)-(6), are perfectly forward looking and hence have no endogenous state variables, and thus we can focus on the budget constraints. As we dicussed above, according to the debt-issuance notation, in period 2, the unique payoff-relevant state variable is the real value of all debt payments

[10] Equivalently, these are what Maskin and Tirole (2001) call the "coarsest history of player actions."

[11] Consider, for example, the case in which (6) is written in terms of the price level instead of inflation; that is $\frac{P_t - P_{t-1}}{P_{t-1}} = \kappa(Y_t - \bar{Y})$. In that case, P_{t-1} is a state variable of a policy game at time t. Since this equation can obviously be written in terms of inflation Π_t, which is determined at time t, the MPE refinement suggests that the price level is not an appropriate state variable of the game.

due at that time summarized by W_1. This variable is the sum of the maturity value of debt issued in period 1 (that is, w_1) and the portion of the maturity value of debt issued at time 0 that was issued in the form of long-term bonds (that is, $x_0 w_0$). Moving to period 1, the state variables are the maturity value of debt issued in period 0 (that is, w_0) and the portion of this debt that is short term (that is, $(1 - x_0)w_0$). As we will see, these two terms play a central role in the analysis and correspond to the balance sheet incentive and the rollover incentive respectively. Finally, in period 0, the state variable is w_{-1}.[12]

The remainder of the model is relatively fairly standard. (3) is a standard asset-pricing condition for one-period risk-free nominal debt, (4) is the asset-pricing condition for two-period risk-free nominal debt, while (5) represents the ZLB, which we impose directly. (6) is the Phillips curve, which applies in periods 0 and 1 because of price indexation.[13] For simplicity, there is no indexation in the terminal period, and thus there is long-run money neutrality with output pinned down by $\bar{Y}$.

2.1.4 Thought Experiment and Parameterization

The following thought experiment is at the heart of our paper. Imagine that in the short run, there is an unexpected shock, $\xi_0 < \bar{\xi}$, which generates a recession. In the medium and long runs, there is no shock, so that $\xi_1 = \xi_2 = \bar{\xi}$. In the baseline parameterization, each period is a year and steady-state output is normalized to 1. The shock ξ_0 is chosen so that the ZLB is binding and output drops by 7.5% in the absence of any debt-maturity changes at the ZLB, while κ is chosen so that the drop in inflation is 2.5%, implying that $\kappa = 0.33$. The initial public debt is $b_{-1} = 1$—that is, 100% of annual output. The discount factor is $\beta = 0.95$. The two welfare weights are $\lambda_y = \lambda_T = 0.01$. The values of these weights are motivated by the parameterization in the quantitative model, where they are discussed in detail using the underlying microfoundations of the quantitative model, along with sensitivity analysis.

2.2 The Deflation Bias

This section reviews optimal monetary policy at the ZLB; it abstracts from fiscal policy, which is equivalent to $\lambda_T = 0$ in (2). Then taxes are lump sum and do not affect welfare. It follows that *both* the level of debt and its composition are irrelevant. While this policy problem is well known, it is a helpful starting point since QE in our model works as a commitment device for

[12] Relative to the debt-issuance notation, the maturity-value notation reduces the number of state variables from five to one in the terminal period, while the number of state variables remains the same in periods 0 and 1. In the language of Maskin and Tirole (2001), the maturity-value notation thus represents a "coarser partition of the history of the players' actions" than the debt-issued notation and thus is the correct specification of the state in an MPE.

[13] The flexible price limit is $\kappa^{-1} = 0$, where no firms index their prices to the past level of aggregate prices.

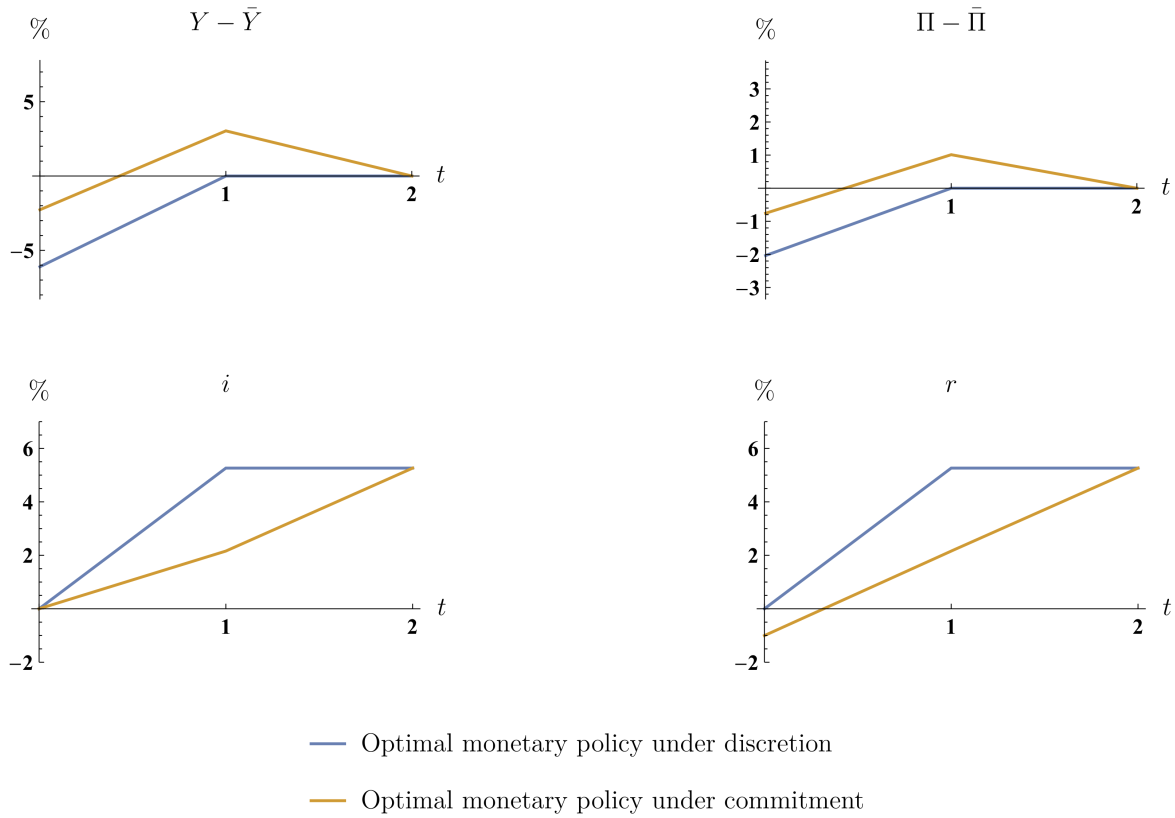

Figure 1: Impulse responses under optimal monetary policy at the zero lower bound

Note: The figure shows the responses of output, inflation, the nominal interest rate, and the real interest rate when a negative demand shock makes the zero lower bound binding in the short run.

a discretionary government in an MPE. Hence, it sets the stage for the commitment problem QE solves. The credibility problem faced by the government at the ZLB has been termed the *deflation bias* of discretionary policy (Eggertsson (2006)).

To understand the deflation bias, consider first the solution in the absence of the ZLB, but taking the shock at time 0 into account. According to objective (2), the first-best outcome is $Y_t = \bar{Y}$ and $\Pi_t = 1$, and it can always be achieved if the ZLB is not binding, as all that is required is that the nominal interest rate satisfy $i_0 = \beta^{-1}\xi_0 - 1$ while remaining at steady-state in periods 1 and 2. If the shock in period 0 is sufficiently large, the ZLB is binding, making the first-best solution infeasible. The yellow line in Figure 1 shows the equilibrium solution under optimal monetary policy at the ZLB when the government can commit in period 0 to a future policy.[14] The government commits to keeping the nominal interest rate low in period 1, resulting in output being above $\bar{Y}$ and inflation overshooting in period 1.

The commitment solution is not time consistent. This is because in periods 1 and 2, the first-best equilibrium is feasible; that is, $Y_1 = Y_2 = \bar{Y}$ and $\Pi_1 = \Pi_2 = 1$. If the government

[14] All commitment problems are solved using a Lagrangian method described in Appendix A.3.

is not constrained by its promise in period 0, it will optimally choose this solution, as it corresponds to the best it can do from that time onward. Taking this solution as given, the time-consistent solution is not the commitment solution that promises an output boom and inflation, but instead the one that can be backed out from (3) and (6) by setting $i_0 = 0$, $Y_1 = \bar{Y}$, and $\Pi_1 = 1$, yielding $Y_0 = \beta^{-1}\xi_0 < \bar{Y}$ and $\Pi_0 = \kappa(\beta^{-1}\xi_0 - \bar{Y}) + 1 < \bar{\Pi}$. This solution is shown in Figure 1 with a blue line. Clearly, the commitment solution is preferable when considering the payoffs for all periods since it ameliorates a severe recession and deflation. The problem is how to make the optimal commitment credible. This is where QE comes in.

2.3 Markov Perfect Equilibrium and Government Debt Maturity

The reason the maturity structure of government debt matters under discretion is that it changes the incentives faced by future governments through the state variables of the game.[15] The major challenge in solving for optimal policy under discretion is modeling expectations. Expectations depend on how the private sector believes future governments will set policy. The period-0 government, for example, can change these expectations because its choice of the duration of its debt, x_0, and its debt's total maturity value, w_0, become state variables in the game played by the government in period 1. The key advantage of assuming a finite horizon, as in this section, is that the policy game can be solved backward to yield solutions for expectations as a function of state variables. We start by solving the government problem in the long run (period 2) and then move backward in time.

2.3.1 The Long Run: The Inflation-Tax Trade-off

Period 2's government solves

$$V^2(W_1) = \max_{T_2,\Pi_2}\{-\frac{1}{2}(\Pi_2 - 1)^2 - \frac{1}{2}\lambda_T(T_2)^2\} \tag{11}$$

subject to (9), where $V^2(W_1)$ is the value function of the government and recall that in the long run we assume that $Y_t = \bar{Y}$.

Combine the optimality conditions (see Appendix A.4) to obtain

$$\underbrace{(\Pi_2 - 1)}_{MC \text{ of } \Pi_2} = \underbrace{\lambda_T T_2\{\frac{W_1}{(\Pi_2)^2}\}}_{MB \text{ of } \Pi_2}. \tag{12}$$

[15] As we discuss in Section 4.1, under monetary and fiscal commitment, while the level of debt matters, the maturity structure of debt is irrelevant.

The left-hand side of (12) is the marginal cost of inflation, while the right-hand side is the marginal benefit. Inflation is beneficial because it reduces the need for taxes. The marginal benefit of inflation has two components. The first term on the right-hand side, $\lambda_T T_2$, is the shadow value of W_1 in period 2; that is, it reflects the marginal benefit of lower debt payments, expressed in period-2 taxes.[16] The second term, the expression in curly brackets, measures the marginal reduction of debt payments resulting from higher inflation. Since debt is nominal, higher inflation reduces the real value of debt payments (and thus the need for taxes).

(12) combined with (9) yields

$$(\Pi_2)^4 - (\Pi_1)^3 - \lambda_T (W_1)^2 = 0. \tag{13}$$

(13) implicitly defines the policy function for inflation,

$$\Pi_2 = \bar{\Pi}^2(W_1), \tag{14}$$

and, via (9), the policy function for taxes,

$$T_2 = \bar{T}^2(W_1). \tag{15}$$

Using the implicit function theorem, it can be shown that $\frac{\partial \bar{\Pi}^2}{\partial W_1} > 0$, $\frac{\partial \bar{T}^2}{\partial W_1} > 0$, and $\frac{\partial V^2}{\partial W_1} < 0$ for $W_1 > 0$. Two important inputs into the the solution in period 1 are $\frac{\partial V^2}{\partial W_1}$ and $\frac{\partial \bar{\Pi}^2}{\partial W_1}$, denoted by V^2_W and $\bar{\Pi}^2_W$ respectively. The envelope theorem implies that

$$V^2_W = -\lambda_T \frac{W_1}{(\Pi_2)^2}, \tag{16}$$

while the implicit function theorem implies that

$$\bar{\Pi}^2_W = 2\lambda_T \frac{W_1}{4(\Pi_2)^3 - 3(\Pi_2)^2}. \tag{17}$$

It is straightforward to compute the nonlinear policy functions (14) and (15) numerically, which are shown in Figure 2 with a solid line for the baseline parameterization.

Moreover, analytical solutions can be derived by approximating the policy functions local to a point at which there are no tax distortions while taking W_1 as given.[17] This approximation leads to the following proposition:

[16] The shadow value is equal to the Lagrange multiplier on (9). See Appendix A.4.

[17] As the expansion is made around the point $\lambda_T = 0$, the approximation is accurate to an order $O(||\lambda_T||^2)$, where W_1 is treated as a parameter.

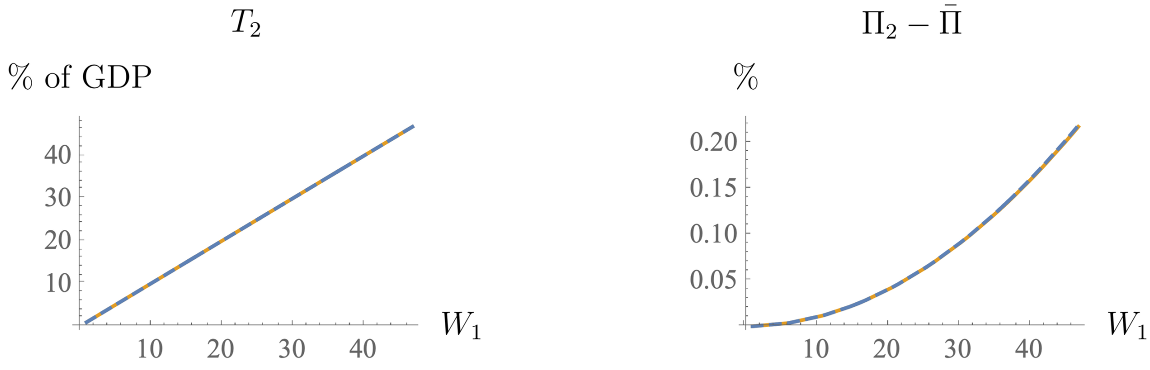

Figure 2: Long-run policy functions

Note: The figure shows policy functions for taxes and inflation in the long run for the model with optimal monetary and fiscal policy under discretion. The only state variable for the long run is the total maturity value of debt (W_1) inherited from the short and medium runs. The dashed lines show the approximate, analytical policy functions discussed in the text.

Proposition 1 *Local to no tax distortions ($\lambda_T = 0$), given W_1, the MPE in period 2 is*

$$\Pi_2 = 1 + (W_1)^2\lambda_T \text{ and} \tag{18}$$

$$T_2 = W_1 - (W_1)^3\lambda_T. \tag{19}$$

Proof. First, at $\lambda_T = 0$, $\Pi_1 = 1$; and by (9), $T_2 = W_1$. (18) then follows directly from a first-order Taylor expansion of (13), and (19) follows from a first-order Taylor expansion of (15) (details are provided in Appendix A.6). ■

Proposition (1) suggests that inflation increases quadratically with W_1 while taxes increase linearly. Hence, in trading off higher taxes vs. higher inflation, as the long-run debt burden increases, the government optimally chooses to rely more and more on inflation. Figure 2 shows that the approximated policy functions shown in Proposition 1, shown by a dashed line, match the nonlinear policy functions almost exactly.

2.3.2 The Medium Run: The Rollover and Balance Sheet Incentives

Period 1's government solves

$$V^1(x_0, w_0) = \max_{\Pi_1, T_1, Y_1, i_1, W_1} \{-\frac{1}{2}(\Pi_1 - 1)^2 - \frac{1}{2}\lambda_y(Y_1 - \bar{Y})^2 - \frac{1}{2}\lambda_T(T_1)^2 + \beta V^2(W_1)\} \tag{20}$$

subject to (5), (6), (8), and

$$1 + i_1 = \beta^{-1}\left(\frac{\bar{Y}}{Y_1}\right)\bar{\Pi}^2(W_1), \tag{21}$$

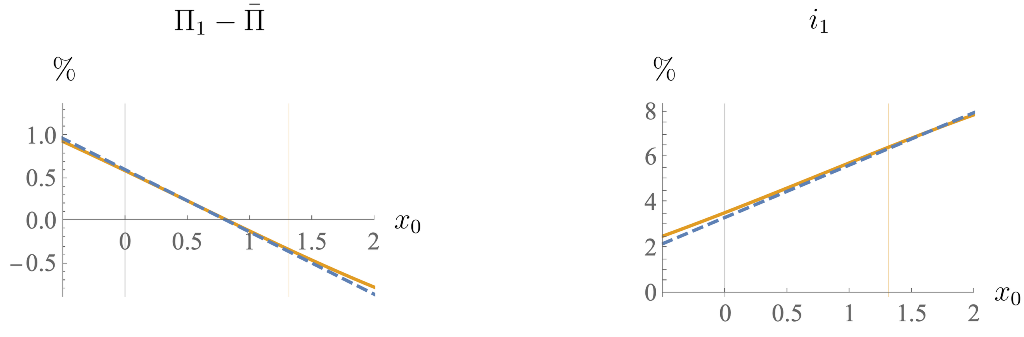

Figure 3: Medium-run policy functions

Note: The figure shows policy functions for inflation and the nominal interest rate in the medium run with optimal monetary and fiscal policy under discretion. The vertical yellow line shows the optimal value of the maturity of debt (x_0= 1.31). The maturity value of debt (w_0) is fixed at the optimal value (72%). The dashed lines show the approximate, analytical policy functions discussed in the text.

where $V^1(x_0, w_0)$ is the value function of the government, $\bar{\Pi}^2(W_1)$ is given by (14), and $V^2(W_1)$ is given by (11). The government treats $\bar{\Pi}^2(W_1)$ and $V^2(W_1)$ as given but understands how they change with the state variable W_1.

The optimality conditions are shown in Appendix A.5. They contain the derivatives V^2_W and $\bar{\Pi}^2_W$, which are given by (16) and (17). The fact that we have an explicit solution for these derivatives is the main advantage of the assumption of a finite horizon. The government's optimality conditions and the constraints (5), (6), (8), and (21) define the equilibrium. These equilibrium conditions then implicitly define the policy function for each of the endogenous variables in period 1 as a function of the *two* state variables x_0 and w_0. That is,

$$Y_1 = \bar{Y}^1(x_0, w_0), \Pi_1 = \bar{\Pi}^1(x_0, w_0), i_1 = \bar{\imath}^1(x_0, w_0), W_1 = \bar{W}^1(x_0, w_0), \text{ and } T_1 = \bar{T}^1(x_0, w_0). \tag{22}$$

The two most economically interesting policy functions are shown in Figure 3 (Appendix A.7.2 contains the full set of policy functions). A lower x_0—that is, a shorter maturity of debt in period 0—triggers a *lower* nominal interest rate and *higher* inflation in period 1. This suggests that the government in period 0 can *credibly commit* the government in period 1 to a lower interest rate and higher inflation by shortening the duration of debt in period 0. This is the key mechanism that gives QE its power. It thus deserves a careful description.

Below we derive a closed-form approximation of the policy functions for inflation and the nominal interest rate shown in Figure 3. The steps to get there clarify the key mechanisms of the model, and we discuss them first. Consider the budget constraint in period 1:

$$W_1 = \underbrace{\frac{w_0}{\Pi_1}}_{\text{Balance sheet incentive}} + \underbrace{i_1(1-x_0)\frac{w_0}{\Pi_1}}_{\text{Rollover incentive}} - \underbrace{(1+i_1)T_1}_{\text{Tax smoothing incentive}}. \tag{23}$$

Recall that W_1 is the state variable of the game in the next period and thus determines the continuation value $\beta V_2(W_1)$ in (20). This variable is determined by three terms, all of which play a key role in determining equilibrium inflation: the *balance sheet incentive*, the *rollover incentive*, and the *tax-smoothing incentive*. We discuss each in detail below.

Combine the optimality conditions of the government's problem in period 1 (see Appendix A.5) to yield a condition analogous to (12) in period 2:

$$\underbrace{(1+\frac{\lambda_y}{\kappa^2})(\Pi_1-1)}_{MC \text{ of } \Pi_1} = \underbrace{\frac{\lambda_T T_1}{1+i_1}\{\underbrace{\frac{w_0}{(\Pi_1)^2} + \kappa^{-1}(1+i_1)(1-x_0)\frac{\Pi_1}{Y_1}\frac{w_0}{(\Pi_1)^2} + i_1(1-x_0)\frac{w_0}{(\Pi_1)^2} - \kappa^{-1}(1+i_1)T_1}_{\frac{\partial W_1}{\partial \Pi_1}}\}.}_{MB \text{ of } \Pi_1} \tag{24}$$

The left-hand side of (24) is the marginal cost of inflation, and the right-hand side is the marginal benefit. Again, inflation is beneficial because it reduces the need for taxes. The first term on the right-hand side, $\frac{\lambda_T T_1}{1+i_1}$, reflects the marginal benefit of reducing future debt payments (that is, W_1), expressed in terms of period-1 taxes. The second term, the expression in curly brackets, is equal to $\frac{\partial W_1}{\partial \Pi_1}$ and reflects how much period-1 inflation reduces the debt that is rolled over to the terminal period.

While the consolidated optimality condition (24) can be used to approximate the policy function for inflation (as we will see below), the key to approximating the policy function for the interest rate is to combine (6) with (21) to yield

$$1 + i_1 = \beta^{-1}\left(\frac{\bar{Y}}{Y_1}\right)\bar{\Pi}^2(W_1) = \beta^{-1}\underbrace{\left(\frac{\bar{Y}}{\kappa^{-1}(\Pi_1-1)+\bar{Y}}\right)}_{\text{Sticky-price effect}}\underbrace{\bar{\Pi}^2(W_1)}_{\text{Fisher effect}}. \tag{25}$$

According to the first term in (25), the nominal interest rate in period 1 declines as inflation rises. The reason, which we term the sticky-price effect, is that higher inflation increases output in period 1 relative to period 2, putting downward pressure on the interest rate (given expectations about future inflation). The second term is the Fisher effect. It captures the fact that higher inflation expectations increase the nominal interest rate.

An approximation of (24) local to the point $\lambda_T = 0$, $\Pi_1 = Y_1 = 1, 1 + i_1 = \beta^{-1}$, and

$T_1 = \frac{w_0}{2}$, taking the state variables x_0 and w_0 as given (see Appendix A.6), yields

$$MC_1 = (1 + \frac{\lambda_y}{\kappa^2})(\Pi_1 - 1) \text{ and} \tag{26}$$

$$MB_1 = \frac{\beta\lambda_T w_0}{2}\{\underbrace{w_0}_{\text{BSI}} + \underbrace{\kappa^{-1}\beta^{-1}(1-x_0)w_0}_{\text{ROI 1}} + \underbrace{\beta^{-1}(1-\beta)(1-x_0)w_0}_{\text{ROI 2}} - \underbrace{\kappa^{-1}\beta^{-1}\frac{w_0}{2}}_{\text{TSI}}\}, \tag{27}$$

where MC_1 is the marginal cost of Π_1 (the left-hand side of (24)) and MB_1 is the marginal benefit of Π_1 (the right-hand side of (24)). Similarly, an approximation of (25) then results in the following proposition:

Proposition 2 *Local to no tax distortion ($\lambda_T = 0$),*

(i) the policy functions for inflation and the interest rate in the MPE are given by

$$\Pi_1 = 1 + \left(\frac{1}{1+\frac{\lambda_y}{\kappa^2}}\right)\{1 - \frac{\kappa^{-1}\beta^{-1}}{2} + \beta^{-1}(1-\beta+\kappa^{-1})(1-x_0)\}\frac{\beta\lambda_T w_0^2}{2} \text{ and} \tag{28}$$

$$i_1 = \beta^{-1}(1-\beta) - \beta^{-1}\kappa^{-1}(\Pi_1 - 1), \tag{29}$$

(ii) $\frac{\partial \Pi_1}{\partial x_0} < 0$, $\frac{\partial i_1}{\partial x_0} > 0$,

(iii) $\frac{\partial \Pi_1}{\partial w_0} > 0$, $\frac{\partial i_1}{\partial w_0} < 0$ *iff* $x_0 < \frac{1+\frac{1}{2}\kappa^{-1}}{1-\beta+\kappa^{-1}}$.

Proof. The first part of the proposition follows from equating (26) and (27) and from approximating (25) (further details are provided in Appendix A.6). The second and third parts of the proposition follow directly by differentiating (28) and (29). ■

Proposition 2 shows in closed form what we showed numerically in Figure 3: a lower x_0 (that is, a shorter maturity of debt in period 0) triggers higher inflation and a lower nominal interest rate in period 1. Figure 3 plots the approximated policy functions (28) and (29) with a dashed line in comparison with the exact policy functions shown by a solid line. Evidently, the approximation is relatively accurate, suggesting that the approximated marginal cost and benefit of inflation capture the key mechanism driving the shape of the policy functions.[18]

Figure 4 plots the approximated marginal cost and benefit of inflation. The marginal cost of inflation increases linearly with inflation. The marginal benefit, however, is independent of inflation, as shown by the horizontal line. The intersection of the two curves gives equilibrium inflation, as given by (28), at point A. Consider comparative statics for x_0 and w_0. A reduction in x_0 increases the marginal benefit of inflation at any level of inflation—thus shifting the curve

[18]Local to no tax distortions, the time path for taxes is indeterminate. Assuming $T_1 = \frac{w_0}{2}$, however, provides a relatively good approximation, as seen in Figure 3.

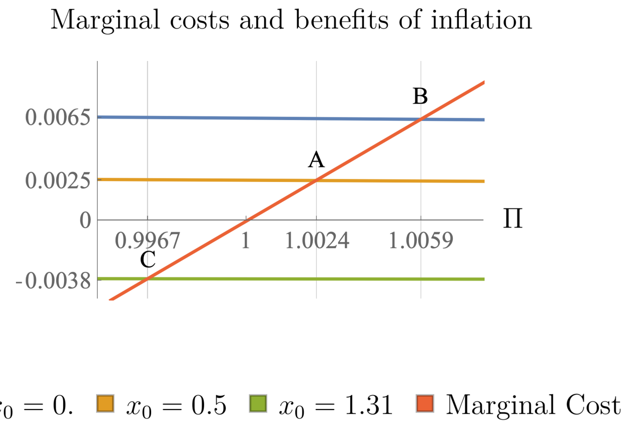

Figure 4: Medium-run marginal cost and benefit of inflation

Note: The figure shows the approximate marginal cost and benefit of inflation in the medium run with optimal monetary and fiscal policy under discretion. The marginal cost of inflation is increasing in inflation. The marginal benefit does not depend on inflation, but instead depends on the maturity of debt (x_0). The intersection of the two lines gives equilibrium inflation.

up—leading to a new equilibrium at point B with higher inflation and a lower nominal interest rate (via (29)).[19]

The forces at play are what we term the rollover incentives 1 and 2 (ROI 1 and ROI 2) in (27). ROI 1 arises because higher inflation reduces the real interest rate of the short-term debt rolled over to the next period. This reduces the government's financing cost. ROI 2 arises because the interest rate cost of short-term debt is nominal. Hence, for a given w_0, increasing the fraction of short-term debt increases the benefit of inflation.[20] ROI 1 and ROI 2 only apply to the fraction of the debt that is short term—that is, $(1 - x_0)w_0$. Intuitively, this is because the interest rate on the long-term debt has already been determined, while the interest rate on the short-term debt that is rolled over is determined in period 1. A reduction in x_0 therefore amplifies these incentives by increasing the share of debt being rolled over.

A reduction in x_0 always increases the marginal benefit of inflation. The same is not true for an increase in w_0. The condition $x_0 < \frac{1+\frac{1}{2}\kappa^{-1}}{1-\beta+\kappa^{-1}}$ must be met for a higher w_0 to be inflationary.[21] To interpret this condition, it is again useful to consider the marginal benefit of inflation (described in (27)). Consider first the balance sheet incentive (BSI). This incentive refers to the fact that since debt is nominal, inflation in period 1 reduces the real value of

[19] An increase in x_0 would shift it down and hence decrease equilibrium inflation at point C.

[20] Note that with flexible prices, as $\kappa^{-1} = 0$, ROI 1 goes away but ROI 2 does not. Thus, because of ROI 2, a lower x_0 triggers higher inflation in period 1, even under flexible prices. The period-1 policy functions under flexible prices are given in Figures A.5 and A.6 in Appendix A.7.4.

[21] In our numerical example, the condition is $x_0 < 0.82$. Appendix A.7.2 contains Figure A.3, which shows the dependence of period-1 variables on w_0 and illustrates how in our baseline parameterization with high x_0, equilibrium inflation is indeed *declining* in w_0.

the debt. Since both long- and short-term debt are nominal, however, this incentive does not depend on x_0. The marginal benefit of inflation via the BSI is always higher the higher is w_0. Conversely, because of the tax-smoothing incentive (TSI), the marginal benefit of inflation is always lower the higher is w_0. The ROI can either increase or decrease the marginal benefit of inflation associated with a higher w_0, depending on whether x_0 is greater than or smaller than 1. Which force prevails, and what is the mechanism?

To understand how the forces interact, it is useful to consider the special case $x_0 = 1$. In that case, the government only issues long-term debt in period 0 and there is no short-term debt to be rolled over in period 1. Yet the government still finds it optimal to levy taxes in period 1 (to smooth tax collection across periods). The tax collection in period 1 is then invested in short-term bonds that are rolled over at interest rate i_1. In this case, the government is a net debtor in long-term bonds and a saver in short-term bonds. Thus, in evaluating whether w_0 increases the marginal benefit of inflation, one needs to take into account that while inflation reduces the real value of the long-term debt (because of the BSI), it reduces the return of the assets (short-term nominal bonds) the government is holding on its balance sheet. The condition $x_0 < \frac{1+\frac{1}{2}\kappa^{-1}}{1-\beta+\kappa^{-1}}$ guarantees that the former force is always stronger so that the marginal benefit of inflation is increasing in w_0. Now consider the case in which $x_0 \neq 1$. When $x_0 > 1$, the ROI reinforces the TSI (because the government enters the period with short-term assets), while when $x_0 < 1$, the ROI reinforces the BSI.

To sum up, a lower x_0 always triggers expectations of higher inflation and a lower interest rate in period 1—and this is the central insight of our paper—while the effect of a higher w_0 depends on the overall balance sheet position of the government. The MPE for periods 1 and 2 can be used to formalize a model of QE in period 0, the subject of Sections 2.3.3 and 2.3.4.

2.3.3 The Short Run: Quantitative Easing as Comparative Statics in Policy

Consider a situation in which a shock in period 0 triggers the ZLB. Suppose that monetary and fiscal policies are set in periods 1 and 2 according to the MPE described in Sections 2.3.1 and 2.3.2. Consider now a policy regime in period 0 in which $i_0 = 0$ and there is some fixed (real) amount of government nominal debt; that is, $D_0 = \frac{B_0}{P_0} + \frac{L_0}{P_0} = \bar{D}$. Now consider a policy in which in period 0 the government shortens the duration of its outstanding debt via open market operations. QE in period 0 can now be thought of as comparative static: what happens if the government purchases, say, \$100 billion worth of long-term government bonds with short-term bonds (or reserves) via QE, holding total government debt, $\bar{D}$, constant? The magnitude of QE directly maps onto the state variables x_0 and, to a smaller degree, w_0. We find this thought experiment to be interesting for it corresponds to the type of question policy maker ask themselves when deciding on the size of QE at a given point in time.

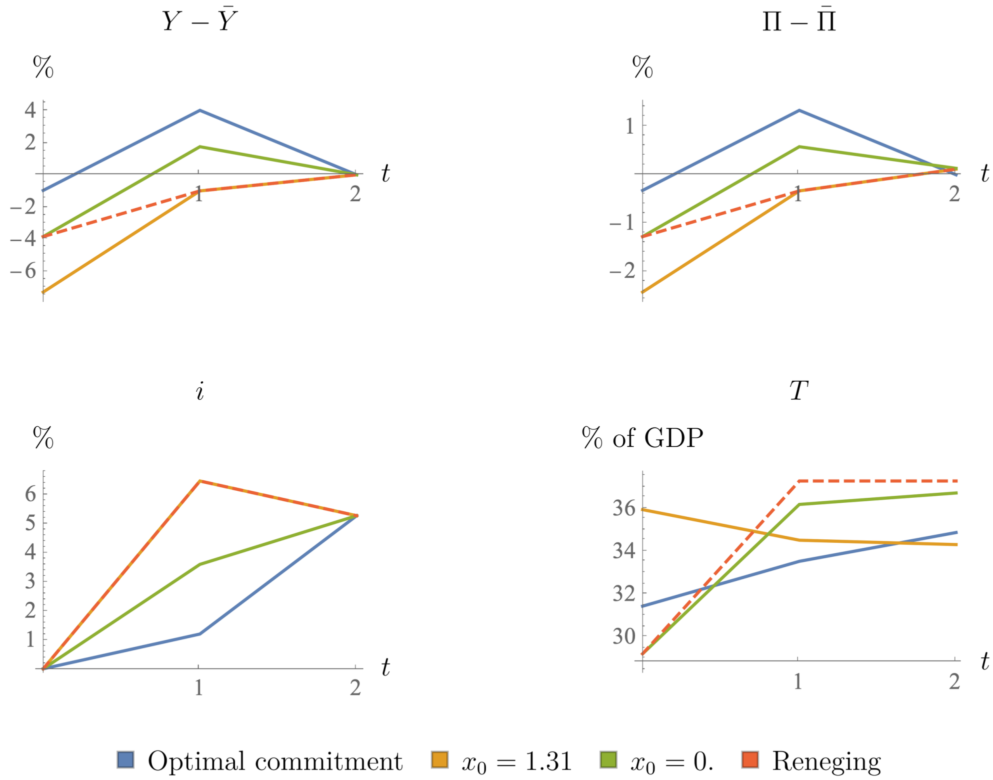

Figure 5: Impulse responses at the zero lower bound under various maturities of debt issued in the short run with otherwise optimal monetary and fiscal policy

Note: The figure shows the impulse response of output, inflation, the nominal interest rate, and taxes when a negative demand shock makes the zero lower bound binding in the short run. The model is solved for two different maturities of debt (x_0) issued in the short run under discretion (indicated by different colors), holding fixed the stock of debt issued ($\bar{D}$). The figure also shows the results for the model with optimal monetary and fiscal policy under commitment and the model in which the government deviates in periods 1 and 2 from the path of the nominal rate and inflation under $x_0 = 0$.

While it is clear enough that QE reduces the duration of the government debt—that is, it lowers x_0—the effect on w_0 is less obvious. Since $i_0 = 0$ and there is perfect foresight after the realization of the shock in period 0, simple manipulations yield $w_0 = \frac{(1+i_1)\bar{D}}{1+i_1(1-x_0)}$. This expression says that a lower x_0 is associated with a smaller w_0 for a given i_1. The reason is that w_0 measures the maturity value of debt at time 0, which includes future interest payments. Less long-term debt implies lower interest rate payments and thus lower maturity value in period 0, unless there is a simultaneous increase in i_1 to compensate. In numerical experiments, however, this effect is trivial, as we will see.

Figure 5 shows the solution of the model for an illustrative QE policy and compares it with some alternative policies. First, the yellow line shows the equilibrium under the assumptions that x_0 and w_0 are fixed at time 0 at the levels that would be optimal in the absence of the ZLB shock—that is $(x_0, w_0) = (1.31{,}0.723)$—and that there is an MPE in periods 1 and 2. Next, the blue line shows the equilibrium for optimal fiscal and monetary policies under commitment.[22] This commitment, however, is not credible in an MPE when the state variables are $(x_0, w_0) = (1.31{,}0.723)$, as depicted by the yellow line.

What can the government do? The green line shows the effect of QE whereby the government shortens the maturity structure of debt so that $x_0 = 0$ (thus issuing only one-period debt) while keeping D_0 fixed. This implies a trivial adjustment of w_0 from 0.723 to 0.721. Comparing the yellow with the green line, the solution illustrates that QE brings about a lower interest rate and a higher inflation rate in period 1; this force arises exclusively because of the shortening of the duration of debt. The expectation of this outcome then reduces the drops in output and inflation in period 0. QE, in other words, enables the government to credibly commit to a monetary expansion.

To understand *why* QE makes a commitment to low future interest rates credible, it is helpful to understand the cost of deviating from the loose monetary policy implied by this policy in period 1. The dashed-red line, labeled "Reneging" in Figure 5, shows the evolution of each variable if, in period 0, the government announces a plan to keep future interest rates low in line with the solution given by the green line (assuming the private sector believes this announcement), implements QE so that $(x_0, w_0) = (0{,}0.721)$, but then deviates from the plan by choosing the equilibrium that had been optimal in the absence of QE (that is, the MPE associated with $(x_0, w_0) = (1.31{,}0.723)$). As in the analysis of the deflation bias, the government obtains better outcomes for output and inflation via this deviation strategy.

What is new is the effect the policy has on taxation: the government needs to raise taxes by more than it otherwise would have (compare the dashed-red line tax path to the green line's path).[23] The reason for the tax cost of reneging on the the optimal discretionary policy is twofold. First, the lower rate of inflation increases the real value of debt. This is the balance sheet incentive. Second, the government needs to pay a higher interest rate on the debt it is rolling over from period 1 to period 2. This is the rollover incentive.

The effect of QE can be characterized analytically if we assume that QE operates only via x_0 (thus abstracting from its effect on w_0, which is quantitatively trivial, as we have just seen), as given in the following proposition:

Proposition 3 *If (i) there is a policy regime with fixed* w_0 *in period 0, (ii) there is an MPE in periods 1 and 2, and (iii) the policy function* $\Pi_1(w_0, x_0)$ *is approximated using Proposition*

[22] Section 4.1 further discusses the solution under commitment, where, in particular, the maturity structure of government debt in period 0 is irrelevant.

[23] In computing the path for taxes when the government reneges, one can only pin down the sum of taxes in periods 1 and 2. We assume taxes are the same in both periods.

2, then

$$\frac{\partial Y_0}{\partial x_0} = -\left(\frac{\lambda_T}{1+\frac{\lambda_y}{\kappa^2}}\right)\beta^{-1}(1-\beta+\kappa^{-1})\kappa^{-1}(2\Pi_1-1)\frac{w_0^2}{2}\xi_0 < 0.$$

Proof. From (3) we obtain $Y_0 = \beta^{-1}Y_1\Pi_1\xi_0 = \beta^{-1}\kappa^{-1}(\Pi_1-1)\Pi_1\xi_0$. Using (28) to approximate Π_1 and taking the partial derivative with x_0 yields the result. ■

Proposition (3) says that when the ZLB is binding in period 0, a reduction in the maturity of government debt in period 0 leads to an increase in output for a given w_0. This effect is higher the larger is the value of outstanding debt; it is increasing in the degree of tax distortions (λ_T); and it is stronger the higher is the degree of nominal rigidity. This statistic, $\frac{\partial Y_0}{\partial x_0}$, can be used to answer an important policy question: what is the output effect at the ZLB of a certain amount of QE? The main objective of Section 3 is to obtain a quantiative answer to this question.

2.3.4 The Short Run: Quantitative Easing as an Optimal Time-Consistent Policy

The MPE in the short run provides an interpretation of QE as an optimal time-consistent policy for government debt and its maturity. Period 0's government solves

$$V^0(w_{-1},\xi_0) = \max_{\Pi_0,T_0,Y_0,i_0,R_0,x_0,w_0}\left\{-\frac{1}{2}(\Pi_0-1)^2 - \frac{1}{2}\lambda_T T_0^2 - \frac{1}{2}\lambda_y\left(Y_0-\bar{Y}\right)^2 + \beta V^1(x_0,w_0)\right\}$$

subject to

$$\begin{aligned}
\frac{x_0 w_0}{1+R_0} + \frac{(1-x_0)\,w_0}{1+i_0} &= \frac{w_{-1}}{\Pi_0} - T_0,\\
\frac{1}{1+i_0} &= \beta\frac{Y_0}{\bar{Y}^1(x_0,w_0)}\frac{1}{\bar{\Pi}^1(x_0,w_0)}\frac{1}{\xi_0}, i_0 \geq 0,\\
\frac{1}{1+R_0} &= \beta^2\frac{Y_0}{\bar{Y}}\frac{1}{\bar{\Pi}^1(x_0,w_0)}\frac{1}{\bar{\Pi}^2(\bar{W}^1(x_{0,}w_0))}\frac{1}{\xi_0}, \text{ and}\\
(\Pi_0-1) &= \kappa\left(Y_0-\bar{Y}\right),
\end{aligned}$$

where $V^0(w_{-1},\xi_0)$ is the value function of the government and the state variables are w_{-1} and ξ_0. Moreover, $V^1(x_0,w_0)$ is given by (20), and the expectation functions—$\bar{\Pi}^1(.)$, $\bar{Y}^1(.)$, $\bar{W}^1(.)$, and $\bar{\Pi}^2(.)$—are given by (22) and (14).[24]

The green line in Figure 6 shows the solution for the MPE if policy is conducted optimally and the ZLB is binding in period 0. The yellow line shows the solution for optimal policy

[24] We take optimality conditions for all variables, other than x_0 and w_0, and then evaluate the objective numerically on a grid of x_0 and w_0 in Mathematica, taking as given the solution derived for periods 1 and 2. We check for a global optimum.

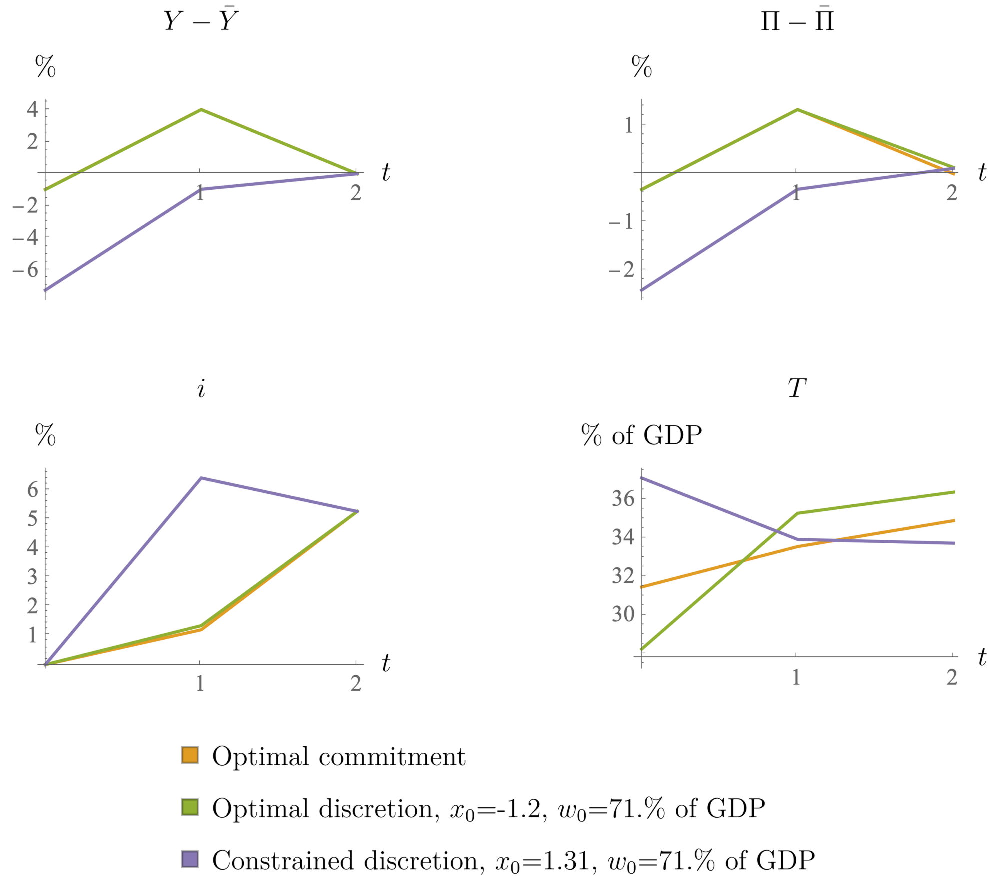

Figure 6: Impulse responses under optimal quantitative easing and other policies at the zero lower bound

Note: The figure shows the responses of output, inflation, the nominal interest rate, and taxes when a negative demand shock makes the zero lower bound binding in the short run. The yellow line shows the solution for optimal policy under commitment. The green line shows the solution for optimal policy under discretion—that is, optimal quantitative easing. The purple line shows the solution for optimal policy under discretion with the maturity of debt in period 0 set to the value that would be optimal with no shock.

under commitment. The purple line shows the solution under discretion if the government keeps the maturity of the debt fixed at time 0 at what would have been optimal in the absence of the shock; $x_0 = 1.31$. In this case, the government is allowed to optimize over w_0; we label this constrained discretion.

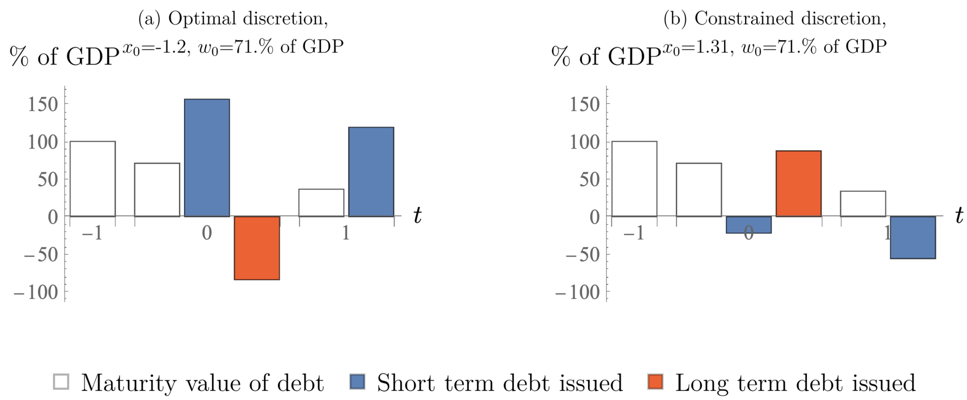

Figure 7: Government balance sheet dynamics under discretion at the zero lower bound

Note: The figure shows the evolution of the government balance sheet when a negative demand shock makes the zero lower bound binding in the short run. Panel (a) shows the solution for optimal policy under discretion—that is, optimal quantitative easing. Panel (b) shows the solution for optimal policy under discretion with the maturity of debt in period 0 set to the value that would be optimal with no shock—that is, constrained discretion.

Constrained discretion is qualitatively similar to the optimal monetary policy under discretion when $\lambda_T = 0$. In contrast, the fully optimal time-consistent solution, in which x_0 is chosen freely, almost completely replicates the optimal-commitment solution (yellow versus green line) in terms of output, inflation, and the interest rate.[25] In the fully optimal solution, the government reduces maturity substantially to $x_0 = -1.2$ instead of keeping it at 1.31, which was the optimal duration in the absence of the ZLB.[26]

Figure 7 shows the balance sheet position of the government—that is, the maturity value of debt (w_{-1}, w_0, W_1) and the long- and short-term debt issued (b_0, l_0, b_1). Panel (a) shows the balance sheet positions of the government when it structures both the level and composition of its debt optimally—that is, under optimal discretion. In period 0, the government issues only short-term debt and buys long-term debt from the private sector, thus closely replicating the commitment solution. Panel (b) shows the asset position of the government when the maturity composition is kept fixed in period 0 as if there were no shock, while the level of

[25]The solutions will not coincide fully, however. Under commitment, there will never be inflation in period 2, while in the MPE, there will always be some inflation in period 2 because of inherited nominal debt. There is also a more pronounced difference in taxes, which further shows that the solutions are not exactly the same. As is natural, for taxes, they are higher in period 1 under optimal discretion compared with commitment. While our overall conclusions are independent of parameterization, in Figure A.4 in Appendix A.7.3 we present an example in which the differences are bigger for interest rates and inflation in all periods.

[26]The negative value of x_0 provides an interpretation of some of the unconventional *credit* policies by the Federal Reserve, often referred to as QE1. A negative x_0 implies that the central bank is printing short-term reserves and buying long-term privately issued bonds. To the extent that the duration of the asset the Federal Reserve took on its balance sheet during the credit-easing operation is longer than that of reserves (which are fully short term), these policies change the inflation incentive of the government via the term structure.

debt is chosen optimally—that is, under constrained discretion. The government issues long-term debt to purchase short-term debt, which is optimal only in the absence of the shock, as it eliminates the inflation bias created by nominal government debt in period 1.

3 A Quantitative Model

This section considers a standard New Keynesian model that has been subjected to quantitative evaluation and can be parameterized based on the existing literature (see, for example, Woodford (2003) and Gali (2007) for textbook treatments). The standard model is extended to include the costs of taxation and long-term government debt. Since the cost of taxation is the key new parameter, it is the main focus of the calibration and sensitivity analyses.

3.1 Environment

A representative household maximizes

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t [u(C_t) + g(G_t) - v(H_t)]\xi_t, \tag{30}$$

subject to a sequence of budget constraints

$$P_t C_t + B_t + S_t L_t \leq N_t H_t + (1 + i_{t-1}) B_{t-1} + (1 + \rho_{t-1} Q_t) L_{t-1} - P_t T_t + \int_0^1 Z_t(i) di, \tag{31}$$

where we use the same notation as in the simple model. Here, $u(.)$ and $g(.)$ are both increasing and concave in their arguments while $v(.)$ is increasing and convex. C_t is private consumption, and G_t is government consumption, where both are Dixit-Stiglitz aggregates of a continuum of varieties of goods.

B_t is a one-period riskless nominal government bond while L_t is a perpetuity nominal government bond with price S_t and decay factor ρ_t. A perpetuity issued in period t pays ρ_t^j dollars $j+1$ periods later for some decay factor $0 \leq \rho_t < \beta^{-1}$. At time t, the household can sell its existing perpetuities, L_{t-1}, which have a decay factor ρ_{t-1}, at price Q_t. The model thus incorporates debt of arbitrary duration. A value of $\rho_t = 0$, for example, implies that L_t reduces to a one-period short-term bond, while $\rho_t = 1$ corresponds to a classic consol bond of infinite duration. More generally, with stable prices, the duration of a perpetuity with decay factor ρ_t is $(1 - \beta\rho_t)^{-1}$.

The maximization problem and optimality conditions are standard, but with an additional pricing condition for existing and new perpetuities. The standard Euler equation, which prices one-period short-term bonds, is reported below in (32) while the asset-pricing equations for current and existing long-term perpetuities are shown in (33).

A continuum of monopolistically competitive firms, indexed by i, produce varieties of goods using a production function that is linear in labor, $y_t(i) = H_t(i)$. Following Rotemberg (1982), we model firms as facing a cost of changing prices given by $d\left(\frac{p(i)}{p_{t-1}(i)}\right)$, where $d' > 0$ and $d''(.) < 0$. The firm maximizes expected discounted profits facing a subsidy τ that is set so that steady-state output is efficient. The firms' problem, after we impose a symmetric equilibrium, implies the nonlinear Phillips curve reported below in (34).

As in Barro (1979), there is an output cost of taxation, captured by the function $s(T_t)$. Total government spending is $F_t = G_t + s(T_t)$.[27] The output cost of taxation implies that more taxation directly reduces government spending, G_t, that contributes to welfare. For simplicity, total government spending is assumed to be constant so that $F_t = \bar{F}$.[28]

At the beginning of each period t, the government buys all existing debt (perpetuities that have duration ρ_{t-1}) at price Q_t and issues new debt of duration ρ_t at price S_t. The government issues no short-term bonds in equilibrium so that $B_t = 0$. If we define $l_t = \frac{L_t}{P_t}$ and Π_t as gross inflation, the government budget constraint is (35). Government policy—which is the choice of the tax T_t, interest rate i_t, debt l_t, and maturity of government debt ρ_t—is also constrained by the ZLB, as reported in (32), and the resource constraint, written in (36).

The equilibrium conditions and constraints based on the problems described above are

$$\frac{1}{1+i_t} = \mathbb{E}_t\left[\beta\frac{u_c(C_{t+1})\xi_{t+1}}{u_c(C_t)\xi_t\Pi_{t+1}}\right], \quad i_t \geq 0, \tag{32}$$

$$S_t = \mathbb{E}_t\left[\beta\frac{u_c(C_{t+1})\xi_{t+1}}{u_c(C_t)\xi_t\Pi_{t+1}}\left(1+\rho_t S_{t+1}\right)\right], \; Q_t = \mathbb{E}_t\left[\beta\frac{u_c(C_{t+1})\xi_{t+1}}{u_c(C_t)\xi_t\Pi_{t+1}}\left(1+\rho_{t-1} Q_{t+1}\right)\right], \tag{33}$$

$$\theta Y_t\left[u_c(C_t) - v_y(Y_t)\right]\xi_t + u_c(C_t)\xi_t d'\left(\Pi_t\right)\Pi_t = \mathbb{E}_t\left[\beta u_c(C_{t+1})\xi_{t+1} d'\left(\Pi_{t+1}\right)\Pi_{t+1}\right], \tag{34}$$

$$S_t l_t = \left(1+\rho_{t-1}Q_t\right) l_{t-1}\Pi_t^{-1} + \left(\bar{F} - T_t\right), \text{ and} \tag{35}$$

$$Y_t = C_t + \bar{F}. \tag{36}$$

[27] Unlike in the simple model, we here assume that the tax-collection costs are in terms of the final good rather than labor.

[28] If government is also allowed to choose F_t, this introduces one additional policy instrument. This does not, however, change the fact that taxes have a welfare effect via diverting resources away from welfare-contributing government consumption, G_t, to tax collection, which contributes nothing to utility. Eggertsson (2006, 2008) considers the case in which both F_t and T_t are optimally chosen in a model with one-period debt. Rather than assuming $F_t = \bar{F}$, the model could alternatively be written assuming that $G_t = \bar{G}$ and would yield identical results. In either case, the interpretation is that taxes have a negative effect on welfare, as they require resources to "produce" the tax revenues. In the former case, higher taxes result in lower government spending, G_t, and lower welfare. In the latter, higher taxes result in lower private consumption, C_t, and lower welfare. We thank an anonymous referee for stressing to us the importance of this alternative interpretation.

3.2 Markov Perfect Equilibrium and Solution Method

Define the expectation variables $f_t^E \equiv \mathbb{E}_t\left[\frac{u_C(C_{t+1})\xi_{t+1}}{\Pi_{t+1}}\right]$, $g_t^E \equiv \mathbb{E}_t\left[\frac{u_C(C_{t+1})\xi_{t+1}}{\Pi_{t+1}}(1+\rho_t S_{t+1})\right]$, $j_t^E \equiv \mathbb{E}_t\left[\frac{u_C(C_{t+1})\xi_{t+1}}{\Pi_{t+1}}(1+\rho_{t-1}Q_{t+1})\right]$, and $h_t^E \equiv \mathbb{E}_t\left[u_C\left(C_{t+1}\right)\xi_{t+1}d'\left(\Pi_{t+1}\right)\Pi_{t+1}\right]$. An examination of conditions (32)-(36) reveals that the physical state variables of the model at time t are ρ_{t-1}, l_{t-1}, and the exogenous shock ξ_t. The assumption of MPE implies that expectations are only a function of the state variables

$$f_t^E = \bar{f}^E(l_t,\rho_t,\xi_t),\ g_t^E = \bar{g}^E(l_t,\rho_t,\xi_t),\ j_t^E = \bar{j}^E(l_t,\rho_t,\xi_t),\ \text{and } h_t^E = \bar{h}^E(l_t,\rho_t,\xi_t), \tag{37}$$

where $\bar{f}^E(.), \bar{g}^E(.), \bar{j}^E(.)$, and $\bar{h}^E(.)$ are unknown functions.

The government's problem is

$$V(l_{t-1},\rho_{t-1},\xi_t) = \max_{i_t,T_t,l_t,\rho_t}\left[U\left(.\right) + \beta\mathbb{E}_t V(l_t,\rho_{t,}\xi_{t+1})\right] \tag{38}$$

subject to

$$1+i_t = \frac{u_c(C_t)\xi_t}{\beta f_t^E},\ i_t \geq 0, \tag{39}$$

$$S_t = \frac{1}{u_c(C_t)\xi_t}\beta g_t^E,\quad Q_t = \frac{1}{u_c(C_t)\xi_t}\beta j_t^E, \tag{40}$$

$$h_t^E = \theta Y_t\left[u_c(C_t)\xi_t - v_y(Y_t)\right]\xi_t + u_c(C_t)\xi_t d'\left(\Pi_t\right)\Pi_t, \tag{41}$$

$$S_t l_t = \left(1+\rho_{t-1}Q_t\right)l_{t-1}\Pi_t^{-1} + \left(\bar{F} - T_t\right),\ \text{and} \tag{42}$$

$$Y_t = C_t + \bar{F} + d\left(\Pi_t\right), \tag{43}$$

where f_t^E, g_t^E, h_t^E, and j_t^E are given by (37) and $V(l_{t-1},\rho_{t-1},\xi_t)$ is the value function.

An equilibrium is defined as a collection of stochastic processes for the endogenous variables, where the processes solve both constraints (32)-(36) and the optimality conditions of the government's problem (38). The optimality conditions, reported in Appendix C.4, contain derivatives of the expectation functions, such as $\bar{f}_l^E$ and $\bar{f}_\rho^E$. The expectation functions are unknown and hence so are their derivatives. The simple model was solved backward from a terminal period in which there were no expectations of the future. The solution of the terminal period could then be used to construct expectations for the second-to-last period, which in turn could be used for the period before. In an infinite-horizon model, this is not feasible.

The solution method follows the literature in making the assumption that the expectation functions are smooth and differentiable (see, for example, Soderlind (1999) and Klein et al. (2008)). This allows for an approximation of the expectation functions by using perturbation methods, which rely on the implicit function theorem, and the method of undetermined coefficients. The solution method is discussed in Appendices C.7 and C.8. A key simplification is that the model is assumed to be efficient in steady state for positive debt at any given maturity (the value of debt can be taken from the data).[29] This allows for a full characterization of the steady state in the MPE in closed form, which is used as an approximation point.

3.3 Thought Experiment

The thought experiment here is the infinite-horizon analog to the thought experiment in Section 2.3.3. The shock follows a two-state Markov process with an absorbing state, as in Eggertsson and Woodford (2003). In period 0 there is an unexpected shock so that $\xi_0 < \bar{\xi}$. In the following periods, there is a probability $1-\mu$ that the shock will revert back to steady state so that $\xi_1 = \bar{\xi}$ and a probability μ that the shock will remain in its low state so that $\xi_1 = \xi_0 = \xi_S < \bar{\xi}$. If the shock reverts to steady state, it stays there forever. If the shock remains, then it behaves exactly the same in the following period; that is, it has a probability $1-\mu$ of reverting to steady state and probability μ of remaining so that $\xi_2 = \xi_1 = \xi_0 < \bar{\xi}$. The shock takes the same structure in all future periods. This means that the expected duration of the shock is $\frac{1}{\mu}$ quarters. The stochastic period in which the shock returns to steady state is denoted by K. The period prior to that, $t < K$, is the short run (denoted by S), while $t \geq K$ is the long run. The shock is chosen to be large enough so that the ZLB is always binding in the short run. In an approximate equilibrium, the shock is defined as $r_s^e = \bar{r} + \sigma^{-1}(\hat{\xi}_t - E_t\hat{\xi}_{t+1})$, interpreted as the real interest rate needed for output to reach its flexible price level.

In the short run—that is, $t < K$—policy is (possibly) suboptimal as in Section 2.3.3. This allows for a straightforward match to particular QE episodes observed in the US in the Great Recession. In the long run, in which $t \geq K$, policy is set according to the MPE. QE shortens the duration of government debt in the short run—that is, it leads to a smaller ρ_t—holding fixed the total value of the debt issued ($S_t l_t = \bar{D}$). This means outcomes can be compared with empirical estimates from asset markets during QE episodes as a test of the model.

Consider a variable x, and denote by Δx the change in this variable in response to QE. A QE episode is mapped onto a reduction of $\Delta\rho$ and a resulting change in variables of interest, Δx. We can compare the model prediction to the data, which we do in Section 3.5.3.

[29] This allows us to characterize the steady state in closed form in the MPE. This is a major simplification relative to Klein et al. (2008), who rely on nontrivial approximations to characterize the steady state. Leeper et al. (2019) similarly consider a time-consistent equilibrium in the New Keynesian model with long-term debt in which the steady state is not efficient, approximating it using collocation methods.

3.4 Parameterization

The model is fully parameterized, with choices of parameters (discussed in detail below), related to (i) fiscal policy and QE ($\rho, \Delta\rho, \psi, \hat{b}_S$); (ii) structural parameters of the model from the private sector (β, κ, σ); (iii) the size and persistence of the shock (r_S^e, μ); and (iv) the welfare-based weights the government puts on variability in output and taxation (λ_y, λ_T). We discuss calibration of each in turn, with special attention to λ_T, which is new.

3.4.1 Calibration of Baseline Parameters

The fiscal policy/QE parameters are directly backed out of the data. The model assumes a consolidated government budget constraint so that, for government debt, the relevant measure is all government debt held by the public (thus, government debt held by the Federal Reserve is netted out). Reserves issued by the Federal Reserve are treated as short-term government debt. The duration of the consolidated government's debt is shown in Figure C.8 in Appendix C.9.1, where different phases of QE, termed QE1, QE2, and QE3, are discussed.[30] The baseline QE experiment is QE2, which started on November 2010.[31] The baseline maturity is 16.87 quarters—the level at the beginning of QE2—resulting in $\rho = 0.9502$. The reductions in maturity due to QE2 and QE3 are shown in Table 1. See Appendix C.9.1 for details on the construction of these statistics.

In addition to debt duration and its change due to QE2 (that is, $\rho(\text{pre-}QE2)$ and $\Delta\rho(QE2)$), two other parameters are directly calibrated in Table 1: ψ and $\hat{b}_S$. The parameter $\psi \equiv \frac{lS}{T} = \frac{D}{T}$ is the steady-state level of the debt-to-taxes ratio. The long-run average of the ratio of the market value of debt to output is computed using data from the Federal Reserve Bank of Dallas, while NIPA is used to measure the ratio of taxes to output, yielding $\psi = 7.2$.[32] Debt in the short run, $\hat{b}_S$, is 30% above its steady-state value based on Dallas Fed data.[33]

The parameters β, κ, and σ are taken from Eggertsson and Woodford (2003), as shown in Table 1. The model implies that $\lambda_y = \kappa/\theta$, where θ is the elasticity of substitution across

[30]QE3 is taken to be the sum of what is commonly referred to as QE3 and the Maturity Extension Program; see Appendix C.9.1.

[31]The reason why QE2 serves as the baseline calibration is that it only included purchases of long-term government bonds. Thus it arguably captures well the basic force the paper formalizes. Meanwhile, QE1 included purchases of various private securities, which is why Bernanke (2009) defines QE1 as credit easing and QE2 as quantitative easing. A credit-easing episode is better modeled by modeling frictions in the credit markets, as done, for example, in Gertler and Karadi (2011, 2012) and Del Negro et al. (2017). Consistent with this perspective, Krishnamurthy and Vissing-Jorgensen (2011) and others find that QE1 largely operated via risk premia, while QE2 instead operated also via expectations of lower future federal funds rates.

[32]We use the average of these two series from 1951-III to 2010-III.

[33]This is calculated by considering debt/GDP in 2008-IV compared with its average value from 1951-III to 2010-III. Moreover, as mentioned previously, we adjust the quantity of debt after QE2 to keep the market value of the debt fixed during the QE2 intervention (it has to be adjusted to 0.297 in our baseline calibration).

Table 1: Calibrated/estimated parameter values in the quantitative model

Parameter	Value	Parameter Description
β	0.99	Discount factor
σ	0.5	Scaled IES
κ	0.02	Slope of Phillips curve
λ_y	0.029	Weight on output
λ_T	0.021	Weight on taxes
r_S^e	-0.01	Shock size
μ	0.88	Shock persistence
$\rho(\text{pre-}QE2)$	0.9502	Baseline debt maturity
$\Delta\rho(QE2)$	0.0024	Change in debt maturity
$\Delta\rho(QE3)$	0.0072	Change in debt maturity
ψ	7.2	Debt-to-taxes ratio
$\hat{b}_S$	0.30	Initial debt

varieties of goods. θ is chosen to be 11, such that it is consistent with a 10% markup. This target for markup is within the range of estimated values in De Loecker and Warzynski (2012).[34] The size of the shock r_t^e and its persistence are chosen to match the fall in real GDP and inflation at the start of the 2008 crisis in the US using the values for these targets from Del Negro et al. (2017) (-7.5% and -2.5% respectively). The resulting values are in Table 1.

The parameter that remains to be chosen is λ_T. This parameter is central to the results, and there is little guidance in the literature on how to choose it. Accordingly, it is the main focus of the calibration and the sensitivity analysis.

3.4.2 Calibration of Tax Distortions (λ_T)

The fundamental role λ_T plays in the model is to determine the *relative* response of inflation and taxes to fiscal shocks that increase debt. To illustrate how we use this model property, together with an estimated empirical counterpart, to calibrate λ_T, we proceed in four steps.

First, Figure 8 shows the model's impulse responses of inflation and taxes to a fiscal shock for the benchmark parameterization of λ_T.[35] In response to a fiscal shock that increases debt, both inflation and taxes increase and then gradually return back to steady state. The shaded areas below the impulse responses represent the total response of inflation and taxes up to sixteen quarters. This is the standard horizon over which impulse responses are cumulated in the empirical literature on fiscal policy (see Ramey and Zubairy (2018), or RZ hereafter).

[34] For instance, Table 2 in De Loecker and Warzynski (2012) shows that depending on the specification, estimated markups vary from 3% to 28%.

[35] Our interpretation of a fiscal shock here is a one-time increase in fiscal outlays, which results in an increase in debt if we keep debt maturity fixed.

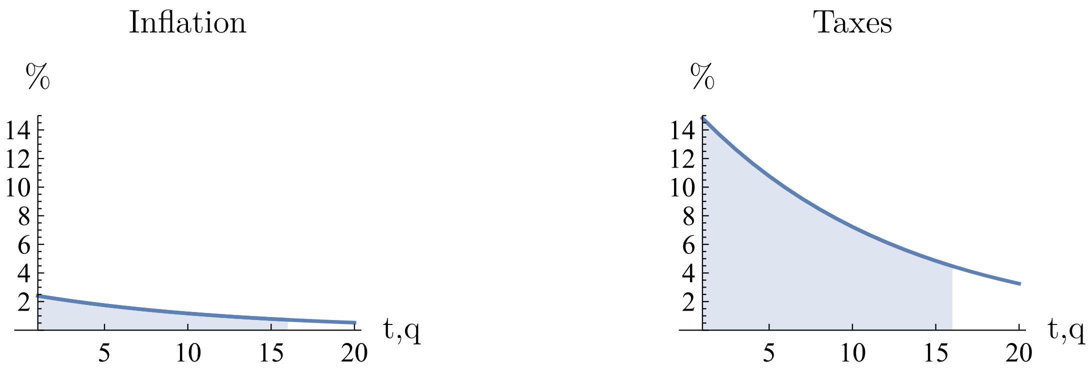

Figure 8: Impulse responses outside of the zero lower bound

Note: The figure shows the impulse responses of inflation and taxes outside the zero lower bound to a shock that increases debt. This is for the benchmark parameterization in Table 1.

The calibration strategy of λ_T relies on the model property that the higher is λ_T, the more the government responds to a fiscal shock by increasing inflation *relative* to taxes in the impulse responses shown above.[36] This is shown in Figure 9, which plots the ratio

$$D_{\pi,T}(\lambda_T) = \frac{IRF_\pi(16)}{IRF_T(16)} \tag{44}$$

as a function of λ_T while using the benchmark values for all other parameters. Here, $IRF_\pi(16)$ and $IRF_T(16)$ represent the shaded areas in Figure 8. Figure 9 shows that the higher is λ_T, the more the government relies on inflation relative to taxes to pay down debt. This theoretical moment is then a natural target with which to identify a value for λ_T. Point B corresponds to the benchmark calibration reported in Table 1 for λ_T, which is obtained by matching (44) to the empirical counterpart of this statistic, which we now turn to.

The empirical counterpart to (44) is generated by replicating and extending the empirical analysis of RZ. RZ collect data on inflation, debt, taxes, and government spending for the period 1889–2018 in the US. In contrast, we truncate our data to end in 2008 to focus on evidence outside of the 2008 crisis.[37] RZ focus on the effect of fiscal shocks on output using local projection methods.[38] We adapt their analysis to analyze cumulative impulse responses for taxes and inflation in response to a fiscal shock—that is, the empirical objects $\widehat{IRF}_\pi(16)$ and $\widehat{IRF}_T(16)$. We then take the ratio of these impulse responses to construct the empirical

[36] This can be seen analytically in period 2 of the simple model, as (12) rewritten gives $(\Pi_2 - 1)\,\Pi_2 = \lambda_T T_2^2$.

[37] We truncated the data based on feedback from referees so that it strictly covers the pre-ZLB period. The results do not change much if we instead use the whole period.

[38] In their analysis, they use Ramey's (2011) military-news variable and Blanchard and Perotti's (2002) shock series as instruments for government spending in local projection regressions; see their Equation (3).

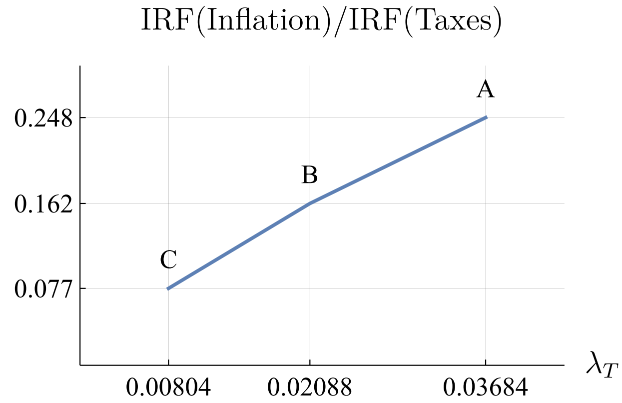

Figure 9: Identification and estimation of tax-smoothing parameter

Note: The figure shows how λ_T is identified and estimated using the response of inflation relative to taxes in the model and matching it to the empirical counterpart. Point B corresponds to the point estimate of λ_T.

target $\widehat{D}_{\pi,T}$, the point estimate of which is 0.162 and the standard error ($\hat{\sigma}$) of which is 0.085. Appendices B.1-B.3 contains more details on the estimation, including how we construct the standard error for the ratio of impulse responses using the delta method.

Then we choose λ_T so that (44) in the model matches this empirical target; that is, we back out the λ_T that solves $D_{\pi,T}(\lambda_T) = \widehat{D}_{\pi,T}$. As shown in Figure 9, there is a solution for λ_T that solves this equation. Point B is thus the point estimate for λ_T, as reported in Table 1. Points A and C correspond to values for λ_T estimated by matching $+/-$ one standard deviation from the point estimate of $\widehat{D}_{\pi,T}$, which we use in our main sensitivity analysis later.

3.5 Main Results

3.5.1 Dynamic Paths

The blue line in Figure 10 shows the evolution of the main variables of interest in the benchmark scenario in which government debt and its duration are fixed throughout the ZLB and an MPE is in place once the shock has subsided. In response to the shock, output and inflation fall by 7.5% and 2.5% respectively, as targeted in the calibration. The figure shows the realization of a shock that reverts back to steady state in period 8, the expected duration of the shock. Once the shock reverts to steady state, the government raises the interest rate to stabilize inflation and output at a level slightly above steady state because government debt is 30% above steady state.

The red line in Figure 10 shows the optimal monetary and fiscal policy under commitment.[39] In response to the shock, the government commits to a lower future interest rate.

[39] It can be shown, using a similar argument as that in the simple model, that under monetary and fiscal

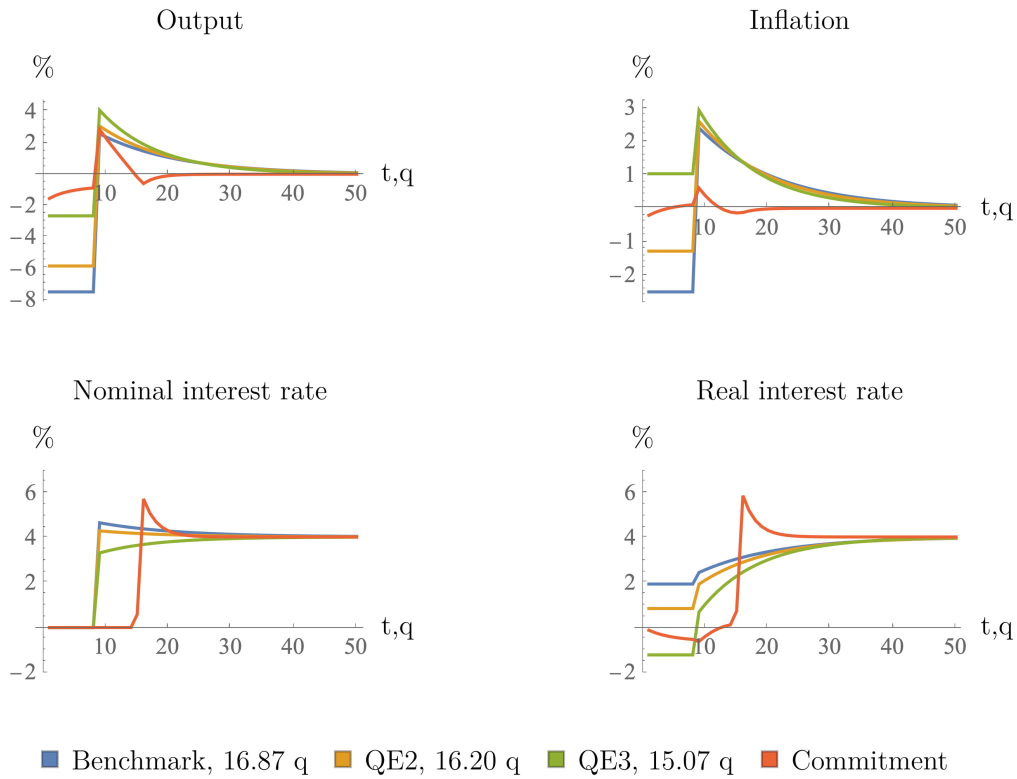

Figure 10: Impulse responses under various maturities of debt at the zero lower bound

Note: The figure shows the impulse responses of output, inflation, the nominal rate, and the real rate when a negative demand shock makes the zero lower bound bind initially and the shock reverts back to steady state in eight periods. The model is solved for three different debt durations at the zero lower bound, indicated by different colors, holding fixed the value of debt issued. This illustrates quantitative easing as a comparative-static experiment. The figure also shows, for comparison, the model with optimal monetary and fiscal policy under commitment.

While the discretionary central bank raises the interest rate in period 8, the central bank under the optimal-commitment policy raises rates six periods later. As shown in the lower-right panel, this brings about a lower real interest rate during the period of the negative shock, which mitigates the recession greatly, with output only dropping by about 2%. Meanwhile, inflation stays within 50 bp of zero throughout the crisis.

While the optimal-commitment solution represents the best equilibrium, a discretionary government, as portrayed by the blue line, is unable to credibly commit to keeping the interest rate low once the shock subsides because it has an incentive to renege on its previous promises and raise interest rates. This is where QE comes in, as in the simple model.

commitment, the maturity structure of debt is irrelevant even in this quantitative model. The commitment solution was computed using the approximation-based toolkit of Eggertsson et al. (2019), which is based on Eggertsson and Woodford (2003). The properties of the approximation at the ZLB in this class of models are discussed in Eggertsson and Singh (2019).

The yellow line shows the solution if the government reduces the duration of debt as in QE2. This results in a commitment to lower the nominal interest rate in the future relative to the blue line, thus more closely corresponding to the optimal commitment. This is made clearer via the real interest rate (shown in the fourth panel), which is what determines output. By shortening the maturity of outstanding debt, the government is able to commit to lower real interest rates once the shock is over, which stimulates output during the period in which the ZLB is binding. Hence QE is an effective commitment device and renders low future interest rates credible. The green line in the figure shows the effect of QE3, which is larger.

To understand why a shorter maturity brings about a commitment to lower future interest rates, and to compare with the simple model, it is helpful to make a few additional assumptions, even if this glosses over some features of the solution discussed in the next paragraph. The endogenous state variables are l_{t-1} and ρ_{t-1}. For simplicity, consider the case $\rho_t = \rho_{t-1}$, so that $S_t = Q_t$. Furthermore, contemplate a small deviation from steady state so that in period $t+1$ the interest rate is back to steady state, in which case $1 + i_{t+1} = \beta^{-1}$ and $S_t = \frac{1}{1-\beta\rho_{t-1}}$. Making the simplification that $\beta = 1$, the budget constraint (35) can then be written as

$$l_t = \underbrace{\frac{l_{t-1}}{\Pi_t}}_{\text{Balance sheet incentive}} + \underbrace{i_t\left(1-\rho_{t-1}\right)\frac{l_{t-1}}{\Pi_t}}_{\text{Rollover incentive}} + \underbrace{\frac{1}{S_t}\left[F - T_t\right]}_{\text{Tax-smoothing incentive}}. \tag{45}$$

That is analogous to (23) in the simple model, where the balance sheet and rollover incentives are defined. The first term on the right-hand side of (45) corresponds to the balance sheet effect, which is independent of the duration of government debt, while the second term corresponds to the rollover incentive, but x_0 has been replaced by ρ_{t-1}. The last piece corresponds to the tax-smoothing incentive as before. The logic is thus the same in the two models.

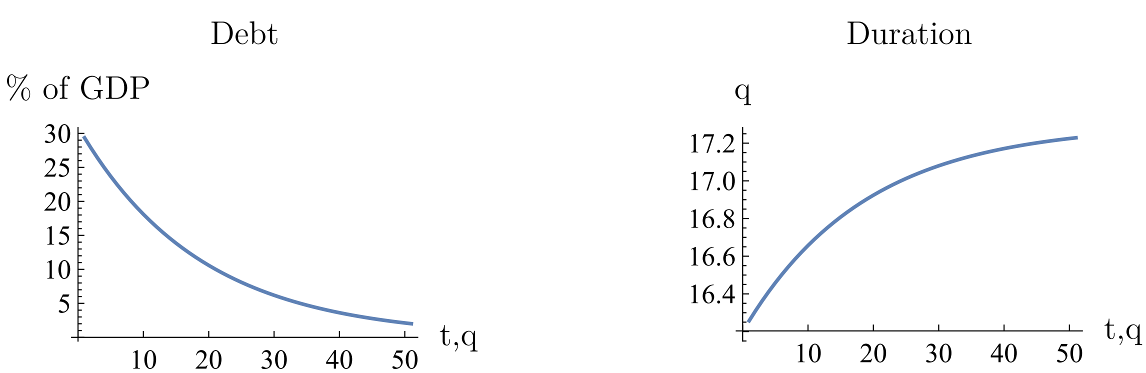

Figure 11: Transition dynamics of the government balance sheet

Note: The figure shows the dynamics of debt and debt duration once the shock is over.

Finally, Figure 11 shows how the transition dynamics of debt and duration are affected by QE once the shock is over. As the figure reveals, once out of the ZLB, the government reduces the debt back to its steady state. Note that as the debt decreases, the government lengthens the duration of its debt, an issue discussed towards the end of Section 4.4.

3.5.2 Output Effects of Quantitative Easing

As a summary of the macroeconomic effects of QE shown in Figure 10, Table 2 shows the output effects of QE in the benchmark calibration. Output at the ZLB increases by about 1.65 percentage points for QE2 and 4.83 percentage points for QE3.

Table 2: Output effects of QE2 and QE3 under baseline calibration

Model Specification	**ΔOutput, QE2**	**ΔOutput, QE3**
Baseline Calibration	1.65%	4.83%

3.5.3 Untargeted Moments

We can now ask: how well does the model account for empirically estimated financial-market behavior in response to QE? To assess this, we compare the model prediction of the changes in the long-term interest rate and in expected inflation to well-known empirical estimates. This is an out-of-sample test, as our calibration did not target any estimates of financial-market effects of QE taken from the literature.

The main comparison is to the estimated effects of QE2 in Krishnamurthy and Vissing-Jorgensen (2011, or KVJ hereafter). Using fed funds futures, they estimate a reduction of $\Delta i\,(8) = -16$ bp in the short-term interest rate's eight-quarters-ahead yield and an increase of $\Delta\pi\,(40) = 5$ bp in expected inflation over ten years. This empirical estimate is particularly attractive for our analysis since KVJ argue that the changes in the fed funds futures are due to signaling by the Fed about future interest rate policy. This is the mechanism the model is designed to capture. These out-of-sample predictions are summarized in Table 3. Since none of the parameters were chosen with this in mind, the match is less than perfect.

A few points are worth highlighting. The way the shock is chosen (that is, its persistence) implies an expected ZLB episode of 8.4 quarters, which is in the range of surveyed expectations of the duration of the ZLB in this period (see, for example, Del Negro et al. (2017)). As already stressed, we pick the shock parameters to generate a recession and drop in inflation and not to match the expected duration of ZLB.

The model accounts for about two-thirds of KVJ's estimated fall in two-year expected future short rates in response to QE2. In addition, the model accounts for about a third

Table 3: Untargeted data and model moments under baseline calibration

Untargeted Moments	**Data**	**Model**
Zero-lower-bound duration	$6-8$ qt	8.4 qt
$\Delta i(8), QE2$	-16 bp	-10.24 bp
$\Delta i(40), QE2$	-30 bp	-11.2 bp
$\Delta i(40), QE3$	-46 bp	-34.9 bp
$\Delta\pi(40)$, $QE2$	5 bp	14.34 bp

of the fall in ten-year Treasury yields as a result of QE2 and about two-thirds of the fall in ten-year Treasury yields that Ehlers (2012) estimates resulted from QE3.[40] Finally, the model overshoots KVJ's estimated ten-year-ahead increase in inflation expectations due to QE2.

To summarize, the model accounts fully for the rise in inflation expectations due to QE2 and accounts for roughly a third to two-thirds of the fall in long-term yields due to QE2 and QE3. Taken at face value, this suggests that alternative theories, such as those based on portfolio rebalancing or financial frictions, may account for the rest of the effects on yields.

3.6 Sensitivity Analyses

3.6.1 Bounds on Output Effects

We consider sensitivity of the results to reasonable variation in λ_T by reporting the results of assuming that λ_T is +/- one standard deviation from the estimated value used in the baseline calibration. That is, it corresponds to points A (labeled λ_T^{high} below) and C (labeled λ_T^{low} below) in Figure 9. The output effects implied from this analysis are shown in Table 4.

Table 4: Output effects of QE2 and QE3 with different degrees of tax smoothing

Model Specification	**ΔOutput, QE2**	**ΔOutput, QE3**
Baseline λ_T	1.65%	4.83%
λ_T^{high}	2.17%	5.74%
λ_T^{low}	0.26%	0.90%

Table 4 shows that for the lower value of λ_T, the output effects of QE2 and QE3 are smaller than in the benchmark, at 0.26 and 0.9 percentage points respectively. Meanwhile, for the higher value of λ_T, the output effects of QE2 and QE3 are higher, at 2.17 and 5.74 percentage points respectively. The intuition for why QE has a smaller effect for a lower value of λ_T is

[40]The ten-year-yield effects of QE2 estimated by Ehlers (2012) are not pure signaling effects; they include a term-premia effect. Thus, it was to be expected that our model would not match that moment closely.

that for low values of this parameter, as we saw in Figure 9, the government relies relatively less on inflation and more on tax increases in order to pay down public debt.

This sensitivity analysis chooses r^e_S to generate an output drop of 7.5%, but leaves inflation free. The value of the persistence of the shock μ is unchanged as λ_T is changed, but recall that μ maps onto the expected duration of the ZLB. This procedure then implies that as λ_T changes, the model generates different inflation drops at the ZLB than that in the data. Appendix C.9.2 documents this and also shows the responses of other key empirical moments.

3.6.2 Bounds on Output Effects Matching Inflation at the Zero Lower Bound

An alternative way of doing the sensitivity analysis with respect to λ_T is to recalibrate μ to hit the 2.5% drop in inflation observed in the data. Doing so yields very high estimates for the output effects of QE2 and QE3. In our view, however, these high estimates are hard to defend. High values of λ_T imply, using this approach, that a very persistent expected slump is needed to generate a 2.5% drop in inflation (high μ). The expected duration of the ZLB according to this procedure is more than double the expected duration in the benchmark calibration. The benchmark calibration implies an expected duration of the ZLB that is very close to market expectations following 2008, while the alternative calibration does not, casting doubt on the plausibility of the large estimated effects of QE using this procedure.

A more informative alternative is to keep μ fixed in line with market expectations and vary λ_T around the point estimate, but recalibrate κ so as to match the drop in inflation at the ZLB. Appendix C.9.3 reports the results of this procedure. The bottom-line is that this alternative procedure results in higher output effects of QE for both high and low λ_T, but they are less extreme than the previous procedure.

3.6.3 Bayesian Approach to Estimating λ_T

In Section 3.4.2, λ_T is estimated by equating the theoretical object $D_{\pi,T}(\lambda_T)$ to $\widehat{D}_{\pi,T}$, and then computing the 68% confidence interval for λ_T by matching $\widehat{D}_{\pi,T}$ +/- one $\hat{\sigma}$, the estimated standard error of $\widehat{D}_{\pi,T}$. The object $D_{\pi,T}(\lambda_T)$, however, is highly nonlinear and a complicated function of λ_T. It is not obvious a priori that if we characterize the entire distribution of $\widehat{D}_{\pi,T}$ and back out the implied λ_T for each point in the distribution, that the resulting confidence interval corresponds closely to the confidence interval we used for λ_T in the previous sections.[41] We show below, however, that this alternative approach leads to almost identical inference.

Drawing from the distribution of $\widehat{D}_{\pi,T}$ and computing the implied distribution of λ_T has a Bayesian interpretation described in detail in Appendix B.4. Figure 12 shows the estimation results in terms of the posterior density of λ_T, where we depict the mean and the 68% credible

[41] We thank the editor for suggesting this alternative estimation procedure.

Posterior density

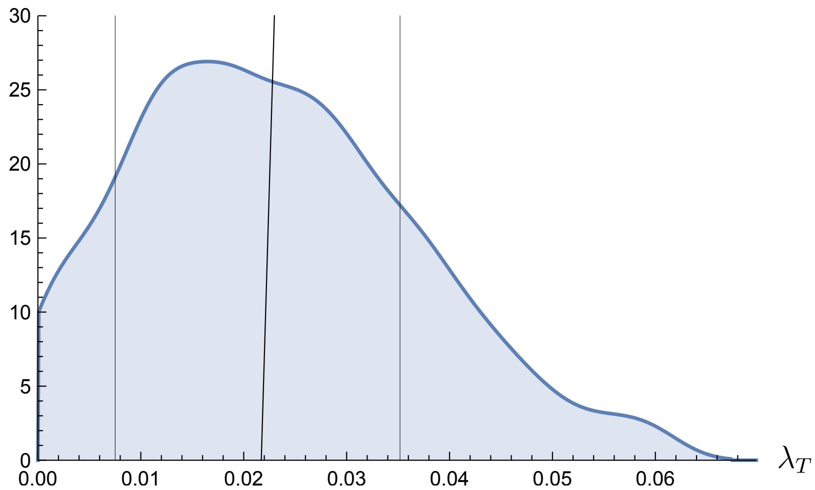

Figure 12: Posterior distribution of the tax-smoothing parameter

Note: The figure shows the posterior distribution of λ_T using a Bayesian estimation approach.

sets with vertical lines.[42] The mean is 0.022 while the 68% probability interval is (0.00787, 0.0351). These estimates are almost identical to those from our baseline approach reported in Figure 9, where the mean is 0.021 and the 68% confidence interval is (0.00804, 0.03684). These new estimates' implied confidence interval for output effects of QE2 is also essentially the same as that in Table 4, see Appendix B.4.

3.6.4 Alternative Calibration Strategies

Recalibrating to match financial-market effects. So far the strategy in the numerical experiment has been to pick the parameters based on existing literature or the data and then to pay special attention to the estimation of λ_T. We then asked, given the parameterization, how well the model could account for financial-market estimates of QE, which we did not directly target. An alternative way of parameterizing the model is to estimate directly more parameters and ask, can the model fully account for the financial-market effects of QE for parameter values that look reasonable? We address this question by calibrating $(\lambda_y,\ \kappa)$ to match KVJ's estimates that QE2 led to $\Delta i^* (8) = -16$ bp and $\Delta \pi^* (40) = 5$ bp.[43] In other words, we estimate $(\lambda_y,\ \kappa, \lambda_T, r_S^e, \mu)$ to match five moments exactly: the drop in inflation and output at the ZLB, the ratio of the inflation response to the tax response to match RZ's

[42] A small fraction of draws, approximately 3.1%, imply negative values of $\widehat{D}_{\pi,T}$. For those draws, we impose the assumption that the estimated λ_T is zero. In the posterior density plot of λ_T, we do not depict the zeros because it is continuous. In computing the mean and probability intervals, however, we do count the zeros.

[43] Note that $\lambda_y = \kappa/\theta$, and thus we are estimating θ when estimating λ_y.

evidence, and the effects of QE2 on yields and expected inflation to match KVJ's evidence.

Appendix C.9.4 contains both the parameter values from this exercise as well as the model's predictions for the output effects of QE2 and QE3. Observe, that while we choose the parameters to match the change in financial-market prices as result of QE2 and QE3, the parameterized model is needed to draw any inference about the effect these policies had on output. The bottom-line is that parameter values are similar to our baseline calibration reported earlier in Table 1 and can thus be interpreted to be relatively reasonable, since the parameters were motivated by the outside literature. The output effects of QE are also very similar to those in the baseline experiment where we did not match the empirically estimated effects of QE2 on yields and expected inflation (the effect of QE2 is 1.65% while that of QE3 is 5.15%).[44]

Sensitivity to structural model: A discounted Euler equation. A recent criticism of the standard New Keynesian model is that current variables are overly sensitive to expectations of future interest rate policy. This is dubbed the "forward guidance puzzle" by Del Negro et al. (2015). McKay et al. (2016), Gabaix (2016), and Michaillat and Saez (2019) have suggested a resolution to this puzzle: the consumption Euler equation has additional discounting, which we denote by α, with $\alpha = 1$ corresponding to the standard model. McKay et al. (2016) suggest that the forward guidance puzzle is resolved with a value of $\alpha = 0.97$. Using this as an input in the model equation directly, we re-estimate the current model using the same procedure as in last subsection. We find that the output effect of QE2 is now 1.46% while that of QE3 is 4.55%. Thus, the output effects of QE are only modestly reduced by this extension.[45]

4 Extensions and Discussion

4.1 Irrelevance of Quantitative Easing under Policy Commitment

We have shown that QE has a natural interpretation when the government cannot fully commit to future policy, i.e., in an MPE. The first fundamental assumption driving this result is that taxation carries some cost, as we made explicit in Section 2.2 by considering the case in which $\lambda_T = 0$. A second fundamental assumption is that the government is unable to commit to

[44]The reason for similar output effects is that in the baseline exercise, the model generated smaller effects on long-term yields but bigger effects on expected inflation compared with KVJ's estimates. These two forces essentially have equal but opposite effects for output in the model.

[45]It may seem surprising that the impact of QE did not decrease more. The reason is that this alternative specification leads to different values for all of the five parameters that were chosen to match empirical moments. Importantly, the experiment is thus not to change α from 1 to 0.97, holding all other parameters fixed, for in this case the model would no longer be matching the empirical moments targeted, such as the drop in output and inflation seen in 2008.

future policy apart from being able to commit to pay back the nominal value of outstanding debt. The role of the second assumption to our results is the focus of this subsection.

If the government can fully commit to future policy, then the maturity structure of government debt is irrelevant. It follows, according to our theory of QE, that QE is irrelevant as well. That full commitment renders the maturity structure of debt irrelevant is a central result in the classic paper of Lucas and Stokey (1983). It is especially simple to show the logic of their argument in the 3 period model of Section 2 but we relegate its derivation to Appendix A.3 along with some additional discussion. What we show there, formally, is that the flow budget constraints of the government, together with the ex-ante asset pricing conditions, allows us to write the government budget constraints as a single intertemporal budget constraint (IBC). The IBC, which is the relevant constraint under the optimal commitment plan, involves neither b_0 nor l_0 (and hence maturity of debt), but instead only the initial condition b_{-1}. Accordingly, the term structure imposes no constraints on the government's optimal plan under commitment.

The bottom-line, then, is that the assumption of taxation costs as well as the assumption that the government cannot commit to future policy are the two most important ingredients driving the results.[46]

4.2 An Independent Central Bank and Its Balance Sheet

A natural objection to our model of QE is that fiscal considerations are often not considered to be part of a central bank's mandate. This subsection shows an equivalence result in the model of Section 2. Suppose an independent central bank cares about its own balance sheet gains/losses in addition to the traditional dual mandate of inflation and output gap. Then, under certain conditions, the same equilibrium can be derived as if the government as a whole sets monetary and fiscal policy jointly, which is the benchmark set-up of this paper.

Consider an independent central bank with the objective:

$$-\sum_{t=0}^{2} \beta^t \{(\Pi_t - 1)^2 + \lambda_Y (Y_t - \bar{Y})^2 + \lambda_v (T_t^c - \bar{T})^2\}, \tag{46}$$

where T_t^c is its transfer to the Treasury and $\bar{T}$ is its target transfer. There are several examples

[46]For completeness, we show in Figure A.1 in Appendix A.7.1, the solution for optimal joint monetary and fiscal policy under commitment vs. optimal monetary policy alone under commitment for the numerical example of Section 2. The simulation shows that the two are close. A similar result also applies in the quantitative model. The figure highlights that not much was missed in the discussion of our paper when we focused on optimal monetary policy alone under commitment. The optimal-policy-commitment solution was also shown in Figure 6 earlier in comparison with the optimal-discretion solution when $\lambda_T > 0$.

of objectives of this form in the literature.[47] One motivation is political economy. In the US context, it seems reasonable that Congress would start asking questions if the Federal Reserve were to sustain large capital losses and not contribute the same amount to the Treasury as before.[48] Another is that the central bank *should care* about capital losses, as they eventually must be followed by taxation (or higher inflation) which in turn affects social welfare.[49]

Recently declassified minutes and memoranda from the Federal Reserve provide evidence suggesting its staff was concerned with balance sheet considerations following the crisis of 2008. One example is a memorandum prepared by Joseph Gagnon, David Lucca, Jonathan McCarthy, Julie Remeche, and Jennifer Roush for the Federal Reserve Open Market Committee (FOMC) on March 11, 2009.[50] On page 9, the authors state:

> However, to the extent that holding a large volume of long-term assets necessitates carrying a large volume of interest-earning liabilities, future Federal Reserve net income is subject to increased interest rate risk. In the example raised above, the extra Federal Reserve net income from holding $1 trillion of MBS [mortgage-backed securities] would shrink to zero if the rate of interest on reserve balances (or on reverse repos) were to rise to 4.25 percent.

Another example is a memorandum to the FOMC on April 21, 2009.[51] Its authors, Eileen Mauskopf and Jae Sim, simulate the Federal Reserve Board's FRB/US model to compute trade-offs related to balance sheet exposure and possible losses. On page 2, the authors state:

> Among the costs that we assume that the Committee will consider are those related to the volume of assets being brought onto the Federal Reserve's balance sheet. For example, holding a large portfolio of long-term securities exposes the Federal Reserve (and thus taxpayers) to appreciable capital losses if interest rates rise quickly as the economy recovers.

[47] For example, Sims (2005) introduces the constraint that central bank transfers to the Treasury cannot fall below a target. Jeanne and Svensson (2007) instead directly introduce into the objective of the central bank one-time losses if the capital of the central bank falls below a certain level. Berriel and Bhattarai (2009) use a quadratic term in central bank net worth in the loss function. See also Del Negro and Sims (2015). Perhaps a more realistic version is an asymmetric objective. That is, we can posit that the central bank is happy to pay more to the Treasury than $\bar{T}$ but does not like T_t^c to be below the target. This extension is possible but not particularly important for the QE question, as one can assume a $\bar{T}$ sufficiently high that $T_t^c < \bar{T}$ in all three periods or, alternatively, that the initial net worth of the central bank is sufficiently low.

[48] Not to mention if, as in the case of Iceland in 2008, the central bank would need a large capital injection in response to enormous capital losses, which led the government to fire the entire board of the central bank.

[49] Jeanne and Svensson (2007) and Berriel and Bhattarai (2009) provide anecdotal evidence from statements from several central bankers before the financial crisis, and they point to some recent central bank legislation.

[50] https://www.federalreserve.gov/monetarypolicy/files/FOMC20090311memo02.pdf

[51] https://www.federalreserve.gov/monetarypolicy/files/FOMC20090421memo02.pdf

Moreover, later in the QE period, other memoranda to the FOMC discussing the effects of further asset purchases not only consider macroeconomic effects, but also include simulation results concerning effects on the Federal Reserve's balance sheet and income. An example is a memorandum to the FOMC on November 30, 2012.[52] This memorandum discusses in detail how, given the simulations' assumptions of interest rate increases in the future, additional asset purchases by the Federal Reserve could lead to capital losses, an increase in interest expense, and a decline in cumulative remittances to the Treasury between 2012 and 2015. We interpret this evidence as being consistent with the idea that avoiding large balance sheet losses was part of the the objective function of the FOMC during the QE period.

Motivated by this evidence, we model the balance sheet of an independent central bank in our 3 period model. In period 0, it has one-period liabilities d_0^c (interest-bearing reserves) and both one-period (b_0^c) and two-period (l_0^c) assets. In period 1, it issues one-period liabilities, d_1^c, and buys one-period assets, b_1^c. The initial net asset position of the central bank is l_{-1}^c.

As in the analysis of a consolidated government, the budget constraints can be rewritten in terms of the minimum set of state variables in *interest-inclusive terms.* The state variable in period 0 is w_{-1}^c; in period 1, the state variables are w_0^c and x_0^c; and in period 2, the state variable is W_1^c defined as:

$$W_1^c \equiv w_1^c + x_0 \frac{w_0^c}{\Pi_1}.$$

The budget constraint of the independent central bank can now be written as:

$$0 = \frac{W_1^c}{\Pi_2} - T_2^c, \tag{47}$$

$$W_1^c = \frac{w_0^c}{\Pi_1} + i_1(1 - x_0)\frac{w_0^c}{\Pi_1} - (1 + i_1)T_1^c, \text{ and} \tag{48}$$

$$x_0 \frac{w_0^c}{1 + R_0} + (1 - x_0)\frac{w_0^c}{1 + i_0} = \frac{w_{-1}^c}{\Pi_0} - T_0^c, \tag{49}$$

where $w_0^c \equiv (1 + R_0)l_0^c + (1 + i_0)\left(b_0^c - d_0^c\right)$, $w_1^c \equiv (1 + i_1)\left(b_1^c - d_1^c\right)$, $w_{-1}^c = l_{-1}^c$, $x_0 \equiv \frac{(1+R_0)l_0^c}{(1+R_0)l_0^c+(1+i_0)\left(b_0^c-d_0^c\right)}$, and $(1-x_0) \equiv \frac{(1+i_0)\left(b_0^c-d_0^c\right)}{(1+R_0)l_0^c+(1+i_0)\left(b_0^c-d_0^c\right)}$. In this notation, w_0^c measures the net asset position of the central bank in period 0, written in terms of maturity value. Meanwhile, x_0 measures the maturity composition of the central bank's net asset position in period 0—the ratio of long-term assets to the net asset position. It thus measures the maturity mismatch in the central bank's balance sheet in period 0. Detailed derivations are in Appendix D.

This leads to the following proposition:

[52] https://www.federalreserve.gov/monetarypolicy/files/FOMC20121130memo05.pdf

Proposition 4 *Under the assumption that an independent central bank maximizes the objective (46) subject to (47)-(49), the problems of the consolidated government and independent central bank are identical if* $l^c_{-1} = -b_{-1}$ *and* $\lambda_v = \lambda_T$.

The bottom-line, then, is that our model of a consolidated government budget constraint can alternatively be interpreted from the perspective of an independent central bank. In Appendix D.3 we discuss in more detail the condition in Proposition 4 that makes the two models exactly equivalent, and how this alternative model should lead researchers to interpret the data in somewhat different way than we have done here.[53]

4.3 Forward Guidance

A policy of forward guidance is usually defined as verbal commitment by a central bank about future interest rate policy. In an MPE, however, verbal commitments have no effect. While it is extreme to assume that central bank statements have no effect on expectations, the MPE formalizes a common argument by policy makers against using forward guidance because the credibility problem reduces its effectiveness.[54]

Even if one does not accept the extreme position that central banks' words carry no weight, the analysis of the MPE is still of interest as it helps isolate what *actions* can be taken to enhance the credibility of forward guidance. There is considerable evidence that policy makers believe their statements alone are not fully credible.[55] This suggests the importance of identifying such a commitment technology.

From a practical point of view, then, the analysis of the MPE in this paper can be interpreted as a study of how QE can be used as a commitment technology to supplement forward guidance. A central bank can make a particular statement about future interest rate policy. If this forward guidance is not believed, then the central bank can engage in QE until market

[53]We make an initial attempt to clarify the relevant data for an independent central bank in our NBER WP version of this paper (No. 21336), but have omitted this extension here to conserve on space.

[54]Nakata (2014) is an analysis of credible plans which incorporates reputation considerations. There is a related literature estimating the effect of forward guidance highlighting that it can either signal change in policy or information about future state of the economy (Campbell et al. (2012) and Andrade and Ferroni (2016)). Since our model assumes full information, it does not capture this interesting feature of forward guidance. If there was incomplete information, our result would still apply under the assumption that the government is committed to revealing the true state of the economy.

[55]For example, John Williams, then president of the Federal Reserve Bank of San Francisco, noted this in an FOMC meeting in 2011 when discussing the possibility of adopting more aggressive forward-guidance language: "In the jargon of academics, our commitment technology is very limited. It is simply impossible for us to set a predetermined course of policy that will bind future Committees." Similarly, the current chair of the Federal Reserve, Jay Powell, noted in the context of policies that involved committing to a future expansion at the ZLB: "Part of the problem is that when the time comes to deliver the inflationary stimulus, that policy is likely to be unpopular, what is known as the time consistency problem in economics."

expectations of future interest rates adjust according to the central bank's intent.[56]

4.4 Empirical Effects of Changing Debt Maturity

The empirical evidence on the effect of changes in the maturity of government debt on interest rates (outside of the financial crisis or ZLB) is relatively small. The existing evidence, however, is largely consistent with the model, even in rough orders of magnitude. A direct mapping to our model, however, is challenging.[57] Previously, we focused on the degree to which the model matched the evidence at the ZLB presented by KVJ, because the mapping of their estimate can be directly translated to the model at the ZLB. Here we focus on the degree to which the model can account for the empirical evidence outside of the ZLB.

This evidence is summarized in Tables 5 and 6. Table 5 compares the implied reduction in long-term rates in response to a one-year maturity reduction, comparing the output of our model to the estimate by Chada et al. (2013), who use a regression analysis to estimate the effect of debt-maturity reduction on interest rates. As the table reveals, the implied reduction in long rates according to our model (at positive interest rates) is less than one-fourth of that predicted by their estimation, suggesting that our calibration is relatively conservative.

The evidence in Table 5 maps relatively cleanly into our analysis as we can compare the reduction in debt maturity in the study in question to our model. Comparison to other studies requires an additional assumption, since they typically only report the size of policy intervention in long-term bonds as a fraction of GDP. Thus, it is only under the assumption that the composition of long-term bonds purchased in the intervention is exactly the same as in the model that the magnitudes are comparable.

Table 5: Effects of debt-maturity reduction on long-term rates outside the zero lower bound

Source	Policy intervention	Change in 5-year forward 10-year rate
Chada et al. (2013)	1-year maturity reduction	−130 to −150 bp
Out of ZLB model prediction	1-year maturity reduction	−32 bp

With this caveat in mind, Table 6 suggests, again, that the reduction in 10-year yields is a bit more conservative in our model relative to empirical studies we consider. Hamilton and

[56] The idea that QE can be used to demonstrate resolve and make the optimal commitment incentive compatible has some history in the literature. For a discussion, see Clouse et al. (2003) and Eggertsson and Woodford (2003), though neither formalizes the idea.

[57] A key difficulty with interpreting the estimates discussed below is that they typically only report the total effect of the changes in maturity on long-term rates, without determining whether this change was associated with changes in expectations about future short-term rates or with changes in term- or risk-premia. A notable exception is KVJ, which is why we used their estimates for the expected short-term rate effects of QE2 as an example of an untargeted moment. It is not obvious, however, whether decomposing the total effect is easy in practice. Changes in expected future short-term rates may endogenously change the term- and risk-premia.

Wu (2012) use an affine term structure model to estimate the effect of purchasing long-term debt on long-term rates using pre-crisis data. As the table suggests, each percentage point of intervention (as a fraction of GDP) leads to a reduction in 10-year yields of about 4.8 bp. Swanson (2011) uses an event-study approach focusing on the Federal Reserve's Operation Twist in 1961. As the table reveals, each percentage point of intervention (as a fraction of GDP) leads to a drop in 10-year yields of 7.6-9.4 bp. The last row in Table 6 considers the effect of QE2 in the case when interest rates are positive in our model. As it indicates, an intervention corresponding to 1 percent of GDP reduces 10-year yields by 2.8 basis points. Thus, the evidence reported in Table 6 is largely consistent with the one presented in Table 5: Our benchmark calibration yields a response of long-term yields that is lower than what these studies document.[58]

Table 6: Effects of Quantitative Easing on long-term yields

Source	**Policy intervention**	**Change in 10-year yields**
Hamilton and Wu (2012)	2.9% of GDP, 2006	-14 bp
Swanson (2011)	1.7% of GDP, 1961	-13-16 bp
Model implied QE2 out of ZLB	4% of GDP, 2010	-11.39 bp

There is also empirical literature related to Figure 11, which shows transition dynamics for debt maturity and debt in the quantitative model. As the figure reveals, the government increases the maturity of its debt as it reduces its debt position. One interpretation of this result is that in a cross-section of countries, those with higher government debt tend to have lower debt maturity and higher inflation, assuming they are in a transition to steady-state. This is the empirical pattern documented in Blanchard and Missale (1994).[59] Finally, Greenwood and Vayanos (2010) consider the effect of the US Treasury buyback program in 1999–2000, which was on the order of $63.5 billion. They argue that this intervention led to a substantial drop in long-term rates at various maturities, even for maturities of Treasuries that the government did not buy back. Such an effect can be rationalized by our model, as QE leads to a decrease in expected short rates.

[58] Bordo and Sinha (2016) study an intervention by the Federal Reserve during the Great Depression which included the purchase of both medium- and long-term government debt and equaled 2% of GDP. They report a reduction in yield for Treasury bonds (with an average maturity of eighteen years) of 19 to 42 bp. As our focus here is on evidence away from the ZLB, we do not include this estimate in Tables 5 and 6.

[59] This is, however, a different interpretation from the one emphasized by Missale and Blanchard (1994). A similar finding is reported by Rose and Spiegel (2015), who show that countries that have a bond market for long-term debt (and thus presumably have a longer debt maturity) tend to experience lower inflation.

4.5 Related Theoretical Literature

A common theme in the literature on inflation and government debt is that more long-term debt gives the government *a stronger incentive to inflate*, not a weaker incentive as this paper shows (see e.g., Missale and Blanchard (1994)). Similarly, empirical analyses often stress that the fiscal benefits of inflation are higher the longer is the maturity of debt (see e.g., Aizenman and Marion (2011), Doepke and Schneider (2008), and Hilscher et. al. (2018)). This line of research does not contradict our result. The reason for the seemingly different findings is that these studies consider the fiscal benefits of inflation over an extended period—longer than the duration of the short-term debt and often of similar duration as the long-term debt.[60] This results in a fundamentally different game than the one in this paper, where the government re-optimizes period by period. In Appendix E we flesh out this argument in detail.

The dynamic inconsistency problem created by the existence of government debt when taxation is costly is a theme of Lucas and Stokey (1983) and the rich subsequent literature.[61] Lucas and Stokey's solution to the problem is a careful manipulation of the maturity structure of debt, i.e., going long, making the optimal solution time consistent. Persson et al. (1987, 2006), Calvo and Obstfeld (1990), and Alvarez et al. (2004) show under what conditions a similar strategy can be implemented (and when it cannot) in a flexible price monetary economy. Calvo and Guidotti (1990,1992) are two other examples that touch on the same themes. Broadly speaking, the key idea in this paper can be interpreted as the Lucas and Stokey (1983) solution in reverse. While their strategy is to lengthen the duration of government debt in order to eliminate the government's incentive to reduce the real interest rate in the future, QE accomplishes the opposite in order to give the government the incentive to keep real interest rates low in the future, as needed at the ZLB. Unlike this literature, this paper considers both the ZLB and price rigidities – these two frictions are fundamental for generating the results.[62]

This paper follows the work cited above by assuming the government never defaults on its debt. This is a reasonable assumption for large countries that issue debt in their own currencies. This assumption, however, is less plausible for small open economies that contract their debt in foreign currency. There is a considerable literature that focuses on small open-economy models which allow for the possibility that the government defaults in certain

[60]Missale and Blanchard (1994) directly assume a reduced-form sensitivity of debt to inflation that is decreasing in the maturity of debt, and the actions of the government in the game they present are best interpreted as corresponding to the actions of a government choosing an inflation policy regime over an extended period (see Appendix E for more details). In Aizenman and Marion (2011), the duration of the inflation policy is the same as that of long-term debt. Hilscher et al. (2018) consider the empirically estimated expected-inflation scenarios extracted from asset markets, while Doepke and Schneider (2008) consider an unexpected increase in inflation for ten years.

[61]Calvo (1978) represents an important early paper in this literature.

[62]There is also a literature emphasizing that, in the absence of state-contingent bonds, the maturity structure of debt can be used to hedge against fiscal shocks; see Angeletos (2002) and Leeper and Zhou (2013).

states of the world; see, e.g., Aguiar et al. (2019) and Arellano and Ramanarayanan (2012). This branch of the literature shows that the possibility of default also leads to a dynamic-inconsistency problem for the government. Moreover, it shows that shortening the maturity structure of government debt alleviates the dynamic inconsistency problem by creating fiscal discipline for future governments. This may seem to contradict the closed economy literature which instead suggests that the government should lengthen its debt maturity to avoid the incentive to inflate. In followup work, Bhattarai et al. (2022), we show how the two literatures can be reconciled. The incentive mechanism explored in the open economy literature is closely related to what we term the *rollover incentive* in Section 2.3.2; see Bhattarai et al. (2022) for details.

Another argument for the effect of QE is that it works through a portfolio balance channel. The most common way of modeling the portfolio balance channel is via preferred habitat motives and market segmentation. Chen et al. (2012) is an example that incorporates such a friction, and finds a small role for the portfolio balance effect, with most effects coming via a commitment to hold future interest rates low, a mechanism formalized here. [63]

Other papers, such as Gertler and Karadi (2011, 2012) and Del Negro et al. (2019), provide frameworks in which asset purchases by the central bank have an effect because of financial imperfections and limits to arbitrage. Those mechanisms can account for a substantial effect of QE1, which included purchases of private securities at a time of considerable market dysfunction. It is more difficult, however, to make such a case for QE2 and QE3, when the market dysfunction was much smaller and the policy mostly involved buying long-term government bonds. This is why QE2 is the benchmark for the model calibration in this paper.

5 Conclusion

This paper develops a model of QE policy in which the maturity of government debt decreases. Faced with a binding ZLB and ensuing recession and deflation, a government policy that shortens government debt maturity improves on outcomes, as QE generates expectations of low future real interest rates. The key ingredient for the mechanism is that when policy is set in a time-consistent manner, there exists a role for manipulating the maturity of government debt to generate a credible commitment to future expansionary policy. We first showed these results theoretically in a simple setting and then assessed their quantitative relevance.

Future work can extend ours both empirically and theoretically. While the focus in this

[63] There is no portfolio balance channel in this paper. This is not, however, because long- and short-term bonds are perfect substitutes. As explained in Eggertsson and Woodford (2003)(see p. 159), the model can be extended to incorporate different risk characteristics of long- and short-term bonds and, thus, a term- and risk-premia. That extension will not change the results in this paper for the same reasons discussed there.

paper was on QE episodes, the mechanism also holds in reverse. For instance, during the "taper tantrum" episode in 2013, when Ben Bernanke announced that the Fed would taper QE in the future, federal funds futures suggested markets expected future short-term rates to rise.[64] We leave a further analysis of such episodes for future research.

[64] We are indebted to Jim Bullard, president of the Federal Reserve Bank of St. Louis, for educating us on this empirical pattern in an insightful discussion of our paper.

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Appendix

A Details on the Simple Model

This section contains detailed microfoundations of the simple model and a derivation showing the utility function of the government used in text is equivalent to a second-order approximation of the household utility. It also contains additional results mentioned in the text.

A.1 Environment

There are three periods, 0, 1, and 2 (short, medium, and long run respectively). A representative household maximizes

$$E_0 \sum_{t=0}^{2} \beta^t \left[\log C_t - \chi H_t\right] \xi_t$$

subject to

$$P_0 C_0 + B_0 + L_0 = B_{-1} + \int_0^1 Z_0(i)di + N_0 H_0 - P_0 T_0, \tag{A.1}$$

$$P_1 C_1 + B_1 = (1+i_0)B_0 + \int_0^1 Z_1(i)di + N_1 H_1 - P_1 T_1, \tag{A.2}$$

$$P_2 C_2 = (1+i_1)B_1 + (1+R_0)L_0 + \int_0^1 Z_2(i)di + N_2 H_2 - P_2 T_2, \tag{A.3}$$

where C_t is consumption, H_t hours, and ξ_t a preference shock. B_{-1} is the initial wealth the household holds in period 0; B_0 and B_1 are one-period nominal risk-free bonds; and L_0 is long-term nominal bonds issued in period 0 and repaid in period 2. The short-term interest rates in periods 0 and 1 are i_0 and i_1, while R_0 is the long-term nominal interest rate. N_t is the nominal wage rate, P_t is the price index, $Z_t(i)$ is firms' profits where (i) indexes firms, and T_t is taxes. The model abstracts from money.[65]

The intertemporal dimension of the household maximization yields first-order conditions for the one-period risk-free bonds:

$$\frac{1}{1+i_t} = \beta E_t \left[\frac{Y_t}{Y_{t+1}} \Pi_{t+1}^{-1} \frac{\xi_{t+1}}{\xi_t}\right] \text{ for } t = 0, 1, \tag{A.4}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ and we substitute the equilibrium condition $C_t = Y_t$, where Y_t is aggregate

[65] Since money and bonds are perfect substitutes at zero interest rates, this abstraction seems less important in the current context than in many other applications.

output derived below.[66] Similarly it yields a first-order condition for the long-term bond:

$$\frac{1}{1+R_0} = \beta^2 E_0 \left[\frac{Y_0}{Y_2}\Pi_1^{-1}\Pi_2^{-1}\frac{\xi_2}{\xi_0}\right]. \tag{A.5}$$

There is a zero bound on the nominal interest rate[67]:

$$i_t \geq 0 \text{ for } t = 0, 1. \tag{A.6}$$

Optimal labor supply satisfies the standard first-order condition:

$$\frac{N_t}{P_t} = \chi Y_t \text{ for } t = 0, 1, 2. \tag{A.7}$$

Consumption is a Dixit-Stiglitz aggregator of the varieties, $C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$, and the optimal price index is

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}, \tag{A.8}$$

where $\theta > 1$ is the elasticity of substitution across goods with (i) indexing good varieties. The intratemporal dimension of the household maximization problem yields a demand for each good variety $c_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} C_t$.

For each good variety, there is a single firm, so $y_t(i) = c_t(i)$. The firm has a linear production function $y_t(i) = h_t(i)$. The firm maximizes profits $Z_t(i) = (1+\tau)p_t(i)y_t(i) - N_t y_t(i)$, facing the demand $y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} Y_t$ where τ is a production subsidy and $Y_t = C_t$.

In periods 0 and 1, a fraction γ of firms set prices flexibly, yielding the first-order condition

$$(1+\tau)\frac{p_t^{opt}}{P_t} = \frac{\theta}{\theta-1}\frac{N_t}{P_t}.$$

Substituting for $\frac{N_t}{P_t}$ from (A.7) and setting $1+\tau = \frac{\theta}{\theta-1}$ to eliminate markups gives

$$\frac{p_t^{opt}}{P_t} = \frac{Y_t}{\bar{Y}},$$

where $\bar{Y} = \chi^{-1}$ is steady-state output.[68]

In periods 0 and 1, a fraction $1-\gamma$ of firms index their prices to the aggregate price level

[66] Thus, all output is consumed.

[67] The lower bound could be different from zero (for example, slightly negative) without changing the results.

[68] The production subsidy is set to abstract from the inflation bias. For an extension incorporating the inflation bias at the ZLB, see Eggertsson (2006). Throughout, a steady state, that is, the value of a variable x in the absence of shocks, is denoted by $\bar{x}$. In the flexible price steady state, then, $\frac{p_t^{opt}}{P_t} = 1$ and $\chi = \bar{Y}^{-1}$.

from last period:

$$\frac{p_t^{ind}}{P_t} = z_t \frac{P_{t-1}}{P_t},$$

where z_t is a term introduced for technical reasons. The price index (A.8) satisfies

$$P_t^{1-\theta} = \gamma (p_t^{opt})^{1-\theta} + (1-\gamma)(p_t^{ind})^{1-\theta}.$$

A fraction of firms, γ, sets prices every period, yielding the first-order condition

$$(1+\tau)\frac{p_t^{opt}}{P_t} = \frac{\theta}{\theta - 1}\frac{N_t}{P_t},$$

where the productive subsidy is set to eliminate markups so that

$$\frac{p_t^{opt}}{P_t} = \chi Y_t,$$

where we have also imposed $Y_t = C_t$. Denote the steady state by $\bar{Y}$. In the steady state, then, $\frac{p_t^{opt}}{P_t} = 1$ so that $\chi = \bar{Y}^{-1}$. Hence the first-order condition is written as

$$\frac{p_t^{opt}}{P_t} = \frac{Y_t}{\bar{Y}}.$$

A fraction $1-\gamma$ index their price to the previous-period price index $z_t P_{t-1,}$ where we introduce z_t for technical reasons to be clarified, yielding

$$\frac{p_t^{ind}}{P_t} = z_t \frac{P_{t-1}}{P_t}.$$

The price index then satisfies

$$P_t^{1-\theta} = \gamma (p_t^{opt})^{1-\theta} + (1-\gamma)(p_t^{ind})^{1-\theta},$$

or

$$1 = \gamma \left(\frac{p_t^{opt}}{P_t}\right)^{1-\theta} + (1-\gamma)\left(\frac{p_t^{ind}}{P_t}\right)^{1-\theta} = \gamma \left(\frac{Y_t}{\bar{Y}}\right)^{(1-\theta)} + (1-\gamma)\left(z_t \frac{P_{t-1}}{P_t}\right)^{1-\theta}.$$

For simplicity, $\theta = 2$. Then,

$$1 = \gamma \left(\frac{Y_t}{\bar{Y}}\right)^{-1} + (1-\gamma)\left(z_t^{-1}\frac{P_t}{P_{t-1}}\right),$$

or

$$(1-\gamma)\left(z_t^{-1}\left(\frac{P_t}{P_{t-1}}\right)-1\right)=\gamma\left(\frac{Y_t-\bar{Y}}{Y_t}\right). \tag{A.9}$$

This finally yields

$$(\Pi_t-1)=\frac{\gamma}{1-\gamma}(Y_t-\bar{Y}) \tag{A.10}$$

for $z_t=\frac{P_t}{Y_t^{-1}(P_t-P_{t-1})+P_{t-1}}$. Relative to (A.9) with $z_t=1$, (A.10) is slightly simpler as it involves $Y_t-\bar{Y}$ rather than $\frac{Y_t-\bar{Y}}{Y_t}$. But these two are the same to a first-order because z_t is 1 in steady state and small numerically.[69] Using (A.10) simplifies the algebra slightly, which is why it is used in the paper.

Thus, we have the simple Phillips curve in periods 0 and 1

$$(\Pi_t-1)=\kappa(Y_t-\bar{Y}) \text{ for } t=0,1, \tag{A.11}$$

where $\kappa\equiv\frac{\gamma}{1-\gamma}$. In period 2, all firms set prices freely:

$$Y_2=\bar{Y}=\chi^{-1}. \tag{A.12}$$

The model abstracts from government spending. The government inherits debt B_{-1}, which needs to be paid back. The key assumption of this paper is that, as in Barro (1979), taxes are not a one-to-one transfer of purchasing power from individuals to the government. Instead, taxes require some collection and/or indirect misallocation costs. More concretely, the "production" of government revenues, T_t, requires hiring labor corresponding to $f(T_t)$ hours, where $f(.)$ is increasing and convex in T_t.[70] Since the production function of the firms is linear in labor, total hours worked are $H_t=Y_t\Delta_t+f\left(T_t\right)$, where Δ_t is a price dispersion term. Because utility is linear in labor, taxes do not have any effect on relative prices (such as wages) and they do not distort allocations. Taxes, however, have a negative effect on social welfare by requiring labor input. The model thus captures a welfare cost of taxes while abstracting from allocation effects.[71]

The government utility function is

$$-\frac{1}{2}E_0\sum_{t=0}^{2}\beta^t\left[\lambda_\pi(\Pi_t-1)^2+\lambda_y(Y_t-\bar{Y})^2+\lambda_T(T_t)^2\right], \tag{A.13}$$

where λ_π, λ_y, and λ_T are weights that are functions of structural parameters. This utility

[69]Imposing $z_t=1$ would imply exact indexation of nonadjusting prices to the past aggregate price.

[70]Here, T_t denotes tax revenues net of government workers' salaries.

[71]In the quantitative model, utility is not assumed to be linear in labor, but the implications for taxes are similar. The reason for this is that taxes only affect allocations up to a second order in the quantitative model.

function is derived via a second-order Taylor expansion of the representative household utility in periods 0 and 1, as shown in Section A.2 below.[72] The term involving taxes appears due to the taxation costs, where $\lambda_T \equiv \chi f''(\bar{T})$ and $\bar{T}$ is steady-state taxes. Going forward, we normalize the weight on inflation to 1.

The flow budget constraints of the government, derived as counterparts to (A.1)-(A.3), are

$$b_0 + l_0 = \frac{b_{-1}}{\Pi_0} - T_0, \tag{A.14}$$

$$b_1 = (1+i_0)\frac{b_0}{\Pi_1} - T_1, \tag{A.15}$$

$$0 = (1+i_1)\frac{b_1}{\Pi_2} + (1+R_0)\frac{l_0}{\Pi_1\Pi_2} - T_2, \tag{A.16}$$

where $\frac{B_{-1}}{P_{-1}} = b_{-1}$, $\frac{B_0}{P_0} = b_0$, $\frac{L_0}{P_0} = l_0$, and $\frac{B_1}{P_1} = b_1$.

A.2 Loss Function

For the welfare-relevant loss function derivation, we need to first derive the production function in "aggregate hours" terms and where a second-order term related to price dispersion will appear as in the standard New Keynesian model. Denote hours used for private production by H_t^p and hours used for tax collection by H_t^g.

We have

$$H_t^p = \int_0^1 h_t(i)di,$$

$$C_t = Y_t.$$

This gives

$$H_t^p = \int_0^1 y_t(i)di = Y_t \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{-\theta} di = Y_t\Delta_t,$$

where price dispersion is

$$\Delta_t \equiv \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{-\theta} di \geq 1.$$

Firms that do not adjust their prices choose

$$p_t^{ind} = z_t P_{t-1},$$

[72] The utility function derived from a second-order approximation, as opposed to the exact utility function, is used because it makes the illustration slightly simpler and more transparent.

so that

$$\Delta_t = \gamma \left(\frac{p_t^{opt}}{P_t}\right)^{-\theta} + (1-\gamma)\left(\frac{\Pi_t}{z_t}\right)^{\theta}.$$

From before, $\frac{p_t^{opt}}{P_t} = \chi Y_t$ such that

$$\Delta_t = \gamma \left(\chi Y_t\right)^{-\theta} + (1-\gamma)\left(\frac{\Pi_t}{z_t}\right)^{\theta}.$$

The aggregate production function is thus

$$H_t^p = Y_t \Delta_t = Y_t \left[\gamma \left(\chi Y_t\right)^{-\theta} + (1-\gamma)\left(\frac{\Pi_t}{z_t}\right)^{\theta}\right].$$

Utility is then

$$U_t = \log C_t - \chi H_t^p - \chi H_t^g = \log Y_t - \chi^{1-\theta}\gamma Y_t^{1-\theta} - \chi(1-\gamma)Y_t\left(\frac{\Pi_t}{z_t}\right)^{\theta} - \chi f(T_t).$$

Finally, substituting for z_t we have

$$\log C_t - \chi H_t^p - f(T_t) = \log Y_t - \chi^{1-\theta}\gamma Y_t^{1-\theta} - \chi(1-\gamma)Y_t\left(Y_t^{-1}\left(\Pi_t - 1\right) + 1\right)^{\theta} - f(T_t).$$

A second-order expansion of this around the efficient steady state yields

$$U_t \approx -\frac{1}{2}\chi^2 (1-\gamma)\,\theta\,(\theta-1)\left[(\Pi_t - 1)^2 + \left(\frac{1-\gamma\theta\,(1-\theta)}{(1-\gamma)\,\theta\,(\theta-1)}\right)(Y_t - \bar{Y})^2\right] - \chi f''\left(\bar{T}\right)T_t^2,$$

which is the loss function we use in the text.

A.3 The Optimal Commitment Problem

To study the optimal monetary and fiscal policy under commitment, we derive a single intertemporal budget constraint of the government. First, combine the three flow budget constraints ((A.14), (A.15), and (A.16)) from Appendix A.1. Then impose ex ante asset-pricing conditions (A.4) and (A.5). Then the government's problem is only subject to the present-value budget constraint

$$\left(\frac{\Pi_2}{1+i_1}\right)\left(\frac{\Pi_1}{1+i_0}\right)T_2 + \left(\frac{\Pi_1}{1+i_0}\right)T_1 + T_0 = \frac{b_{-1}}{\Pi_0};$$

terms involving b_0 and l_0 separately, which determine the maturity of the debt in period 0, are eliminated, thus playing no role in the optimal plan. What can be determined is the total stock of debt, b_0+ l_0, using the flow budget constraint (A.14). This is a restatement of the finding by Lucas and Stokey (1983) that optimal policy under commitment pins down a time path for taxes but leaves the maturity of debt indeterminate.[73]

What is key here, and differentiates the optimal commitment problem from the MPE, is that we can substitute out the asset pricing conditions (A.4) and (A.5) to derive the present value budget constraint, that is, the intertemporal budget constraint (IBC) we refer to in the main text. These conditions depend on expectations about future endogenous variables, which under optimal commitment, the government controls (subject to all the constraints in the model). In the MPE, however, the government can only affect these variables indirectly via the expectation functions that depends on the endogenous state variables, and were a key part of deriving the MPE.

The monetary and fiscal commitment problem then is solved by formulating the Lagrangian, where ϕ_i and γ_i are Lagrange multipliers

$$L_0 = \left\{ \begin{array}{c} \frac{1}{2}\left(\Pi_0 - 1\right)^2 + \frac{1}{2}\lambda_T T_0^2 + \frac{1}{2}\lambda_y \left(Y_0 - \bar{Y}\right)^2 \\ +\beta\frac{1}{2}\left(\Pi_1 - 1\right)^2 + \beta\frac{1}{2}\lambda_T T_1^2 + \beta\frac{1}{2}\lambda_y\left(Y_1 - \bar{Y}\right)^2 + \beta^2\frac{1}{2}\left(\Pi_2 - 1\right)^2 + \beta^2\frac{1}{2}\lambda_T T_2^2 \end{array} \right\}$$

$$+\phi_1\left\{\left(\frac{\Pi_2}{1+i_1}\right)\left(\frac{\Pi_1}{1+i_0}\right)T_2 + \left(\frac{\Pi_1}{1+i_0}\right)T_1 + T_0 - \frac{w_{-1}}{\Pi_0}\right\}$$

$$+\phi_2\left\{\frac{1}{1+i_0} - \beta_1\frac{Y_0}{Y_1}\frac{1}{\Pi_1}\right\} + \phi_3\left\{\frac{1}{1+i_1} - \beta\frac{Y_1}{Y}\frac{1}{\Pi_2}\right\}$$
$$+\phi_4\left\{\left(\Pi_0 - 1\right) - \kappa\left(Y_0 - \bar{Y}\right)\right\} + \phi_5\left\{\left(\Pi_1 - 1\right) - \kappa\left(Y_1 - \bar{Y}\right)\right\} + \gamma_0\{i_0\} + \gamma_1\{i_1\}$$

and computing the derivatives with respect to L_0 for all the endogenous variables $\{\Pi_0$, Y_0, T_0, Π_1, Y_1, T_1, i_0, i_1, Π_2, $T_2\}$ and setting them to zero. For the two interest rates, there are complementary slackness conditions:

$$\gamma_j \geq 0,\ i_j \geq 0,\ i_j\gamma_j = 0.$$

This presentation also nests the case of optimal monetary commitment only, where we set $\lambda_T = 0$, that is discussed in Section 2.2 in the text.

[73]Important for their result, but not ours, is that the government can issue state-contingent bonds that can be used to react to stochastic variations in government spending. The simple model assumes perfect foresight, and so we do not need state-contingent bonds. Generally, in a model with uncertainty, one needs to assume that the government can issue state-contingent bonds for the maturity structure of debt to be irrelevant.

A solution is defined as the set of endogenous variables that solve the first-order conditions of the government's problem, the constraints, and the complementary slackness condition. The solution is computed by guessing and verifying when the ZLB is binding, in which case it boils down to finding a solution to a system of nonlinear equations that can be solved numerically. Mathematica codes that do this are available online.

Note that immediately from the problem, it is clear that $\Pi_2 = 1$, as there is no reason for the government to inflation in period 2, once it can commit ex-ante in period 0 to future course of policy. This holds as long as i_1 is not constrained by ZLB, which is what we consider in the paper. For this reason, as discussed in text, the optimal discretionary and commitment solution will never coincide fully, as in the MPE with nominal debt, there will be some inflation in period 2.

A.4 The MPE in period 2

Government's problem is to

$$V^2(W_1) = \max_{T_2,\Pi_2}\{-\frac{1}{2}(\Pi_2 - 1)^2 - \frac{1}{2}\lambda_T(T_2)^2\}$$

s.t.

$$T_2 = W_1 \left(\Pi_2\right)^{-1}.$$

The Lagrangian is

$$L = \left\{\frac{1}{2}\left(\Pi_2 - 1\right)^2 + \frac{1}{2}\lambda_T T_2^2\right\} + \phi_2 \left\{T_2 - W_1 \Pi_2^{-1}\right\}$$

where ϕ_2 is the Lagrange multiplier. The FOCs are

$$\frac{\partial L}{\partial \Pi_2} = (\Pi_2 - 1) + \phi_2 W_1 \left(\Pi_2\right)^{-2} = 0,$$

$$\frac{\partial L}{\partial T_2} = \lambda_T T_2 + \phi_2 = 0.$$

Consolidate the two FOCs above to get

$$(\Pi_2 - 1) = \lambda_T T_2 W_1 \left(\Pi_2\right)^{-2},$$

which is (12) in text. Next, we can use the flow budget constraint to replace for T_2 and get

$$\left(\Pi_2\right)^3 (\Pi_2 - 1) - \lambda_T \left(W_1\right)^2 = 0,$$

which is (13) in text, as well as the "targeting rule" in period 2

$$\Pi_2\left(\Pi_2-1\right)=\lambda_T T_2^2.$$

The envelope theorem gives

$$\frac{\partial V^2}{\partial W_1}=\lambda_T\frac{W_1}{(\Pi_2)^2},$$

while the implicit function theorem gives

$$\frac{\partial \Pi_2}{\partial W_1}=2\lambda_T\frac{W_1}{4(\Pi_2)^3-3(\Pi_2)^2},$$

which are (16) and (17) in text.

A.5 The MPE in period 1

The problem is

$$V^1(x_0,w_0)=-\frac{1}{2}(1+\frac{\lambda_y}{\kappa^2})(\Pi_1-1)^2-\frac{1}{2}\lambda_T(T_1)^2+\beta V^2(W_1)$$

s.t.

$$\begin{aligned}
W_1&=\frac{w_0}{\Pi_1}+i_1(1-x_0)\frac{w_0}{\Pi_1}-(1+i_1)T_1,\\
1+i_1&=\beta^{-1}(\frac{\bar{Y}}{\kappa^{-1}\Pi_1-\kappa^{-1}+\bar{Y}})\bar{\Pi}^2(W_1),
\end{aligned}$$

where the Phliips curve $(\kappa^{-1}\left(\Pi_1-1\right)=\left(Y_1-\bar{Y}\right))$ has already been imposed in the loss function and the Euler equation. We rewrite the constraints as a single constraint

$$\begin{aligned}
W_1=\frac{w_0}{\Pi_1}&+\{\beta^{-1}(\frac{\bar{Y}}{\kappa^{-1}\Pi_1-\kappa^{-1}+\bar{Y}})\bar{\Pi}^2(W_1)-1\}(1-x_0)\frac{w_0}{\Pi_1}\\
&-\beta^{-1}(\frac{\bar{Y}}{\kappa^{-1}\Pi_1-\kappa^{-1}+\bar{Y}})\bar{\Pi}^2(W_1)T_1.
\end{aligned}$$

The Lagrangian is

$$\begin{aligned}
L=&-\frac{1}{2}(1+\frac{\lambda_y}{\kappa^2})(\Pi_1-1)^2-\frac{1}{2}\lambda_T(T_1)^2+\beta V^2(W_1)\\
&-\phi_{w,1}\left[\begin{array}{c}W_1-\frac{w_0}{\Pi_1}-\{\beta^{-1}(\frac{\bar{Y}}{\kappa^{-1}\Pi_1-\kappa^{-1}+\bar{Y}})\bar{\Pi}^2(W_1)-1\}(1-x_0)\frac{w_0}{\Pi_1}\\+\beta^{-1}(\frac{\bar{Y}}{\kappa^{-1}\Pi_1-\kappa^{-1}+\bar{Y}})\bar{\Pi}^2(W_1)T_1\end{array}\right]
\end{aligned}$$

The FOCs are

$$
\begin{aligned}
\frac{\partial L}{\partial \Pi_1} &= -(1+\frac{\lambda_y}{\kappa^2})(\Pi_1-1) - \phi_{w,1}\{\frac{w_0}{\Pi_1^2} + \beta^{-1}\left(\frac{\bar{Y}\kappa^{-1}}{(\kappa^{-1}\Pi_1-\kappa^{-1}+\bar{Y})^2}\right)\bar{\Pi}^2(W_1)(1-x_0)\frac{w_0}{\Pi_1}\} \\
&\quad - \phi_{w,1}\{\{\beta^{-1}(\frac{\bar{Y}}{\kappa^{-1}\Pi_1-\kappa^{-1}+\bar{Y}})\bar{\Pi}^2(W_1)-1\}(1-x_0)\frac{w_0}{(\Pi_1)^2}\} \\
&\quad - \phi_{w,1}\{-\beta^{-1}\left(\frac{\bar{Y}\kappa^{-1}}{\left(\kappa^{-1}\Pi_1-\kappa^{-1}+\bar{Y}\right)^2}\right)\bar{\Pi}^2(W_1)T_1\} \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial W_1} &= \beta V_W^2(W_1) \\
&\quad - \phi_{w,1}\{1-\beta^{-1}(\frac{\bar{Y}}{\kappa^{-1}\Pi_1-\kappa^{-1}+\bar{Y}})\bar{\Pi}_W^2(W_1)(1-x_0)\frac{w_0}{\Pi_1} + \beta^{-1}(\frac{\bar{Y}}{\kappa^{-1}\Pi_1-\kappa^{-1}+\bar{Y}})\bar{\Pi}_W^2(W_1)T_1\} \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial T_1} &= -\lambda_T T_1 - \phi_{w,1}(\beta^{-1}(\frac{\bar{Y}}{\kappa^{-1}\Pi_1-\kappa^{-1}+\bar{Y}})\bar{\Pi}^2(W_1)) \\
&= 0
\end{aligned}
$$

where note that from the period 2 problem we have

$$(\Pi_2)^4 - (\Pi_1)^3 - \lambda_T(W_1)^2 = 0,$$

and

$$V_W^2 = \lambda_T\frac{W_1}{(\Pi_2)^2},\ \bar{\Pi}_W^2 = 2\lambda_T\frac{W_1}{4(\Pi_2)^3-3(\Pi_2)^2}.$$

Equilibrium in period 1, $\{\Pi_1, T_1, W_1, \phi_{w,1}\}$, is then the solution to the three first-order conditions above and the budget constraint (where again, the Phillips curve and Euler equation have already been imposed), for given w_0, x_0. $\{i_1, Y_1\}$ can then be solved for using the Phillips curve and Euler equations.

Consolidating the two FOCs wrt Π_1 and T_1 gives

$$\{1+\frac{\lambda_y}{\kappa^2}\}(\Pi_1-1) = \frac{\lambda_T T_1}{(1+i_1)}\{\frac{w_0}{\Pi_1^2}+\frac{\kappa^{-1}\Pi_1}{Y_1}(1+i_1)(1-x_0)\frac{w_0}{\Pi_1^2}+(i_1)(1-x_0)\frac{w_0}{\Pi_1^2}-\frac{\kappa^{-1}}{Y_1}(1+i_1)T_1\}$$

which is (24) in text.

A.6 Proofs

This section contains the proofs of the Propositions in the simple model mentioned in the text that rely on an approximation around a no tax-smoothing point.

A.6.1 Period 2

The approximation point is $\lambda_T = 0$, $\Pi_2 = 1$, and $T_2 = W_1$. Start with the equilibrium condition in period 2,

$$\Pi_2^4 - \Pi_2^3 - \lambda_T (W_1)^2 = 0.$$

First-order Taylor expansion, taking W_1 as given, yields

$$\Pi_2 = 1 + {W_1}^2 \lambda_T.$$

Next, the budget constraint in period 2 is

$$T_2 = \frac{W_1}{\Pi_2}.$$

First-order Taylor expansion of it, together with $\Pi_2 = 1 + {W_1}^2 \lambda_T$, gives

$$T_2 = W_1 - W_1^3 \lambda_T.$$

This completes the proof of Proposition 1 in text.

A.6.2 Period 1

The approximation point is $\lambda_T = 0$, $\Pi_1 = 1$, $Y_1 = 1$, $(1 + i_1) = \beta^{-1}$, and $T_1 = \frac{w_0}{2}$. Start with the equilibrium condition in period 1, (24),

$$\{1 + \frac{\lambda_y}{\kappa^2}\}(\Pi_1 - 1) = \frac{\lambda_T T_1}{(1 + i_1)}\{\frac{w_0}{\Pi_1^2} + \frac{\kappa^{-1}\Pi_1}{Y_1}(1 + i_1)(1 - x_0)\frac{w_0}{\Pi_1^2} + i_1(1 - x_0)\frac{w_0}{\Pi_1^2} - \frac{\kappa^{-1}}{Y_1}(1 + i_1) T_1\}.$$

First-order Taylor expansion, taking w_0 and x_0 as given, yields

$$(1 + \frac{\lambda_y}{\kappa^2})(\Pi_1 - 1) = \frac{\lambda_T T^*}{1 + i_1^*}\{w_0 + \kappa^{-1}(1 + i^*)(1 - x_0)w_0 + i^*(1 - x_0)w_0 - \kappa^{-1}(1 + i^*)T^*\} \quad \text{(A.17)}$$

which is (26)-(27) in the text once we impose $T^* = \frac{w_0}{2}$ and $1 + i_1^* = \beta^{-1}$.

For proof of Proposition 2, first, impose $T^{*}=\frac{w_0}{2}$ and $1+i_1^{*}=\beta^{-1}$ in (A.17) to get

$$\Pi_1 = 1+\left(\frac{\lambda_T}{1+\frac{\lambda_y}{\kappa^2}}\right)\{1-\frac{\kappa^{-1}\beta^{-1}}{2}+\beta^{-1}(1-\beta+\kappa^{-1})(1-x_0)\}\frac{w_0^2}{2\beta^{-1}}.$$

Next, approximate (25), that is,

$$1+i_1=\beta^{-1}(\frac{\bar{Y}}{\kappa^{-1}\Pi_1-\kappa^{-1}+\bar{Y}})\bar{\Pi}^2(W_1)$$

to get

$$i_1=\beta^{-1}\left(1-\beta\right)-\beta^{-1}\kappa^{-1}(\Pi_1-1),$$

as $\bar{\Pi}^2(W_1)=0$ at $\lambda_T=0$.

Then it follows that

$$\frac{\partial\Pi_1}{\partial x_0}=-\left(\frac{\lambda_T}{1+\frac{\lambda_y}{\kappa^2}}\right)\{\beta^{-1}(1-\beta+\kappa^{-1})\}\frac{w_0^2}{2\beta^{-1}}<0,$$

$$\frac{\partial i_1}{\partial x_0}=-\beta^{-1}\kappa^{-1}\frac{\partial\Pi_1}{\partial x_0}>0,$$

$$\frac{\partial\Pi_1}{\partial w_0}=w_0\{1+\beta^{-1}\left(\kappa^{-1}+1-\beta\right)(1-x_0)-\kappa^{-1}\beta^{-1}\frac{1}{2}\}\frac{1}{\beta^{-1}}\frac{d\lambda_T}{\{\lambda_\Pi+\frac{\lambda_y}{\kappa^2}\}},$$

$$\frac{\partial i_1}{\partial w_0}=-\beta^{-1}\kappa^{-1}\frac{\partial\Pi_1}{\partial w_0}.$$

This implies that,

$$\frac{\partial\Pi_1}{\partial w_0}>0 \text{ and } \frac{\partial i_1}{\partial w_0}<0 \text{ iff } x_0<\frac{1+\frac{1}{2}\kappa^{-1}}{1-\beta+\kappa^{-1}}.$$

This completes the proof of Proposition 2 in text.

A.7 Additional Results and Figures

We report some additional results mentioned in the main text below.

A.7.1 Optimal monetary commitment with and without fiscal policy

Figure A.1 compares the optimal monetary policy commitment with $\lambda_T=0$ to the one with $\lambda_T>0$ that is referenced in the text. Relative to the optimal monetary policy commitment, the joint commitment policy is similar, but, slightly more expansionary, yielding both higher inflation and output across all periods. The reason is that government debt is nominal, so there are fiscal gains from inflation, as inflation reduces the real value of debt. Moreover, there

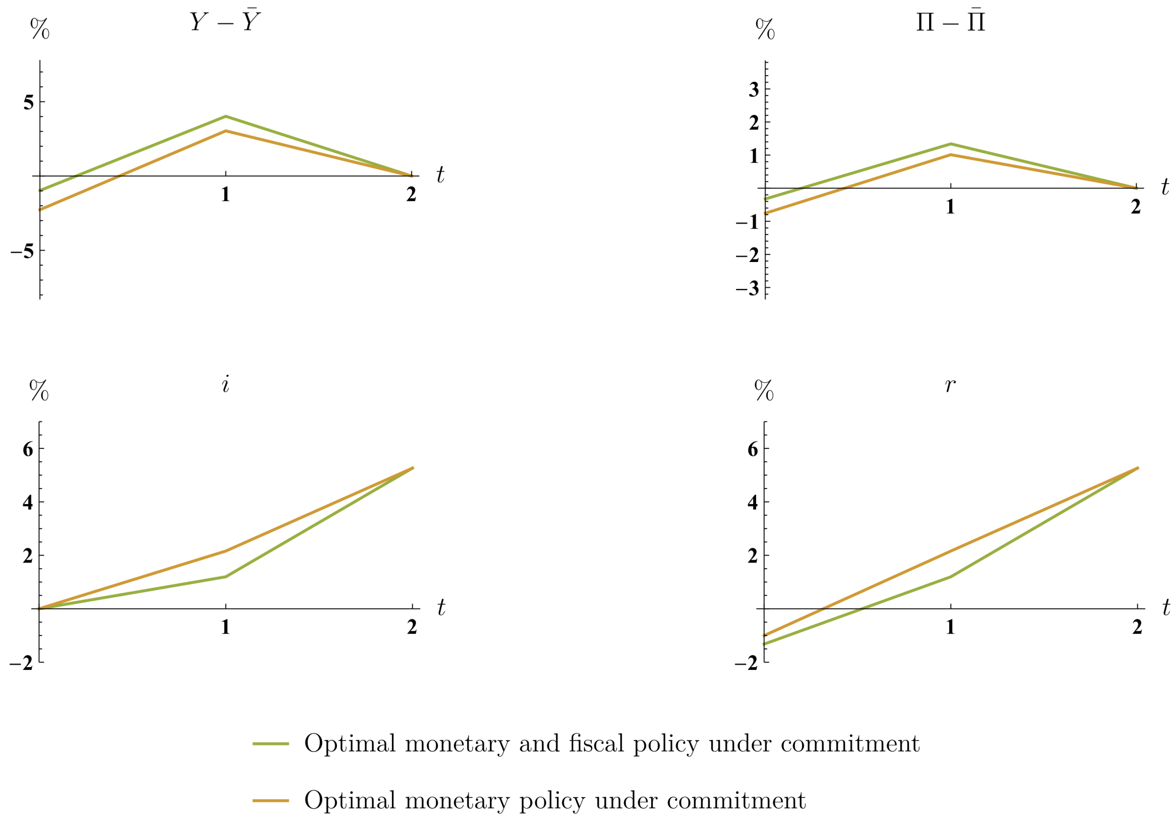

Figure A.1: Impulse responses under optimal policy with commitment at the ZLB

Note: The figure shows the responses of output, inflation, nominal interest rate, and real interest rate when a negative demand shock makes the ZLB bind in the short run. The green line shows the solution for optimal monetary and fiscal policy under commitment. The yellow line shows the solution for optimal monetary policy under commitment.

are fiscal benefits of keeping the real interest rate low as this affects the cost of rolling over the debt, an important feature of the environment that will play a central role in the MPE.

A.7.2 Policy functions

Figure A.2 shows the period 1 policy functions for all variables with respect to x_0 in the MPE in the numerical example.

Figure A.3 shows the period 1 policy function with respect to w_0 in the MPE in the numerical example. Note how the policy functions in period 1 with respect to the maturity value of debt (w_0) depend on the maturity of debt (x_0), as discussed in the text and also given analytically in the approximated solution in Proposition (2). In particular, while with only short-term debt (x_0), increases in the maturity value of debt lead to higher inflation and lower interest rates, the reverse is the case for longer maturity debt.

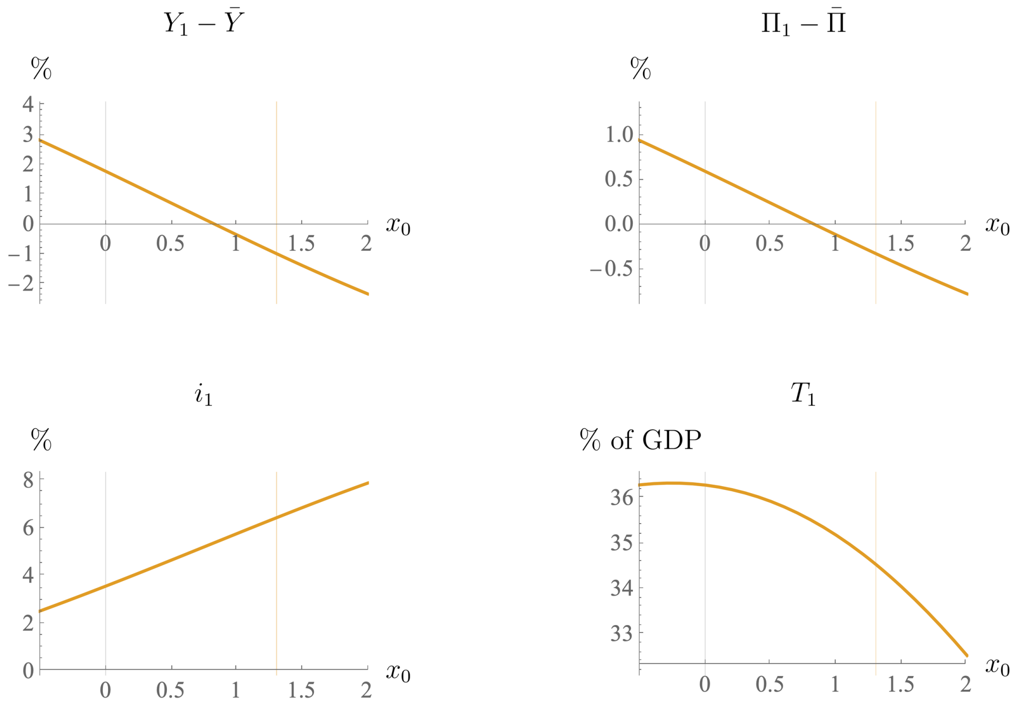

Figure A.2: Medium-run policy functions

Note: The figure shows policy functions for output, inflation, nominal interest rate, and taxes (with respect to x_0) in the medium run with optimal monetary and fiscal policy under discretion. The vertical yellow line shows the optimal value of maturity of debt ($x_0 = 1.31$). The maturity value of debt (w_0) is fixed at the optimal value (72%). The dashed lines show the approximate, analytical policy functions discussed in the text.

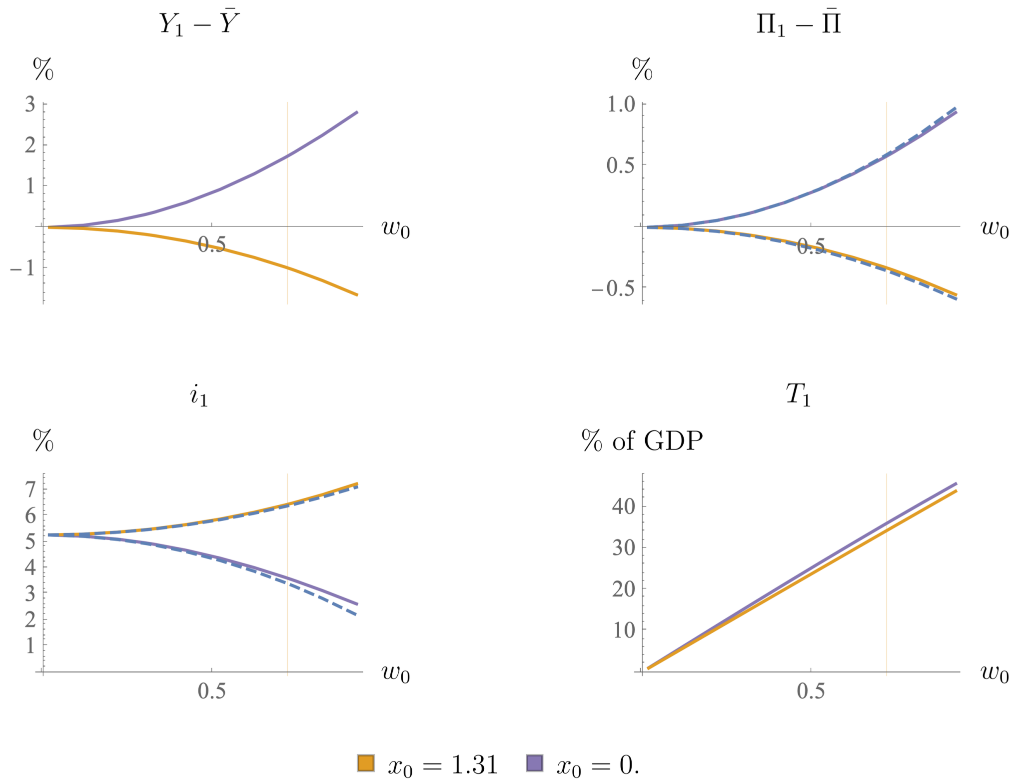

Figure A.3: Medium-run policy functions

Note: The figure shows policy functions for output, inflation, nominal interest rate, and taxes (with respect to w_0) in the medium run in the model with optimal monetary and fiscal policy under discretion. The results are shown for two different maturities of debt (x_0). $x_0 = 1.31$ constitutes the optimal maturity. The vertical yellow line shows the optimal maturity value of debt (w_0=72%). The dashed lines show the approximate, analytical policy functions discussed in the text.

A.7.3 Optimal QE: Alternative calibration

Figure A.4 present results on optimal quantitative easing for a calibration different from the baseline. Here, all parameter values are the same as the baseline reported in text except that initial outstanding debt (w_{-1}) is 0.5. The green line shows the solution for the MPE if monetary and fiscal policy is conducted optimally under discretion. The yellow line shows optimal monetary and fiscal policy under commitment. The purple line show the government policy if the government acts under discretion but keeps the maturity of the debt fixed at time 0 at what would have been optimal in the absence of the shock, that is, $x_0 = 0.2$. This case is labeled constrained discretion. The three solutions in this case are visibly different also for interest rates and inflation.

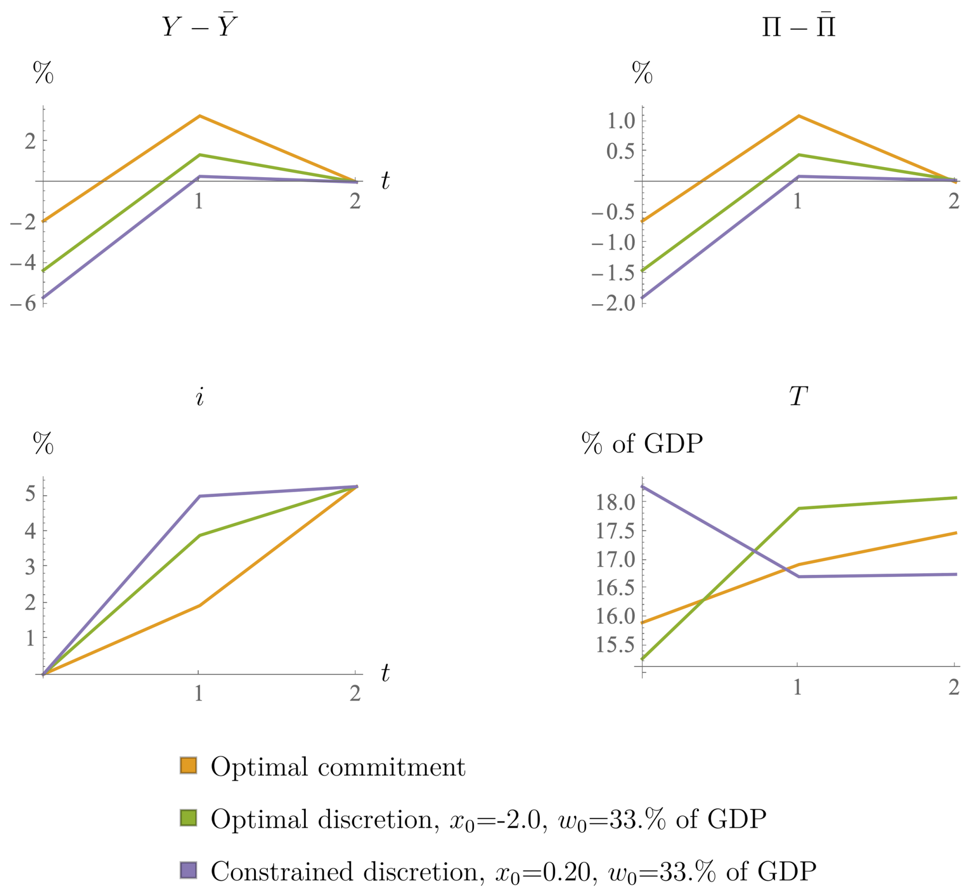

Figure A.4: Impulse responses under optimal quantitative easing and other policies at the ZLB

Note: The figure shows the responses of output, inflation, nominal interest rate, and taxes when a negative demand shock makes the ZLB bind in the short run. The calibration is different from the one in the main text in that the initial outstanding debt (w_{-1}) is 0.5. The yellow line shows the solution for optimal policy under commitment. The green line shows the solution for optimal policy under discretion. The purple line shows the solution for optimal policy under discretion, but with maturity of debt in period 0 at the value optimal with no shock.

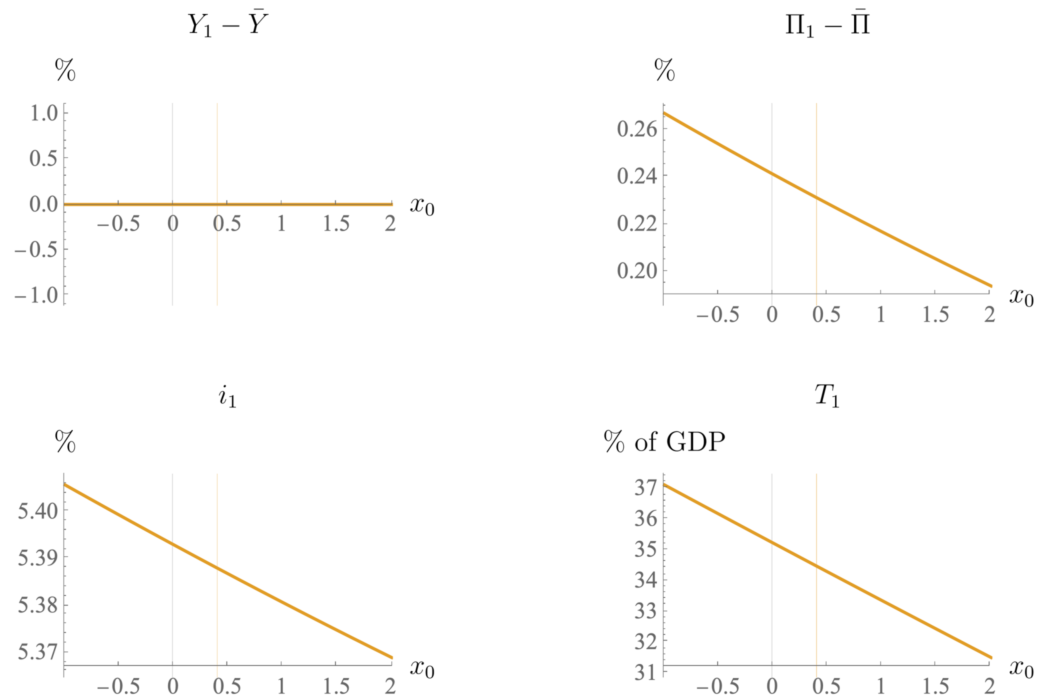

Figure A.5: Medium-run policy functions with flexible prices

Note: The figure shows policy functions for output, inflation, nominal interest rates, and taxes (with respect to x_0) in the medium run with optimal monetary and fiscal policy under discretion when prices are fully flexible in period 1. The vertical yellow line shows the optimal value of maturity of debt (x_0= 0.4). The maturity value of debt (w_0) is fixed at the optimal value (0.6875).

A.7.4 Policy functions: Fully flexible prices

Figure A.5 shows, for the case of flexible prices in period 1, the most relevant policy function, that is, how the endogenous variables change with x_0. The key result is that period 1 inflation is decreasing with maturity of debt inherited from period 0, even under fully flexible prices. With flexible prices, output in period 1 is exogenous. Unlike the sticky price baseline case presented in the main text, here, due to the Fisher effect, the nominal rate in period 1 is decreasing in x_0, because period 2 inflation decreases with x_0. Note that Proposition 2 shows that the effect on nominal rate of x_0 is second-order under flexible prices, as the Fisher effect is not present in Proposition 2. As a result, the nonlinear solution here is capturing a quantitatively small effect. The effect on inflation however, is first-order, as shown in Proposition 2, and also clear here.

Figure A.6 shows, for the case of flexible prices in period 1, the period 1 policy function with respect to w_0. As the optimal debt maturity is lower here, and the condition for the case of flexible prices in Proposition 2 much weaker, the policy functions for inflation and interest rates are upward sloping here for both levels of x_0, unlike the sticky price baseline case above

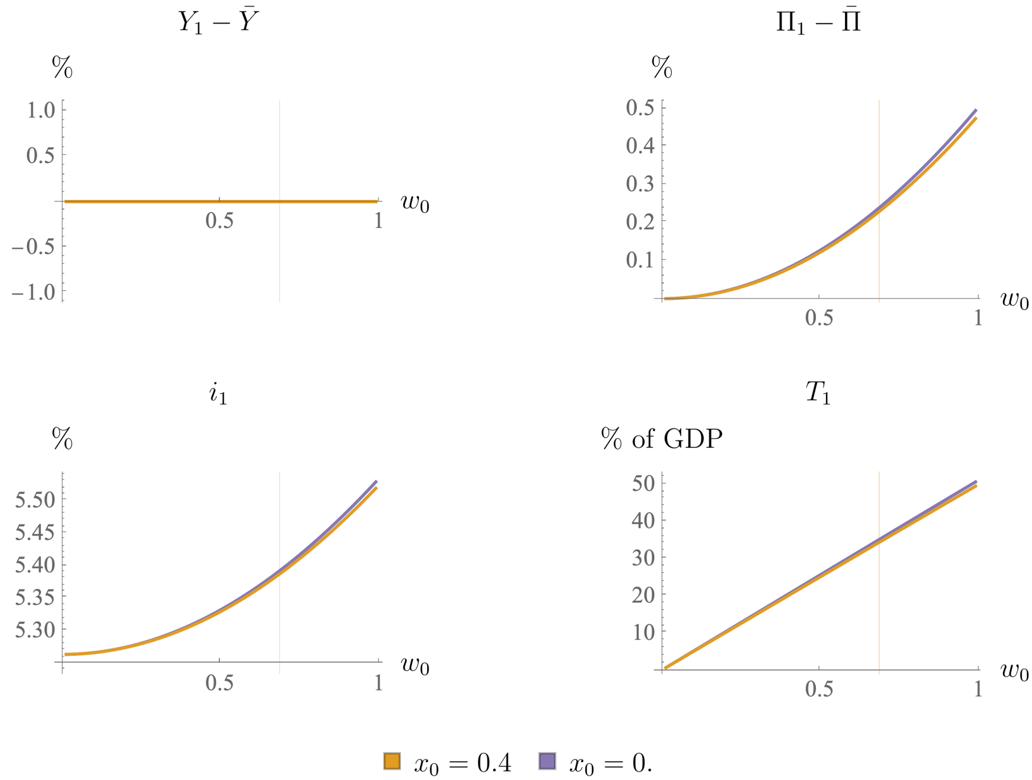

Figure A.6: Medium-run policy functions with flexible prices

Note: The figure shows policy functions for output, inflation, nominal interest rate, and taxes (with respect to w_0) in the medium run in the model with optimal monetary and fiscal policy under discretion when prices are fully flexible in period 1. The results are shown for two different maturities of debt (x_0). x_0= 0.4 constitutes the optimal maturity. The vertical yellow line shows the optimal maturity value of debt ($w_0 = 0.6875$).

in Figure A.3.

A.7.5 Nonlinear Utility

We have as the exact nonlinear utility

$$
\begin{aligned}
\log C_t - \chi L_t &= \log Y_t - \chi^{1-\theta}\gamma Y_t^{1-\theta} - \chi(1-\gamma)Y_t\left(\frac{\Pi_t}{z_t}\right)^{\theta} - \chi f\left(T_t\right) \\
&= \log Y_t - \chi^{1-\theta}\gamma Y_t^{1-\theta} - \chi(1-\gamma)Y_t\left(\frac{\Pi_t}{\frac{\Pi_t}{Y_t^{-1}(\Pi_t-1)+1}}\right)^{\theta} - \chi f\left(T_t\right) \\
&= \log Y_t - \chi^{1-\theta}\gamma Y_t^{1-\theta} - \chi(1-\gamma)Y_t\left(Y_t^{-1}\left(\Pi_t-1\right)+1\right)^{\theta} - \chi f\left(T_t\right) \\
&\equiv g(Y_t, \Pi_t) - \chi f\left(T_t\right)
\end{aligned}
$$

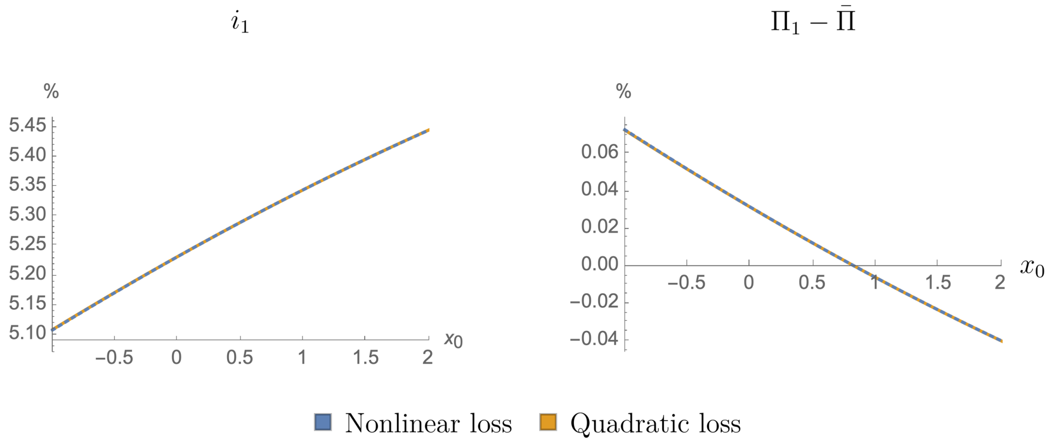

Figure A.7: Period 1 policy functions with and without utility approximation

Note: The figure shows policy functions for inflation and nominal interest rate (with respect to x_0) in the medium run in the model without and without approximated loss function for the government.

while we use the approximation in the main text

$$g\left(Y_t, \Pi_t\right) - g\left(\bar{Y}, \bar{\Pi}\right) = -\frac{1}{2}\chi^2\left(1-\gamma\right)\theta\left(\theta-1\right)\left[\tilde{\Pi}_t^2 + \left(\frac{1-\gamma\theta\left(1-\theta\right)}{\left(1-\gamma\right)\theta\left(\theta-1\right)}\right)\tilde{Y}_t^2\right]$$
$$= -\frac{1}{2}\left[\tilde{\Pi}_t^2 + \lambda_y\tilde{Y}_t^2\right],$$

and for taxes

$$\chi f''\left(\bar{T}\right)T_t^2,$$

to write the quadratic objective as

$$-\frac{1}{2}\left[\tilde{\Pi}_t^2 + \lambda_y\tilde{Y}_t^2\right] - \frac{1}{2}\lambda_T T_t^2.$$

We show in Figure A.7 policy functions for period 1 from an example where we compare the solutions using the exact nonlinear utility (where for $f\left(T_t\right)$ we use a quadratic function) and the quadratically approximated utility. As can be seen, the results are the same.

B Details on estimation of λ_T

B.1 LP-IV estimator

Following Ramey and Zubairy (2018) and their notation, consider a local projection (LP)-IRF of inflation and taxes to a fiscal shock at some horizon h,

$$\sum_{j=0}^{h} y_{t+h} = m_{y,h} \sum_{j=0}^{h} g_{t+j} + \gamma_h + \phi_h(L)\zeta_{t-1} + \omega_{t+h},$$

where $\sum_{j=0}^{h} y_{t+h}$ is cumulative inflation or tax (π or T), $\sum_{j=0}^{h} g_{t+j}$ is cumulative fiscal shock variable, ζ_{t-1} is the vector of controls, and ω_{t+h} is the regression residual.

The object of interest is the estimator $m_{y,h}$ (with $y = \pi$ or T). As in Ramey and Zubairy (2018), we consider an IV estimator $m_{y,h}$, where we use the Ramey news variables and the Blanchard-Perotti shock as IVs for the fiscal shock. We use the data for all variables as provided by Ramey and Zubairy (2018). Our focus in the main text is on the horizon of 16 quarters ($h = 16$).

These LP-IV estimators can be correspondingly represented as

$$\hat{m}_{\pi,h} = e_1'(X'PX)^{-1}X'P\pi, \tag{B.1}$$

$$\hat{m}_{T,h} = e_1'(X'PX)^{-1}X'PT, \tag{B.2}$$

where X is the matrix of regressors with the fiscal shock placed first; $P = W(W'W)^{-1}W'$, where W is the matrix with instruments and exogenous controls; $e_1 = (1, 0, \ldots, 0)'$; and π and T are vectors with the time series corresponding to cumulative inflation and cumulative taxes with a lead h periods ahead.

Let the sample size be N. Under appropriate regularity conditions, these estimators are $\sqrt{N}$ consistent: $\hat{m}_{\pi,h} = m_{\pi,h} + O_p\left(\frac{1}{\sqrt{N}}\right)$ and $\hat{m}_{T,h} = m_{T,h} + O_p\left(\frac{1}{\sqrt{N}}\right)$. Moreover, $\hat{m}_{\pi,h}$ and $\hat{m}_{T,h}$ have a joint asymptotically normal distribution with some covariance matrix Ω.

B.2 Asymptotic representation of ratio of estimators

We are interested in the asymptotic variance of the ratio $g(m_{\pi,h}, m_{T,h}) = \frac{m_{\pi,h}}{m_{T,h}}$.[74] First, Taylor theorem applied to $g(m_{\pi,h}, m_{T,h})$ gives

$$\frac{\hat{m}_{\pi,h}}{\hat{m}_{T,h}} - \frac{m_{\pi,h}}{m_{T,h}} = \frac{(\hat{m}_{\pi,h} - m_{\pi,h})}{m_{T,h}} - \frac{m_{\pi,h}(\hat{m}_{T,h} - m_{T,h})}{m_{T,h}^2} + o_p\left(\frac{1}{\sqrt{N}}\right), \tag{B.3}$$

since $\partial g(m_{\pi,h}, m_{T,h}) = (\frac{1}{m_{T,h}}, -\frac{m_{\pi,h}}{\hat{m}_{T,h}^2})'$. Second, by the Delta theorem, $\frac{\hat{m}_{\pi,h}}{\hat{m}_{T,h}}$ is also asymptotically normal with mean $\frac{m_{\pi,h}}{m_{T,h}}$ and asymptotic variance $\sigma_h^2 = \partial g(m_{\pi,h}, m_{T,h})\Omega(\partial g(m_{\pi,h}, m_{T,h}))'$.

B.3 A simple method to compute standard errors for ratios of LP-estimators

The conventional Delta method approach to inference would require estimation of Ω, using a *joint* Newey-West (NW) estimator for $(\hat{m}_{\pi,h}, \hat{m}_{T,h})$. Such a general estimator would give more information than needed for our purposes. We therefore, propose an alternative way of estimating σ_h^2 that only uses readily available Newey-West estimators for the *individual* LP estimators. In particular, we show that the (asymptotic) standard error for the estimated ratio at horizon h, i.e. $\frac{\hat{m}_{\pi,h}}{\hat{m}_{T,h}}$, coincides with the NW error for IRF of Z_h, a suitably chosen linear combination of π, T that is defined below.

B.3.1 Equivalence with standard Delta method

While our method is simpler and easier to implement than the Delta method, it leads to the same answer as the Delta method. To see the equivalence between our method and the Delta

[74] Here, we use the notation on local projection coefficients to be consistent with that used in Ramey and Zubairy (2018) for clear comparison. In text, we refer to them as $\widehat{IRF}_\pi(j)$ and $\widehat{IRF}_T(j)$ to connect them better to the model.

method, first note that Equation (B.3) implies

$$\begin{aligned}
Var\left[\frac{\hat{m}_{\pi,h}}{\hat{m}_{T,h}} - \frac{m_{\pi,h}}{m_{T,h}}\right] &= Var\left[\frac{\hat{m}_{\pi,h} - m_{\pi,h}}{m_{T,h}} - \frac{m_{\pi,h}(\hat{m}_{T,h} - m_{T,h})}{m_{T,h}^2}\right] \\
&+ 2Cov\left(O_p\left(\frac{1}{\sqrt{N}}\right), o_p\left(\frac{1}{\sqrt{N}}\right)\right) + o\left(\frac{1}{N}\right), \\
&= Var\left[\frac{\hat{m}_{\pi,h}}{m_{T,h}} - \frac{m_{\pi,h}\hat{m}_{T,h}}{m_{T,h}^2}\right] + o\left(\frac{1}{N}\right), \\
&= Var[e_1'(X'PX)^{-1}X'P(\frac{1}{m_{T,h}}\pi - \frac{m_{\pi,h}}{m_{T,h}^2}T)] + o\left(\frac{1}{N}\right), \\
&= Var[e_1'(X'PX)^{-1}X'PZ_h] + o\left(\frac{1}{N}\right),
\end{aligned}$$

where $Z_h = \frac{1}{m_{T,h}}\pi - \frac{m_{\pi,h}}{m_{T,h}^2}T$.

Since by definition, $\lim_{N\to\infty} NVar[\frac{\hat{m}_{\pi,h}}{\hat{m}_{T,h}}] = \sigma^2$, the asymptotic variance of

$$\hat{m}_{Z,h} = e_1'(X'PX)^{-1}X'PZ_h$$

is σ_h^2. That is, the asymptomatic variance of the LP IRF estimator for outcome variable Z_h with the same controls, is also σ_h^2.

Thus, our method indeed leads to the same answer as the standard Delta method and they are equivalent for our purposes.

B.3.2 Computing the standard deviation of $\hat{m}_{Z,h}$

To estimate asymptotic standard deviation of $\hat{m}_{Z,h}$, one can use the Newey-West estimator of IRF of the shock on $\hat{Z}_h$ for each horizon h, where

$$\hat{Z}_h = \left(\frac{1}{\hat{m}_{T,h}}\pi - \frac{\hat{m}_{\pi,h}}{\hat{m}_{T,h}^2}T\right) = Z_h + O_P\left(\frac{1}{\sqrt{N}}\right).$$

Thus, our method first constructs the variable $\hat{Z}_h = \frac{1}{\hat{m}_{T,h}}\pi - \frac{\hat{m}_{\pi,h}}{\hat{m}_{T,h}^2}T$ using the estimates $\hat{m}_{T,h}$ and $\hat{m}_{\pi,h}$. Then, we estimate LP-IV impulse response of the variable $\hat{Z}_h$ to the fiscal shock.[75] The standard error of the LP-IV coefficient $\hat{m}_{Z,h}$ then gives $\hat{\sigma}_h^2$, an estimate of the standard error σ_h^2.

[75] Note that we use the variable $\hat{Z}_h$ instead of Z_h. This is however, not an issue as it is a consistent estimator.

B.4 Bayesian interpretation of the estimator

Let θ define the vector of all the calibrated parameters that determine the behaviour of the model outside of the ZLB. Let $r(\lambda_T, \theta) = \frac{\pi_b(\lambda_T,\theta)}{T_b(\lambda_T,\theta)}$ be the ratio of the cumulative IRF for inflation and taxes, outside of the ZLB, implied by the model. Define $f_\theta(\frac{m_{\pi,h}}{m_{T,h}})$ as a function that maps the IRF ratio $\frac{m_{\pi,h}}{m_{T,h}}$ to λ_T,

$$\lambda_T = f_\theta(\frac{m_{\pi,h}}{m_{T,h}}) = \arg\min_{\lambda}(\frac{m_{\pi,h}}{m_{T,h}} - r(\lambda, \theta))^2. \tag{B.4}$$

We want to compute a posterior distribution of λ_T given time series observations outside of ZLB (W, X, π, T) conditional on calibrated parameters θ. Let us assume that the prior on λ_T is uninformative. Since $f_\theta(\frac{m_{\pi,h}}{m_{T,h}})$ is a differentiable function of the ratio $\frac{m_{\pi,h}}{m_{T,h}}$, the implied prior distribution for $\frac{m_{\pi,h}}{m_{T,h}}$ has no mass points and is absolutely continuous at the true parameter value $\frac{m_{\pi,h}}{m_{T,h}}$. Such a prior meets the conditions for Bernstein-von-Mises theorem (see Theorem 10.4 in Van der Vaart (2000)).

By this result, for a large sample N, the posterior distribution for the ratio $\frac{m_{\pi,h}}{m_{T,h}}$ can be well approximated by a normal distribution with mean $\frac{\hat{m}_{\pi,h}}{\hat{m}_{T,h}}$ and variance $\hat{\sigma}^2$. One can then estimate the posterior distribution of λ_T using Monte Carlo computations:

1. Generate sample $\{\frac{m^b_{\pi,h}}{m^b_{T,h}}\}_{b=1}^B$ using i.i.d. draws from $N(\frac{\hat{m}_{\pi,h}}{\hat{m}_{T,h}}, \hat{\sigma}^2)$.

2. Replace negative draws of $\frac{m^b_{\pi,h}}{m^b_{T,h}}$ with zeros. (We are imposing the uninformative prior with non-negative support on the ratio.)

3. Compute the mean, the median, and the standard deviation of the sample $\{f_\theta(\frac{m^b_{\pi,h}}{m^b_{T,h}})\}_{b=1}^B$.

4. Plot a kernel density/histogram for $f_\theta(\frac{m^b_{\pi,h}}{m^b_{T,h}})$ using only draws with positive values of $\frac{m^b_{\pi,h}}{m^b_{T,h}}$.

In our application, $B = 1000$ and the number of draws that had negative values of $\frac{m^b_{\pi,h}}{m^b_{T,h}}$ were 31, or 3.1%.

Estimated this way, the posterior mean of λ_T is 0.0057 while the 68% probability interval is (0.00187, 0.00879). These estimates are almost identical to those from our baseline approach reported in Figure 9, where the mean is 0.0052 and the 68% confidence interval is (0.00201, 0.00921). These new estimates' implied confidence interval for output effects of QE2 is also essentially the same as that in Table 4, at (0.21%, 2.17%).

C Details on the Quantitative Model

This section details the derivation of the quantitative model and describes the numerical solution method.

C.1 Functional forms and notation

Functional forms for utility are given by

$$u\left(C,\xi\right)=\xi\bar{C}^{\frac{1}{\sigma_c}}\frac{C^{1-\frac{1}{\sigma_c}}}{1-\frac{1}{\sigma_c}},$$

$$v\left(H_t,\xi\right)=\xi\lambda\frac{H_t^{1+\omega}}{1+\omega},$$

$$g\left(G,\xi\right)=\xi\bar{G}^{\frac{1}{\sigma_c}}\frac{G^{1-\frac{1}{\sigma_c}}}{1-\frac{1}{\sigma_c}}.$$

Variables in the steady state are denoted by a bar. In the steady state, $\bar{\xi}=1$ and hours are scaled so that $\bar{Y}=1$. Recall that the production function of the firms in linear in labor so that $y_t(i)=H_t(i)$. In a symmetric equilibrium this implies that $H_t=Y_t$ so that $v(H_t,\xi_t)=v(Y_t,\xi_t)$. With some abuse of notation we will use the notation v_y for v_h.

The following notation and normalizations are also used:

$$\sigma=\frac{C}{Y}\sigma_c$$
$$\kappa=\theta\frac{\left(\sigma^{-1}+\omega\right)}{d''}$$

C.2 Efficient equilibrium

The efficient equilibrium is an important benchmark in the model. Using $G_t=F_t-s(T_t)=F-s(T_t),$ the social planner's problem can be written as

$$\max u\left(C_t,\xi_t\right)+g\left(F-s(T_t)\right)-v\left(Y_t\right)$$

subject to

$$Y_t=C_t+F.$$

Formulate the period Lagrangian

$$\begin{aligned} L_t &= u\left(C_t, \xi_t\right) + g\left(F - s(T_t)\right) - v\left(Y_t\right) \\ &+ \phi_{1t}\left(Y_t - C_t - F\right). \end{aligned}$$

First-order conditions (where all the derivatives should be equated to zero) are

$$\begin{aligned} \frac{\partial L_t}{\partial Y_t} &= -v_Y + \phi_{1t}, \\ \frac{\partial L_t}{\partial C_t} &= u_C(.) + \phi_{1t}\left[-1\right], \\ \frac{\partial L_t}{\partial T_t} &= -g_G(.)s'(T_t), \end{aligned}$$

which yields

$$u_C = v_Y,$$

$$g_G\left(-s'(T_t)\right) = 0.$$

The assumption is that in steady state

$$s'(\bar{T}) = 0,$$

but $s''(0) > 0$. Efficient allocation thus requires

$$u_C = v_Y, T_t = \bar{T}.$$

C.3 Commitment Solution in the Quantitative Model

The optimal monetary and fiscal policy under commitment problem is relatively standard and can be solved by formulating a Lagrangian, similar to that in Section A.3. Again, a key property of the solution under commitment is that the maturity structure of debt is irrelevant. Thus, using similar arguments as in the simple model, it can be shown that ρ_t does not affect the solution in the quantitative model under monetary and fiscal commitment. The solution was computed via first-order approximation of the equilibrium conditions. The solution algorithm used was based on Eggertsson and Woodford (2003) using the toolkit in Eggertsson et al. (2019).

C.4 MPE in the Quantitative Model

The policy problem is

$$J\left(l_{t-1},\rho_{t-1},\xi_t\right)=\max\left[U\left(\Lambda_t,\xi_t\right)+\beta E_t J\left(l_t,\rho_t,\xi_{t+1}\right)\right]$$

subject to

$$S_t(\rho_t)l_t=\left(1+\rho_{t-1}W_t(\rho_{t-1})\right)l_{t-1}\Pi_t^{-1}+\left(F-T_t\right),$$

$$1+i_t=\frac{u_C\left(C_t,\xi_t\right)}{\beta f_t^e},$$

$$i_t\geq 0,$$

$$S_t(\rho_t)=\frac{1}{u_C\left(C_t,\xi_t\right)}\beta g_t^e,$$

$$W_t(\rho_{t-1})=\frac{1}{u_C\left(C_t,\xi_t\right)}\beta j_t^e,$$

$$\theta Y_t\left[u_C\left(C_t,\xi_t\right)-v_y\left(Y_t,\xi_t\right)\right]+u_C\left(C_t,\xi_t\right)d'\left(\Pi_t\right)\Pi_t=\beta h_t^e,$$

$$Y_t=C_t+F+d\left(\Pi_t\right),$$

$$f_t^e=E_t\left[u_C\left(C_{t+1},\xi_{t+1}\right)\Pi_{t+1}^{-1}\right]=\bar{f}^e\left(l_t,\rho_t,\xi_t\right),$$

$$g_t^e=E_t\left[u_C\left(C_{t+1},\xi_{t+1}\right)\Pi_{t+1}^{-1}\left(1+\rho_t S_{t+1}(\rho_t)\right)\right]=\bar{g}^e\left(l_t,\rho_t,\xi_t\right),$$

$$h_t^e=E_t\left[u_C\left(C_{t+1},\xi_{t+1}\right)d'\left(\Pi_{t+1}\right)\Pi_{t+1}\right]=\bar{h}^e\left(l_t,\rho_t,\xi_t\right),$$

$$j_t^e=E_t\left[u_C\left(C_{t+1},\xi_{t+1}\right)\Pi_{t+1}^{-1}\left(1+\rho_{t-1}W_{t+1}(\rho_{t-1})\right)\right]=\bar{j}^e\left(l_t,\rho_t,\xi_t\right).$$

Formulate the period Lagrangian:

$$\begin{aligned}
L_t &= u\left(C_t,\xi_t\right)+g\left(F-s(T_t-T)\right)-v\left(Y_t\right)+\beta E_t J\left(l_t,\rho_t,\xi_{t+1}\right)\\
&+\phi_{1t}\left(S_t(\rho_t)l_t-\left(1+\rho_{t-1}W_t(\rho_{t-1})\right)l_{t-1}\Pi_t^{-1}-\left(F-T_t\right)\right)+\phi_{2t}\left(\beta f_t^e-\frac{u_C\left(C_t,\xi_t\right)}{1+i_t}\right)\\
&+\phi_{3t}\left(\beta g_t^e-u_C\left(C_t,\xi_t\right)S_t(\rho_t)\right)+\phi_{4t}\left(\beta j_t^e-u_C\left(C_t,\xi_t\right)W_t(\rho_{t-1})\right)\\
&+\phi_{5t}\left(\beta h_t^e-\theta Y_t\left[u_C\left(C_t,\xi_t\right)-v_y\left(Y_t,\xi_t\right)\right]-u_C\left(C_t,\xi_t\right)d'\left(\Pi_t\right)\Pi_t\right)\\
&+\phi_{6t}\left(Y_t-C_t-F-d\left(\Pi_t\right)\right)+\psi_{1t}\left(f_t^e-\bar{f}^e\left(l_t,\rho_t,\xi_t\right)\right)+\psi_{2t}\left(g_t^e-\bar{g}^e\left(l_t,\rho_t,\xi_t\right)\right)\\
&+\psi_{3t}\left(h_t^e-\bar{h}^e\left(l_t,\rho_t,\xi_t\right)\right)+\psi_{4t}\left(j_t^e-\bar{j}^e\left(l_t,\rho_t,\xi_t\right)\right)+\gamma_{1t}\left(i_t-0\right).
\end{aligned}$$

First-order conditions (where all the derivatives should be equated to zero) are

$$\frac{\partial L_s}{\partial \Pi_t} = \phi_{1t}\left[\left(1+\rho_{t-1}W_t(\rho_{t-1})\right)l_{t-1}\Pi_t^{-2}\right] + \phi_{5t}\left[-u_C d^{''}\Pi_t - u_C d^{'}\right] + \phi_{6t}\left[-d'\right],$$

$$\frac{\partial L_s}{\partial Y_t} = -v_Y + \phi_{5t}\left[-\theta u_C + \theta Y_t v_{yy} + \theta v_y\right] + \phi_{6t},$$

$$\frac{\partial L_s}{\partial i_t} = \phi_{2t}\left[u_C\left(1+i_t\right)^{-2}\right] + \gamma_{1t},$$

$$\frac{\partial L_s}{\partial S_t} = \phi_{1t}\left[l_t\right] + \phi_{3t}\left[-u_C\right],$$

$$\frac{\partial L_s}{\partial W_t} = \phi_{1t}\left[\rho_{t-1}l_{t-1}\Pi_t^{-1}\right] + \phi_{4t}\left[-u_C\right] \ ,$$

$$\frac{\partial L_s}{\partial C_t} = u_C + \phi_{2t}\left[-u_{CC}\left(1+i_t\right)^{-1}\right] + \phi_{3t}\left[-u_{CC}S_t(\rho_t)\right] + \phi_{4t}\left[-u_{CC}W_t(\rho_{t-1})\right]$$
$$+ \phi_{5t}\left[-\theta Y_t u_{CC} - u_{CC}d'\Pi_t\right] + \phi_{6t}\left[-1\right],$$

$$\frac{\partial L_s}{\partial T_t} = g_G\left(-s'(T_t)\right) + \phi_{1t},$$

$$\frac{\partial L_s}{\partial l_t} = \beta E_t J_l\left(l_t, \rho_t, \xi_{t+1}\right) + \phi_{1t}\left[S_t(\rho_t)\right] + \psi_{1t}\left[-\bar{f}_l^e\right] + \psi_{2t}\left[-\bar{g}_l^e\right] + \psi_{3t}\left[-\bar{h}_l^e\right] + \psi_{4t}\left(-\bar{j}_l^e\right),$$

$$\frac{\partial L_s}{\partial \rho_t} = \beta E_t J_\rho\left(l_t, \rho_t, \xi_{t+1}\right),$$

$$\frac{\partial L_s}{\partial f_t^e} = \beta\phi_{2t} + \psi_{1t},$$

$$\frac{\partial L_s}{\partial g_t^e} = \beta\phi_{3t} + \psi_{2t},$$

$$\frac{\partial L_s}{\partial j_t^e} = \beta\phi_{4t} + \psi_{4t},$$

$$\frac{\partial L_s}{\partial h_t^e} = \beta\phi_{5t} + \psi_{3t}.$$

The complementary slackness conditions are

$$\gamma_{1t} \geq 0,\ i_t \geq 0,\ \gamma_{1t}i_t = 0.$$

The envelope conditions are

$$J_l\left(l_{t-1}, \rho_{t-1}, \xi_t\right) = \phi_{1t}\left[-\left(1+\rho_{t-1}W_t(\rho_{t-1})\right)\Pi_t^{-1}\right],$$

$$J_\rho\left(l_{t-1}, \rho_{t-1}, \xi_t\right) = \phi_{1t}\left[-W_t(\rho_{t-1})l_{t-1}\Pi_t^{-1}\right].$$

C.5 Steady state

A MPE steady state is nontrivial to characterize because, generally, the first order conditions involve derivatives of the unknown expectation functions, as can be seen from the above. The solution method used here relies on the production subsidy, and the assumption that $s'(\bar{T}) = 0$ discussed in section C.2, so that the MPE steady state is efficient. Define the real value of debt as $D_t = S_t l_t$.

Proposition 5 *There is a MPE steady state in which* $\bar{\Pi} = 1, \bar{Y} = 1, \bar{\imath} = \beta^{-1} - 1, \bar{\gamma}_1 = \bar{\phi}_1 = \bar{\phi}_2 = \bar{\phi}_3 = \bar{\phi}_4 = \bar{\phi}_5 = \bar{\psi}_1 = \bar{\psi}_2 = \bar{\psi}_3 = \bar{\psi}_4 = 0, \bar{\phi}_6 = \bar{u}_c = \bar{v}_Y$ *and* $T = \bar{T}, D_t = \bar{D} = \frac{1}{1-\beta}(\bar{F} - \bar{T})$.

Proof. The proof of this proposition is simply an application of guess-and-verify, that is, it is easy to verify that the steady state stated above satisfies all the first-order conditions of the government's problem from the last section as well as the constraints. ■

An important element of the proposition above is that it defines the steady state debt as $\bar{D} = Sl = \frac{\beta}{1-\rho\beta} l$ which implies that the MPE does not provide a theory of the optimal maturity in the efficient steady state; it can be any ρ. While an obvious limitation of the theory, it is a major convenience as a practical matter, for it implies that we can choose ρ based directly on the data when approximating the dynamics of the model.

The fact that the model has a simple closed-form solution for the steady state allows us to side step the computational problem addressed in Klein et al. (2008). The problem is that the steady state will, in general, depend on the unknown expectation functions. Klein et al. (2008) solve this problem by relying on a successive set of approximations to the decision rules with polynomial functions. No such approximation is required here, for in the efficient steady state the Lagrange multipliers on the relevant constraints are zero and so, they drop out. The key to this simplification is the full efficiency of the steady state.

The most important assumption for the computation of the MPE is that we assume the existence of an efficient steady state at a positive level of public debt. This allows us to capture the most important nonlinearity embedded in the government budget constraint to a first order and allows for a straightforward approach to approximate the expectation functions as described above. As shown in the text, in the MPE, if debt goes above this value, taxes rise and debt converges back to the steady state over time and debt maturity similarly converges to a finite value. This assumption implies that, up to the steady state value of debt and taxes, there are no taxation costs; however, once above steady state, the cost of taxation increases.

One interpretation of this assumption is that it is a reduced-form way of capturing that there is the optimal level of debt is positive, as has been found in the literature that emphasizes the liquidity role of government debt (see, for example, Woodford (1990) and Aiyagari and McGrattan (1998)). Hence, under this interpretation, the function $s(.)$ is a reduced form way of capturing the cost and benefits of public debt, and at the steady state they are exactly offsetting each other. Integrating this motivation for holding public debt explicitly remains an important extension of the model. Another important limitation of the approach is that, since the steady state is efficient, it does not provide a theory of optimal steady state debt maturity as we noted above.

C.6 Loss Function

We can derive the loss function for the government in the quantitative model as well using a quadratic approximation of utility. This derivation here makes clear the comparison of the weights in the loss function that is mentioned in the text. For household utility, we need to approximate the following three components:

$$u\left(Y_t - F - d(\Pi_t), \xi_t\right); \; g\left(F - S(T_t - T), \xi_t\right); \; v\left(Y_t, \xi_t\right).$$

Standard manipulations that are prevalent in the literature give, as a second-order approximation to household utility,

$$\begin{aligned}
&\frac{1}{2}\frac{-\sigma_c\left(\hat{T}_t^2\left(F-\bar{Y}\right)s''\left(\bar{T}\right)+Fd''(1)\hat{\Pi}_t^2\right)+2\sigma_c\hat{Y}_t\left(-\bar{Y}+F\hat{\xi}_t+F\right)+\hat{Y}_t^2}{\sigma_c\left(F-\bar{Y}\right)}+\\
&\frac{1}{2}\bar{Y}\left(\frac{d''(1)\hat{\Pi}_t^2}{F-\bar{Y}}+\hat{\xi}_t\left(\frac{2\sigma_c}{\sigma_c-1}-\frac{2\hat{Y}_t}{F-\bar{Y}}\right)+\frac{2\sigma_c}{\sigma_c-1}\right)\\
&-\frac{1}{2}2\lambda\left(\hat{\xi}_t+1\right)\hat{Y}_t\bar{Y}^{\omega}-\frac{1}{2}\frac{2\lambda\left(\hat{\xi}_t+1\right)\bar{Y}^{\omega+1}}{\omega+1}-\frac{1}{2}\lambda\omega\hat{Y}_t^2\bar{Y}^{\omega-1}+tip,
\end{aligned}$$

which in turn is given as

$$\frac{1}{2}\left(\hat{Y}_t^2\left(\frac{1}{F\sigma_c-\sigma_c\bar{Y}}-\lambda\omega\bar{Y}^{\omega-1}\right)+\hat{T}_t^2\left(-s''\left(\bar{T}\right)\right)-2\left(\hat{\xi}_t+1\right)\hat{Y}_t\left(\lambda\bar{Y}^{\omega}-1\right)-d''(1)\hat{\Pi}_t^2\right)+tip.$$

Multiply everything by $\frac{1}{\omega+\sigma^{-1}}$ and consider efficient equilibrium in the steady state ($u_C = v_Y$), together with the scaling that $\lambda\bar{Y}^{\phi} = 1$ and $\bar{Y} = \bar{C} + F = 1$, to obtain

$$-\frac{\sigma\hat{T}_t^2 s''\left(\bar{T}\right)}{2\left(\omega\sigma+1\right)}-\frac{\sigma d''(1)\hat{\Pi}_t^2}{2\left(\omega\sigma+1\right)}-\frac{\hat{Y}_t^2}{2}.$$

So, finally, the approximation of utility yields

$$-\frac{1}{2}\left[\lambda_\pi\pi_t^2+\hat{Y}_t^2+\lambda_T\hat{T}_t^2\right],$$

where

$$\lambda_T = \frac{s''\left(\bar{T}\right)}{\omega + \sigma^{-1}}, \lambda_\pi = \frac{d''(1)}{(\omega + \sigma^{-1})} = \frac{\theta}{\kappa}.$$

In the calibration of the quantitative model, we normalize the weight on inflation λ_π to 1 and instead parameterize the weight on output, λ_y.

C.7 First-order Approximation of MPE at Positive Interest Rates

The approximation method relies on log-linear approximation of the FOCs of the government's problem, as well as the equilibrium constrains around the MPE steady state in the proposition above. Below, the log-linear approximation is reported at positive interest rate where the government's first-order conditions have been combined in the first two conditions:

$$\lambda_\pi \pi_t + \kappa^{-1}\hat{Y}_t = [\kappa^{-1}(1-\rho)\,\sigma^{-1} + \beta^{-1}]\frac{\bar{D}}{\bar{T}}\lambda_T \hat{T}_t,$$

$$[1-\beta\pi_l\kappa^{-1}(1-\rho)\,\sigma^{-1} - (Y_l + \sigma\pi_l)(1-\rho)\,\sigma^{-1} + \rho\beta S_l(1-\rho)]\hat{T}_t = -(\psi)\,\lambda_T^{-1}\beta\pi_l\kappa^{-1}Y_t + E_t\hat{T}_{t+1},$$

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(\hat{\imath}_t - E_t\pi_{t+1} - \hat{r}_t^e),$$

$$\pi_t = \kappa\hat{Y}_t + \beta E_t\pi_{t+1},$$

$$\hat{l}_t + \frac{\rho\beta}{1-\rho\beta}\hat{\rho}_t = \beta^{-1}\hat{l}_{t-1} + \beta^{-1}\frac{\rho\beta}{1-\rho\beta}\hat{\rho}_{t-1} - \beta^{-1}\pi_t - (1-\rho)\hat{S}_t - \psi\hat{T}_t,$$

$$\hat{S}_t = -\hat{\imath}_t + \rho\beta E_t\hat{S}_{t+1},$$

$$\hat{S}_t - \hat{W}_t = \hat{\rho}_t - \hat{\rho}_{t-1}.$$

The challenge is that the expectation functions π_l, S_l, y_l are unknown. Provided that the expectation functions are differentiable, the solution of the model is of the form

$$\pi_t = \pi_l l_{t-1} + \pi_\rho\hat{\rho}_{t-1} + \pi_r r_t^e,\ \hat{Y}_t = Y_l l_{t-1} + Y_\rho\hat{\rho}_{t-1} + Y_r r_t^e,\ \hat{S}_t = S_l l_{t-1} + S_\rho\hat{\rho}_{t-1} + S_r r_t^e,$$
$$\hat{\imath}_t = i_l l_{t-1} + i_\rho\hat{\rho}_{t-1} + i_r r_t^e,\ \hat{T}_t = T_l l_{t-1} + T_\rho\hat{\rho}_{t-1} + T_r r_t^e,\ \text{and } l_t = l_l l_{t-1} + l_\rho\hat{\rho}_{t-1} + l_r r_t^e,$$

where π_l, Y_l, S_l, i_l, l_l, T_l, π_r, Y_r, S_r, i_r, l_r, and T_r are unknown coefficients to be determined. The final step to computing the solution is then to plug in the conjectured solution and to match coefficients along with the requirement that expectations are rational.

C.8 Solution at ZLB

Debt is kept fixed at the ZLB at l_L . At the zero lower bound, $\hat{\imath}_t = 1 - \beta^{-1}$. Given the specific assumptions on the two-state Markov shock process, the equilibrium is described by the following system of equations:

$$
\begin{aligned}
\hat{Y}_t &= -\sigma\left(-\pi_l(1-\mu)l_t - r^e_t - \mu\pi_S + \hat{\imath}_t\right) + (1-\mu)Y_l l_t + \mu Y_S, \\
\hat{\pi}_t &= \beta\left(\pi_l(1-\mu)l_t + \mu\pi_S\right) + \kappa Y_t, \\
\hat{\imath}_t &= 1 - \beta^{-1}, \\
l_t &= l_S, \\
\hat{S}_t &= -\hat{\imath}_t + \rho\beta\left((1-\mu)S_l l_S + \mu S_S\right),
\end{aligned}
$$

where variables with a l subscript denote the solution we compute at positive interest rates, and variables with a S subscript denote values at the ZLB. The solution to this system is

$$
\begin{aligned}
\hat{Y}_t = Y_S =& \frac{\beta\pi_l(\mu-1)l_S}{\kappa} - \frac{(1-\beta\mu)\left(\beta\pi_l(1-\mu)^2 l_S - \kappa\left(l_S\left(\pi_l(\mu-1)\sigma + (\mu-1)Y_l\right) - \sigma r^e_S + \sigma i_S\right)\right)}{\kappa(\kappa\mu\sigma - (1-\mu)(1-\beta\mu))}, \\
\hat{\pi}_t = \pi_S =& -\frac{\beta\pi_l(1-\mu)^2 l_S - \kappa\left(l_S\left(\pi_l(\mu-1)\sigma + (\mu-1)Y_l\right) - \sigma r^e_S + \sigma i_L\right)}{\kappa\mu\sigma - (1-\mu)(1-\beta\mu)}, \\
\hat{\imath}_t = i_S =& 1 - \beta^{-1}, \; l_t = l_S, \\
\hat{S}_t = S_S =& \frac{\rho\beta(1-\mu)S_l l_S - \hat{\imath}_t}{1-\rho\beta\mu},
\end{aligned}
$$

where the solution for variables with a l subscript has already been provided above.

C.9 Calibration and Sensitivity Analysis

C.9.1 Debt maturity data

The model assumes a consolidated government budget constraint so that, for government debt, the relevant measure is all government debt held by the public (thus, government debt held by the Federal Reserve is netted out). Reserves issued by the Federal Reserve are treated as short-term government debt. The duration of the consolidated government's debt is given in Figure C.8.[76] The vertical dashed lines are important events associated with the Federal Reserve buying long-term treasury bonds. Around those dates, the maturity of outstanding government debt declined. In generating this figure, we first use estimates from Chadha et al. (2013) on the duration of Treasury debt held outside the Federal Reserve, which we then augment with data on reserves issued by the Federal Reserve that is available from public sources (FRED).

The baseline calibration of the model relies on the QE2 program of November 2010. Base-

[76]In generating this figure, we first use estimates from Chadha et al. (2013) on the duration of Treasury debt held outside the Federal Reserve, which we then augment with data on reserves issued by the Federal Reserve that is available from public sources (FRED).

line maturity is 16.87 q, which is the level at the beginning of the QE2 program, resulting in $\rho = 0.9502$. As a measure of the effects on the average maturity of outstanding government debt from QE2, the difference is taken between the start of QE2 and the start of MEP (thus, the difference between the third and fourth lines in Figure C.8). That is, according to this measure, the reduction in maturity due to QE2 was from 16.87 q to 16.2 q, with a difference of 0.67 q. This implies a change in ρ from 0.9502 to 0.9477 as documented in the table in the main text. QE3 is defined as the sum of the MEP and QE3 programs (given by the fourth, fifth, and sixth lines in Figure C.8). Thus, as a measure of the effects on the average maturity of outstanding government debt from QE3, we consider the end of the sample in Figure C.8, where the maturity is at 15.07 q. Compared to the baseline maturity, the difference is then 1.8 q. Moreover, this implies a change in ρ from 0.9502 to 0.9430 as documented in the table in the main text.

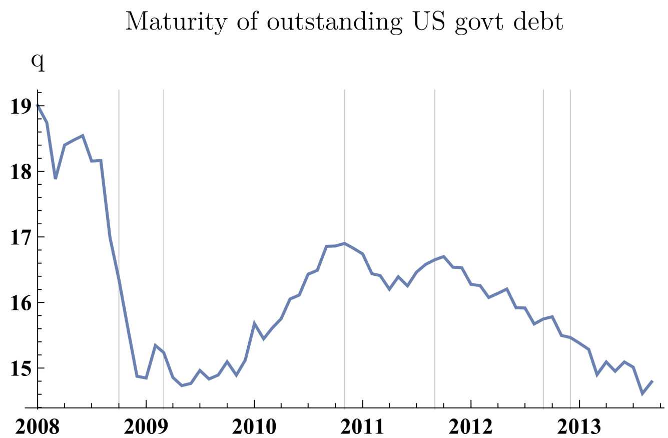

Figure C.8: Consolidated US Government Debt Maturity

Note: The figure presents the maturity of outstanding US government debt adjusted for reserves issued by the Federal Reserve. The vertical dashed lines are important events associated with the Federal Reserve buying long-term Treasury bonds: November 2008 and March 2009 (Quantitative Easing 1); November 2010 (Quantitative Easing 2 (QE 2)); September 2011 (Maturity Extension Program (MEP)); and September 2012 and December 2012 (Quantitative Easing 3 (QE3)).

C.9.2 Bounds on output effects

For the sensitivity analysis on bounds on output effects , we chose r^e_S to generate an output drop of 7.5%.[77] We keep, however, the same value for the persistence of the shock μ as we vary λ_T, which maps onto the expected duration of the ZLB. This latter choice implies that as we vary λ_T, the model generates different inflation drops at the ZLB. Table C.1 shows this

[77]The numbers reported are essentially unchanged even if we maintain a fixed r^e_S when we do this sensitivity analysis—up to a first decimal point.

result and the responses of several other key empirical moments (recall that in the baseline calibration, other than the drops in output and inflation at the ZLB, other moments are out-of-sample/untargeted). Thus, inflation at the ZLB is -4.3% for λ_T^{low} while it is -0.04% for λ_T^{high}. Furthermore, the estimated effects of QE on various yields and on inflation compensation are also affected by the choice of λ_T.

Table C.1: Untargeted data and model moments with different degrees of tax smoothing

Moments	**Data**	**Baseline λ_T**	λ_T^{high}	λ_T^{low}
Zero-lower-bound output drop	-7.50%	-7.50%	-7.50%	-7.50%
Zero-lower-bound inflation drop	-2.50%	-2.50%	-0.04%	-4.30%
Zero-lower-bound duration	$6-8$ qt	8.4 qt	8.4 qt	8.4 qt
$\Delta i(8)$, QE2	-16 bp	-10.24 bp	-33.5 bp	1.45 bp
$\Delta i(40)$, QE2	-30 bp	-11.2 bp	-20.81 bp	1.62 bp
$\Delta i(40)$, QE3	-46 bp	-34.9 bp	-57.10 bp	3.84 bp
$\Delta \pi(40)$, QE2	5 bp	14.34 bp	9.85 bp	6.30 bp

C.9.3 Bounds on output effects matching inflation at the ZLB

In this sensitivity, we keep μ fixed as we vary λ_T around the point estimate but recalibrate κ so as to match the drop in inflation at the ZLB. We report results from this exercise for the output effects of QE2 in the first row of Table C.2 and effects on future inflation and yields in the rest of the rows of Table C.2. As can be seen, this tends to make the effect of QE somewhat larger.

Table C.2: Output effects and untargeted data and model moments when recalibrating to match zero-lower-bound inflation drop

Moments	**Data**	**Baseline λ_T**	λ_T^{high}	λ_T^{low}
QE2 ΔOutput		1.65%	4.00%	0.40%
Zero-lower-bound output drop	-7.50%	-7.50%	-7.50%	-7.50%
Zero-lower-bound inflation drop	-2.50%	-2.50%	-2.50%	-2.50%
Zero-lower-bound duration	$6-8$ qt	8.4 qt	8.4 qt	8.4 qt
$\Delta i(8)$, QE2	-16 bp	-10.24 bp	-20 bp	-0.04 bp
$\Delta i(40)$, QE2	-30 bp	-11.2 bp	-18 bp	-1.30 bp
$\Delta \pi(40)$, QE2	5 bp	14.34 bp	57.7 bp	3.74 bp

C.9.4 Recalibrating to match financial market effects

Here, we estimate $(\lambda_y,\, \kappa, \lambda_T, r_S^e, \mu)$ to match five moments exactly: the drop in inflation and output at the ZLB, the ratio of the inflation response to the tax response to match RZ's evidence, and the effects of QE2 on yields and expected inflation to match KVJ's evidence.

Table C.3 contains the parameter values from this exercise while Table C.4 shows the model's predictions for the output effects of QE2 and QE3. The parameter values are similar to our baseline calibration reported earlier in Table 1. The output effects of QE are also very similar to those in the baseline exercise reported earlier in Table 2, where we did not match the empirically estimated effects of QE2 on yields and expected inflation. The reason is that in the baseline exercise, the model generated smaller effects on long-term yields but bigger effects on expected inflation compared with KVJ's estimates. These two forces essentially have equal but opposite effects for output in the model.

Table C.3: Calibrated/estimated parameter values in the quantitative model that match the financial-market effects of quantitative easing

Parameter	Value	Parameter Description
β	0.99	Discount factor
σ	0.5	Scaled IES
κ	0.027	Slope of Phillips curve
λ_y	0.0029	Weight on output
λ_T	0.0052	Weight on taxes
r_S^e	-0.016	Shock size
μ	0.84	Shock persistence
$\rho(\text{pre-}QE2)$	0.9502	Baseline debt maturity
$\Delta\rho(QE2)$	0.0024	Change in debt maturity
$\Delta\rho(QE3)$	0.0072	Change in debt maturity
ψ	7.2	Debt-to-taxes ratio
$\hat{b}_S$	0.30	Initial debt

Table C.4: Output effects of QE2 and QE3 in the quantitative model that matches financial-market effects

Model Specification	ΔOutput, QE2	ΔOutput, QE3
Matching financial-market effects	1.65%	5.15%

D Independent central bank model

D.1 Central bank balance sheet

We discuss in more detail the balance sheet of an independent central bank. In period 0, the central bank has one-period liability, d_0^c (central bank interest bearing reserves), and both one-period, b_0^c, and two-period, l_0^c, assets. In period 1, the central bank issues one-period liability, d_1^c, and buys one-period assets, b_1^c. Central bank transfers to the Treasury are denoted by T^c. The initial net asset position of the central bank is l_{-1}^c.

The budget constraints in the three periods are then

$$l_0^c + \{b_0^c - d_0^c\} = \frac{l_{-1}^c}{\Pi_0} - T_0^c, \tag{D.1}$$

$$\{b_1^c - d_1^c\} = (1 + i_0)\frac{\{b_0^c - d_0^c\}}{\Pi_1} - T_1^c, \tag{D.2}$$

$$0 = (1 + R_0)\frac{l_0^c}{\Pi_1\Pi_2} + (1 + i_1)\frac{\{b_1^c - d_1^c\}}{\Pi_2} - T_2^c. \tag{D.3}$$

For one period asset and liability, only the net position matters, which are denoted by the curly brackets as $\{b_0^c - d_0^c\}$ and $\{b_1^c - d_1^c\}$.

D.2 Maturity value characterization

The budget constraints (D.1)-(D.3) above can be rewritten in terms of the minimum set of state variables in interest inclusive terms, which in period 0 is w_{-1}^c, in period 1, w_0^c *and* x_0^c, and in period 2, W_1^c:

$$W_1^c \equiv w_1^c + x_0\frac{w_0^c}{\Pi_1},$$

$$0 = \frac{W_1^c}{\Pi_2} - T_2^c,$$

$$W_1^c = \frac{w_0^c}{\Pi_1} + i_1(1 - x_0)\frac{w_0^c}{\Pi_1} - (1 + i_1)T_1^c,$$

$$x_0\frac{w_0^c}{1 + R_0} + (1 - x_0)\frac{w_0^c}{1 + i_0} = \frac{w_{-1}^c}{\Pi_0} - T_0^c,$$

where, $w_0^c \equiv (1 + R_0)l_0^c + (1 + i_0)\left(b_0^c - d_0^c\right)$, $w_1^c \equiv (1 + i_1)\left(b_1^c - d_1^c\right)$, $w_{-1}^c = l_{-1}^c$, $x_0 \equiv \frac{(1+R_0)l_0^c}{(1+R_0)l_0^c+(1+i_0)\left(b_0^c-d_0^c\right)}$, and $(1 - x_0) \equiv \frac{(1+i_0)\left(b_0^c-d_0^c\right)}{(1+R_0)l_0^c+(1+i_0)\left(b_0^c-d_0^c\right)}$.

In this notation, w_0^c measures the *net asset position* of the central bank, written in terms of maturity value (that is inclusive of interest payments). Moreover, x_0 measures the maturity composition of the central bank net asset position in period 0 – the ratio of long-term assets in the net asset position. x_0 thus measures the *maturity mismatch* in the central bank balance sheet. If $x_0 = 0$, there is no maturity mismatch. If the central bank issues d_0^c to buy l_0^c, this variable changes while if the central bank issues d_0^c to buy b_0^c, this variable does not change. w_1^c is the net asset position of the central bank in period 1, again written in terms of maturity value. The state variable in period 0 is w_{-1}^c, in period 1, the state variables are w_0^c and x_0^c, and in period 2, the state variable is W_1^c.

These are the budget constraints that appear in the main text. The loss function for the central bank is already discussed in text.

D.3 Discussion

Recall that for the consolidated government, the game analyzed was one in which the government starts out with debt but does not have government expenditures. The government thus has to concern itself with how to collect taxes over the three periods. To obtain an analog for an independent central bank, consider a central bank that has a negative asset position, which it needs to improve on in order to reach a balanced position in the terminal period. This amounts to assuming that $l_{-1}^c = -b_{-1}$, as in the equivalence proposition. More realistically, one could assume that the central bank starts with a positive capital position. The general point is that it faces essentially the same policy problem, except that we differently interpret the variables that enter the budget constraints.

The equivalence proposition stipulates that the model can alternatively be interpreted as referring to an independent central bank with balance sheet concerns. An important consideration, which we leave for future research to explore, is how to empirically identify λ_v under this alternative interpretation since here it is not appropriate to use the consolidated government's budget constraint to parameterize the model, as we did in the quantitative analysis.[78]

E Addendum to Section 4.5 with a numerical example

This Section expands upon the discussion in the main text in Section 4.5 and provides a numerical example that helps clarify the arguments made. A considerable literature (see for example, Missale and Blanchard (1994)) assumes that if more debt is long-term then there is

[78] In an earlier version of this paper, Bhattarai, Eggertsson, and Gafarov (2015), we attempted this by using data on the size and composition of the Federal Reserve balance sheet to parameterize the budget constraints in a similar model. λ_v, however, was calibrated within the context of the model and without using any additional evidence as we did in the main analysis.

more to gain from an inflation policy (for a similar assumption applied more recently, see for example Aizenman and Marion (2011) and references therein). The main text of the paper claims that the reason this literature comes to an apparently different result from the analysis of MPE in this paper and Calvo and Guidotti (1990, 1992) is that it typically considers the consequence of an inflation policy over an extended period of time while the MPE contemplates the benefit of inflation in a period-by-period game in which each period is of the same duration as the short-term debt.

E.1 Revisiting Missale and Blanchard (1994)

It is helpful to review the model by Missale and Blanchard (1994) to clarify why their result may appear to contradict the forces in the MPE analyzed in this paper. Missale and Blanchard (1994) assume directly in their debt accumulation equation that higher inflation reduces the real value of debt by more if the maturity of outstanding debt is longer. Their debt accumulation is

$$D_t = (1 + r_t)\{1 - m_t(\pi_t - E_{t-1}\pi_t)\}D_{t-1}, \tag{E.1}$$

where D_t is the real value of debt, r_t is the real interest rate, and m_t is maturity – a lower m_t denotes shorter maturity.[79]

To see how this relates to the model of this paper, consider the simple model outlined in Section 2 in the medium and long runs, and for simplicity, suppose that $T_1 = 0$. Combining the government budget constraints in periods 1 and 2 yields an expression for taxes in period 2,

$$\begin{aligned} T_2 &= (1 + i_1)(1 + i_0)\frac{b_0}{\Pi_1\Pi_2} + (1 + R_0)\frac{l_0}{\Pi_1\Pi_2}, \\ &= \frac{w_0}{\Pi_1\Pi_2} + i_1(1 - x_0)\frac{w_0}{\Pi_1\Pi_2}, \end{aligned} \tag{E.2}$$

using the notation where b_0 is the real value of short-term nominal debt and l_0 is the real value of long-term nominal debt. The second line expresses the budget constraint in terms of maturity value of the debt w_0 and its duration, x_0, which is a natural way of expressing the minimum set of state variables as discussed in the text.

Consider a new inflation policy chosen in period 1 that may extend over both period 1 and 2 or only over period 1. This change in policy is unexpected so that the variables i_0 and

[79] Note that this debt accumulation equation is not derived from first principles in a model with short-term and long-term nominal debt, as we do in this paper. As Missale and Blanchard (1994) note, this budget constraint is akin to thinking of long-term nominal debt directly as *indexed* debt. Here, we simply show what is used by Missale and Blanchard (1994) and how our model can be related (or not) to the intuition obtained from this debt accumulation equation.

R_0 are fixed – which in the second line amounts to the assumption that w_0 and x_0 are given. (E.2) reveals that T_2 is decreasing in inflation in either period 1 or 2, that is T_2 is declining in both. Π_1 and Π_2. The interest rate, i_1, can also play an important role for the effect of inflation on T_2, depending on how much of the debt is short term and other assumptions as soon will be clear.

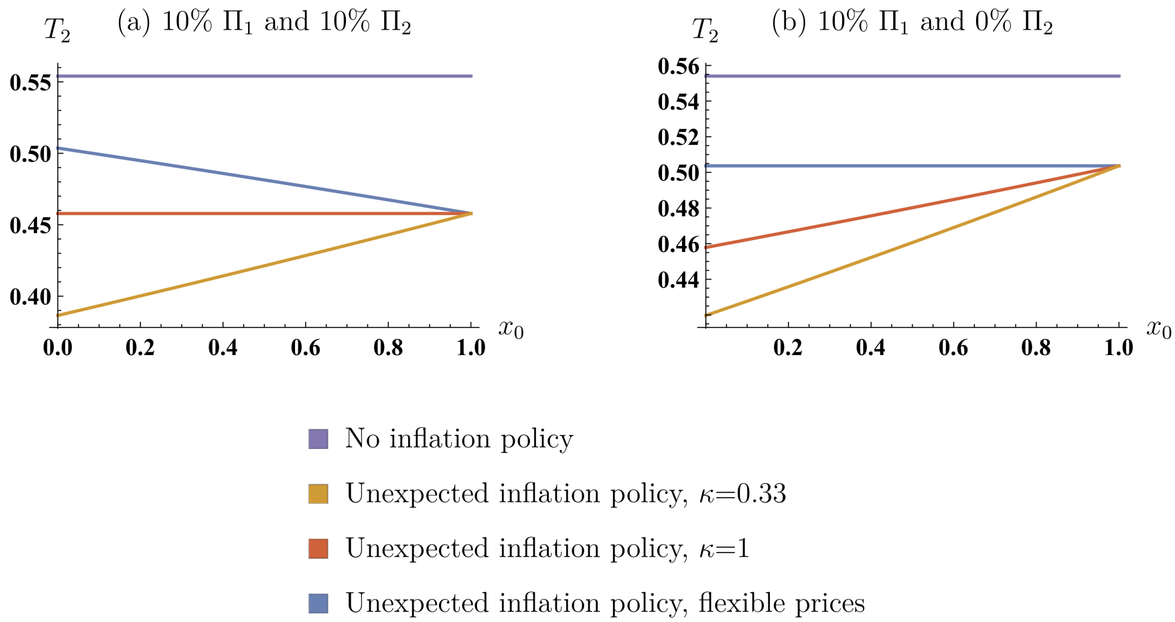

Figure E.9: Fiscal Benefit and Debt Duration

Note: The figure shows how the tax burden depends on debt duration for various inflation policies and extent of price rigidities.

The claim made in the literature is that the benefit of inflation is higher, when more debt is long-term, that is, a higher amount of l_0 relative to b_0. It was argued in the main text that this result fundamentally relies on (i) the horizon of the inflation policy and (ii) the degree of price rigidities. This argument is made more explicit below by first considering an inflation policy that applies both to periods 1 and 2.

E.2 Higher inflation in period 1 and 2 and flexible prices

Consider the consequences of the government choosing a new policy in period 1: It announces a higher inflation for *both* period 1 and 2, that is Π_1 and Π_2 and denote the new inflation rate by $\Pi_1 = \Pi_2 = \Pi^*$. The new policy is unexpected and the expectation about gross inflation in period 0 is 1: $E_0\Pi_1 = E_0\Pi_2 = 1$. This implies that $1 + i_0 = \beta^{-1}$ and $1 + R_0 = \beta^{-2}$.

If prices are flexible then it follows from the consumption Euler equation that $1 + i_1 =$

$\beta^{-1}\Pi_2 = \beta^{-2}\Pi^*$. Substituting this into (E.2) yields

$$T_2 = \beta^{-2}\frac{b_0}{\Pi^*} + \beta^{-2}\frac{l_0}{(\Pi^*)^2}. \tag{E.3}$$

Figure E.9, panel (a), shows the effect on T_2 of 10% inflation in periods 1 and 2 assuming different values for the maturity of the debt x_0 – holding fixed the total value of debt issued in period 0, that is, $l_0 + b_0 = D = 0.5$ and $\beta = 0.95$, as in the text.[80]

Consider first the purple line which shows that in the absence of the inflation policy, then $T_2 = \beta^{-2}B = 0.554$ and T_2 is independent of the duration of debt, that is x_0 which shown on the x-axis. The blue line shows that the tax burden is always lower as a result of the unexpected inflation policy for all x_0. As the figure reveals, however, the fiscal benefit is lower (T_2 is higher) the more debt is short term (that is x_0 is low) as suggested by the Missale and Blanchard (1994) debt accumulation (E.1).

The reason for this result is straight-forward. If the government has short run debt, then the interest rate will rise on the debt that is rolled over, that is $1 + i_1 = \beta^{-1}\Pi_2 = \beta^{-1}\Pi^*$. Thus the inflation in period 2 will not depreciate the real value of the debt. The way this shows up in (E.3) is that the long-term debt l_0 is divided by $(\Pi^*)^2$ while b_0 is only divided by (Π^*) — this is because the inflation in period 2 will not reduce the real value of the short-term debt as it is rolled over on the higher interest rate that fully adjusts to the increase in inflation in period 2. This is precisely the logic for why inflation has bigger benefits the more debt is long term in Missale and Blanchard (1994), as assumed in their debt accumulation equation (E.1). As will be shown below, however, this result depends both on assuming flexible prices and on the horizon of the inflation policy. Consider first the issue of price rigidities.

E.3 Higher inflation in period 1 and 2 and sticky prices

The main text of the paper suggested that this result depended on the assumption that the interest rate immediately adjusts – as it does under the assumption of fully flexible prices. If prices are rigid, however, interest rates may or may not immediately change. To see this, consider the same thought experiment as before, that is, the government announces an increase in inflation in period 1 for both period 1 and 2. As earlier assume this is unexpected so that $1 + i_0 = \beta^{-1}$ and $1 + R_0 = \beta^{-2}$. Now, however, let the solution for $1 + i_1$ take into account

[80] A little bit of algebra shows that the expression for w_0 as you change x_0 holding $\mathrm{b}_0 + l_0 = D$ fixed is

$$w_0 = D(1 + i_0) + x_0\frac{D(1 + i_0)(R_0 - i_0)}{(1 + R_0) - x_0(R_0 - i_0)}$$

prices rigidities of the form studied in Section 2 given by the Phillips curve, that is

$$1 + i_1 = \beta^{-1}\left(\frac{\bar{Y}}{Y_1}\right)\Pi^* = \beta^{-1}\left(\frac{\bar{Y}}{\kappa^{-1}\left(\Pi^* - 1\right) + \bar{Y}}\right)\Pi^*, \tag{E.4}$$

using the consumption Euler equation, which together with the Phillips curve, changes (E.2) to become

$$T_2 = \beta^{-2}\left(\frac{\bar{Y}}{\kappa^{-1}\left(\Pi_1 - 1\right) + \bar{Y}}\right)\frac{b_0}{\Pi^*} + \beta^{-2}\frac{l_0}{(\Pi^*)^2}. \tag{E.5}$$

As (E.4) shows, with rigid prices, the nominal interest may decrease as a result of the increase in inflation. In particular, the condition for the interest rate to decrease is that $\kappa < 1$. The implication for T_2 in (E.5) is that if $\kappa < 1$, then the fiscal gain from inflation policy is higher, the more short-term debt is being held, the opposite conclusion from the accumulation equation assumed by Missale and Blanchard (1994).

This is illustrated by the yellow line in panel (a) in Figure E.9, where $\kappa = 0.33$.[81] As shown in the figure, T_2 is lower the more short term debt the government holds when entering period 1 and while announcing the new inflation regime. The logic is that the government is able to roll the debt over at a lower rate than before the policy was announced, and thus the benefits of inflation are higher the more short-term debt the government holds. The red line shows a case in which the duration of debt is irrelevant for T_2. This happens when $\kappa = 1$. The perfectly flexible prices already analyzed in last subsection corresponds to $\kappa- > \infty$.[82]

E.4 Higher inflation only in period 1

The text suggested that a more fundamental difference with the literature such as Missale and Blanchard (1994) was that the MPE only considers the benefit of inflation in a single period, while the thought experiment above considers an increase in inflation over an extended period of time – of the same duration as the long-term debt. The inflation incentives associated with an extended period of inflation is fundamentally different from the one considered in the MPE. There, the government is choosing inflation every period holding its expectation about future inflation via some expectation function as shown below. And while the inflation incentive for

[81] As in the main text $\bar{Y} = 1$.

[82] It is worth pointing out that, while useful to illustrate this point, the two period model – provided prices are rigid enough – may exaggerate how higher inflation reduces the nominal interest rate , even if we leave a full study of this to future work. More generally, over a longer horizon, most models predict that interest rates should ultimately rise together with the inflation rate. A stripped down New Keynesian model with, for example, a perfectly forward looking Phillips curve is at the other extreme. It predicts that the nominal rate jumps immediately with a higher inflation announcement, if it is fully credible. Note that none of the results are relying on whether nominal interest rate rise quickly or not with inflation expectations, since we show that the same insights hold true in the New Keynesian model in the quantitative section, where the nominal interest rate would increase immediately with higher inflation expectations in the thought experiment above.

an extended inflationary regime depended fundamentally on the degree of price rigidities, it turns out that the degree of price rigidities is irrelevant for the mechanism once we consider the MPE. Before moving to the MPE, however, consider first a slightly simpler case, which also helps to understand the mechanics of the rollover incentive.

Suppose inflation in period 2 is given by some expectation function $\Pi^2(.)$ where the dot refers to a state variable that might be influenced by policy. Then (E.2) becomes

$$T_2 = \beta^{-1}(1+i_1)\frac{b_0}{\Pi_1\Pi^2(.)} + \beta^{-2}\frac{l_0}{\Pi_1\Pi^2(.)}. \tag{E.6}$$

Consider an inflation policy announced in period 1 that only implies that inflation in period 1 is higher, that is $\Pi_1 = \Pi^*$. Consider first an example in which the function $\Pi^2(.)$ is unaffected by Π_1 – the most simple case – so that $\Pi_2 = 1$. Assume that all prices are flexible so that $i_1 = \beta^{-1}\Pi_2 = 1$. Then

$$T_2 = \beta^{-2}\frac{b_0}{\Pi^*} + \beta^{-2}\frac{l_0}{\Pi^*}. \tag{E.7}$$

In this case, the fiscal benefit of inflation is independent of the duration of debt – it reduces the real value of short and long-term debt in exactly the same way as seen in (E.7). Panel (b) of Figure E.9 shows with the purple line, as before, T_2 for the case when there is no inflation. The blue line shows the effect of a 10 percent increase of inflation in period 1. Both lines are horizontal, since as argued above, T_2 is independent of x_0 as the effect of the inflation is the same on short- and long-term debt.

As before, however, things change with price rigidities. Following the same steps as before by substituting for i_1 yields

$$T_2 = \beta^{-2}\left(\frac{\bar{Y}}{\kappa^{-1}\left(\Pi^* - 1\right) + \bar{Y}}\right)\frac{b_0}{\Pi^*} + \beta^{-2}\frac{l_0}{\Pi^*}.$$

For an inflation policy that only implies higher inflation in period 1 ($\Pi_1 = \Pi^* = 1.10$), with inflation in period 2 unaffected ($\Pi_2 = 1$), there are unambiguously larger benefits of holding short-term debt (lower x_0) because the increase in inflation reduces the nominal interest rate, i_1, via higher output. And it is only if the government holds short term debt that it can take advantage of the new lower interest rate. The benefit of inflation (for any $x_0 < 1$) is greater the more rigid prices are, as shown by comparing the yellow line ($\kappa = 0.3$) to the red line ($\kappa = 1$). This effect is related to the rollover incentive in the MPE but it is not, however, exactly the same as will be shown next.

E.5 The MPE and the rollover incentive

The benefit of holding short-term debt in response to inflation in period 1 only (holding Π_2 fixed), does not correspond to the rollover incentive outlined in the main text of the paper. In contrast to this effect, the rollover incentive operates regardless of whether prices are sticky or flexible. We showed this in terms of period 1 policy functions for the fully flexible price case in Figure A.5. This is because the computation of the MPE takes into account not only the effect of output on i_1 but also how $\Pi^2(.)$ is changed by the government's choice of Π_1(which in turn has an additional effect on i_1).

The MPE gives a precise theory of this relationship documented in the paper. Higher inflation in period 1 in the MPE reduces inflation in period 2 through the state variable W_1 which appears as an argument in $\Pi^2(.)$. Importantly W_1 goes down with higher inflation in period 1, thus reducing $\Pi^2(.)$ which is accordingly decreasing in Π_1. Similarly, higher Π_1, via lower $\Pi^2(.)$, reduces i_1-this is the Fisher effect mentioned in the paper. If the government has a lot of short-term debt, the benefit of inflation in period 1 is thus larger than in the example above, because not only does higher Π_1 raise Y_1 and thereby reduce i_1, it also reduces $\Pi^2(.)$ (thus further reducing i_1 via the Fisher effect).

E.6 Conclusion

The purpose of this Section is to explicitly show the difference between the inflation incentive in the MPE and in some of the existing literature that has tended to emphasize that long-term debt gives the government higher inflation incentive. It was shown that the result (or typically a direct assumption) in this literature does not contradict the MPE. The main reason for the seeming divergence is that the literature emphasizing that long term debt generates higher inflation incentive is typically considering an inflation policy over an extended period of time, and is thus perhaps best interpreted as analyzing the incentive for a "regime change" in which case a central bank moves from a low inflation regime towards a high one over an extended period of time.[83]

The MPE, in contrast, is concerned with how short-term debt changes the inflation incentive of the government period by period, and thus may better capture the incentive effects of debt-policy within a single policy regime. Another important point is that the assumption of price rigidities can have an important effect on the inflation incentive, especially when considering an inflation policy over a long period of time, as some of the literature cited has in mind. A point highlighted, however, and also stressed in the main text, is that the qualitative conclusion of the MPE does not depend on the degree of price rigidities. For either sticky or

[83] To be clear, we did not explicitly formalize how one should model a regime change, but we think this is the most reasonable interpretation of this literature.

fully flexible prices, reducing the duration of government debt increases its inflation incentive in the MPE via the rollover incentive. The rollover incentive is thus the main mechanism behind the results in the paper.